

PLUMI PRACTICE PACK

Plumi Practice Pack: A Level Mathematics (Edexcel-style, 9MA0)

Mark scheme | Total 298 marks

Q	Section	Level	Topic	Marks
1	Pure	Easy	Laws of logarithms	5
2	Pure	Medium	Integration by substitution	8
3	Pure	Medium	Numerical methods: Newton-Raphson	9
4	Pure	Medium	Harmonic form $R \sin(x - \alpha)$	11
5	Pure	Hard	Connected rates of change	11
6	Pure	Hard	Parametric equations: tangents and area	11
7	Pure	Hard	Differential equations and modelling	12
8	Pure	Super hard	Tangents, integration by parts, a parameter	13
9	Pure	Super hard	Differential equations: draining a cone	12
10	Pure	Super hard	Trigonometric identities, cubic equations, proof	14
11	Statistics	Easy	Binomial distribution	5
12	Statistics	Medium	The normal distribution	9
13	Statistics	Medium	Hypothesis testing: binomial	9
14	Statistics	Medium	Regression and correlation	7
15	Statistics	Hard	Probability: Venn diagrams, independence	11
16	Statistics	Hard	Hypothesis testing: normal mean	10
17	Statistics	Hard	The normal distribution: quartiles and independence	10
18	Statistics	Super hard	Calculus, the normal distribution and a binomial test	12
19	Statistics	Super hard	Conditional probability with normal distributions	11
20	Statistics	Super hard	Discrete distributions and a binomial test	11
21	Mechanics	Easy	Vectors: velocity and position	5
22	Mechanics	Medium	Constant acceleration: vertical motion	8
23	Mechanics	Medium	Connected particles: Newton's laws	7
24	Mechanics	Medium	Moments	7
25	Mechanics	Hard	Friction on an inclined plane	10
26	Mechanics	Hard	Connected particles on a rough plane	11
27	Mechanics	Hard	Projectiles: a range of possible speeds	12
28	Mechanics	Super hard	Statics: ladder with friction at both ends	13
29	Mechanics	Super hard	Variable acceleration: distance and displacement	11
30	Mechanics	Super hard	Connected particles: rough slope, slack string	13
Total (A: 106, B: 95, C: 97)				298

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General marking guidance

- All candidates must receive the same treatment. Mark the first candidate in exactly the same way as the last.
- Marks must be awarded for what candidates have shown they can do, not penalised for omissions. Where a candidate has made more than one attempt and has not crossed any out, mark the last complete attempt.
- Correct answers with no working: in questions that say 'show that', 'use algebra' or 'solutions relying entirely on calculator technology are not acceptable', a bald answer scores no marks. Elsewhere, a correct answer with no working scores full marks unless the scheme says otherwise; an incorrect answer with no working scores no marks.
- Where a final answer is given and then rounded incorrectly, the accuracy mark may be awarded if the correct value was seen first (isw applies).

Abbreviations

Code	Meaning
M	Method mark: a correct method applied to appropriate values, allowing slips.
A	Accuracy mark: dependent on the associated M mark(s) being earned.
B	Mark for a correct result or statement, independent of method.
dM	Dependent method mark: can only be awarded if the preceding M mark has been earned.
ft	Follow through: credit for a correct method using an incorrect earlier value.
* (e.g. A1*)	Given answer (AG): the working must be complete and correct, with no errors seen.
cso / cao	Correct solution only / correct answer only.
awrt	Answers which round to the stated value.
oe / isw	Or equivalent / ignore subsequent working.
CR / H_0 , H_1	Critical region / null and alternative hypotheses.



SECTION A: PURE MATHEMATICS

Question 1 Pure · Easy · Laws of logarithms

Question	Scheme	Marks
1	$2 \log_3 x = \log_3 x^2$	M1
	$\log_3 x^2 - \log_3(x - 2) = \log_3 \frac{x^2}{x - 2}$	M1
	$\frac{x^2}{x - 2} = 9$ or $x^2 = 9(x - 2)$	A1
	$x^2 - 9x + 18 = 0 \Rightarrow (x - 3)(x - 6) = 0$	dM1
	$x = 3, x = 6$	A1
		(5 marks)

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Notes

M1: Uses the power law correctly on $2 \log_3 x$ at any stage. M1: Uses the subtraction law correctly; $\frac{\log_3 x^2}{\log_3(x - 2)}$ is M0.

A1: Either form of the equation with the logarithms removed. Using 2^3 or 3×2 in place of 3^2 is A0.

Alternative: writes $2 = \log_3 9$, so $\log_3 x^2 = \log_3 9 + \log_3(x - 2) = \log_3 9(x - 2)$ and equates arguments. Mark in the same way.

dM1: Dependent on both M marks. Solves the three-term quadratic by any valid method.

A1: Both $x = 3$ and $x = 6$. Both satisfy $x > 2$, so a candidate who rejects either value scores A0.

Question 2 Pure · Medium · Integration by substitution

Question	Scheme	Marks
2(a)	$x = 7 \Rightarrow u = 3$ and $x = 14 \Rightarrow u = 4$	B1
	$x = u^2 - 2 \Rightarrow dx = 2u du$ (or $\frac{du}{dx} = \frac{1}{2\sqrt{x+2}}$)	M1
	Full substitution: $\int (u^2 - 3) u (2u) du$, with no x remaining	M1
	$= \int (2u^4 - 6u^2) du$ with no errors seen	A1* (4)
2(b)	$\int (2u^4 - 6u^2) du: u^n \rightarrow u^{n+1}$ for at least one term	M1
	$[\frac{2}{5}u^5 - 2u^3]$	A1
	$(\frac{2048}{5} - 128) - (\frac{486}{5} - 54)$: substitutes the u limits 4 and 3 and subtracts	dM1
	Area = $\frac{1192}{5}$	A1 (4)
		(8 marks)



Question 2 notes continued

Notes

(a) B1: may be awarded for limits seen anywhere in (a), including on the final integral. A1*: given answer; the integrand must be seen as $(u^2 - 3)(u)(2u)$ or equivalent before the printed form.

(b) dM1: depends on the previous M1. Using the x limits 7 and 14 with an integral in u is dM0.

(b) A1: $\frac{1192}{5}$; 238.4 is the same exact value and is also accepted. A correct answer with no integration seen scores 0 marks.

Question 3 Pure · Medium · Numerical methods: Newton-Raphson

Question	Scheme	Marks
3(a)	$f(2) = -0.386$ and $f(3) = 3.80$	M1
	Change of sign and f continuous on $[2, 3]$, so there is a root in $[2, 3]$	A1 (2)
3(b)	$f'(x) = 2x - \frac{2}{x}$	B1
	$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{-0.3863}{3}$	M1
	$x_1 = 2.129$	A1 (3)
3(c)	$f'(1) = 2 - 2 = 0$	M1
	The Newton-Raphson formula would require division by $f'(1) = 0$ (the tangent at $x = 1$ is horizontal and does not meet the x -axis)	A1 (2)
3(d)	$f(2.1225) = -0.00018$ and $f(2.1235) = 0.0031$	M1
	Change of sign, f continuous, so $2.1225 < \alpha < 2.1235$ and $\alpha = 2.123$ to 3 d.p.	A1 (2)
		(9 marks)

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Notes

(a) M1: attempts both $f(2)$ and $f(3)$. A1: both values correct (awrt -0.39 and 3.8), with sign change, continuity and a conclusion.

(b) M1: correct Newton-Raphson formula with their f' . A1: awrt 2.129.

(d) M1: chooses $[2.1225, 2.1235]$ or a tighter interval that contains α and rounds to 2.123 at both ends, and evaluates f at both ends. A1: both values correct (awrt -0.0002 and 0.003), with sign change, continuity and a conclusion.

Question 4 Pure · Medium · Harmonic form $R \sin(x - \alpha)$

Question	Scheme	Marks
4(a)	$R = \sqrt{7^2 + 4^2} = \sqrt{65}$	B1
	$\tan \alpha = \frac{4}{7}$ (or $\sin \alpha = \frac{4}{\sqrt{65}}$ or $\cos \alpha = \frac{7}{\sqrt{65}}$)	M1

Question 4 continued

Question	Scheme	Marks
	$\alpha = 29.74^\circ$	A1 (3)
4(b)	Maximum when $\sin(15t - \alpha) = 1$: $H_{\max} = 18 + \sqrt{65}$	M1
	$= 18 + \sqrt{65}^\circ\text{C}$	A1
	$15t - 29.74 = 90 \Rightarrow t = \frac{90 + 29.74}{15}$	M1
	$t = 7.98$	A1 (4)
4(c)	$\sin(15t - \alpha) = \frac{20 - 18}{\sqrt{65}} \Rightarrow 15t - \alpha = 14.36^\circ$	M1
	$t = 2.94$	A1
	Second solution: $15t - \alpha = 180^\circ - 14.36^\circ = 165.64^\circ$	dM1
	$2.94 \leq t \leq 13.03$	A1 (4)
		(11 marks)

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Notes

(a) M1: $\tan \alpha = \pm \frac{4}{7}$; $\tan \alpha = \frac{7}{4}$ leading to 60.26° is M0. A1: awrt 29.74.

(b) First A1: exact, $18 + \sqrt{65}$ (26.06 alone is A0). Second A1: awrt 7.98. Setting $15t = 90$ (ignoring α) is M0.

(c) dM1: depends on the previous M1; uses the symmetry of the sine curve with their α . A1: awrt 2.94 and 13.03; accept strict inequalities or 'between $t = 2.94$ and $t = 13.03$ '.

Question 5 Pure · Hard · Connected rates of change

Question	Scheme	Marks
5(a)	Radius of surface r : $r^2 = 10^2 - (10 - h)^2$	M1
	$r^2 = 20h - h^2 \Rightarrow A = \pi h(20 - h)$	A1* (2)
5(b)	$V = \pi \left(10h^2 - \frac{1}{3}h^3\right) \Rightarrow \frac{dV}{dh} = \pi(20h - h^2)$	M1
	$= \pi h(20 - h) = A$	A1* (2)
5(c)	$\frac{dh}{dt} = \frac{50}{\pi h(20 - h)}$	M1
	At $h = 4$: $\frac{50}{64\pi}$	M1



Question 5 continued

Question	Scheme	Marks
	$= 0.249 \text{ cm s}^{-1}$	A1 (3)
5(d)	$\frac{dA}{dt} = \pi(20 - 2h) \times \frac{50}{\pi h(20 - h)} = \frac{100(10 - h)}{h(20 - h)}$	M1
	$\frac{100(10 - h)}{h(20 - h)} = 5 \Rightarrow h^2 - 40h + 200 = 0$	M1
	$h = 20 \pm 10\sqrt{2}$	dM1
	$h = 20 - 10\sqrt{2} (= 5.86)$ since $0 < h \leq 10$	A1 (4)
		(11 marks)

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Notes

Difficulty features: several topics connected (circle geometry, then differentiation, then connected rates); cases and restrictions (one root of the quadratic is outside $0 < h \leq 10$); hidden method in (d) (the rate condition must be turned into an equation in h).

(a) M1: Pythagoras in the cross-section, using the centre of the hemisphere. A1*: given answer, cso.

(c) First M1: chain rule with $\frac{dV}{dt} = 50$ and their $\frac{dV}{dh}$. A1: awrt 0.249; accept $\frac{25}{32\pi}$.

(d) First M1: $\frac{dA}{dh} \times \frac{dh}{dt}$ in terms of h . Second M1: sets equal to 5 and forms a three-term quadratic. dM1: solves. A1: $20 - 10\sqrt{2}$ only, with $20 + 10\sqrt{2}$ rejected (greater than 10); the decimal 5.86 alone is A0 as the exact depth is required.

Question 6 Pure · Hard · Parametric equations: tangents and area

Question	Scheme	Marks
6(a)	$\frac{dy}{dx} = \frac{9 - 3t^2}{2t}$	M1
	$t = 2$: point (4, 10), gradient $-\frac{3}{4}$, $y - 10 = -\frac{3}{4}(x - 4)$	M1
	$3x + 4y - 52 = 0$	A1* (3)
6(b)	$t = \sqrt{x}$ (as $t \geq 0$) substituted into y	M1
	$y = 9\sqrt{x} - x\sqrt{x} = \sqrt{x}(9 - x)$	A1* (2)
6(c)	$Q: x = \frac{52}{3}$	B1
	Area under C from P : $\int_2^3 (9t - t^3)(2t) dt$ or $\int_4^9 \left(9x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$	M1
	$\left[6t^3 - \frac{2}{5}t^5\right]$ or $\left[6x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]$	A1



Question 6 continued

Question	Scheme	Marks
	Substitutes the limits ($t = 2, 3$ or $x = 4, 9$) and subtracts: $\frac{148}{5}$	dM1
	Area of S = triangle under l from $x = 4$ to $x = \frac{52}{3}$ minus area under C : $\frac{1}{2} \times \frac{40}{3} \times 10 - \frac{148}{5}$, i.e. $\frac{200}{3} - \frac{148}{5}$	M1
	$= \frac{556}{15}$	A1 (6)
		(11 marks)

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Notes

Difficulty features: several topics connected (parametric differentiation, Cartesian form, composite area); little scaffolding (the area of S needs a triangle minus an area under the curve, not prompted); unfamiliar presentation (the region is bounded partly by the tangent, not only by the curve).

(a) A1*: given answer, cso. Alternative for the gradient: $\frac{dy}{dt} = 9 - 3t^2$ and $\frac{dx}{dt} = 2t$ seen separately.

(b) A1*: cso; the restriction $t \geq 0$ (so $t = \sqrt{x}$, not $\pm$) should be clear.

(c) Second M1: requires the triangle (or $\int_4^{52/3}$ of the line) and the area under C between $x = 4$ and $x = 9$ combined correctly. Using limits $t = 0$ to 3 for the curve area is M0 for this mark. dM1: dependent on the previous M1. A1: $\frac{556}{15}$ (awrt 37.1 alone is A0).

Question 7 Pure · Hard · Differential equations and modelling

Question	Scheme	Marks
7(a)	$\frac{1}{x(6-x)} \equiv \frac{A}{x} + \frac{B}{6-x}$ and finds one constant	M1
	$\frac{1}{6x} + \frac{1}{6(6-x)}$	A1 (2)
7(b)	$\int \frac{1}{x(6-x)} dx = \int k dt$	M1
	$\pm \lambda \ln x \pm \mu \ln(6-x)$ from their partial fractions	M1
	$\frac{1}{6} \ln x - \frac{1}{6} \ln(6-x) = kt + c$	A1
	$t = 0, x = 1: c = \frac{1}{6} \ln \frac{1}{5}$, i.e. $c = -\frac{1}{6} \ln 5$	M1
	$t = 1, x = 2: \frac{1}{6} \ln \frac{2}{4} = k - \frac{1}{6} \ln 5 \Rightarrow 6k = \ln \frac{5}{2}$ (so $k = 0.1527$)	M1
	$\frac{x}{6-x} = \frac{1}{5} e^{6kt} = \frac{1}{5} \left(\frac{5}{2}\right)^t \Rightarrow \frac{6}{x} - 1 = 5 \left(\frac{2}{5}\right)^t \Rightarrow x = \frac{6}{1 + 5 \left(\frac{2}{5}\right)^t}$	A1* (6)
7(c)	$\frac{dx}{dt} = k(6x - x^2)$ is greatest when $x = 3$	M1



Question 7 continued

Question	Scheme	Marks
	$\frac{6}{1 + 5\left(\frac{2}{5}\right)^t} = 3 \Rightarrow \left(\frac{2}{5}\right)^t = \frac{1}{5} \Rightarrow t = \frac{\ln 5}{\ln 2.5}$	dM1
	$t = 1.756$ weeks	A1
	$1.756 \times 7 = 12.3$, so 12 days	A1 (4)
		(12 marks)

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Notes

Difficulty features: long chain of reasoning (partial fractions, separate, two conditions, rearrange); hidden method in (c) ('increasing most rapidly' means maximising $\frac{dx}{dt}$ as a function of x); modelling interpretation (convert weeks to days).

(a) Accept equivalent forms such as $\frac{1}{6} \left(\frac{1}{x} + \frac{1}{6-x} \right)$.

(b) First M1: separates the variables (may be implied). Second M1: integrates to logarithms; condone missing modulus signs, and sign slips on $\ln(6-x)$ for this mark only.

(b) Third and fourth M1: use the two conditions to find c and k (or $e^{6k} = \frac{5}{2}$), in either order. A1*: given answer, cso; the step $e^{6kt} = \left(\frac{5}{2}\right)^t$ must be seen or implied. Verifying the printed answer only (substituting $t = 0$ and $t = 1$) scores 0 marks in (b).

(c) M1: any valid reason for $x = 3$ (completing the square, differentiating $kx(6-x)$ with respect to x , or $\frac{d^2x}{dt^2} = k(6-2x)\frac{dx}{dt} = 0$). First A1: awrt 1.76 weeks. Second A1: 12 days cao.

Question 8 Pure · Super hard · Tangents, integration by parts, a parameter

Question	Scheme	Marks
8	$\frac{dy}{dx} = 2xe^{-kx} - kx^2e^{-kx}$	M1 A1
	Tangent at $x = a$ passes through O : $(2a - ka^2)e^{-ka} = \frac{a^2e^{-ka}}{a}$	M1
	$2 - ka = 1 \Rightarrow a = \frac{1}{k}$	dM1
	$A \left(\frac{1}{k}, \frac{1}{k^2e} \right)$ and $l: y = \frac{x}{ke}$	A1
	$\int x^2e^{-kx} dx = -\frac{x^2}{k}e^{-kx} + \frac{2}{k} \int xe^{-kx} dx$	M1 A1
	$= -e^{-kx} \left(\frac{x^2}{k} + \frac{2x}{k^2} + \frac{2}{k^3} \right)$	M1 A1
	Area $= \int_0^{1/k} \frac{x}{ke} dx - \int_0^{1/k} x^2e^{-kx} dx = \frac{1}{2k^3e} - \frac{1}{k^3} \left(2 - \frac{5}{e} \right)$	M1
	$= \frac{11 - 4e}{2k^3e}$	A1



Question 8 continued

Question	Scheme	Marks
	$\frac{11 - 4e}{2k^3e} = \frac{11 - 4e}{16e} \Rightarrow k^3 = 8$	dM1
	$k = 2$	A1
		(13 marks)

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Notes

Difficulty features: parameters instead of numbers (the curve contains an unknown constant k throughout); hidden method (the touching point must be found from 'tangent through the origin'); little scaffolding (one 13-mark part); long chain of reasoning (gradient condition, line, two integrations by parts with k , area in terms of k , solve for k).

First M1: product rule giving the form $\alpha x e^{-kx} \pm \beta x^2 e^{-kx}$. Second M1: any complete method linking the gradient at $x = a$ to a line through O . dM1: cancels e^{-ka} and solves with $a \neq 0$. A1: A or l correct in terms of k .

Integration by parts (twice) with the constant k : M1 A1, M1 A1. Alternative: the substitution $u = kx$ reduces the area to $\frac{1}{k^3}$ times the area for $k = 1$, $\frac{11 - 4e}{2e}$; a complete, correct argument scores all integration marks.

Area M1: area under the line (or the triangle $\frac{1}{2} \times \frac{1}{k} \times \frac{1}{k^2e}$) minus area under the curve, limits 0 and $\frac{1}{k}$. A1: area $\frac{11 - 4e}{2k^3e}$ or exact equivalent.

Final dM1: dependent on all previous M marks; equates and solves for k . A1: $k = 2$ cao. Verifying $k = 2$ by substitution only scores at most the derivative, tangent and area marks evaluated at $k = 2$ (maximum 7).

Question 9 Pure · Super hard · Differential equations: draining a cone

Question	Scheme	Marks
9(a)	Similar triangles: radius of surface $r = \frac{h}{2}$	B1
	$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$	M1
	$\frac{dV}{dh} = \frac{\pi h^2}{4}$	A1
	$\frac{dV}{dt} = -k\sqrt{h}$ and $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$	M1
	$\frac{dh}{dt} = -\frac{4k\sqrt{h}}{\pi h^2} = -\lambda h^{-\frac{3}{2}}$ with $\lambda = \frac{4k}{\pi}$, so $n = -\frac{3}{2}$	A1 (5)
9(b)	$\int h^{\frac{3}{2}} dh = -\int \lambda dt \Rightarrow \frac{2}{5}h^{\frac{5}{2}} = -\lambda t + c$	M1 A1
	$t = 0, h = 16: c = \frac{2}{5}(1024) = \frac{2048}{5}$	M1
	$t = 10, h = 9: \frac{2}{5}(243) = -10\lambda + \frac{2048}{5} \Rightarrow \lambda = 31.24$	M1
	Empty when $h = 0: t = \frac{2048}{5\lambda} = \frac{10240}{781} = 13.11$ minutes	dM1
	13 minutes 7 seconds	A1 (6)



Question 9 continued

Question	Scheme	Marks
9(c)	As $h \rightarrow 0$, $\frac{dh}{dt} \rightarrow -\infty$: the water level would fall infinitely fast, which is impossible	B1 (1)
		(12 marks)

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Notes

Difficulty features: several topics connected (similar triangles, volume, connected rates, separable differential equation); long chain of reasoning (derive the model, solve it, use two conditions, interpret); little scaffolding (the exponent is to be found, and the total time is asked for directly); modelling decisions (criticise the model as $h \rightarrow 0$).

(a) B1: may be implied by $V = \frac{\pi h^3}{12}$. A1: $n = -\frac{3}{2}$, with the proportionality constant introduced and λ identified as positive.

(b) First M1: separates variables correctly; A1: correct integration. Second and third M1: use both conditions (in either order) to find c and λ . dM1: dependent on all previous M marks in (b); sets $h = 0$. A1: 13 min 7 s cao.

(b) Alternative: from $h^{\frac{5}{2}}$ decreasing linearly, $\frac{t}{10} = \frac{1024}{1024 - 243}$ scores full marks if the linearity is derived from the integration.

Question 10 Pure · Super hard · Trigonometric identities, cubic equations, proof

Question	Scheme	Marks
10(a)	$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$	M1
	$= (2 \cos^2 \theta - 1) \cos \theta - 2(1 - \cos^2 \theta) \cos \theta$	M1
	$= 4 \cos^3 \theta - 3 \cos \theta$	A1* (3)
10(b)	$8 \cos^3 \theta - 6 \cos \theta - 1 = 0 \Rightarrow 2(4 \cos^3 \theta - 3 \cos \theta) = 1$	M1
	$\cos 3\theta = \frac{1}{2}$	A1
	$0 \leq 3\theta \leq 540^\circ: 3\theta = 60^\circ, 300^\circ, 420^\circ \Rightarrow \theta = 20^\circ, 100^\circ, 140^\circ$	M1
	$x = 1.879, -0.347, -1.532$	A1
	A cubic equation has at most three roots, and these three are distinct, so there are no others	B1 (5)
10(c)	$x^3 - 3x - 1 \equiv (x - 2 \cos 20^\circ)(x - 2 \cos 100^\circ)(x - 2 \cos 140^\circ)$; constant terms: $-8 \cos 20^\circ \cos 100^\circ \cos 140^\circ = -1$	M1
	$\cos 100^\circ = -\cos 80^\circ, \cos 140^\circ = -\cos 40^\circ$	M1
	$8 \cos 20^\circ \cos 40^\circ \cos 80^\circ = 1 \Rightarrow \cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$	A1* (3)
10(d)	With roots x_1, x_2, x_3 : coefficients of x^2 and x give $x_1 + x_2 + x_3 = 0$ and $x_1 x_2 + x_2 x_3 + x_3 x_1 = -3$	M1



Question 10 continued

Question	Scheme	Marks
	$x_1^2 + x_2^2 + x_3^2 = (x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_2x_3 + x_3x_1) = 6$, so $4(\cos^2 20^\circ + \cos^2 100^\circ + \cos^2 140^\circ) = 6$	M1
	$\cos^2 100^\circ = \cos^2 80^\circ$, $\cos^2 140^\circ = \cos^2 40^\circ$: value = $\frac{3}{2}$	A1 (3)
		(14 marks)

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Notes

Difficulty features: hidden method in (c) and (d) (the student must think of factorising the cubic and comparing coefficients); unfamiliar presentation (a cubic solved with a trigonometric substitution; trigonometric results read from a polynomial); proof and justification (the identity, why there are no other roots, the exact product); cases and restrictions (extended range for 3θ with $0 \leq \theta \leq 180^\circ$).

(a) Second M1: uses $\cos 2\theta = 2\cos^2 \theta - 1$, $\sin 2\theta = 2\sin \theta \cos \theta$ and $\sin^2 \theta = 1 - \cos^2 \theta$. A1*: cso.

(b) Second M1: at least two values of 3θ in the extended range. A1: all three roots, awrt. B1: the 'at most three roots' argument; 'because $|x| \leq 2$ ' alone is B0.

(c) First M1: the factorised form follows from (b) because a monic cubic with three distinct roots is the product of the three linear factors; compares constant terms. A1*: both sign changes clearly shown.

(d) Alternative: $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ gives $\frac{3}{2} + \frac{1}{2}(\cos 40^\circ + \cos 80^\circ - \cos 20^\circ)$, and the bracket is 0 from the x^2 coefficient (or sum-to-product); M1 M1 A1. Accept 1.5. A value from a calculator with no working scores 0 marks.



SECTION B: STATISTICS

Question 11 Statistics · Easy · Binomial distribution

Question	Scheme	Marks
11(a)	$X \sim B(25, 0.08)$	B1 (1)
11(b)	$\binom{25}{3} (0.08)^3 (0.92)^{22}$	M1
	$= 0.1881$	A1 (2)
11(c)	$1 - P(X \leq 3) = 1 - 0.8649$	M1
	$= 0.1351$	A1 (2)
		(5 marks)

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Notes

(a) B1: binomial with both parameters.

(b) M1: alternative $P(X \leq 3) - P(X \leq 2)$. A1: awrt 0.188.(c) M1: writing or using $1 - P(X \leq 3)$; $1 - P(X \leq 4)$ is M0. A1: awrt 0.135.**Question 12** Statistics · Medium · The normal distribution

Question	Scheme	Marks
12(a)	$\frac{30 - \mu}{\sigma} = -1.0364$ or $\frac{48 - \mu}{\sigma} = 1.2816$	M1
	Both z values correct: ± 1.0364 and ± 1.2816 with correct signs at some stage	B1
	Solves $30 - \mu = -1.0364\sigma$ and $48 - \mu = 1.2816\sigma$ simultaneously	dM1
	$\mu = 38.0$	A1
	$\sigma = 7.77$	A1 (5)
12(b)	Standardises 35 or 45 with their μ and σ (or calculator normal function)	M1
	0.467	A1 (2)
12(c)	(their part (b)) ²	M1
	0.218	A1 (2)
		(9 marks)



Question 12 notes continued

Notes

(a) M1: standardises once with a z -value (not a probability); condone one sign slip. B1: awrt 1.04 and 1.28. A1 A1: awrt 38.0 and awrt 7.77.

(b) A1: awrt 0.467; accept 0.466 to 0.468 from rounded earlier values.

(c) A1: awrt 0.218; accept 0.217 to 0.219.

Question 13 Statistics · Medium · Hypothesis testing: binomial

Question	Scheme	Marks
13(a)	$X \sim B(40, 0.45): \binom{40}{18} (0.45)^{18} (0.55)^{22}$	M1
	$= 0.1260$	A1 (2)
13(b)	$H_0 : p = 0.45, \quad H_1 : p \neq 0.45$	B1 (1)
13(c)	$P(X \leq 11) = 0.0179$ [$P(X \leq 12) = 0.0386$]; $P(X \geq 25) = 0.0196$ [$P(X \geq 24) = 0.0405$]	M1
	$X \leq 11$	A1
	$X \geq 25$	A1 (3)
13(d)	$0.0179 + 0.0196 = 0.0375$	B1 (1)
13(e)	25 lies in the critical region	M1
	Reject H_0 : there is evidence that the proportion of players who complete the first level at their first attempt is not 45%	A1 (2)
		(9 marks)

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Notes

(b) B1: both hypotheses in terms of p (or π). Hypotheses in words only, or in terms of X , are B0.

(c) M1: a relevant cumulative probability for either tail. A1 A1: accept $X < 12$ and $X > 24$; set notation is acceptable.

(d) B1: awrt 0.0375.

(e) M1: alternatively $P(X \geq 25) = 0.0196 < 0.025$. A1: dependent on M1; a conclusion in context that is not definite, e.g. 'the claim is not supported'. 'The claim is wrong' is A0.

Question 14 Statistics · Medium · Regression and correlation

Question	Scheme	Marks
14(a)	$N = 10^{2.15+0.0792t}$ (or $\log_{10} N = \log_{10} a + t \log_{10} b$)	M1
	$N = 10^{2.15} \times (10^{0.0792})^t$, which is of the form ab^t	A1



Question 14 continued

Question	Scheme	Marks
	$a = 141, b = 1.20$	A1 (3)
14(b)	The number of bacteria increases by 20% each hour (is multiplied by 1.2 each hour)	B1 (1)
14(c)	$H_0 : \rho = 0, H_1 : \rho < 0$	B1
	Critical value -0.5822	B1
	$-0.6013 < -0.5822$ so reject H_0 : there is evidence of negative correlation between time and $\log_{10} N$, which supports the biologist's belief	A1 (3)
		(7 marks)

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Notes

(a) Second A1: both values, awrt 141 and 1.20.

(b) B1: must refer to the rate of growth per hour in context; 'b is the gradient' is B0.

(c) First B1: hypotheses in terms of ρ . Second B1: ± 0.5822 . A1: correct comparison and a conclusion in context mentioning correlation (or the relationship) between time and $\log N$ or the number of bacteria.

Question 15 Statistics · Hard · Probability: Venn diagrams, independence

Question	Scheme	Marks
15(a)	$\frac{0.08}{0.08 + r} = 0.5$	M1
	$r = 0.08$	A1 (2)
15(b)	$p + q + 0.08 + 0.12 + 0.08 + 0.3 = 1 \Rightarrow p + q = 0.42$	M1
	$(p + 0.12)(q + 0.2) = 0.12$	M1
	$(p + 0.12)(0.62 - p) = 0.12 \Rightarrow p^2 - 0.5p + 0.0456 = 0$, solved	dM1
	$p = 0.12, q = 0.3$ or $p = 0.38, q = 0.04$	A1
	$P(A) < P(B)$ gives $p = 0.12, q = 0.3$	A1 (5)
15(c)	$\frac{P(A \cap (B \cup C))}{P(B \cup C)} = \frac{0.12}{0.58}$	M1
	$= \frac{6}{29}$	A1 (2)
15(d)	$P(B) \times P(C) = 0.5 \times 0.16$	M1
	$= 0.08 = P(B \cap C)$ so B and C are independent	A1 (2)



Question 15 continued

Question	Scheme	Marks
		(11 marks)

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Notes

Difficulty features: crucial rearrangement (independence gives a quadratic in p); cases and restrictions (two solutions; the condition $P(A) < P(B)$ selects one); several conditions to combine (exclusivity, independence, conditional probability).

(b) Second M1: uses $P(A \cap B) = P(A) \times P(B)$ with expressions for both probabilities. dM1: dependent on both previous M marks; eliminates one variable and solves the quadratic (equivalently $q^2 - 0.34q + 0.012 = 0$).

(b) First A1: both pairs. Second A1: selects the correct pair with a reason: $p = 0.38$ gives $P(A) = 0.5 > P(B) = 0.24$.

(c) A1: $\frac{6}{29}$ or awrt 0.207.

(d) M1: alternatively compares $P(B | C) = 0.5$ with $P(B) = 0.5$. A1: correct values and a conclusion.

Question 16 Statistics · Hard · Hypothesis testing: normal mean

Question	Scheme	Marks
16(a)	$H_0 : \mu = 24, \quad H_1 : \mu < 24$	B1
	$\frac{32.82 - 24}{\sigma} = 1.96$	M1
	$\sigma = 4.5$	A1
	$\bar{y} = \frac{442}{20} = 22.1$ and $\bar{Y} \sim N\left(24, \frac{4.5^2}{20}\right)$	M1
	$z = \frac{22.1 - 24}{4.5/\sqrt{20}}$ or $P(\bar{Y} < 22.1)$	M1
	$z = -1.888 < -1.6449$ or $P(\bar{Y} < 22.1) = 0.0295 < 0.05$	A1
	Reject H_0 : there is evidence that the mean time with the new instructions is less than 24 minutes, which supports the designer's belief	A1 (7)
16(b)	$\frac{22.1 - 24}{4.5/\sqrt{n}} < -2.3263$	M1
	$\sqrt{n} > \frac{2.3263 \times 4.5}{1.9} = 5.510 \Rightarrow n > 30.36$	M1
	$n = 31$	A1 (3)
		(10 marks)

Notes

Difficulty features: hidden step (the standard deviation must be deduced from a percentage point, unprompted); parameters instead of numbers (the sample size n is the unknown in (b)); cases and restrictions (the smallest integer n , not the boundary value).



Question 16 notes continued

(a) B1: hypotheses in terms of μ . First M1: standardises 32.82 with $z = 1.96$ (from the tables). A1: $\sigma = 4.5$. Second M1: uses $\bar{y} = 22.1$ and divides the variance by 20. Third M1: standardises. First A1: awrt -1.89 or awrt 0.0295 with a correct comparison. Final A1: dependent on all M marks; conclusion in context, not definite.

(a) Critical region route: $\bar{Y} < 24 - 1.6449 \times \frac{4.5}{\sqrt{20}} = 22.34$ and 22.1 lies in it; marks as above.

(b) First M1: correct inequality or equation with ± 2.3263 (awrt 2.33). Second M1: rearranges for $\sqrt{n}$ or n . A1: $n = 31$ cao; 30 or 30.36 is A0.

Question 17 Statistics · Hard · The normal distribution: quartiles and independence

Question	Scheme	Marks
17(a)	By symmetry $Q_3 = \mu + 6$, so $\frac{6}{\sigma} = 0.6745$	M1
	$z = 0.6745$	B1
	$\sigma = 8.90$	A1
	$\frac{32 - \mu}{\sigma} = -1.2816$	M1
	$\mu = 43.4$	A1 (5)
17(b)	$(1 - p)^8 = 0.9$, where $p = P(L < h)$	M1
	$p = 1 - 0.9^{1/8} = 0.01308$	A1
	$\frac{h - 43.4}{8.90} = -2.224$	M1
	$h \leq 23.6$	A1
	$h = 23$ (rounding down: $h = 24$ gives probability $0.889 < 0.9$; $h = 23$ gives 0.916)	A1 (5)
		(10 marks)

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Notes

Difficulty features: hidden method (the interquartile range and symmetry give σ); unfamiliar presentation in (b) (a guarantee about a whole pack must be turned into a probability for one battery); cases and restrictions (the whole number h must be rounded down, not to the nearest integer, and checked).

(a) First M1: uses the interquartile range with symmetry about μ ($2 \times 0.6745\sigma = 12$). B1: awrt 0.674 or 0.675 . Second M1: standardises 32 with $z = \pm 1.2816$ (awrt 1.28). A1 A1: awrt 8.90 and awrt 43.4 .

(b) First M1: forms an equation or inequality for all 8 lasting at least h hours (independence). A1: awrt 0.0131 . Second M1: standardises with their p and a negative z (awrt -2.22). A1: awrt 23.6 . Final A1: 23 , with a reason (larger h lowers the probability, or the check at 24); 24 is A0. Using $0.9 \div 8$ or $1 - 0.9 = 0.1$ for p is M0.



Question 18 Statistics · Super hard · Calculus, the normal distribution and a binomial test

Question	Scheme	Marks
18(a)	$\frac{d\mu}{dt} = 0.2 - 0.04t = 0 \Rightarrow t = 5, \mu_{\max} = 50 + 1 - 0.5 = 50.5$	M1
	Greatest proportion when μ is greatest: $P(L > 51.5) =$ $P\left(Z > \frac{51.5 - 50.5}{0.8}\right) = P(Z > 1.25)$	M1
	$= 0.1056$	A1 (3)
18(b)	$H_0 : p = 0.1056, H_1 : p > 0.1056$	B1
	$Y \sim B(50, 0.1056): P(Y \geq 10) = 1 - P(Y \leq 9)$	M1
	$= 0.0342 < 0.05$ (or critical region $Y \geq 10$), so reject H_0	A1
	There is evidence that the proportion of bolts rejected 5 hours after switch-on is greater than the model predicts	A1 (4)
18(c)	More than 5% rejected when $\frac{51.5 - \mu}{0.8} < 1.6449$, i.e. $\mu > 51.5 - 1.6449 \times 0.8 = 50.184$	B1
	$50 + 0.2t - 0.02t^2 = 50.184 \Rightarrow t^2 - 10t + 9.2 = 0$, solved	M1
	$t = 1.03$ or $t = 8.97$	A1
	Proportion exceeds 5% for $1.03 < t < 8.97$; 8.97 lies outside $0 \leq t \leq 8$, so the interval is $1.03 < t \leq 8$	M1
	Total time $= 8 - 1.03 = 6.97$ hours	A1 (5)
		(12 marks)

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Notes

Difficulty features: hidden method (the greatest proportion needs the maximum mean, found by differentiating; the student is not told to); several topics connected (differentiation, normal distribution, binomial hypothesis test, quadratic equations); cases and restrictions (in (c) the proportion exceeds 5% between the roots, and the interval must be cut at $t = 8$); long chain of reasoning (maximise, convert to a proportion, test, then invert the model for a total time).

(a) First M1: differentiates μ (or completes the square, $\mu = 50.5 - 0.02(t - 5)^2$), sets the derivative to zero and finds the greatest mean. Second M1: standardises 51.5 with their greatest mean. A1: awrt 0.106 (0.1056).

(b) B1: $p =$ their (a), one-tailed; $p = 0.106$ is acceptable. First A1: awrt 0.0342 (0.0343 from unrounded p ; 0.0350 if $p = 0.106$ is used) compared with 0.05, or a correct critical region, and H_0 rejected. Second A1: dependent on the M mark; conclusion in context, not definite. Critical region route: $P(Y \geq 9) = 0.0763$ and $P(Y \geq 10) = 0.0342$ (0.0778 and 0.0350 with $p = 0.106$), so CR $Y \geq 10$ and 10 lies in it; full marks.

(c) B1: inverse normal with $z = 1.6449$ (awrt 1.64) giving 50.184 (awrt 50.18). First M1: forms and solves the three-term quadratic. A1: both roots, awrt 1.03 and 8.97 (using $\mu = 50.18$ gives $t = 1$ and 9; accept). Second M1: uses the interval between the roots, restricted to $0 \leq t \leq 8$, stating that 8.97 lies outside the domain. A1: awrt 6.97 (accept 7.0 from $t = 1$). Answering $8.97 - 1.03 = 7.94$ is M0 A0 for the last two marks.



Question 19 Statistics · Super hard · Conditional probability with normal distributions

Question	Scheme	Marks
19(a)	$P(D A) = P\left(Z > \frac{20.3 - 20.0}{0.15}\right)$ or $P(D B) = P\left(Z > \frac{20.3 - 20.1}{0.2}\right)$	M1
	$P(D A) = 0.0228$ and $P(D B) = 0.1587$	A1
	$\frac{0.1587(1 - q)}{0.0228q + 0.1587(1 - q)} = 0.8$	M1
	$0.03174(1 - q) = 0.01820q$, solved for q	dM1
	$q = 0.635$	A1 (5)
19(b)	Original $P(D) = 0.635 \times 0.0228 + 0.365 \times 0.1587 = 0.0723$	M1
	$0.635 \times 0.0228 + 0.365 P(D B) = \frac{1}{2} \times 0.0723$	M1
	$P(D B) = 0.0595$	A1
	$z = 1.559$	M1
	$\frac{20.3 - \mu_B}{0.2} = 1.559$	M1
	$\mu_B = 19.99$	A1 (6)
		(11 marks)

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Notes

Difficulty features: parameters instead of numbers (the proportion of rods made by machine A is unknown and must be deduced from a conditional probability); several topics connected (two normal models, total probability, conditional probability, inverse normal); hidden method in (b) (halving the overall rate must be turned into a tail probability for machine B alone, using the proportion found in (a)); long chain of reasoning.

(a) Second M1: Bayes (or a tree) with q and $1 - q$ set equal to 0.8. dM1: solves the linear equation. A1: awrt 0.635.

(b) First M1: total probability with their q (awrt 0.0723). Second M1: halves it with machine A unchanged; halving only machine B 's rate is M0. Third M1: inverse normal for their probability (awrt 1.56). Fourth M1: standardises with the unknown mean. A1: awrt 20.0 (19.99).

Question 20 Statistics · Super hard · Discrete distributions and a binomial test

Question	Scheme	Marks
20(a)	$k + \frac{k}{2} + \frac{k}{3} + \frac{k}{6} = 1 \Rightarrow k = \frac{1}{2}$	M1 A1
	Total > 6: (1, 6), (6, 1), (2, 6), (6, 2), (3, 6), (6, 3), (6, 6)	M1
	$P(\text{win}) = 2\left(\frac{1}{2}\right)\left(\frac{1}{12}\right) + 2\left(\frac{1}{4}\right)\left(\frac{1}{12}\right) + 2\left(\frac{1}{6}\right)\left(\frac{1}{12}\right) + \left(\frac{1}{12}\right)^2 = \frac{23}{144}$	A1



Question 20 continued

Question	Scheme	Marks
	$H_0 : p = \frac{23}{144}, \quad H_1 : p > \frac{23}{144}$	B1
	$Y \sim B\left(n, \frac{23}{144}\right)$; requires $P(Y \geq 11) \leq 0.05$	M1
	Trials: $n = 40$: $P(Y \geq 11) = 0.0449$; $n = 41$: $P(Y \geq 11) = 0.0528$	M1
	Both values correct	A1
	Largest $n = 40$	A1 (9)
20(b)	For fixed 11 wins, $P(Y \geq 11)$ increases as the number of games increases (more games make 11 wins more likely under H_0)	M1
	So with fewer than 40 games $P(Y \geq 11) < 0.0449 < 0.05$: there is evidence that she wins more often than the model predicts	A1 (2)
		(11 marks)

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Notes

Difficulty features: parameters instead of numbers (the number of games n is the unknown); hidden method (the student must find k , then the model probability of a win, before any test can be set up); cases (careful enumeration of the pairs of scores with a total greater than 6); proof and justification (part (b) needs an argument about how the tail probability changes with n).

(a) Second M1: identifies the pairs with a total greater than 6 (at least five correct, no extras). A1: $\frac{23}{144}$ or awrt 0.160.

B1: one-tailed, with their p . Third M1: the correct condition on $P(Y \geq 11)$. Fourth M1: tries values of n either side of the boundary. A1: awrt 0.0449 and 0.0528 (with $p = 0.16$: 0.045 and 0.053). Final A1: 40 cao.

(b) M1: a correct reason about how the tail probability depends on n . A1: correct conclusion in context; a conclusion with no reason is M0 A0.



SECTION C: MECHANICS

Question 21 Mechanics · Easy · Vectors: velocity and position

Question	Scheme	Marks
21(a)	$\tan^{-1}\left(\frac{6}{8}\right) = 36.9^\circ$ west of north (or $\tan^{-1}\left(\frac{8}{6}\right) = 53.1^\circ$ north of west)	M1
	Bearing 323°	A1 (2)
21(b)	$\mathbf{r} = (2\mathbf{i} - 5\mathbf{j}) + 2.5(-6\mathbf{i} + 8\mathbf{j})$	M1
	$= -13\mathbf{i} + 15\mathbf{j}$	A1
	$\sqrt{13^2 + 15^2} = \sqrt{394} = 19.8 \text{ km}$	A1 (3)
		(5 marks)

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Notes

(a) A1: 323 cao, e.g. $360^\circ - 36.9^\circ$ or $270^\circ + 53.1^\circ$; 037, 053, 143 and 307 are A0.(b) M1: uses $t = 2.5$ and adds the initial position; condone one arithmetic slip. $t = 2.3$ is M1 A0 A0. A1: awrt 19.8; accept $\sqrt{394}$.

Question 22 Mechanics · Medium · Constant acceleration: vertical motion

Question	Scheme	Marks
22(a)	$0^2 = 7^2 - 2 \times 9.8 \times s$	M1
	$s = 2.5$	A1
	Greatest height $= 2.5 + 23.1 = 25.6 \text{ m}$	A1 (3)
22(b)	$-23.1 = 7t - 4.9t^2$ (or a complete two-stage method)	M1
	$4.9t^2 - 7t - 23.1 = 0 \Rightarrow 7t^2 - 10t - 33 = 0 \Rightarrow (7t + 11)(t - 3) = 0$	A1
	$t = 3 \text{ s}$	A1 (3)
22(c)	$v = 7 - 9.8 \times 3$ (or $v^2 = 7^2 + 2 \times 9.8 \times 23.1$)	M1
	Speed $= 22.4 \text{ m s}^{-1}$	A1 (2)
		(8 marks)

Notes

(a) Second A1: 25.6 (accept 26).

(b) M1: condone sign errors in a three-term equation with $s = \pm 23.1$, $u = \pm 7$, $a = \pm 9.8$. Two-stage method: time to the top $\frac{5}{7} \text{ s}$ plus time to fall 25.6 m from rest $\frac{16}{7} \text{ s}$. A1: 3 cao; any negative root must be rejected.

Question 22 notes continued

(c) A1: 22.4 (accept 22); a negative final answer for a speed is A0.

Question 23 Mechanics · Medium · Connected particles: Newton's laws

Question	Scheme	Marks
23(a)	Whole system: $8200 + 600 + 200 = 3000a$	M1
	Deceleration 3 m s^{-2}	A1 (2)
23(b)	Trailer: $F + 200 = 600 \times 3$ (or van: $8200 + 600 - F = 2400 \times 3$)	M1
	Correct equation with their deceleration	A1ft
	$F = 1600 \text{ N}$	A1
	Thrust (the tow bar pushes the trailer backwards)	B1 (4)
23(c)	1600 N, forwards (in the direction of motion)	B1ft (1)
		(7 marks)

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Notes

(a) M1: Newton's second law for the whole system with the three external forces and mass 3000; condone sign errors. Separate equations for van and trailer solved for a also score M1.

(b) M1: equation for the trailer alone (or the van alone) including the tow bar force and the correct mass. A1ft: follow through their deceleration. A1: 1600 cao. B1: thrust, with a reason from the direction of the force (or a negative tension).

(c) B1ft: their magnitude from (b) with the direction; both needed.

Question 24 Mechanics · Medium · Moments

Question	Scheme	Marks
24(a)	On the point of tipping about D : reaction at C is zero	B1
	Moments about D : $60g \times 1.5 = 30g \times (4.5 - x)$	M1
	$90 = 135 - 30x \Rightarrow x = 1.5 \text{ m}$	A1* (3)
24(b)	On the point of tipping about C , reaction at D is zero; man $y \text{ m}$ from A	B1
	Moments about C : $60g(1 - y) = 30g \times 0.5$	M1
	$y = 0.75 \text{ m}$	A1 (3)
24(c)	The man's weight acts at a single point (where he stands)	B1 (1)
		(7 marks)



Question 24 notes continued

Notes

(a) M1: moments about D (or any point, with the reaction at D found or eliminated); condone sign slips. A1*: given answer, cso.

(b) Once the man has left B the plank cannot tip about D , so the only possible tipping is about C , when he is between A and C . M1: correct moments equation about C with $R_D = 0$. A1: 0.75 m.

(c) B1: 'the man's mass is concentrated at a point' is also acceptable; 'the man has no dimensions' alone is B0.

Question 25 Mechanics · Hard · Friction on an inclined plane

Question	Scheme	Marks
25(a)	$R + 110 \sin 20^\circ = 12g \cos \alpha$	M1
	$R + 37.62 = 94.08$	A1
	$R = 56.5 \text{ N}$	A1 (3)
25(b)	$F = 0.3R (= 16.9)$	M1
	$110 \cos 20^\circ - 12g \sin \alpha - F = 12a$	M1
	$103.37 - 70.56 - 16.94 = 12a$	A1ft
	$a = 1.32 \text{ m s}^{-2}$	A1 (4)
25(c)	$R' = 12g \cos \alpha - P \sin 20^\circ$ used in $F' = 0.3R'$	M1
	$P \cos 20^\circ = 12g \sin \alpha + 0.3(12g \cos \alpha - P \sin 20^\circ)$	dM1
	$P = 94.8$	A1 (3)
		(10 marks)

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Notes

Difficulty features: modelling decisions (the normal reaction changes when the tension changes); several forces resolved in two directions with a non-standard angle.

(a) M1: resolves perpendicular to the slope with all three forces; condone sin/cos confusion and sign errors. M0 if the component of the tension is missing. A1: awrt 56 or 56.5.

(b) Second M1: all four terms, friction acting down the slope. A1ft: with their F . A1: awrt 1.3.

(c) M1: a new normal reaction in terms of P (M0 if the reaction from (a) is reused). dM1: zero acceleration, all three terms. A1: awrt 95 or 94.8.

Question 26 Mechanics · Hard · Connected particles on a rough plane

Question	Scheme	Marks
26(a)	B in equilibrium: $T = mg$	M1
	$T = kmg \sin \alpha + \mu R$, $R = kmg \cos \alpha$	M1

Question 26 continued

Question	Scheme	Marks
	$mg = \frac{3}{5}kmg + \frac{4}{5}\mu kmg$	A1
	$5 = 3k + 4\mu k \Rightarrow \mu = \frac{5 - 3k}{4k}$	A1* (4)
26(b)	Least k : A about to move up, friction limiting down the plane: $1 = \frac{3}{5}k + \frac{1}{2} \cdot \frac{4}{5}k$	M1
	Greatest k : A about to move down, friction limiting up the plane: $1 = \frac{3}{5}k - \frac{1}{2} \cdot \frac{4}{5}k$	M1
	$k = 1$ and $k = 5$	A1
	$1 \leq k \leq 5$	A1 (4)
26(c)	$k = 6 > 5$ so A moves down the plane and B moves up: $6mg \sin \alpha - T - \frac{1}{2}(6mg \cos \alpha) = 6ma, T - mg = ma$	M1
	Eliminates T : $3.6mg - 2.4mg - mg = 7ma$	M1
	$a = \frac{g}{35}$	A1 (3)
		(11 marks)

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Notes

Difficulty features: cases (friction limiting up the plane at one extreme and down the plane at the other); parameters instead of numbers (the mass ratio k); modelling decisions (direction of motion and of friction must be decided in (c)).

(a) Second M1: all terms present, condone sign errors and sin/cos confusion. A1*: given answer, cso.

(b) For $1 < k < 5$ friction is not limiting. Accept strict inequalities in the final answer.

(c) The string stays taut: $T = m(g + \frac{a}{35}) = \frac{36}{35}mg > 0$. A1: accept awrt 0.0286g (0.28 m s⁻² is A0 as the answer is required in terms of g).

Question 27 Mechanics · Hard · Projectiles: a range of possible speeds

Question	Scheme	Marks
27(a)	At the wall: $21 = 0.6Ut$ and $y = 0.8Ut - 4.9t^2$	M1
	$y = 28 - \frac{6002.5}{U^2}$	A1
	$28 - \frac{6002.5}{U^2} > 8.4 \Rightarrow U^2 > 306.25$	dM1
	$U > 17.5$	A1
	Time of flight $\frac{1.6U}{9.8}$, horizontal distance $0.6U \times \frac{1.6U}{9.8} = \frac{0.96U^2}{9.8}$	M1
	Correct range expression	A1



Question 27 continued

Question	Scheme	Marks
	$\frac{0.96U^2}{9.8} \leq 33 \Rightarrow U^2 \leq 336.875$	dM1
	$17.5 < U \leq 18.35$	A1 (8)
27(b)	At the wall: $t = \frac{21}{10.8}$, vertical velocity $14.4 - 9.8 \times \frac{21}{10.8} = -4.66$	M1
	Speed = $\sqrt{10.8^2 + 4.66^2}$	M1
	$= 11.8 \text{ m s}^{-1}$	A1 (3)
27(c)	e.g. the model ignores air resistance, which would reduce the range (or the ball has a size, so it could clip the wall)	B1 (1)
		(12 marks)

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Notes

Difficulty features: parameters instead of numbers (the speed U is unknown and the answer is a set); cases and restrictions (two conditions give two inequalities, one strict and one not); modelling decisions (a limitation that affects the answer).

(a) First M1: time to reach the wall and the height there (or the trajectory equation at $x = 21$). First dM1: inequality (or equation) with 8.4, solved. Second M1: time of flight and range. Second dM1: inequality (or equation) with 33, solved. Final A1: both limits, awrt 18.35, with $>$ at 17.5 (to clear the wall) and $\leq$ at 18.35 (accept $<$ at either end for this mark only if both are found).

(a) The ball automatically lands beyond the wall when it clears it; candidates need not show this.

(b) A1: awrt 11.8.

(c) B1: e.g. air resistance would reduce the range, so a larger U would be needed; or the ball has a size, so it needs to pass more than 8.4 m above the ground.

Question 28 Mechanics · Super hard · Statics: ladder with friction at both ends

Question	Scheme	Marks
28(a)	Resolving horizontally (friction at A towards the wall): $\frac{1}{2}R = S$	M1
	Resolving vertically (friction at B upwards): $R + \frac{1}{3}S = 4W$	M1
	Both correct, with both frictions limiting and in the correct directions	A1
	Moments about A , man x m from A : $S \times 4 \sin \theta + \frac{1}{3}S \times 4 \cos \theta = W \times 2 \cos \theta + 3W \times x \cos \theta$	M1
	$3.2S + 0.8S = 1.2W + 1.8Wx$	A1
	Solves the resolving equations: $R = \frac{24W}{7}$, $S = \frac{12W}{7}$	dM1
	$S = \frac{12W}{7}$ (or $R = \frac{24W}{7}$)	A1



Question 28 continued

Question	Scheme	Marks
	$4 \times \frac{12W}{7} = 1.2W + 1.8Wx$, solved for x	dM1
	$x = \frac{22}{7} \text{ m } (= 3.14 \text{ m})$	A1 (9)
28(b)	Man at B : moments $4S = 1.2W + 7.2W \Rightarrow S = 2.1W$	M1
	Resolving vertically with limiting friction at the wall: $R = 4W - \frac{1}{3}S = 3.3W$	M1
	Horizontally: $\mu R = S \Rightarrow \mu = \frac{2.1}{3.3}$	M1
	$\mu = \frac{7}{11}$	A1 (4)
		(13 marks)

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Notes

Difficulty features: modelling decisions (the directions of both friction forces must be chosen); little scaffolding (the greatest distance is asked for directly); long chain of reasoning (resolve twice, take moments, solve for the position); parameters instead of numbers (an unknown coefficient of friction in (b)).

R and S are the normal reactions at A and B . When the ladder is about to slip, A tends to move away from the wall (friction at A towards the wall) and B tends to move down (friction at B upwards).

(a) Moments may be taken about any point. First dM1: dependent on the first two M marks. Second dM1: dependent on all previous M marks. A1: $\frac{22}{7}$ or awrt 3.14.

(b) Third M1: uses limiting friction at the ground with their R and S . A1: $\frac{7}{11}$ or awrt 0.636. As $\frac{1}{2} < \frac{7}{11}$, this is consistent with (a), where the man could not reach the top.

Question 29 Mechanics · Super hard · Variable acceleration: distance and displacement

Question	Scheme	Marks
29(a)	$s = \int (t^2 - 9t + 14) dt$	M1
	$s = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 14t$ (no constant, as $s = 0$ when $t = 0$)	A1
	$v = 0: (t - 2)(t - 7) = 0$	M1
	$t = 2$ and $t = 7$; the robot moves away from W until $t = 2$, then towards W until $t = 7$, then away again	A1
	Least displacement at $t = 7$: $s(7) = \frac{343}{3} - \frac{441}{2} + 98$	M1
	$s(7) = -\frac{49}{6} = -8.17 \text{ m}$; $8.17 < 9$, so the robot does not reach the wall	A1 (6)
29(b)	Splits at $t = 2$ and $t = 7$: $s(2) = \frac{38}{3}$, $s(7) = -\frac{49}{6}$, $s(8) = -\frac{16}{3}$	M1



Question 29 continued

Question	Scheme	Marks
	Distances $\frac{38}{3}$, $\frac{38}{3} + \frac{49}{6}$ and $\frac{49}{6} - \frac{16}{3}$	A1
	Adds the magnitudes	dM1
	$\frac{109}{3} = 36.3 \text{ m}$	A1 (4)
29(c)	$36.3 > 35$, so the model suggests the battery is not sufficient (it would run out before $t = 8$)	B1ft (1)
		(11 marks)

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Notes

Difficulty features: hidden method (the robot changes direction: the times when $v = 0$ must be found, though the question never mentions them); cases and restrictions (distance and displacement differ; the integral must be split at each change of direction); long chain of reasoning (integrate, locate the turning points, compare with the wall, add magnitudes); modelling decisions (use the distance to judge whether the battery is sufficient).

(a) Third M1: the decision must use the turning point at $t = 7$ (and the value at $t = 8$, $-\frac{16}{3}$, which is nearer O). Using only $s(8)$ is M0. Final A1: correct value and conclusion.

(b) Typical error: $\int_0^8 v \, dt = -\frac{16}{3}$ gives the displacement (5.33 m from O), not the distance; this scores M0. Integrating from 0 to 8 in one step scores 0 marks in (b).

(b) A1: $\frac{109}{3}$ or awrt 36.3. (c) B1ft: follow through their distance compared with 35.

Question 30 Mechanics · Super hard · Connected particles: rough slope, slack string

Question	Scheme	Marks
30(a)	Before B lands: $1.4 = \frac{1}{2}a(1)^2 \Rightarrow a = 2.8$, speed $v = 2.8 \text{ m s}^{-1}$	M1
	After B lands (string slack): $0 = 2.8^2 - 2a'(0.4) \Rightarrow a' = 9.8$	M1
	$2g \sin \alpha + \mu(2g \cos \alpha) = 2a'$, i.e. $0.6g + 0.8\mu g = 9.8$	M1
	$\mu = 0.5$	A1 (4)
30(b)	$A: T - 2g \sin \alpha - \mu(2g \cos \alpha) = 2 \times 2.8$	M1
	$B: Mg - T = 2.8M$	M1
	Eliminates T : $9.8M - 19.6 = 2.8M + 5.6$	dM1
	$M = 3.6$	A1 (4)
30(c)	Component of weight down the plane $2g \sin \alpha = 11.76 \text{ N}$; maximum friction $\mu(2g \cos \alpha) = 7.84 \text{ N}$	M1
	$11.76 > 7.84$, so A slides back down the plane	A1



Question 30 continued

Question	Scheme	Marks
	Friction now up the plane: $2g \sin \alpha - \mu(2g \cos \alpha) = 2a''$, $a'' = 1.96$	M1
	String taut again after A slides back the 0.4 m of slack: $v^2 = 2 \times 1.96 \times 0.4$	M1
	$v = 1.25 \text{ m s}^{-1}$	A1 (5)
		(13 marks)

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Notes

Difficulty features: parameters instead of numbers (both the mass of B and the coefficient of friction are unknown and must be found from motion data); modelling decisions (the string goes slack; friction reverses direction when A starts to slide back); cases (whether A stays at rest must be decided); long chain of reasoning (three stages of motion linked by speeds and distances).

(a) M1: constant acceleration for the first stage, giving a and v . M1: constant acceleration for the second stage, giving the deceleration. M1: Newton's second law for A with the string slack, weight component and friction both down the plane. A1: 0.5.

(b) M1 M1: condone sign errors; the whole-system equation $Mg - 2g \sin \alpha - \mu(2g \cos \alpha) = (M + 2)(2.8)$ scores both. Tension check: $T = 3.6(9.8 - 2.8) = 25.2 \text{ N} > 0$.

(c) First M1: compares the two forces (or $\tan \alpha = 0.75 > \mu$). Second M1: friction must act up the plane. Third M1: recognises that the string is slack for 0.4 m, the extra distance A moved after B landed. A1: awrt 1.25.





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