GEOMETRIC DISTRIBUTION

Question 1 **(**+)**

The discrete random variable X is modelled as being geometrically distributed with parameter 0.2.

a) State two conditions that must be satisfied by X, so that the geometric model is valid.

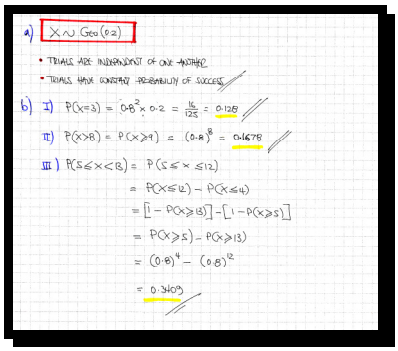
b) Showing full workings, where appropriate, calculate the value of …

i. … $\mathrm{P}(X=3)$.

ii. … $\mathrm{P}(X>8)$.

iii. … $\mathrm{P}(5\leq X<13)$.

$\boxed{}$, $\boxed{0.128}$, $\boxed{0.1678}$, $\boxed{0.3409}$



Question 2 (+)**

It is known that in a certain town 30% of the people own an *Apfone*.

A researcher asks people at random whether they own an *Apfone*.

The random variable X represents the number of people asked up to and including the first person who owns an *Apfone*.

Determine that …

a) … $\mathrm{P}(X=4)$.

b) … $\mathrm{P}(X>4)$.

c) … $\mathrm{P}(X<6)$.

$\boxed{0.1029}$, $\boxed{0.2401}$, $\boxed{0.8319}$

$X \sim \mathrm{Geo}(0.3)$, where X = first person who owns an Apfone

a) $\mathrm{P}(X=4) = 0.7\times0.7\times0.7\times0.3 = 0.1029$

b) $\mathrm{P}(X>4) = \mathrm{P}(X\geqslant 5) = \mathrm{P}$(first 4 people do not own an Apfone)
$= 0.7\times0.7\times0.7\times0.7 = 0.2401$

c) $\mathrm{P}(X<6) = \mathrm{P}(X\leqslant 5) = \mathrm{P}(X=1,2,3,4,5)$
$= 0.3 + 0.3\times0.7 + 0.3\times0.7^2 + 0.3\times0.7^3 + 0.3\times0.7^4$
$= 0.3\left[1+0.7+0.7^2+0.7^3+0.7^4\right]$
$= 0.8319$

Question 3 (+)**

Arthur and Henry are rolling a fair six sided die and the winner of their game will be the first person to get a "six".

Arthur rolls the die first.

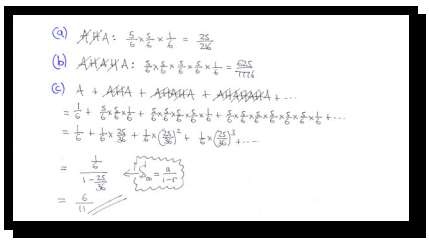
Determine the probability that …

a) … Arthur wins on his second throw.

b) … Arthur wins on his third throw.

c) … Arthur wins the game.

$$\boxed{\frac{25}{216}}\,,\quad \boxed{\frac{625}{7776}}\,,\quad \boxed{\frac{6}{11}}$$



Question 4 (***)

Nigel is playing tournament chess against a computer program and the probability he wins against the program at any given game is 0.25.

Nigel is playing several practice games every day, one after the other.

a) Find the probability that on a given day ...

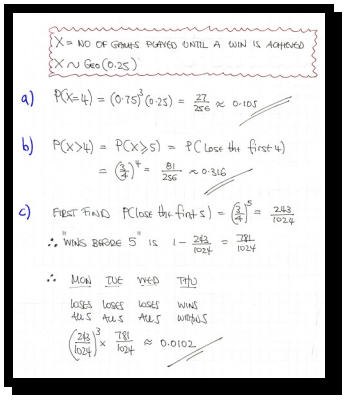
i. ... Nigel wins for the first time on the 4^{th} game played.

ii. … Nigel has to play more than 4 games before he wins for the first time.

If Nigel does not win any of the first 5 games played in a given day, he plays no more games in that day. Nigel starts training on a Monday on a given week.

b) Determine the probability that Nigel wins his first game on the Thursday of that week.

$$\boxed{\frac{27}{256}\approx 0.105}\,,\quad \boxed{\frac{81}{256}\approx 0.316}\,,\quad \boxed{\approx 0.0102}$$



Question 5 (***)

In a statistical experiment, a token is placed at the origin $(0,0)$ of a square grid.

A fair six sided die is rolled repeatedly until a "*six*" is obtained.

- Every time a "*six*" **is not** obtained, the token is moved by one unit in the positive x direction.

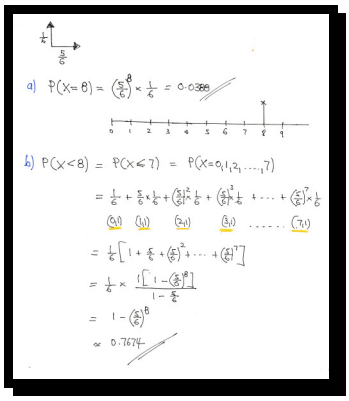
- When a "*six*" **is** obtained, the token is moved by one unit in the positive y direction and the experiment is over, with the token at the point with coordinates $(X,1)$.

Determine …

a) … $\mathrm{P}(X=8)$.

b) … $\mathrm{P}(X<8)$

$\boxed{\approx 0.0388}$, $\boxed{\approx 0.7674}$



Question 6 (*+)**

The small central section on a standard dart board is called the bull's eye.

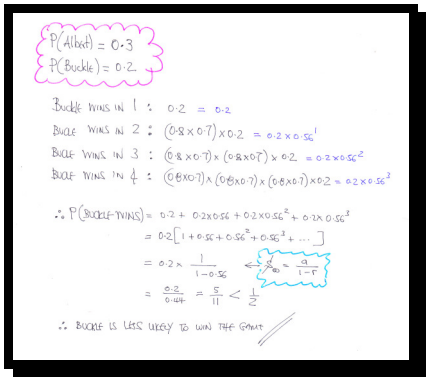
When Albert aim for the bull's eye the probability he hits it is 0.3.

When Buckle aim for the bull's eye the probability he hits it is 0.2.

One day the two players decide to play a game aiming a single dart at the bull's eye in alternative fashion, starting with Buckle. The winner is the first to hit the bull's eye.

Assuming that all probabilities are constant, show that Buckle is less likely to win the game compared with Albert.

$$\boxed{\mathrm{P}(\text{Buckle})=\frac{5}{11}<\frac{1}{2}}$$



Question 7 (****)

Two cricket players, Markus and Dean, decide to throw balls at a wicket, in alternate fashion, starting with Markus. The winner is the player who is first to hit the wicket.

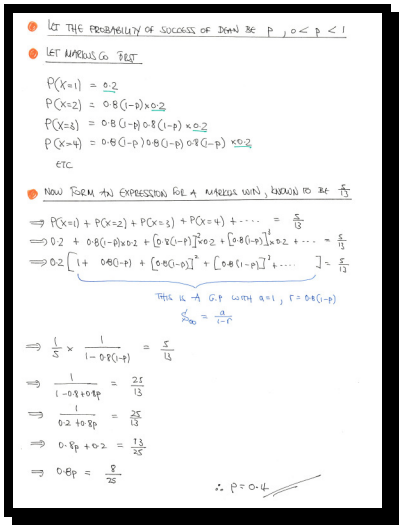
The probability that Markus hits the wicket is 0.2 for any of his throws.

The probability that Dean hits the wicket is p for any of his throws.

If Markus throws first, the probability he wins the game is $\frac{5}{13}$.

Determine the value of p.

$\boxed{}$, $\boxed{p=0.4}$



Question 8 (****)

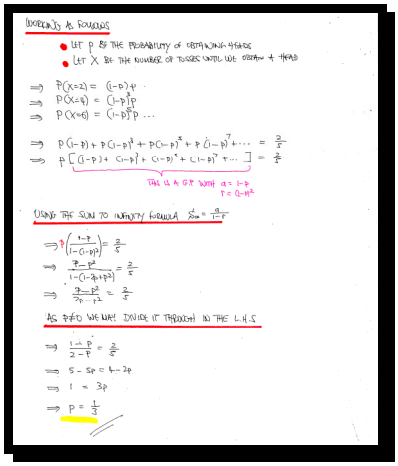
A coin is biased so that the probability of obtaining "heads" in any toss is p, $p \neq \frac{1}{2}$.

The coin is tossed repeatedly until a "head" is obtained.

The probability of obtaining "heads" after an even number of tosses is $\frac{2}{5}$.

Determine the value of p.

$\square$, $\boxed{p = \frac{1}{3}}$



Question 9 (****)

A bag contains a large number of coins, of which some are pound coins and some are two pound coins. A coin is selected at random from the bag with replacement, **until a two pound coin** is selected.

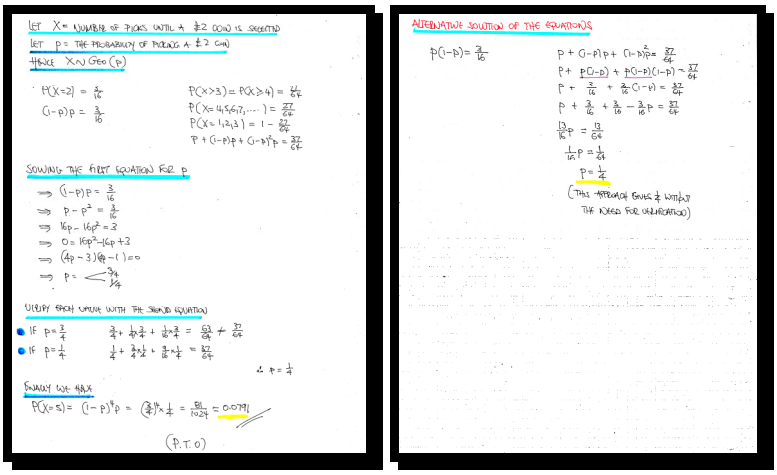
It is given that the probability it will take …

… exactly 2 attempts until a two pound coin is selected is $\frac{3}{16}$.

… more than 3 attempts until a two pound coin is selected is $\frac{27}{64}$.

Determine the probability that a two pound coin will be selected for the first time on the fifth attempt.

$\boxed{}$, $\boxed{\frac{81}{1024} \approx 0.0791}$



Question 10 **(*****)**

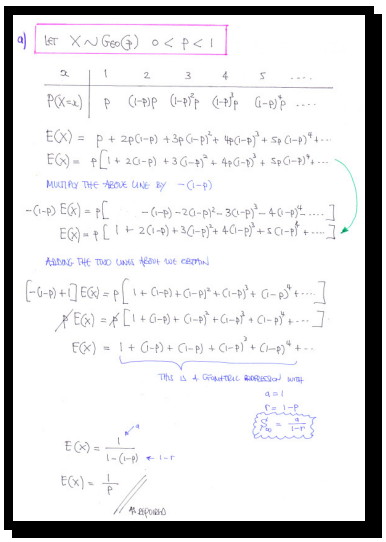
A discrete random variable X is geometrically distributed with parameter p.

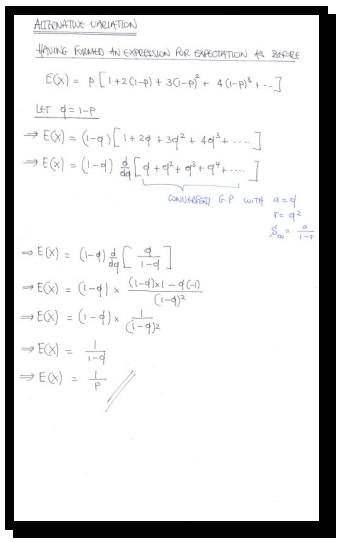
Show that …

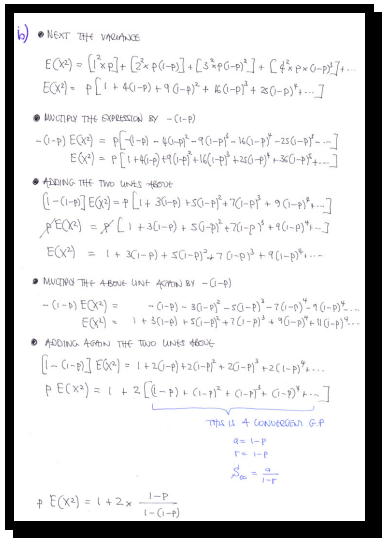
a) $\ldots \mathrm{E}(X)=\dfrac{1}{p}$.

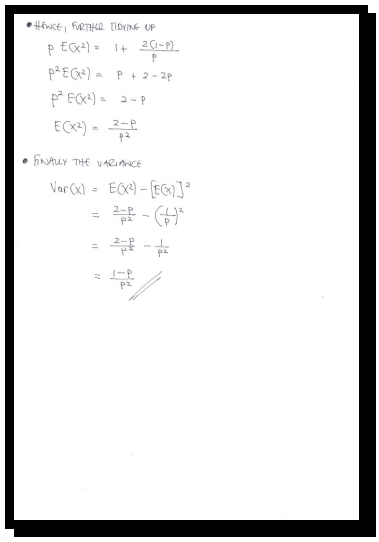
b) $\ldots \mathrm{Var}(X)=\dfrac{1-p}{p^2}$.

[ ], [proof]









NEGATIVE BINOMIAL DISTRIBUTION

Question 1 **(**)**

A discrete random variable X has negative binomial distribution, with 6 successes required each with probability of success 0.4.

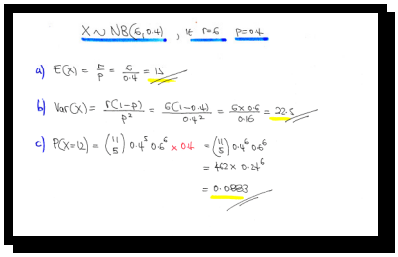
Determine the value of …

a) … $\mathrm{E}(X)$.

b) … $\mathrm{Var}(X)$.

c) … $\mathrm{P}(X=12)$.

$\square$, $\boxed{\mathrm{E}(X)=15}$, $\boxed{\mathrm{Var}(X)=22.5}$, $\boxed{\mathrm{P}(X=12)\approx 0.0883}$



Question 2 (+)**

Justin is playing "*Alien Shooter*" on the internet. It is a game where you battle against randomly drawn opponents where the reward of a battle is a "*game ticket*".

The probability of Justin winning a battle is thought to be 0.4.

a) Showing detailed workings where appropriate, calculate the probability of Justin …

i. … winning his first battle on his third attempt.

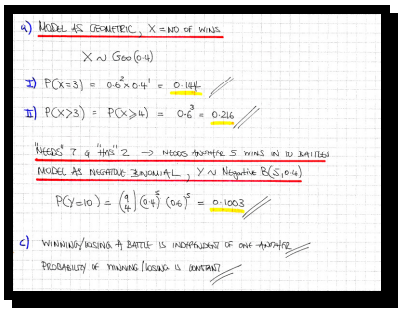
ii. … winning his first battle after his third attempt.

In Alien Shooter when you collect 7 game tickets you can upgrade your spaceship. Justin has already collected 2 game tickets from the previous day's play. He starts playing today, hoping to upgrade his spaceship.

b) Determine the probability he will have to have 10 battles by the time he is able to upgrade his spaceship.

c) State two conditions that must be satisfied in this scenario if the calculations are to be valid.

$\boxed{}$, $\boxed{0.144}$, $\boxed{0.216}$, $\boxed{0.1003}$

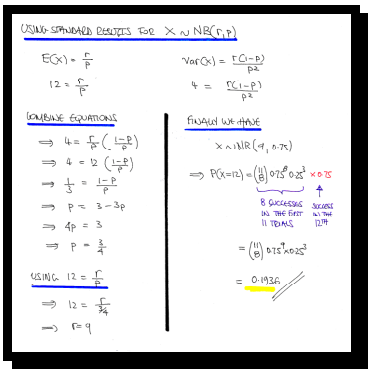


Question 3 **(**+)**

A discrete random variable X has negative binomial distribution, with $\mathrm{E}(X)=12$ and $\mathrm{Var}(X)=4$.

Determine the value of $\mathrm{P}(X=12)$.

$\boxed{}$, $\boxed{\mathrm{P}(X=12)\approx 0.1936}$



Question 4 (*+)**

It has been established over a long period of time that the probability that an electricity company operative will be able to take a reading from a house meter, due to the resident being at home, is 0.4.

a) Determine the probability that the first reading to be taken will be on, or before, the 7^{th} house visit.

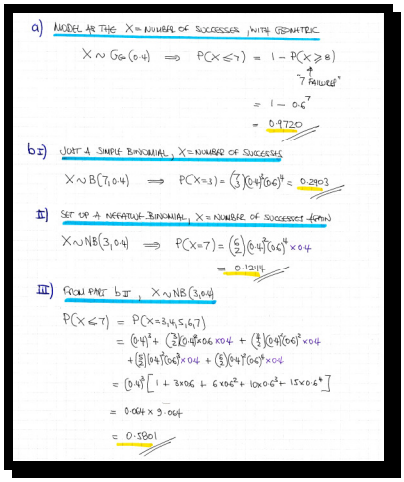
b) Find the probability that the operative will be able to take …

i. … exactly 3 readings in his first 7 visits.

ii. … his 3^{rd} reading on his 7^{th} visit.

iii. … his 3^{rd} reading on, or before his 7^{th} visit.

$\boxed{}$, $\boxed{\approx 0.9720}$, $\boxed{\approx 0.2903}$, $\boxed{\approx 0.1244}$, $\boxed{\approx 0.5801}$



Question 5 **(***+)**

Steve is filming and uploading videos on the internet, and **every week** he plans to upload 5 videos.

The probability that a video has no significant faults and so it is deemed to be suitable for uploading to the internet, is 0.25. Once Steve has uploaded 5 videos he stops filming for that week.

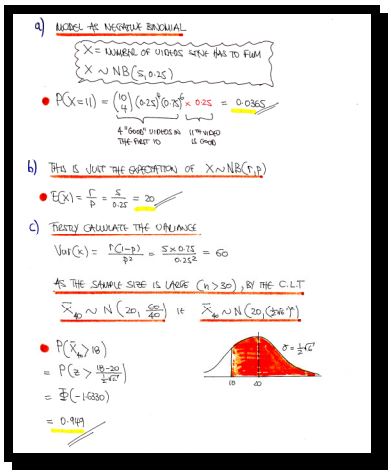
a) Find the probability that, in a given week, Steve will have to film 11 videos.

b) Determine the number of videos Steve will be expected to film in a given week in order to meet his weekly uploading target of 5 videos.

In a season Steve plans to film for 40 weeks.

c) Estimate the probability that the **mean** number of videos Steve has to film per week, is greater than 18.

$\boxed{}$, $\boxed{\approx 0.0365}$, $\boxed{\mathrm{E}(X)=20}$, $\boxed{\approx 0.949}$



Question 6 (**+)**

The discrete random variable X represents the number of times a biased coin must be tossed until 3 "*heads*" have been obtained.

It is further given that $\mathrm{Var}(X)=36$.

a) Calculate $\mathrm{P}(X=9)$.

b) If the first "head" was obtained on the second toss, find $\mathrm{P}(X\leq 9)$

$\boxed{}$, $\boxed{\approx 0.0779}$, $\boxed{\approx 0.5551}$

