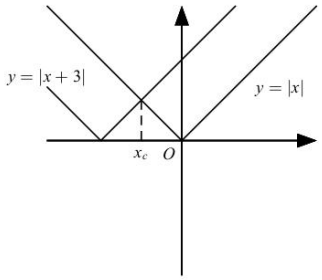
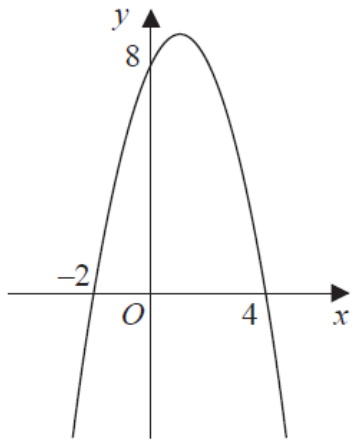
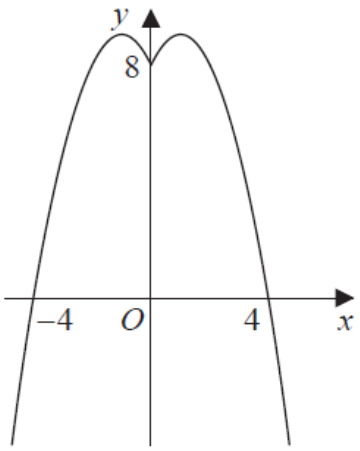
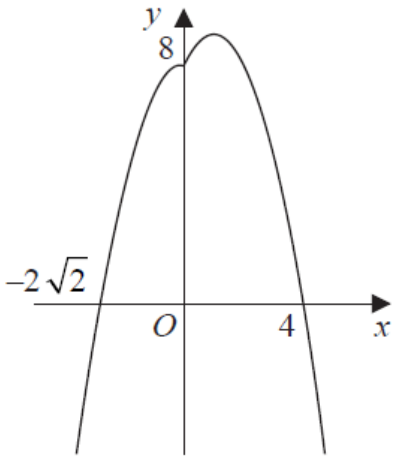
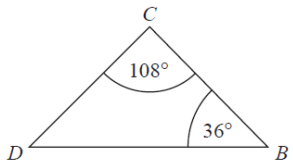
AEA Mark scheme

Question	Scheme		Marks	AOs
1(a)	$(1-y)^{-2} = 1 + 2y + 3y^2 + 4y^3 + \ldots$		**B1**	AO1
			(1)	
(b)	$S = 1 + 2\left(\frac{x}{x+3}\right) + 3\left(\frac{x}{x+3}\right)^2 + \cdots$	Identifies $y = \frac{x}{x+3}$	**M1**	AO1
	$\Rightarrow S = 1 + 2y + 3y^2 + \cdots = \left(1 - \frac{x}{x+3}\right)^{-2}$	Uses (a) to form correct expression.	**A1**	AO2
	$= \left(\frac{x+3-x}{x+3}\right)^{-2} = \left(\frac{x+3}{3}\right)^2$	Combines terms and reciprocates.	**M1**	AO2
	$= \left(1 + \frac{1}{3}x\right)^2$ (so $a = \frac{1}{3}$ and $n = 2$)	Correct answer.	**A1**	AO2
			(4)	
(c)	Need $\left\|\frac{x}{x+3}\right\| < 1$ or $\|x\| < \|x+3\|$	Correct condition.	**B1**	AO1
	 Critical value is when $x + 3 = -x$, so $x = \ldots$	Any suitable strategy for critical value.	**M1**	AO3
	So convergent on $\left\{x \in \mathbb{R} : x > -\frac{3}{2}\right\}$ Accept equivalent set notation forms, e.g. $\left\{x : x \in \mathbb{R}, x > -\frac{3}{2}\right\}$ or $\left\{x : x > -\frac{3}{2}\right\}$ or $\left(-\frac{3}{2}, \infty\right]$		**A1**	AO2
			(3)	
				(8 marks)

Question	Scheme		Marks	AOs
2(a)	**(i)** 	**(ii)** 		
(i)	Correct shape and (0, 8)		**B1**	AO1
	Crossing x-axis at -2 and 4		**B1**	AO1
(ii)	Symmetrical shape with 2 maxima and crosses at ± 4		**B1**	AO1
	Correct shape at (0, 8) - not curved as a minimum		**B1**	AO3
(iii)		Correct for $x > 0$ and (4, 0) marked	**B1**	AO1
		Correct for $x \leqslant 0$ with $-2\sqrt{2}$ marked and zero gradient at (0, 8)	**B1**	AO3
		Clear "kink" at (0, 8)	**B1**	AO3
			(7)	
(b)	Considers $x \geqslant 0 : 0 \leqslant x \leqslant 4$ Correct inequality with both end points included.		**B1**	AO3
	Considers $x < 0$: attempts $-(8 - x^2) = 8 - 2x - x^2 \Rightarrow x = \ldots$		**M1**	AO3
	$x < 0, 2x^2 + 2x - 16 = 0 \Rightarrow x = \dfrac{-1-\sqrt{33}}{2}$ only (withhold A if second root given).		**M1** **A1**	AO3 AO3
			(4)	
				(11 marks)

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{AC} = k\mathbf{q}$	**B1**	AO1
		(1)	
(b)	$\overrightarrow{BO} = \overrightarrow{BD} + \overrightarrow{DO} = -k\mathbf{p} - \mathbf{q}$ or other suitable vector equation leading to $\overrightarrow{BO}$ used.	**M1**	AO1
	$\overrightarrow{BO} = k\overrightarrow{CD}$ or $\overrightarrow{CD} = \frac{1}{k}\overrightarrow{BO}$ or equivalent used (or implied by use).	**B1**	AO3
	$\overrightarrow{CD} = \frac{1}{k}(-k\mathbf{p} - \mathbf{q}) = -\mathbf{p} - \frac{1}{k}\mathbf{q}$ *	**A1***	AO2
		(3)	
(c)	Use of $\mathbf{p} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{q}$ or equivalent vector equation used.	**B1**	AO3
	$\Rightarrow \mathbf{p} + \overrightarrow{AC} + \overrightarrow{CD} = \mathbf{q}$ $\Rightarrow \mathbf{p} + k\mathbf{q} - \mathbf{p} - \frac{1}{k}\mathbf{q} = \mathbf{q} \Rightarrow (k^2 - k - 1)\mathbf{q} = \mathbf{0}$	**M1**	AO3
	$\Rightarrow k^2 - k - 1 = 0 \Rightarrow k = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$	**M1**	AO3
	(So as $k > 0$, we must have) $k = \frac{1+\sqrt{5}}{2}$ **(S+)**	**A1**	AO2
		(4)	
(d)	 Use the symmetry of the shape to deduce relevant angle e.g. $\angle BCD = 180° - 72° = 108°$. (May use another triangle within the shape.)	**B1**	AO1
	Hence $\sin 54° = \frac{\frac{1}{2}DB}{BC} = \frac{\frac{1}{2}kOA}{OA} = \frac{1}{2}k$ — Correct work in their triangle to reduce to an expression in k	**M1**	AO3
	$= \frac{1+\sqrt{5}}{4}$	**A1**	AO3
		(3)	
S1	S1 mark: Award S1 for a clear and concise solution that is either - 11 marks with correct notation for vectors in the main. or - that scores 10+ and includes the S+ point but may have some poor notation.	**(1)**	AO2
			(11 + 1 marks)

Notes:

(c) S+ for clear consideration of which root is needed.

Question	Scheme		Marks	AOs
4(a)	(Parallel to plane): $1.2W\cos\theta = 0.6W$	Resolves parallel to plane with correct value of $\sin\alpha$ used. **(S+)**	**M1**	AO3
	$\theta = \frac{\pi}{3}$	Accept 60° **(S−)**	**A1**	AO3
			(2)	
(b)	(Parallel to plane): $F = 1.2W\cos\theta - 0.6W$	M resolves in both directions. A for both correct. **(S+)**	**M1**	AO3
	(Perpendicular to plane): $R = 0.8W + 1.2W\sin\theta$		**A1**	AO3
	Minimum value of θ when $F = \mu R$ $\Rightarrow 1.2\cos\theta - 0.6 = 0.4 + 0.6\sin\theta$	Uses $F = \mu R$ to form equation in θ only.	**M1**	AO3
	$\Rightarrow 2\cos\theta - \sin\theta = \frac{5}{3} \Rightarrow \sqrt{5}\cos(\theta + \gamma) = \frac{5}{3}$	Applies $R\cos(\theta + \alpha)$	**M1**	AO2
	$\Rightarrow \theta = \arccos\left(\frac{\sqrt{5}}{3}\right) - \arctan\left(\frac{1}{2}\right)$*		**A1***	AO2
			(5)	
(c)	(Parallel to plane): $F = 0.6W - 1.2W\cos\theta$ (Perpendicular to plane): $R = 0.8W + 1.2W\sin\theta$	Resolves both directions.	**M1**	AO3
	$F = \mu R \Rightarrow 0.6W - 1.2W\cos\theta = 0.4W + 0.6W\sin\theta$	Correct equation using $F = \mu R$	**A1**	AO3
	$\Rightarrow 2\cos\theta + \sin\theta = \frac{1}{3} \Rightarrow \sqrt{5}\cos(\theta - \gamma) = \frac{1}{3}$ OR $\frac{6t}{1+t^2} + \frac{6(1-t^2)}{1+t^2} = 1 \Rightarrow 7t^2 - 6t - 5 = 0$	Correct strategy for finding θ	**M1**	AO3
	Greatest value of $\theta = \arccos\left(\frac{\sqrt{5}}{15}\right) + \arctan\left(\frac{1}{2}\right)$ OR $\theta = 2\arctan\left(\frac{3+2\sqrt{11}}{7}\right)$	Either form for θ	**A1**	AO3

Question	Scheme		Marks	AOs
4(c) *continued*	So range is $\frac{\pi}{3} < \theta \leqslant \arccos\left(\frac{\sqrt{5}}{15}\right) + \arctan\left(\frac{1}{2}\right)$ (oe)	Allow `least value' of θ is $\frac{\pi}{3}$ $(\frac{\pi}{3} \leqslant \theta)$ i.e lower strict inequality not required for this mark but incorrect inequality is. **(S−)**	**A1**	AO2
			(5)	
S1	S1 mark: Award S1 for a clear and concise solution that is either - fully correct and does not include an S− point. or - that scores 11/12 and includes an S+ point but not an S− point.		**(1)**	AO2
				(12 + 1 marks)
Notes:				
(a)+(b)	S+ for good reasoning for the angle, good use of diagrams etc. S− for giving answer in degrees.			

Question	Scheme		Marks	AOs
5	$\tan^2 x = 4\cos 2x - 1$ $\Rightarrow \sec^2 x - 1 = 4(2\cos^2 x - 1) - 1$ OR $\dfrac{\sin^2 x}{\cos^2 x} = 4(2\cos^2 x - 1) - 1$	Reduces RHS to single angle and adapts $\tan^2 x$ suitably.	**M1**	AO1
	$\Rightarrow 1 - \cos^2 x = 8\cos^4 x - 5\cos^2 x$	Achieve equation in just $\cos x$	**M1**	AO1
	$\Rightarrow 8\cos^4 x - 4\cos^2 x - 1 = 0$	Correct quartic.	**A1**	AO1
	$\Rightarrow \cos^2 x = \dfrac{4 \pm \sqrt{16 - 4\times 8\times -1}}{16} = \ldots$	Attempts roots.	**M1**	AO3
	Intersection point α satisfies $\cos^2 \alpha = \dfrac{1}{4}\left(1 + \sqrt{3}\right)$	Correct root identified. **(S+)**	**A1**	AO3
	So $\sin^2 \alpha = 1 - \dfrac{1}{4}\left(1 + \sqrt{3}\right) = \dfrac{1}{4}\left(3 - \sqrt{3}\right)$	Correct expressions for $\sin^2 \alpha$ or $\sin \alpha$ and $\tan^2 \alpha$ or $\tan \alpha$ seen anywhere in solution.	**B1**	AO2
	And $\tan^2 \alpha = \dfrac{3 - \sqrt{3}}{1 + \sqrt{3}} = 2\sqrt{3} - 3$		**B1**	AO2
	Area $= 2\times \displaystyle\int_0^{\alpha} \left(4\cos 2x - 1 - \tan^2 x\right) \mathrm{d}x$	Correct strategy for the area, use of 0 and α with 2×, or with $-\alpha$ and α. May integrate separately and combine later. **(S+)(S−)**	**M1**	AO3
	$= 2\times \displaystyle\int_0^{\alpha} \left(4\cos 2x - 1 - (\sec^2 x - 1)\right) \mathrm{d}x$	Uses suitable identity to set up integral (ignore 2× and limits here).	**M1**	AO3
	$= 2\times \left[2\sin(2x) - \tan x\right]_0^{\alpha}$	Correct integral (ignore limits etc.)	**A1**	AO3
	$= 2\times\left(4\sin\alpha\cos\alpha - \tan\alpha\right)$ $= 2\times\left(4\times\dfrac{1}{2}\sqrt{3-\sqrt{3}}\times\dfrac{1}{2}\sqrt{1+\sqrt{3}} - \sqrt{2\sqrt{3}-3}\right)$	Uses the expression in $\cos\alpha$, $\sin\alpha$ and $\tan\alpha$ in their integral.	**M1**	AO3

Question	Scheme		Marks	AOs
5 *continued*	$= 2\sqrt{2\sqrt{3}} - 2\sqrt{2\sqrt{3}-3}$ *	Completes to given answer.	**A1***	AO2
			(12)	
S1	S1 mark: Award S1 for a clear and concise solution that is either - fully correct with no S− point. or - that scores 11/12 and includes an S+ point and no S−.		**(1)**	AO2
				(12+1 marks)

Notes:

S+ for showing consideration and rejection of the negative root for $\cos^2 x$

S+ for good use of symmetry and/or odd and even function nature of the trig functions.

S− if the strategy is cumbersome – e.g. Separate integrals, no use of symmetry etc.

Question	Scheme		Marks	AOs
6(i)	$(\log_5 p)^2 = \log_5(p^2) \Rightarrow (\log_5 p)^2 = 2\log_5 p$	Applies power rule.	**M1**	AO1
	$\Rightarrow \log_5 p(\log_5 p - 2) = 0 \Rightarrow \log_5 p = 0 \therefore p = 1$		**A1**	AO2
	or $\Rightarrow \log_5 p = 2 \ \therefore \ p = 25$		**A1**	AO2
	$\log_5(q-p) = \log_5 q - \log_5 p$ $\Rightarrow \log_5(q-p) = \log_5\left(\frac{q}{p}\right)$	Applies correct rule.	**M1**	AO1
	$\Rightarrow q - p = \frac{q}{p}$		**A1**	AO2
	$(p = 1 \Rightarrow q - 1 = q$ is impossible$)$ so $q = \frac{p^2}{p-1}$	Makes q the subject, no need to see $p = 1$ rejected for M **(S+)**	**M1**	AO3
	So $p = 25$ and $q = \frac{625}{24}$ (only)		**A1**	AO3
			(7)	
(ii)(a)	$\log_r(A) = \log_{r^2}(B) \Rightarrow (r^2)^{\log_r(A)} = (r^2)^{\log_{r^2}(B)}$	Adopts a correct strategy, raise both sides to base r^2 or use change of base.	**M1**	AO2
	$\Rightarrow r^{2\log_r(A)} = B \Rightarrow r^{\log_r(A^2)} = B \Rightarrow A^2 = B$*	Achieves result correctly.	**A1***	AO2
			(2)	
(ii)(b)	$2 = \log_2(4)$ or $2 = \log_4(16)$	Seen anywhere.	**B1**	AO1
	$\therefore \log_4(3x^3 + 26x^2 + 40x) = \log_2(4(x+2))$ (oe)	Combines to just two log terms.	**M1**	AO3
	$\therefore 3x^3 + 26x^2 + 40x = 16(x+2)^2$	Applies the result of (ii)(a)	**M1**	AO3
	e.g. $x(3x+20)(x+2) = 16(x+2)^2$ or $3x^3 + 10x^2 - 24x - 64 = 0$ leading to $(x+2)(3x^2 + kx \pm 32) = 0$	Identifying a linear factor.	**M1**	AO3
	$(x+2)(\underline{3x^2 + 4x - 32}) = 0$	Correct quadratic reached (no need for $(x+2)$ for A1 **(S+)**	**A1**	AO3

Question	Scheme		Marks	AOs
6(ii)(b) *continued*	$(x + 2)(3x - 8)(x + 4) = 0$	Factorises	**M1**	AO3
	But equation not defined for ($x = -2$ and) $x = -4$ $\left(\text{but equation works for } x = \frac{8}{3}\right)$. So $x = \frac{8}{3}$ is only solution. **(S+)**		**A1**	AO3
S2			**(7)**	
	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for either - a fully correct solution that is succinct but does not mention any S+ points. or - a fully correct solution that is slightly laboured but includes an S+ point. or - part (i) or (ii) fully correct and contains an S+ point.		**(2)**	AO2
				(16 + 2 marks)

Notes:

(i) **S+** for giving reason $p \neq 1$

(ii) **S+** for dealing correctly with $(x + 2)$ and not just "cancelling" – if statement $x \neq -2$ is given, cancelling is fine.

S+ for correct reasoning for the only valid root with check that equation does work for the given value.

Question	Scheme	Marks	AOs
7(a)	$x^2 + y^2 = \pi^2$ or equivalent.	**B1**	AO1
		(1)	
(b)	$OA = \theta$ seen or implied.	**B1**	AO1
	$AG = \pi - \theta$ seen or implied.	**B1**	AO1
	Clear method for x or y e.g. Let X be the point vertically below G such that angle $GXA = 90°$ OR identifies $x = \sin\theta + GA\cos\theta$ or $y = GA\sin\theta + 1 - \cos\theta$ **(S+)**	**M1**	AO2
	So $x = TA\sin\theta + AX = \sin\theta + (\pi - \theta)\cos\theta$ *	**A1***	AO2
	and $y = 1 - TA\cos\theta + GX = 1 - \cos\theta + (\pi - \theta)\sin\theta$ *	**A1***	AO2
		(5)	
(c)	$\frac{dx}{d\theta} = \cos\theta - (\pi - \theta)\sin\theta - \cos\theta$ (allow one slip)	**M1**	AO1
	So Area $\int_{x=0}^{x=\pi} y\frac{dx}{d\theta}d\theta = \int_{\pi}^{0}\left[1 - \cos\theta + (\pi - \theta)\sin\theta\right]\left[-(\pi - \theta)\sin\theta\right] d\theta$ (ignore limits)	**A1**	AO1
	Identifies suitable substitution: Let $u = \pi - \theta$	**M1**	AO3
	Applies $\cos(\pi - \theta) = -\cos\theta$ and $\sin(\pi - \theta) = \sin\theta$ to their integral.	**M1**	AO3
	Simplifies to given answer, with limits correctly derived. Area $= -\int_0^{\pi}\left(-u\sin u - u\cos u\sin u - u^2\sin^2 u\right) du \rightarrow$ ans* **(S+)**	**A1***	AO2
		(5)	
(d)	$\int_0^{\pi} u^2\sin^2 u\ du = \int_0^{\pi}\frac{u^2}{2}du - \int_0^{\pi}\frac{u^2}{2}\cos 2u\ du$ — Use of $\sin^2 u$ in terms of $\cos 2u$	**M1**	AO2
	$= \frac{\pi^3}{6}, -\left\{\left[\frac{u^2}{4}\sin 2u\right]_0^{\pi} - \int_0^{\pi} u\frac{\sin 2u}{2}\ du\right\}$ — achieves $\frac{\pi^3}{6}$	**A1cso**	AO2
	Use integration by parts on second integral.	**M1**	AO2
	$= \frac{\pi^3}{6} + \int_0^{\pi} u\sin u\cos u\ du$ * — Shows [...] = 0 and simplifies to answer.	**A1***	AO2
		(4)	

Question	Scheme		Marks	AOs
7(d) *continued*	**Alternative**			
	$\int \sin^2 u \, \mathrm{d}u = \int \frac{1}{2} - \frac{1}{2}\cos 2u \, \mathrm{d}u = \frac{1}{2}u \pm \frac{1}{4}\sin 2u$	Use of $\sin^2 u$ in terms of $\cos 2u$ and integrates to use in parts.	**M1**	AO2
	$\int_0^{\pi} u^2 \sin^2 u \, \mathrm{d}u = \left[u^2\left(\frac{1}{2}u - \frac{1}{4}\sin 2u\right)\right]_0^{\pi} - \int_0^{\pi} 2u\left(\frac{1}{2}u - \frac{1}{4}\sin 2u\right) \mathrm{d}u$			
	$= \frac{\pi^3}{2} - \int_0^{\pi} u^2 \, \mathrm{d}u + \int_0^{\pi} \frac{1}{2}\sin 2u \, \mathrm{d}u$	achieves $\frac{\pi^3}{6}$	**A1cso**	AO2
	$= \frac{\pi^3}{6}$	Having attempted parts directly.	**M1**	AO2
	$= \frac{\pi^3}{6} + \int_0^{\pi} u \sin u \cos u \, \mathrm{d}u$ *	Correct work and simplifies to answer.	**A1***	AO2
			(4)	
(e)	$\int_0^{\pi} u \sin u \, \mathrm{d}u = \left[-u\cos u\right]_0^{\pi} + \int_0^{\pi} \cos u \, \mathrm{d}u$	Use of parts on this integral (ignore limits).	**M1**	AO2
	$= \pi$		**A1**	AO1
	$\int_0^{\pi} u \sin u \cos u \, \mathrm{d}u = \int_0^{\pi} u \frac{\sin 2u}{2} \mathrm{d}u = \left[-u\frac{\cos 2u}{4}\right]_0^{\pi} + \int_0^{\pi} \frac{\cos 2u}{4} \, \mathrm{d}u$		**M1**	AO2
	$= -\frac{\pi}{4}$	M1 – use of parts A1 – correct result	**A1**	AO1
	Area between curve and +ve axes $= \frac{\pi^3}{6} + \pi - 2 \times \frac{\pi}{4} = \frac{\pi^3}{\underline{6}} + \frac{\pi}{\underline{2}}$		**A1**	AO2
	Total area available = – tower + semicircle	Suitable strategy. **(S+)**	**M1**	AO3
	Area of semicircle $= \frac{\pi^3}{2}$ <u>or</u> area of tower's base = π		**B1**	AO2
	So area reachable is $\underline{\frac{5\pi^3}{6}}$ $\underline{\mathrm{m}^2}$		**A1**	AO3
			(8)	

Question	Scheme	Marks	AOs
7(e) *continued* **S2**	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for either - a fully correct solution that is succinct but does not mention any S+ points. or - a fully correct solution that is slightly laboured but includes an S+ point. or - a score of >20 but solution is otherwise succinct or contains an S+ point.	**(2)**	AO2
			(23 + 2 marks)

Notes:

(b) **S+** for a clear and well expressed explanation or clear diagram with accompanying work.
(c) **S+** for clear demonstration of the change of limits.
(e) **S+** for a concise strategy using symmetry.