HYPOTHESIS TESTING

INTRODUCTION

BINOMIAL DISTRIBUTION HYPOTHESIS INTRODUCTION

Question 1

In a craft activity in a primary school, kids use beads which are kept in a bag. The bag contains a large number of beads of different colours. The beads are not replaced into the bag at the end of the activity.

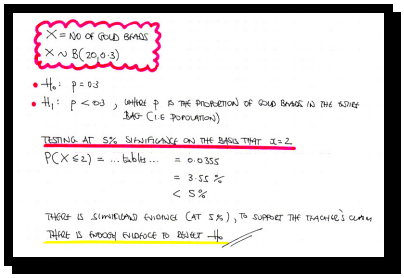
It is known that $\frac{3}{10}$ of the beads are coloured gold.

The teacher claims that children use more gold beads during the activity and checks a random sample of 20 beads out of the bag, after the end of the activity.

She finds just two gold beads in the sample.

Test, at the 5% level of significance, whether or not there is evidence to support the teacher's claim.

significant evidence, $3.55\% < 5\%$



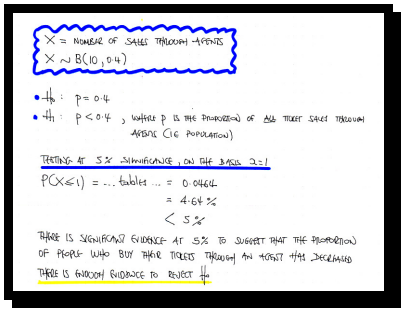
Question 2

A theatre company finds from its records that 40% of its customers book their tickets through agents. The company redesigns its website and then carries out a survey with 10 randomly chosen customers.

The result of the survey is that 1 of these 10 customers booked their tickets through an agent.

Test, at the 5% level of significance, whether the percentage of customers who book their tickets through an agent has decreased.

significant, $4.64\% < 5\%$



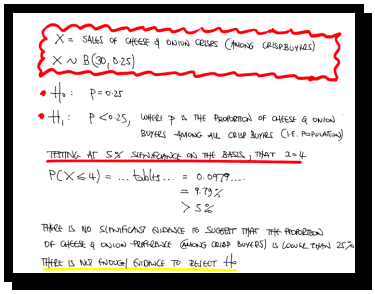
Question 3

The owner of a corner shop believes that 25% of the customers who buy crisps will buy the "cheese and onion" variety.

He finds that in the last 30 customers who bought crisps, only 4 customers bought the "cheese and onion" variety.

Test, at the 5% level of significance, whether there is evidence to suggest that the proportion of customers who choose the "cheese and onion" variety is lower than 25% .

not significant evidence, $9.79\% > 5\%$



Question 4

The owner of a "Fish and Chips" shop believes that 35% of the customers who buy chips like vinegar with their chips.

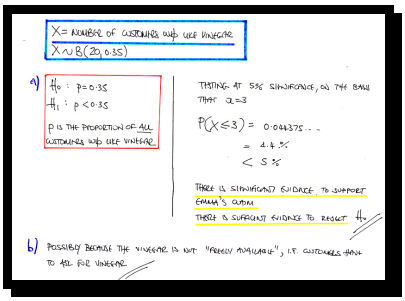
His wife Emma, claims that this proportion is lower and decides to test it by checking whether the next 20 customers who buy chips will ask for vinegar.

Only 3 customers in Emma's sample asked for vinegar.

a) Test, at the 5% level of significance, whether or not there is evidence to support Emma's claim.

b) State a reason as to why this method of testing might produce a biased result.

significant evidence, $4.44\% < 5\%$, customers have to ask



Question 5

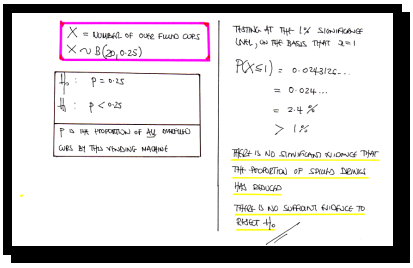
The probability that a coffee vending machine will spill the drink is 25%.

The machine is now serviced, and after the service the next twenty dispenses of drinks are recorded.

It is found that only one drink is now spilled.

Test, at the 1% level of significance, whether the proportion of spilled drinks has reduced.

not significant evidence, $2.43\% > 1\%$



Question 6

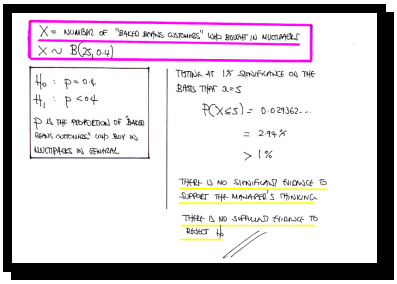
It was suggested to the new manager of a supermarket that 40% of the customers who buy baked beans will buy beans in "multi-packs".

The manager thinks that this figure is too high and decides to test it.

He finds that out of the next 25 customers who bought baked beans, only 5 customers bought baked beans in "multi-packs".

Test, at the 1% level of significance, whether or not there is evidence to support the supermarket's manager's thinking.

not significant evidence, $2.94\% > 1\%$



Question 7

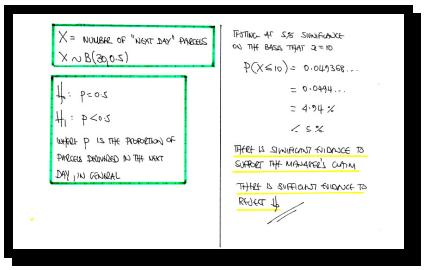
A parcel dispatch company has established that the probability of a parcel being delivered in the next working day is 0.5.

The company implements changes and the manager of a depot feels that fewer parcels are now delivered in the next working day.

He monitors a random sample of 30 parcels left to be delivered and finds 10 were delivered in the next working day.

Test, at the 5% level of significance, whether or not there is evidence to support the manager's claim.

significant evidence, $4.94\% < 5\%$



Question 8

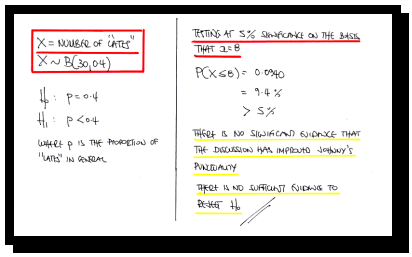
Johnny is often late for his lessons and according to his Head of Year this proportion is unacceptably high at 0.4.

After a formal discussion for this matter, the Head of Year decides to check Johnny's punctuality by looking at a random sample of 30 lessons.

Johnny was late on 8 occasions in the sample.

Test, at the 5% level of significance, whether there is evidence that the discussion with the Head of Year has improved Johnny's punctuality.

not significant evidence, $9.4\% > 5\%$



Question 9

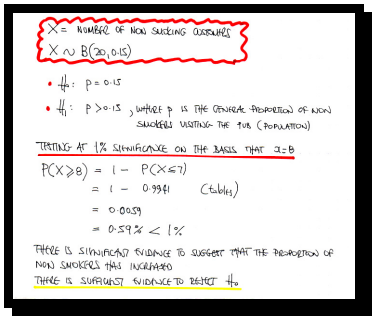
A pub manager feels that since the introduction of the "smoking ban" in his pub, the proportion of the non smoking customers visiting his pub has increased.

It had been established that before the "smoking ban" 15% of the customers visiting his pub were non smokers.

He now finds 8 non smokers in a random sample of 20 customers.

Test, at the 1% level of significance, whether there is evidence to suggest that the proportion of the non smoking customers visiting his pub has increased.

significant evidence, $0.59\% < 1\%$



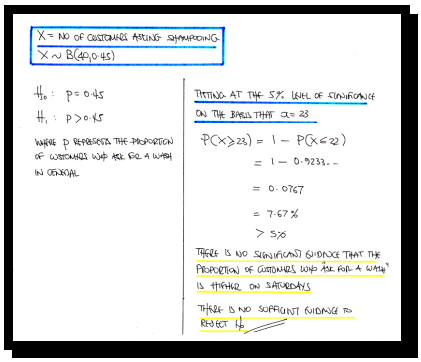
Question 10

In a salon 45% of the customers ask for the hair to be washed before it is cut.

During a busy Saturday 40 customers visited the salon and 23 of these customers asked for a wash.

Test, at the 5% level of significance, whether the proportion of customers who ask for their hair to be washed is higher on Saturdays.

not significant evidence, $7.67\% > 5\%$



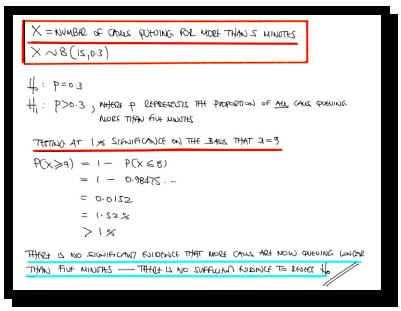
Question 11

In a certain bank, the probability that a phone call is in a queue for more than five minutes is 0.3.

A new telephone console is installed in order to improve efficiency, however when a sample of 15 calls is checked, 9 calls were found to be queuing for more than five minutes.

Test, at the 1% level of significance, whether there is evidence that more calls are now queuing for more than five minutes.

not significant evidence, $1.52\% > 1\%$



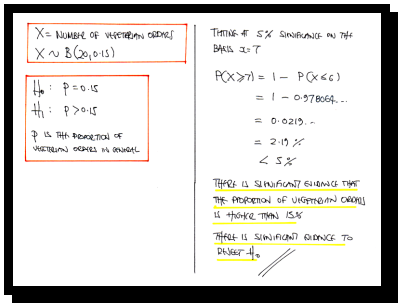
Question 12

The proportion of vegetarian orders in a restaurant is thought to be 15% .

During lunch on a given day, 7 diners out of 20 , ordered vegetarian food.

Test, at 5% level of significance, whether the proportion of diners who order vegetarian food is higher than 15% .

significant evidence, $2.19\% < 5\%$



Question 13

A tomato plant is thought to produce a good yield if it produces more than 3 kg of fruit.

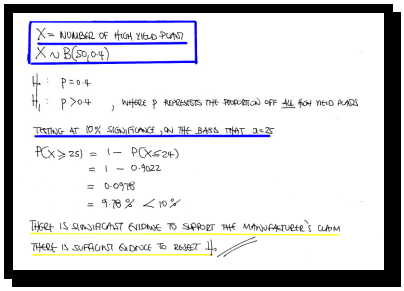
A farmer has established over a long period of time that the probability that a tomato plant will produce a good yield is 0.4.

The makers of a new fertilizer claim that their product will increase the yield of the tomato plants.

The farmer uses the fertilizer in a random plot of 50 plants and finds half the plants in this plot produce a good yield.

Test, at the 10% level of significance, whether there is evidence to support the claim made by the makers of the fertilizer.

significant evidence, $9.78\% < 10\%$



Question 14

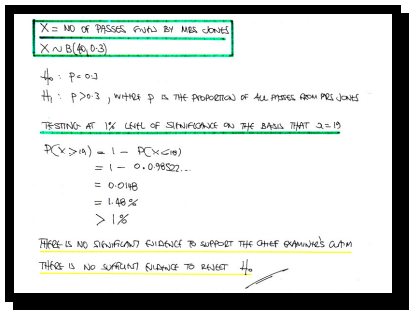
It had been established over a long period of time that 30% of the driving test candidates examined by Mrs Jones will pass their driving test.

The chief examiner of the centre, on his return after a year long break, feels that a higher proportion of candidates pass their driving test when examined by Mrs Jones.

In a random sample of 40, of Mrs Jones' recent candidates, he finds 19 passed their test.

Test, at the 1% level of significance, whether or not there is evidence to support the chief examiner's claim.

not significant evidence, $1.48\% > 1\%$



Question 15

In a craft activity in a primary school, kids use beads which are kept in a bag. The bag contains a large number of beads of different colours. It is known that 0.3 of the beads are coloured gold.

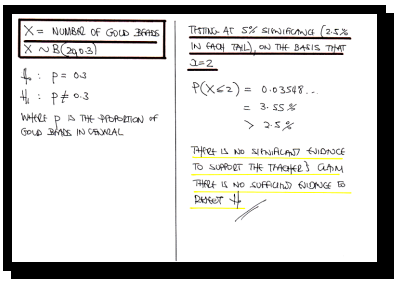
The teacher claims the proportion of gold beads in the bag has changed after the activity.

She checks a random sample of 20 beads out of the bag, after the end of the activity

She finds two gold beads in the sample.

Test, at the 5% level of significance, whether or not there is evidence to support the teacher's claim.

not significant evidence, $3.55\% > 2.5\%$



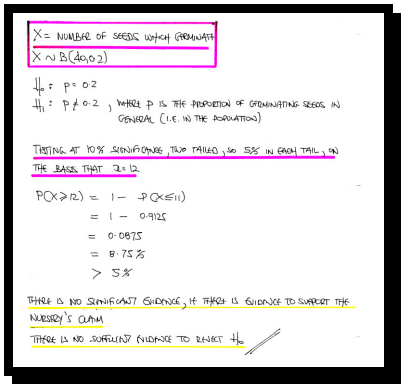
Question 16

It is claimed that 20% of the seeds sold by a nursery will germinate.

Ann buys 40 such seeds and finds that 12 of them germinated.

Test, at the 10% level of significance, whether or not there is evidence to support the nursery's claim.

not significant evidence, $8.75\% > 5\%$



BINOMIAL DISTRIBUTION HYPOTHESIS

EXAM QUESTIONS

Question 1 (***)

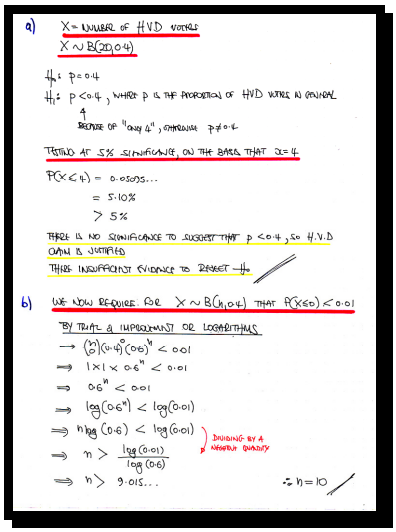
A mayoral candidate, Hans Van Dyke, claims that 40% of the electoral will vote for him in the next election. In a recent opinion poll of 20 recently selected voters it was found that **only 4** people will vote for Hans Van Dyke.

a) Test, at the 5% level of significance whether, or not, the opinion poll supports Hans Van Dyke's claim.

In a second opinion poll of n randomly selected people, it was found that no one will be voting for Hans Van Dyke. As a result of this poll, Hans Van Dyke's claim is rejected at 1% significance.

b) Determine the smallest value of n.

$\boxed{}$, $\boxed{\text{not significant evidence, } 5.10\% > 5\%}$, $\boxed{n = 10}$



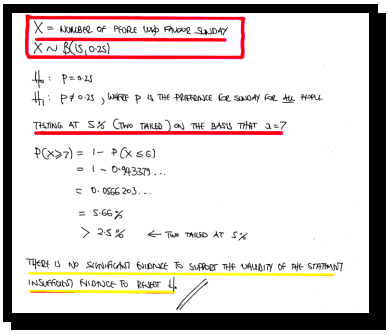
Question 2 (***)

When people are asked "what is your favourite day of the week?", it is thought that on average one person in four replies "Sunday".

To test this assertion 15 people were asked this question and 7 replied "Sunday".

Carry out a significance test, at the 5% level, of whether the statement "on average the preferred day of the week is Sunday, for one in four persons".

[ ], [not significant evidence, 5.66% > 2.5%]



Question 3 (***)

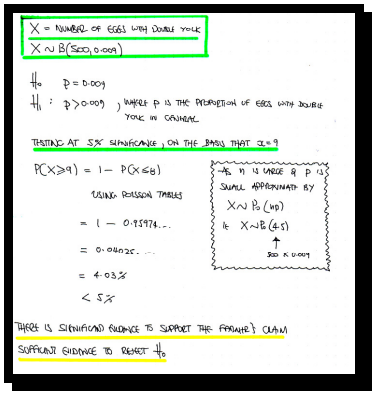
A chicken farmer supplies a local restaurant with eggs. The owner of the restaurant feels that the eggs having a double yolk is 0.009.

The farmer however claims that the proportion of her eggs having a double yolk is higher than 0.009.

In the next batch of 500 eggs the restaurant chef records 9 eggs with a double yolk.

Using a distributional approximation, and further assuming that the batch of 500 eggs is a random sample, test at the 5% level of significance whether the farmer's claim is justified.

$\boxed{}$, $\boxed{\text{claim justified, } 4.03\% < 5\%}$



Question 4 **(***+)**

A London Mayoral candidate thinks that 35% of the voters will vote for him.

His campaign manager thinks that this percentage is in fact higher.

A random sample of 40 voters is polled and it is found that it contained 19 voters indicating their intention to vote in favour of this candidate.

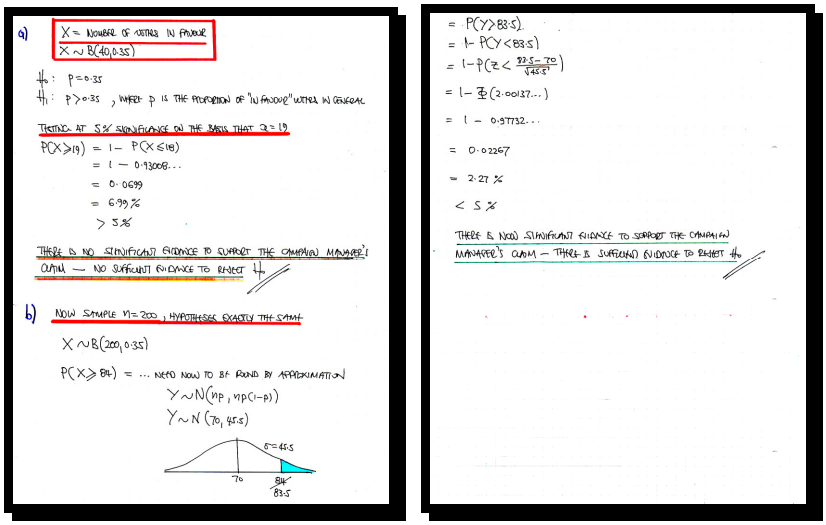
a) Test at the 5% significance level the claim of the campaign manager.

A larger random sample of 200 is then polled. It is found that it contained 84 voters indicating their intention to vote in favour of this candidate.

b) Use a distributional approximation, to retest at the 5% significance level the claim of the campaign manager.

$\boxed{}$, $\boxed{\text{not significant evidence, } 6.99\% > 5\%}$,

$\boxed{\text{significant evidence, } 2.27\% < 5\%}$



Question 5 **(***+)**

Drawing pins are sold in boxes of 20 and it is thought that 10% of these drawing pins have flaws.

a) If a box of these drawing pins is examined at random, find the probability that it will contain ...

i. ... 3 drawing pins with flaws.

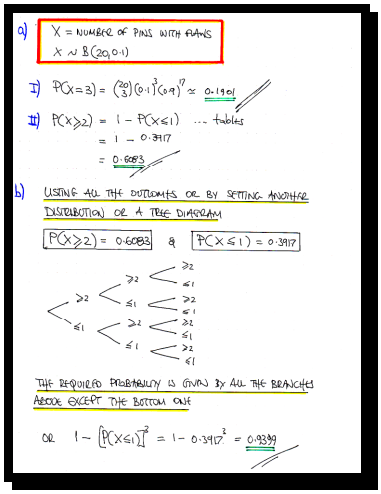
ii. ... at least 2 drawing pins with flaws.

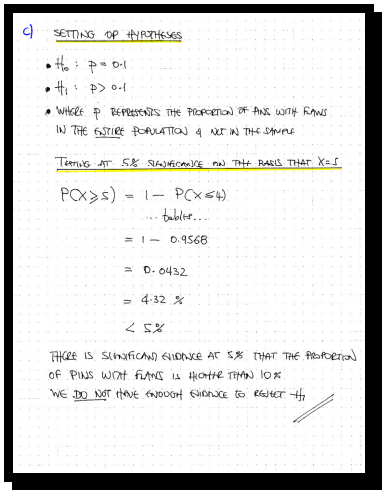
b) If 3 boxes of these drawing pins are examined at random, determine the probability that at least one of these boxes will contain at least 2 drawing pins with flaws.

A single box of these drawing pins is picked at random and found to contain exactly 5 drawing pins with flaws.

c) Test, at the 5% level of significance, whether this constitutes evidence that the proportion of drawing pins with flaws is higher than 10% .

$\boxed{}$, $\boxed{0.1901}$, $\boxed{0.6083}$, $\boxed{0.9399}$, $\boxed{\text{significant, } 4.32\% < 5\%}$





Question 6 (*+)**

The discrete random variable X represents the number of households with satellite TV subscriptions.

It is assumed that X follows a binomial distribution $\mathrm{B}(n,0.35)$.

a) If $n=25$, find the probability ...

i. ... $\mathrm{P}(X=12)$

ii. ... $\mathrm{P}(X>12)$.

b) If $n=25$, determine the probability

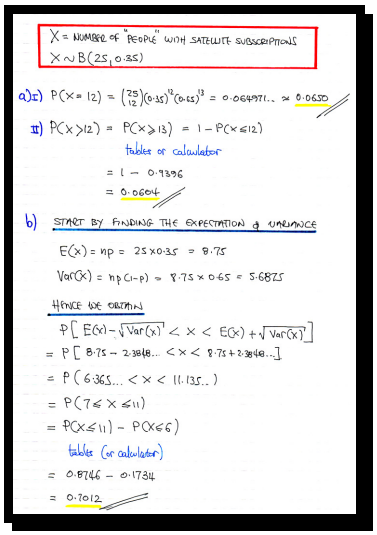
$$\mathrm{P}\left[\mathrm{E}(X)-\sqrt{\mathrm{Var}(X)}<X<\mathrm{E}(X)+\sqrt{\mathrm{Var}(X)}\right].$$

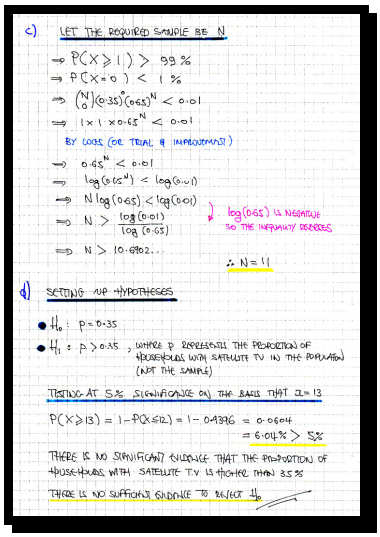
c) Find the smallest number of households that must be sampled so that the probability of having at least a household with satellite TV subscription is greater than 99%.

An analyst believes that the proportion of households with satellite TV subscription is higher, because in a sample of 25 households 13 had a satellite TV subscription.

d) Using a clear method, test the analyst's belief, at the 5% level of significance.

$\boxed{}$, $\boxed{0.0650}$, $\boxed{0.0604}$, $\boxed{0.7012}$, $\boxed{n=12}$, $\boxed{\text{not significant, } 6.04\% > 5\%}$





Question 7 (****)

William has been established over a long period of time that when he shoots an arrow at a target the probability of hitting it, is 0.35.

William buys a professional bow as he believes it will increase the probability of hitting the target.

Let X define the number of successful hits of the target, and assume that successful or non successful hits are independent of one another.

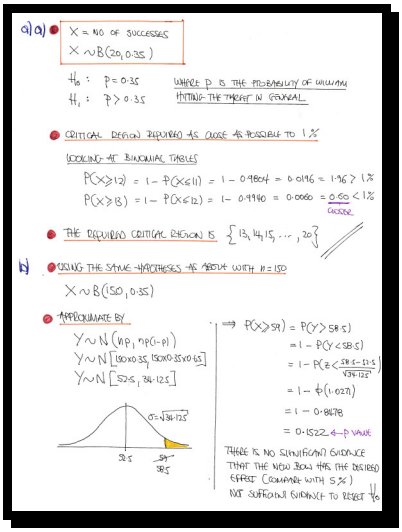
a) Determine a critical region based on 20 shots with the new bow.

The significance level must be as close as possible to 1%.

William decides to carry out a significance test by shooting 150 arrows at the target using the new bow. He finds that 59 arrows hit the target.

b) Using a distributional approximation, calculate an approximate p-value and hence state the conclusion in context.

$\boxed{}$, $\boxed{\{13,14,15,...,20\}}$, $\boxed{p=0.1522}$, $\boxed{\text{not significant evidence, } 15.22\% > 5\%}$



Question 8 (****)

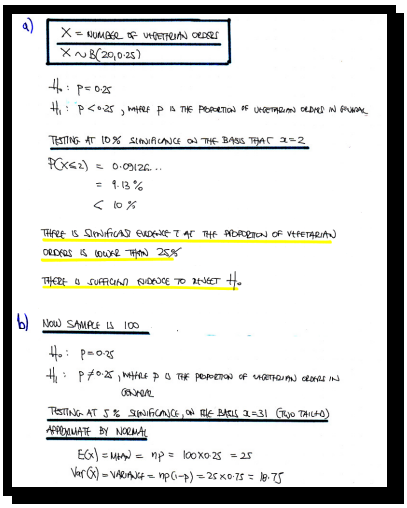
At "Stavros Restaurant" the owner is told by his chef that 25% of the customers order vegetarian food. The owner wants to check the validity of the chef's assertion so he checks a random sample of 20 orders, only to find 2 vegetarian orders.

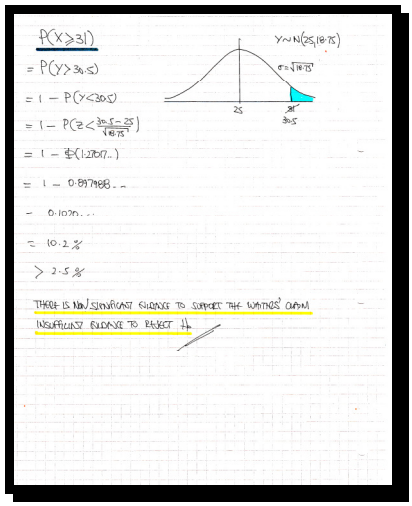
a) Is there evidence, at the 10% level of significance, that the proportion of vegetarian orders is lower than 25%?

At "Mavros Restaurant" the owner is told by his waiters that 25% of the customers order vegetarian food. The owner wants to check the validity of the waiters' belief so he checks a random sample of 100 orders.

b) Given that there are 31 vegetarian orders in the sample, use a distributional approximation, to test at the 5% level of significance, the belief of the waiters at "Mavros Restaurant".

$\boxed{}$, $\boxed{\text{significant, } 9.13\% < 10\%}$, $\boxed{\text{"not significant!!!", } 10.20\% > 2.5\%}$





BINOMIAL DISTRIBUTION CRITICAL REGIONS

Question 1 (**)

A nutritional expert believes that children ate healthier in the past. This is measured by the number of kids that eat five portions of fruit and vegetables per day. The proportion of children that ate "five a day" was 40% in the 1950's.

The nutritional expert believes that the proportion is now lower.

He carries out an investigation with a random sample of 15 children.

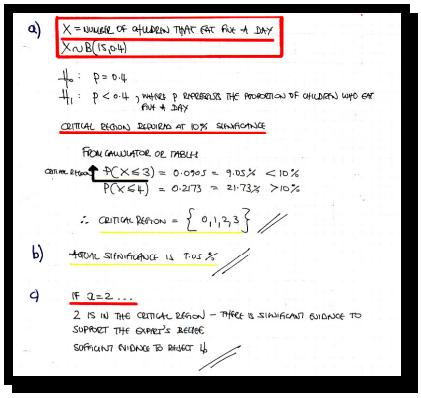
a) Find the critical region to test at the 10% level of significance the expert's belief.

b) State the actual significance level for a test using the critical region of part **(a)**.

Two children, who ate "five a day", were found in the sample.

c) Complete the test.

$\boxed{\text{C.R.}=\{0,1,2,3\}}$, $\boxed{9.05\%}$, $\boxed{\text{significant evidence, 2 is in C.R.}}$



Question 2 (**)

The proportion of tiles with minor faults produced in a factory is thought to be 10% .

The factory manager believes that the proportion is higher due to the old machinery.

He inspects a random sample of 20 tiles.

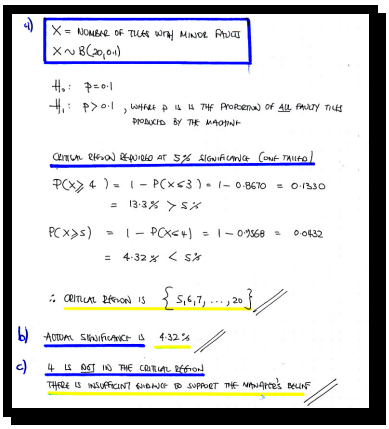
a) Find the critical region to test the manager's belief, at the 5% level of significance.

b) State the actual significance level for a test using the critical region of part **(a)**.

Four faulty tiles were found in the sample.

c) Complete the test.

$\boxed{}$, $\boxed{\text{C.R.}=\{5,6,7,...,20\}}$, $\boxed{4.32\%}$, $\boxed{\text{not significant evidence, } 4 \text{ not in C.R.}}$



Question 3 (**)

The recruitment director of a large accounting firm believes that maths graduates are more successful when applying for positions in his firm compared with graduates of other subjects.

One in five job applicants to this accounting firm is successful.

The recruitment director selects a random sample of 25 maths graduate applicants.

a) Find the critical region to test at the 5% level of significance the director's belief.

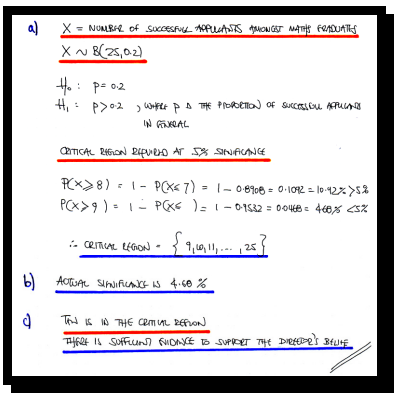
State your hypotheses clearly.

b) State the probability of incorrectly rejecting the null hypothesis in a test, using the critical region obtained in part **(a)**.

Ten successful maths graduate applicants were found in the sample.

c) Complete the test.

$\boxed{\quad}$, $\boxed{\text{C.R.} = \{9,10,11,...,25\}}$, $\boxed{4.68\%}$, $\boxed{\text{significant evidence, } 10 \text{ is in C.R.}}$



Question 4 (**)

Ten years ago the residents in a car congested area were asked whether they were in favour of a residents' parking scheme. The proportion of residents who supported the parking permit scheme was 30%. The scheme was never implemented due to lack of funding.

The funding is now available and a new councillor believes that the support for the scheme is different now.

The replies to questionnaires of twenty current residents are considered.

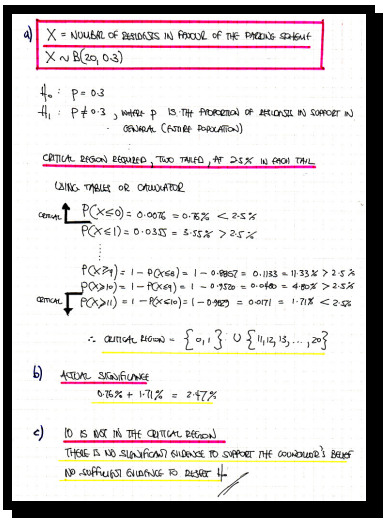
a) Determine the critical region to test, at the 5% level of significance, the councillor's belief.

b) State the actual significance level for a test using the critical region of part **(a)**.

Ten residents in support of the scheme were found in the sample.

c) Complete the test.

$\boxed{\text{C.R.} = \{0,1\} \cup \{11,12,13,...,20\}}$, $\boxed{2.47\%}$, $\boxed{\text{not significant evidence, 10 is not in C.R.}}$



Question 5 **(**)**

In a small factory, the quality of ceramic tiles is monitored daily by checking a random sample of 50 tiles from the day's production. It is required to find whether the proportion of tiles with minor faults is different from 10% .

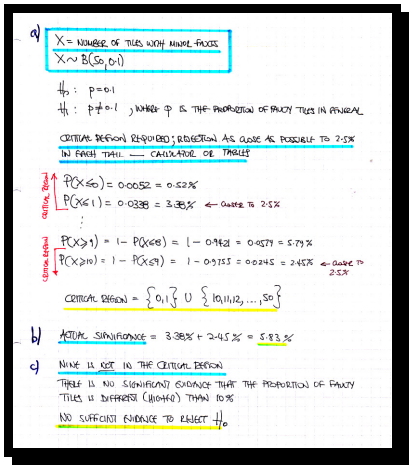
a) Find the critical region for this test.

The probability of rejecting at either tail must be as close as possible to 2.5% .

b) State the actual significance level for a test using the critical region of part **(a)**.

c) If nine faulty tiles are found in the sample complete the test.

$\boxed{\text{C.R.}=\{0,1\}\cup\{10,11,12,...,50\}}$, $\boxed{5.83\%}$, $\boxed{\text{not significant evidence, } 9 \text{ is not in C.R.}}$



Question 6 (**)

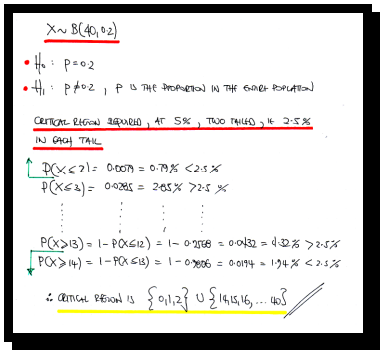
A test statistic has distribution $\mathrm{B}(40, p)$.

Given that

$$\mathrm{H}_0 : p = 0.2, \quad \mathrm{H}_1 : p \neq 0.2,$$

find the critical region for the test statistic at the 5% significance level.

$\square$, $\{0,1,2\} \cup \{14,15,16,\ldots,40\}$



Question 7 (**)

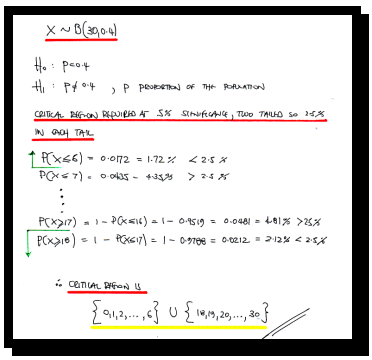
A test statistic has distribution $\mathrm{B}(30, p)$.

Given that

$$\mathrm{H}_0 : p = 0.4, \quad \mathrm{H}_1 : p \neq 0.4,$$

find the critical region for the test statistic at the 5% significance level.

$\boxed{}$, $\boxed{\{0,1,2,3,4,5,6\} \cup \{18,19,20,...,30\}}$



Question 8 **(**+)**

A test statistic has distribution $\mathrm{B}(25, p)$.

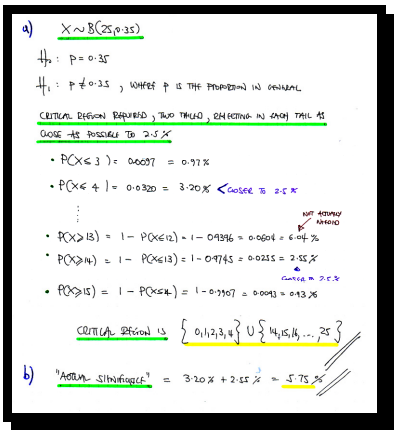
a) Given that

$$\mathrm{H}_0 : p = 0.35, \quad \mathrm{H}_1 : p \neq 0.35,$$

find the critical region for the test statistic such that the probability of rejecting in each tail is as close as possible to 2.5%.

b) State the probability of incorrectly rejecting H_0 using this critical region.

$\boxed{\quad}$, $\boxed{\{0,1,2,3,4\} \cup \{14,15,16,\ldots,25\}}$, $\boxed{5.75\% \text{ or } 0.0575}$



BINOMIAL DISTRIBUTION CRITICAL REGIONS

EXAM QUESTIONS

Question 1 **(***+)**

The records in a dentist's surgery show that 15% of the patients that make an appointment fail to turn up.

a) In a day with 15 appointments determine the probability that …

i. … exactly 2 patients will fail to turn up.

ii. … at least 2 patients will fail to turn up.

b) Use a distributional approximation to find the probability that in a month with 400 appointments, more than 45 but at most 65 patients will fail to turn up.

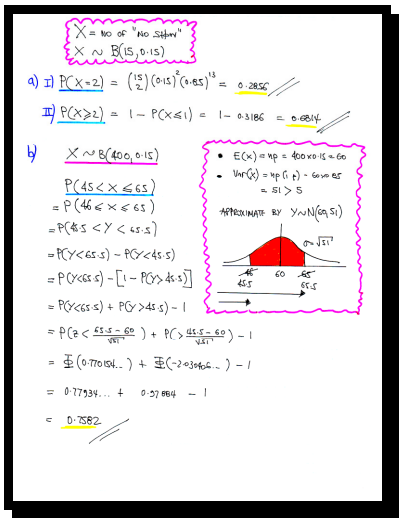
The surgery administrator feels that the percentage of patients that make an appointment and fail to turn up is likely to change in the future. The surgery tries an automated system of generating message reminders on patients' mobile phones.

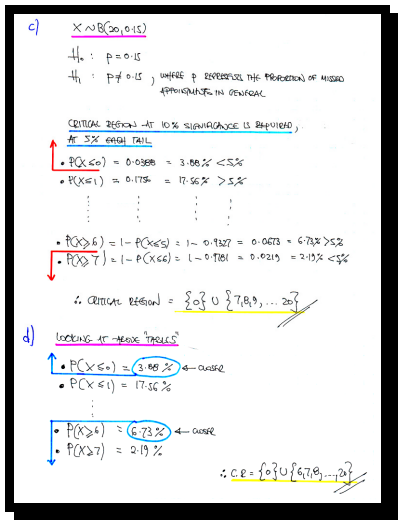
It is required to find whether the proportion of patients that fail to turn up is different from 15% , by monitoring the next 20 appointments.

c) Determine the critical region for this test, at the 10% level of significance.

d) Write down the critical region for the same test if the probability of rejecting at either tail is as close as possible to 5% .

$\boxed{}$, $\boxed{0.2856}$, $\boxed{0.6814}$, $\boxed{0.7582}$, $\boxed{\{0\}\cup\{7,8,9,\ldots,20\}}$, $\boxed{\{0\}\cup\{6,7,8,9,\ldots,20\}}$





Question 2 (+)**

A teacher is investigating the students' method of getting back home and is told that 15% of the students gets back home by car.

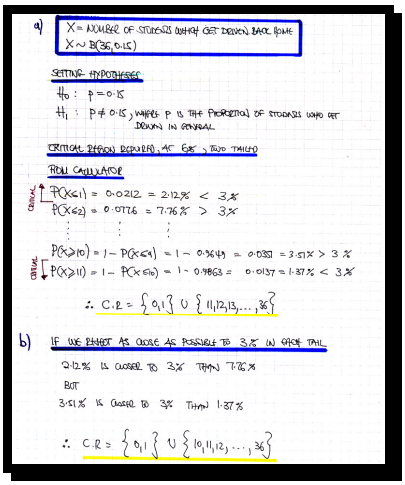
He decides to investigate this fact further and decides to use a random sample of 36 students across all the school's year groups.

Stating your hypotheses clearly, find the critical region …

a) … for his test at the 6% level of significance.

b) … for a similar test where the probability of rejecting at either tail must be as close as possible to 3%.

$\boxed{}$, $\boxed{x \leq 1 \text{ or } x \geqslant 11}$, $\boxed{x \leqslant 1 \text{ or } x \geqslant 10}$



Question 3 (+)**

A teacher is investigating the students' after school activities and is told that 1 in 20 students attends martial arts classes.

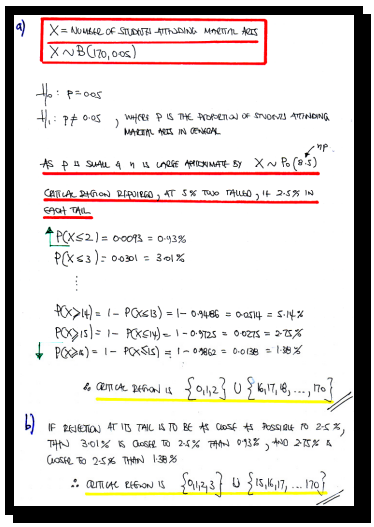
He decides to investigate this fact further and decides to use a random sample of 170 students across all the school's year groups.

By using a distributional approximation, and stating your hypotheses clearly, find the critical region …

a) … for his test at the 5% level of significance.

b) … for a similar test where the probability of rejecting at either tail must be as close as possible to 2.5% .

$\boxed{\quad}$, $\boxed{x \leqslant 2 \text{ or } x \geqslant 16}$, $\boxed{x \leqslant 3 \text{ or } x \geqslant 15}$



Question 4 (+)**

Hooks produced in a factory are packed in boxes of sixty. It is thought that 5% of the hooks produced are defective.

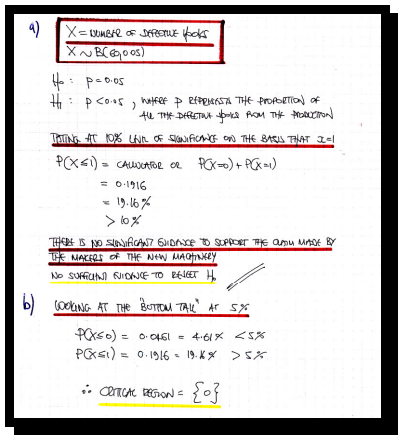
The main production machine is replaced and the makers of the new machine claim that their machine will produce less defective hooks.

A box from the production of the new machine is inspected and is found to contain one defective hook.

a) Test at the 10% level of significance the claim made by the makers of the new machine.

b) If the test was carried out at the 5% level of significance find the critical region for the test.

$\boxed{}$, $\boxed{\text{not significant evidence, } 19.16\% > 10\%}$, $\boxed{\text{C.R.} = \{0\}}$



Question 5 (+)**

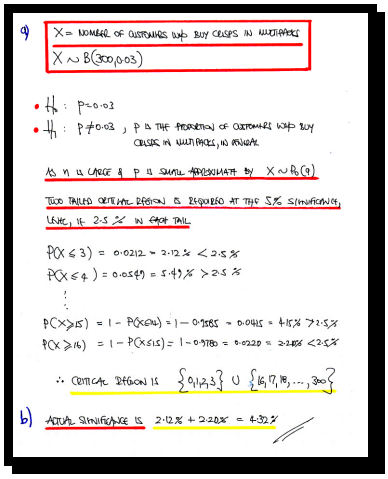
The manager of a supermarket believes that of the customers who buy crisps, 3% buy them in "Mega Packs".

He decides to test his belief by recording how many of the next 300 customers who buy crisps, bought them in "Mega Packs".

a) Using a distributional approximation, find the critical region to test the manager's belief, at the 5% level of significance.

b) State the actual significance level for a test using the critical region of part **(a)**.

$\boxed{}$, $\boxed{C.R. = \{0,1,2,3\} \cup \{16,17,18,...,300\}}$, $\boxed{4.32\%}$



POISSON DISTRIBUTION HYPOTHESIS TESTING

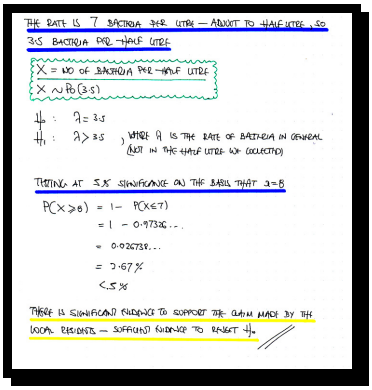
Question 1

Bacteria are randomly distributed in the water of a lake at the rate of 7 per litre.

A new factory opens next to the lake and the local residents feel that the factory is polluting the lake. To test this claim, a random sample of half a litre of water from the lake is examined and found to contain 8 bacteria.

Test at the 5% level of significance, whether there is evidence to support the claim of the local residents.

$\boxed{}$, $\boxed{\text{significant evidence, } 2.67\% < 5\%}$



POISSON DISTRIBUTION HYPOTHESIS TESTING

EXAM QUESTIONS

Question 1 (**)

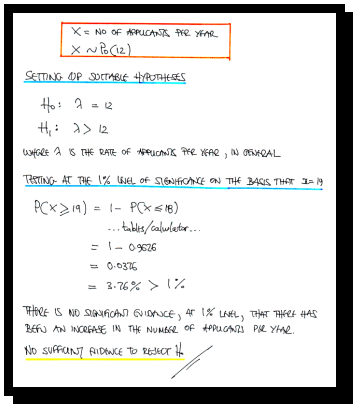
A company advertises for a summer job every year.

It has been established over a long period of years that the number of applicants per year follows a Poisson distribution with mean 12.

This year there were 19 applicants for this summer job.

Test, at the 1% level of significance, whether there is evidence of an increase of the mean number of the applicants for the job.

[ ], [not significant, $3.74\% > 1\%$]



Question 2 (+)**

Minor imperfections occur in the cloth manufacture in a certain factory at the rate of 1.2 per square metre of cloth.

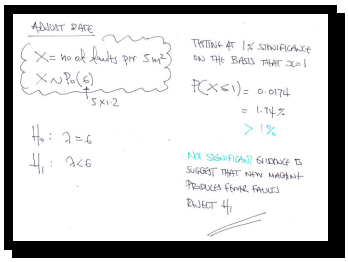
The factory owner buys a new cloth manufacturing machine.

The makers of the new machine claim that their product will reduce these minor imperfections.

A random 5 square metre piece of cloth produced by the new machine is examined and found to contain 1 minor imperfection.

Test at the 1% level of significance, whether there is evidence to support the claim of the makers of the new machine.

not significant evidence, $1.74\% > 1\%$



Question 3 (***)

George works in a shoe shop on Saturdays.

On average he serves 8 customers every hour.

a) Find the probability that, in a given hour on a Saturday, George will serve more than 10 customers.

b) Find the probability that, in a given three hour interval on a Saturday, George will serve exactly 24 customers.

The manager of the shop claims that George has recently become complaisant and serves fewer customers.

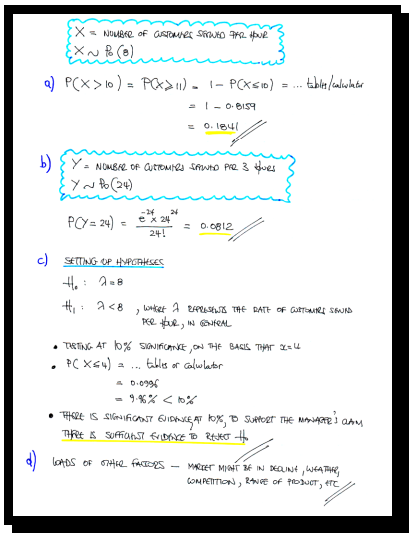
During a given hour on a particular Saturday George serves 4 customers.

c) Test at the 10% level of significance, whether there is evidence to support the manager's claim.

d) Comment on whether other factors might affect the reliability of the test.

$\boxed{}$, $\boxed{0.1841}$, $\boxed{0.0812}$, $\boxed{\text{significant evidence, } 9.96\% < 10\%}$,

$\boxed{\text{not reliable; maybe business is not brisk}}$



Question 4 (*+)**

Minor imperfections occur in the cloth manufacture in a certain factory at the rate of 1.6 per square metre of cloth.

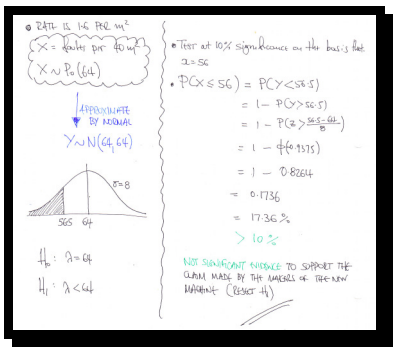
The factory owner buys a new cloth manufacturing machine.

The makers of the new machine claim that their product will reduce the number of these minor imperfections.

A random 40 square metre piece of cloth produced by the new machine is examined and found to contain 56 minor imperfections.

Test at the 10% level of significance, whether there is evidence to support the claim of the makers of the new machine.

not significant evidence, $17.36\% > 10\%$



Question 5 **(***+)**

It is thought that calls arrive at a company telephone switchboard at the steady rate of 7 per minute.

a) Find the probability that, in a given minute, there will be more than 5 but at most 10 calls arriving the company's telephone switchboard.

A 5 minute interval is divided into 10 equal 30 second intervals.

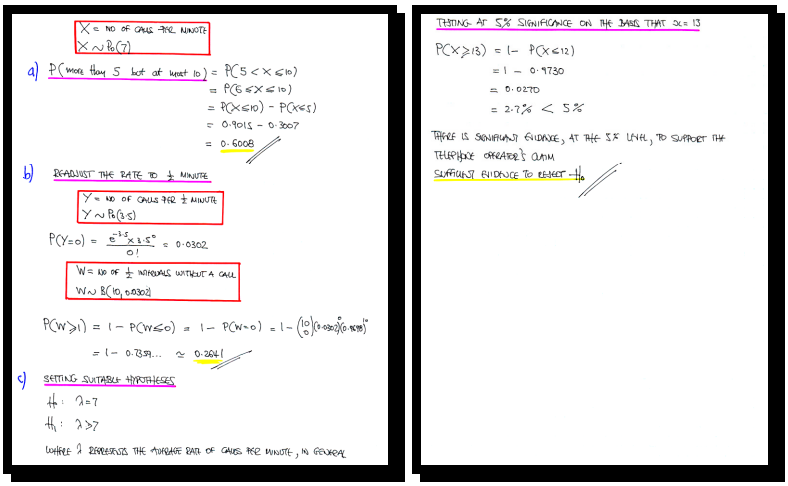
b) Find the probability that there will be at least one 30 second interval without a single call arriving the company's telephone switchboard.

The telephone operator claims that the rate of calls has risen recently, and asks her manager for a helper.

The manager investigates this matter and finds that in a randomly chosen minute, 13 calls reached the telephone switchboard.

c) Test at the 5% level of significance, whether there is evidence to support the telephone operator's claim.

$\boxed{}$, $\boxed{0.6008}$, $\boxed{0.2641}$, $\boxed{\text{significant evidence, } 2.7\% < 5\%}$



Question 6 (****)

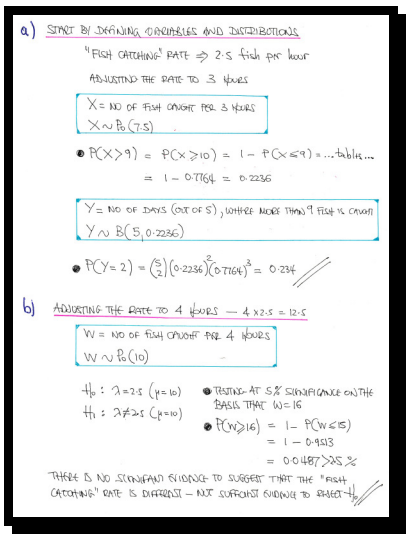
Since his retirement, Fred goes fishing Monday to Friday, for 3 hours on each of these 5 days. The number of fish he catches every hour follows a Poisson distribution with mean 2.5.

a) Find the probability that Fred catches more than 9 fish on exactly 2 of the days, in a given 5 day fishing week.

Fred buys a new type of bait and decides to test whether there is any difference to the rate at which he catches fish. He tries his new bait by going fishing on a Sunday and ends up catching 16 fish in 4 hours.

b) Carry out a significance test, at the 5% level, stating your hypotheses clearly.

$\boxed{}$, $\boxed{0.234}$, $\boxed{\text{not significant evidence, } 4.87\% > 2.5\%}$



Question 7 (****)

The number of accidents occurring in a certain stretch of motorway is modelled by a Poisson distribution with mean of 0.7 per day. The traffic police introduce speed camera in an effort to reduce this rate. They decide to monitor this stretch of motorway for 60 days after the introduction of the cameras.

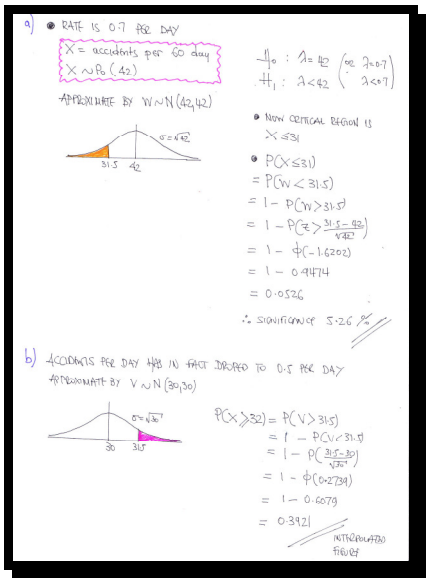
A hypothesis test is to be carried out afterwards. It is decided that the cameras would have had the desired effect if 31 or less accidents occur during these 60 days.

a) Determine the significance level for this test. State the hypotheses clearly.

It is now given that the introduction of cameras has in fact reduced the mean accident rate to 0.5 per day.

b) Find the probability that the traffic police will conclude that the cameras have not reduced the accident rate.

significant level = 5.26% , 0.3921



Question 8 (**+)**

It is known that during the first hour of trading, customers arrive at a garden centre at the rate of 3 customers every 10 minutes.

a) State two conditions, so that a Poisson distribution could be used to model the number of arrivals in the garden centre.

b) State one reason as to why the Poisson model might not be suitable.

Using a suitable Poisson model, and ignoring the answer to part **(b)**, determine the probability that...

c) … exactly 4 customers arrive in the first 10 minutes of trading.

d) … exactly 8 customers arrive in the first 20 minutes of trading.

e) … exactly 4 customers arrive in the first 10 minutes of trading and a further 4 customers arrive in the next 10 minutes.

f) … the **first** customer will arrive 7 minutes after opening.

The first hour after opening is subdivided into 6 equal 10 minute intervals.

Calculate the probability that...

g) … exactly 4 customers arrive in **none** of these 10 minute intervals.

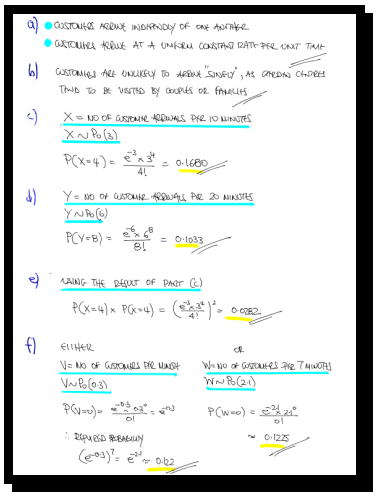
h) … there will be a **single** 10 minute interval, where **no** customers arrive to the garden centre.

The garden centre claims that 98% of its seeds will germinate. It was found that in a random sample of 125 seeds, 117 seeds germinated.

i) Use a suitable approximation, to test at the 1% level of significance, whether the garden centre overstates thr germination proportion of its seeds. You must state your hypotheses clearly in this part.

$\boxed{}$, $\boxed{0.1680}$, $\boxed{0.1033}$, $\boxed{0.0282}$, $\boxed{0.1225}$, $\boxed{0.3316}$, $\boxed{0.2314}$,
$\boxed{\text{significant evidence, } 0.42\% < 1\%}$

[solution overleaf]



a) • CUSTOMERS ARRIVE INDEPENDENTLY OF ONE ANOTHER.
• CUSTOMERS ARRIVE AT A UNIFORM CONSTANT RATE PER UNIT TIME.

b) CUSTOMERS ARE UNLIKELY TO ARRIVE "SINGLY", AS GARDENS CENTRES TEND TO BE VISITED BY COUPLES OR FAMILIES.

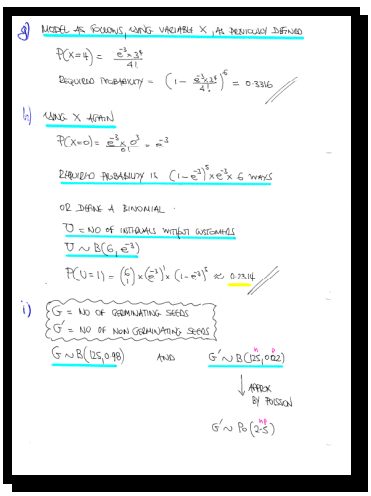
c) X = NO OF CUSTOMER ARRIVALS PER 10 MINUTES
$X \sim \text{Po}(3)$
$\text{P}(X=4) = \dfrac{e^{-3} \times 3^4}{4!} = 0.1680$

d) Y = NO OF CUSTOMER ARRIVALS PER 20 MINUTES
$Y \sim \text{Po}(6)$
$\text{P}(Y=8) = \dfrac{e^{-6} \times 6^8}{8!} = 0.1033$

e) USING THE RESULT OF PART (c)
$\text{P}(X=4) \times \text{P}(X=4) = \left(\dfrac{e^{-3} \times 3^4}{4!}\right)^2 \approx 0.0282$

f) EITHER
V = NO OF CUSTOMERS PER HOUR
$V \sim \text{Po}(0.3)$
$\text{P}(V=0) = \dfrac{e^{-0.3} \times 0.3^0}{0!} = e^{-0.3}$
∴ REQUIRED PROBABILITY
$(e^{-0.3})^7 = e^{-2.1} \approx 0.122$

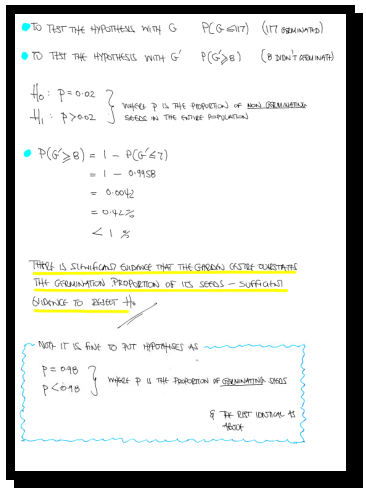
OR
W = NO OF CUSTOMERS PER 7 HOURS
$W \sim \text{Po}(2.1)$
$\text{P}(W=0) = \dfrac{e^{-2.1} \times 2.1^0}{0!} \approx 0.1225$



g) MODEL AS BINOMIAL USING VARIABLE X, AS PREVIOUSLY DEFINED
$\text{P}(X=4) = \dfrac{e^{-3} \times 3^4}{4!}$
REQUIRED PROBABILITY $= \left(1 - \dfrac{e^{-3} \times 3^4}{4!}\right)^6 = 0.3316$

h) USING X AGAIN
$\text{P}(X=0) = \dfrac{e^{-3} \times 3^0}{0!} = e^{-3}$
REQUIRED PROBABILITY IS $(1-e^{-3})^5 \times e^{-3} \times 6$ WAYS
OR DEFINE A BINOMIAL
U = NO OF INTERVALS WITHOUT CUSTOMERS
$U \sim \text{B}(6, e^{-3})$
$\text{P}(U=1) = \binom{6}{1} \times (e^{-3})^1 \times (1-e^{-3})^5 \approx 0.2314$

i) G = NO OF GERMINATING SEEDS
G' = NO OF NON GERMINATING SEEDS
$G \sim \text{B}(125, 0.98)$ AND $G' \sim \text{B}(125, 0.02)$
↓ APPROX BY POISSON
$G' \sim \text{Po}(2.5)$



• TO TEST THE HYPOTHESIS WITH G $\text{P}(G \leqslant 117)$ (117 GERMINATED)
• TO TEST THE HYPOTHESIS WITH G' $\text{P}(G' \geqslant 8)$ (8 DIDN'T GERMINATE)

$H_0 : p = 0.02$
$H_1 : p > 0.02$
WHERE p IS THE PROPORTION OF NON GERMINATING SEEDS IN THE ENTIRE POPULATION

• $\text{P}(G' \geqslant 8) = 1 - \text{P}(G' \leqslant 7)$
$= 1 - 0.9958$
$= 0.0042$
$= 0.42\%$
$< 1\%$

THERE IS STRONG EVIDENCE THAT THE GARDEN CENTRE OVERSTATES THE GERMINATION PROPORTION OF ITS SEEDS – SUFFICIENT EVIDENCE TO REJECT H_0

NOW IT IS FINE TO PUT HYPOTHESES AS
$p = 0.98$
$p < 0.98$
WHERE p IS THE PROPORTION OF GERMINATING SEEDS
& THE REST IDENTICAL AS ABOVE

POISSON DISTRIBUTION CRITICAL REGIONS

Question 1

The new chief of the traffic police was told that in a busy stretch of motorway, under his jurisdiction, accidents occur at the rate of 7 per month.

The new chief wants to know whether this figure is up to date and decides to test it to see if this rate is different from 7 accidents per month.

a) Find the critical region for this test.

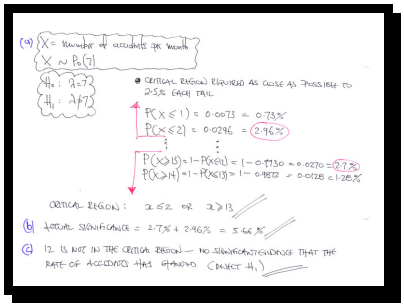
The probability of rejecting at either tail must be as close as possible to 2.5%.

b) State the actual significance level for a test using the critical region of part **(a)**.

In the month following the new chief's appointment there were 12 accidents in that stretch of motorway.

c) Complete the test.

$\boxed{\text{C.R.} = \{0,1,2\} \cup \{13,14,15,\ldots\}}$, $\boxed{5.66\%}$, $\boxed{\text{not significant evidence, } 12 \text{ is not in C.R.}}$



Question 2

A test statistic has distribution $\text{Po}(\lambda)$.

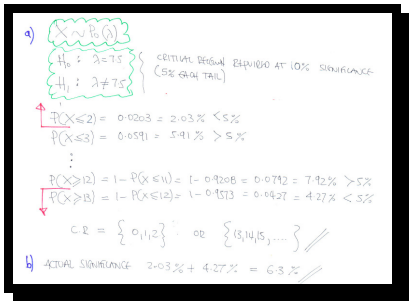
a) Given that

$$\text{H}_0 : \lambda = 7.5, \quad \text{H}_1 : \lambda \neq 7.5,$$

find the critical region for the test statistic at the 10% significance level.

b) State the actual level of significance in a test using this critical region.

$\{0,1,2\} \cup \{13,14,15,16,...\}$, 6.3%



Question 3

A test statistic has distribution $\text{Po}(\lambda)$.

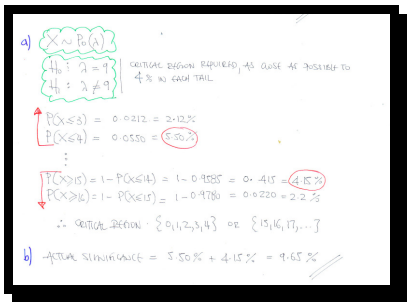
a) Given that

$$\text{H}_0 : \lambda = 9, \quad \text{H}_1 : \lambda \neq 9,$$

find the critical region for the test statistic such that the probability of rejecting in each tail is as close as possible to 4%.

b) State the probability of incorrectly rejecting H_0 using this critical region.

$\{0,1,2,3,4\} \cup \{15,16,17,18,...\}$, 9.65% or 0.0965



POISSON DISTRIBUTION CRITICAL REGIONS

EXAM QUESTIONS

Question 1 (+)**

During Saturday afternoons, customers are known to walk into a certain clothes shop at the rate of 8 every 10 minutes.

The new shop manager wants to find if that rate has changed since he took over.

a) Find the critical region, at the 5% level of significance, to investigate whether the rate of 8 customers walking into the shop every 10 minutes has changed.

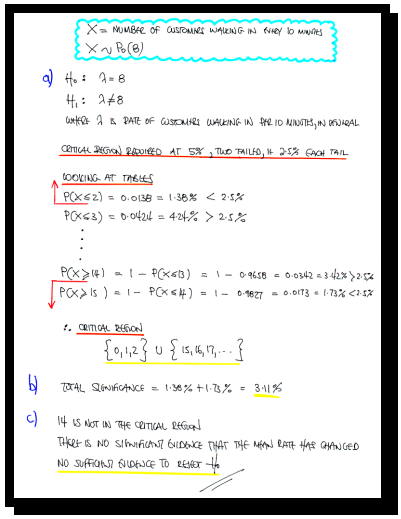
b) State the actual significance level for a test using the critical region of part **(a)**.

During a Saturday afternoon, 14 customers walked into the shop in a random 10 minute interval.

c) Complete the test.

$\boxed{\quad}$, $\boxed{\text{C.R.} = \{0,1,2\} \cup \{15,16,17,\ldots\}}$, $\boxed{3.11\%}$,

$\boxed{\text{not significant evidence, 14 is not in C.R.}}$



Question 2 (*+)**

From a Poisson distribution with parameter λ, a single observation w is taken and is to be used to test, at the 5% level of significance,

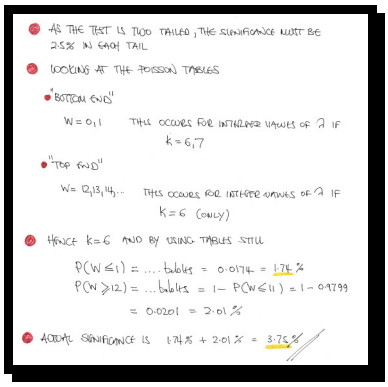
$$\mathrm{H}_0 : \lambda = k \qquad \text{against} \qquad \mathrm{H}_1 : \lambda \neq k ,$$

where k is a positive integer.

Find the actual significance for the test if the critical region is

$$W \leq 1 \ \cup \ W \geq 12 .$$

$\boxed{}$, $\boxed{3.75\%}$



Question 3 (*+)**

The proportion of customers who buy alcoholic drinks with their shopping from a certain supermarket is 3%.

The new manager of the supermarket believes that this proportion is different from 3%.

He decides to test his belief using a random sample of 300 customers.

a) Find the critical region at the 5% level of significance to test the manager's belief.

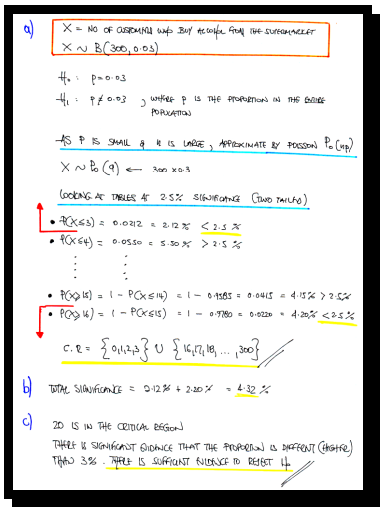
b) State the actual significance for a test using the critical region of part **(a)**.

Twenty shoppers in the sample bought alcoholic drinks.

c) Complete the test.

$\boxed{\quad}$, $\boxed{\text{C.R.}=\{0,1,2,3\}\cup\{16,16,18,\ldots,300\}}$, $\boxed{4.32\%}$,

$\boxed{\text{significant evidence, 20 is in C.R.}}$



Created by T. Madas