DISCRETE RANDOM VARIABLES

Question 1 (**)

The probability distribution of a discrete random variable X is given by

x	0	1	2	3	4
$\mathrm{P}(X=x)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	a	$\frac{1}{24}$

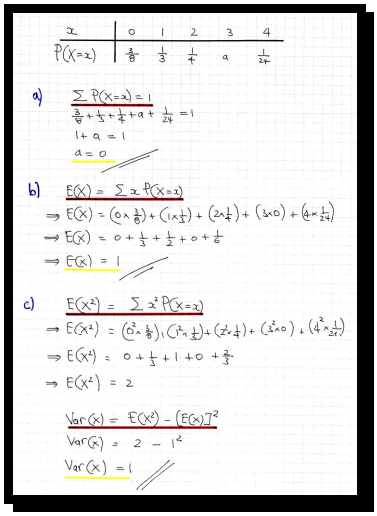
where a is a positive constant.

a) Explain why $a=0$.

b) Find the value of $\mathrm{E}(X)$.

c) Calculate $\mathrm{Var}(X)$.

$\square$, $\boxed{\sum \mathrm{P}(X=x)=1}$, $\boxed{\mathrm{E}(X)=1}$, $\boxed{\mathrm{Var}(X)=1}$



Question 2 **(**)**

The probability distribution of a discrete random variable X is given by

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$

Find, showing full workings where appropriate, the value of

a) $\mathrm{P}(1<X\leq 3)$.

b) $\mathrm{F}(1.8)$.

c) $\mathrm{E}(X)$.

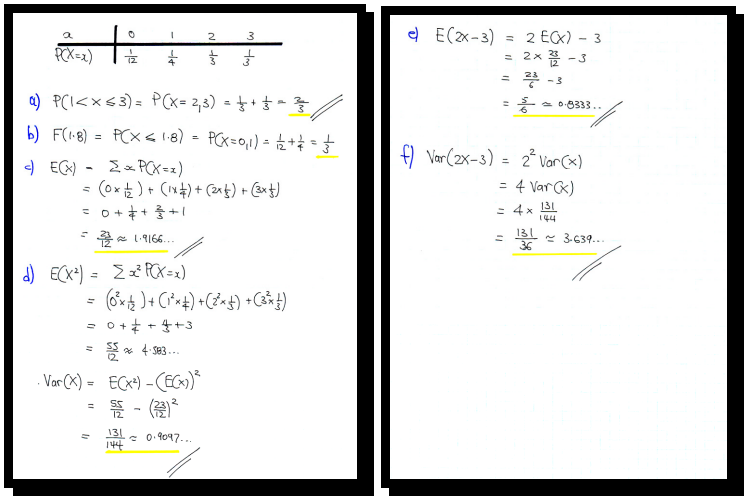
d) $\mathrm{Var}(X)$.

e) $\mathrm{E}(2X-3)$.

f) $\mathrm{Var}(2X-3)$.

$\square$, $\boxed{P(1<X\leq 3)=\frac{2}{3}}$, $\boxed{\mathrm{F}(1.8)=\frac{1}{3}}$, $\boxed{\mathrm{E}(X)=\frac{23}{12}}$, $\boxed{\mathrm{Var}(X)=\frac{131}{144}\approx 0.910}$,

$\boxed{\mathrm{E}(2X-3)=\frac{5}{6}}$, $\boxed{\mathrm{Var}(2X-3)=\frac{131}{36}\approx 3.639}$



Question 3 **(**)**

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} kx^2 & x=3,4,5 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the value of the constant k.

b) Find the value of …

i. … $\mathrm{E}(X)$.

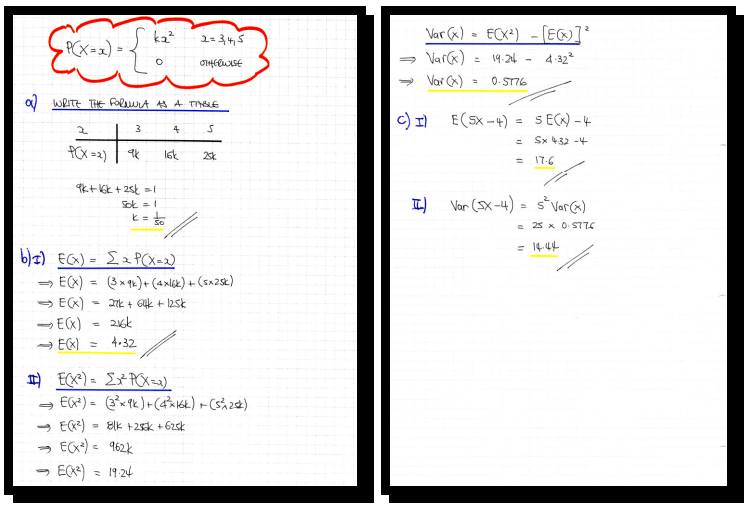
ii. … $\mathrm{Var}(X)$.

c) Determine …

i. … $\mathrm{E}(5X-4)$.

ii. … $\mathrm{Var}(5X-4)$.

$\square$, $\boxed{k=\frac{1}{50}}$, $\boxed{\mathrm{E}(X)=4.32}$, $\boxed{\mathrm{Var}(X)=0.5776}$, $\boxed{\mathrm{E}(5X-4)=17.6}$, $\boxed{\mathrm{Var}(5X-4)=14.44}$



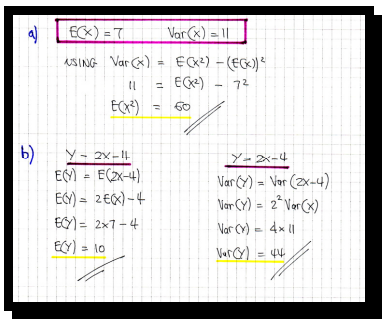
Question 4 **(**)**

The discrete random variable X has mean 7 and variance 11.

a) Calculate $\mathrm{E}\left(X^2\right)$.

b) Given that $Y=2X-4$, determine the mean and variance of Y.

$\boxed{}$, $\boxed{\mathrm{E}\left(X^2\right)=60}$, $\boxed{\mathrm{E}(Y)=10}$, $\boxed{\mathrm{Var}(Y)=44}$



Question 5 **(**)**

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} k(2-x)^2 & x=-2,-1,0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the value of the constant k.

b) Find the value of …

i. … $\mathrm{E}(X)$.

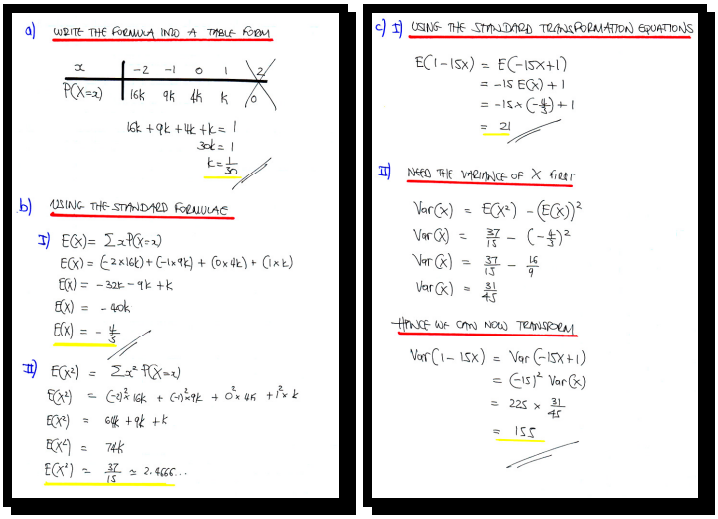
ii. … $\mathrm{E}\left(X^2\right)$.

c) Determine …

i. … $\mathrm{E}(1-15X)$.

ii. … $\mathrm{Var}(1-15X)$.

$\square$, $\boxed{k=\frac{1}{30}}$, $\boxed{\mathrm{E}(X)=-\frac{4}{3}}$, $\boxed{\mathrm{E}\left(X^2\right)=\frac{37}{15}}$, $\boxed{\mathrm{E}(1-15X)=21}$, $\boxed{\mathrm{Var}(1-15X)=155}$



Question 6 **(**)**

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} kx(5-x) & x=1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

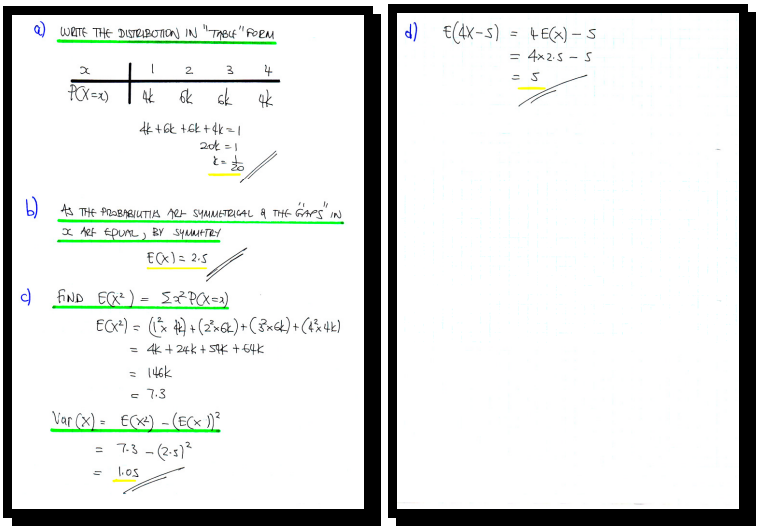
a) Determine the value of the constant k.

b) State the value of $\mathrm{E}(X)$.

c) Calculate $\mathrm{Var}(X)$.

d) Determine the value of $\mathrm{E}(4X-5)$.

$\boxed{}$, $\boxed{k=\frac{1}{20}}$, $\boxed{\mathrm{E}(X)=2.5}$, $\boxed{\mathrm{Var}(X)=1.05}$, $\boxed{\mathrm{E}(4X-5)=5}$



Question 7 (***)

The probability distribution of a discrete random variable X is given by

x	0	1	2	3
$\mathrm{P}(X=x)$	0.5	0.35	a	b

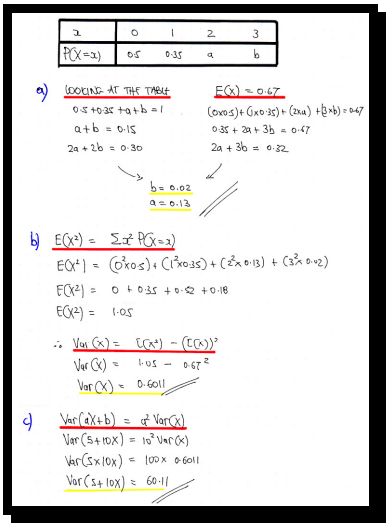
where a and b are positive constants.

a) Given that $\mathrm{E}(X)=0.67$, find the value of a and the value of b.

b) Determine the variance of X.

c) Calculate $\mathrm{Var}(5+10X)$.

$\square$, $\boxed{a=0.13,\ \ b=0.02}$, $\boxed{\mathrm{Var}(X)=0.6011}$, $\boxed{\mathrm{Var}(5+10X)=60.11}$



Question 8 (***)

The probability distribution of a discrete random variable X is given by

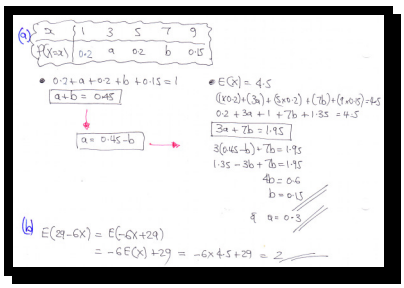
x	1	3	5	7	9
$\mathrm{P}(X=x)$	0.2	a	0.2	b	0.15

where a and b are positive constants.

a) Given that $\mathrm{E}(X)=4.5$, find the value of a and the value of b.

b) Determine $\mathrm{E}(29-6X)$.

$\boxed{a=0.3,\ \ b=0.15}$, $\boxed{\mathrm{E}(29-6X)=2}$



Question 9 **(***)**

Two fair spinners, both numbered with 0, 1, 2 and 3, are spun together and the **product** of their scores is recorded.

The discrete random variable X represents the product of the scores of these spinners and its probability distribution is summarized in the table below

x	0	1	2	3	4	6	9
$\mathrm{P}(X=x)$	$\frac{7}{16}$	$\frac{1}{16}$	a	b	c	$\frac{2}{16}$	$\frac{1}{16}$

a) Find the value of a, b and c.

b) Determine $\mathrm{E}(X)$.

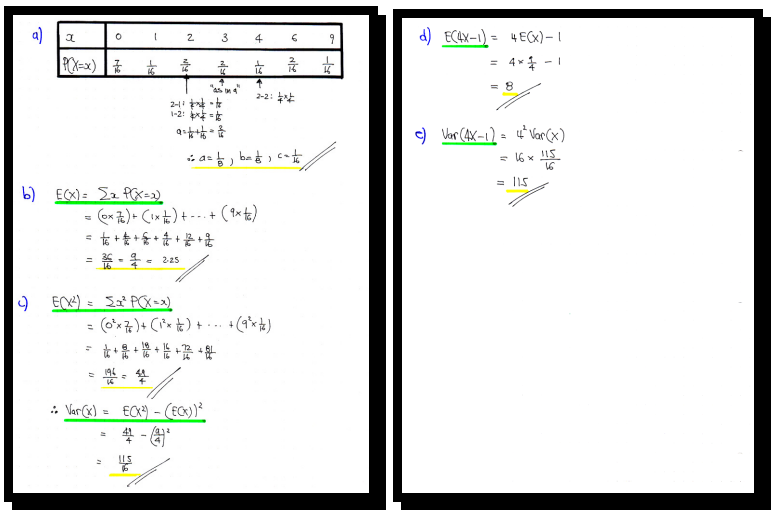
c) Find the value of $\mathrm{Var}(X)$.

d) Calculate $\mathrm{E}(4X-1)$.

e) Calculate $\mathrm{Var}(4X-1)$.

$\square$, $\boxed{a=\frac{1}{8},\ b=\frac{1}{8},\ c=\frac{1}{16}}$, $\boxed{\mathrm{E}(X)=\frac{9}{4}}$, $\boxed{\mathrm{Var}(X)=\frac{115}{16}}$, $\boxed{\mathrm{E}(4X-1)=8}$,

$\boxed{\mathrm{Var}(4X-1)=115}$



Question 10 (***)

The probability distribution of a discrete random variable X is summarised in the table below.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.2	0.1	0.2	0.25	0.25

a) Find the value of …

i. … $\mathrm{E}(X)$.

ii. … $\mathrm{E}\left(X^2\right)$.

iii. … $\mathrm{Var}(X)$.

b) Calculate …

i. … $\mathrm{E}(3-X)$.

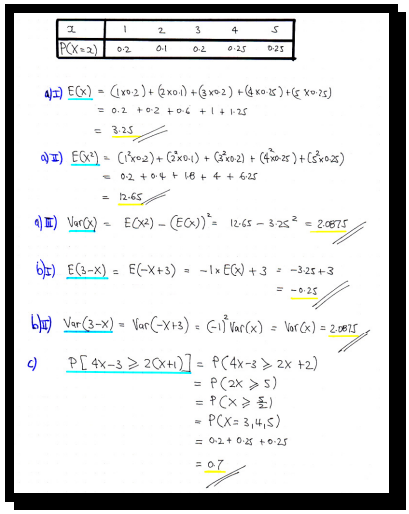
ii. … $\mathrm{Var}(3-X)$

c) Determine the value of

$$\mathrm{P}\left[4X-3\geq 2(X+1)\right].$$

$\boxed{}$, $\boxed{\mathrm{E}(X)=3.25}$, $\boxed{\mathrm{E}\left(X^2\right)=12.65}$, $\boxed{\mathrm{Var}(X)=2.0875}$, $\boxed{\mathrm{E}(3-X)=-0.25}$,

$\boxed{\mathrm{Var}(3-X)=2.0875}$, $\boxed{\mathrm{P}\left[4X-3\geq 2(X+1)\right]=0.7}$



Question 11 (***)

The cumulative distribution $\mathrm{F}(x)$, of a discrete random variable X is given by

x	1	2	3	4	5	6	7	8
$\mathrm{F}(x)$	0.25	0.40	0.55	0.65	0.75	0.85	0.95	1

a) Find the value …

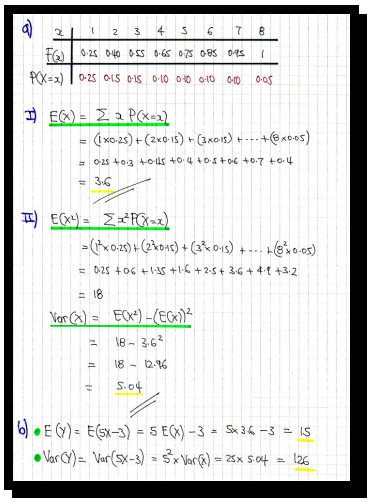
i. … $\mathrm{E}(X)$.

ii. … $\mathrm{Var}(X)$.

The discrete random variable Y is defined as $Y = 5X - 3$.

b) Determine the mean and variance of Y.

$\boxed{}$, $\boxed{\mathrm{E}(X) = 3.6}$, $\boxed{\mathrm{Var}(X) = 5.04}$, $\boxed{\mathrm{E}(Y) = 15}$, $\boxed{\mathrm{Var}(Y) = 126}$



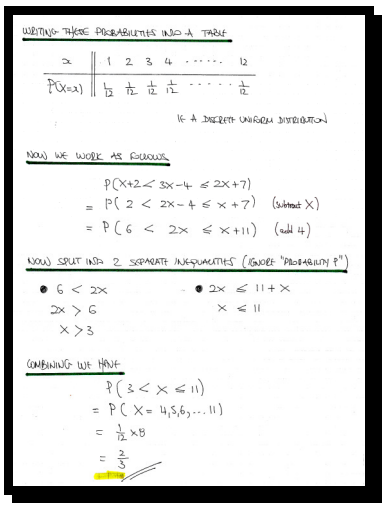
Question 12 (***)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases}\frac{1}{12} & x=1,\,2,\,3,\,\ldots,\,12\\ 0 & \text{otherwise}\end{cases}$$

Determine $\mathrm{P}(X+2<3X-4\leq 2X+7)$.

$\boxed{}$, $\boxed{\mathrm{P}(X+2<3X-4\leq 2X+7)=\frac{2}{3}}$



Question 13 **(***+)**

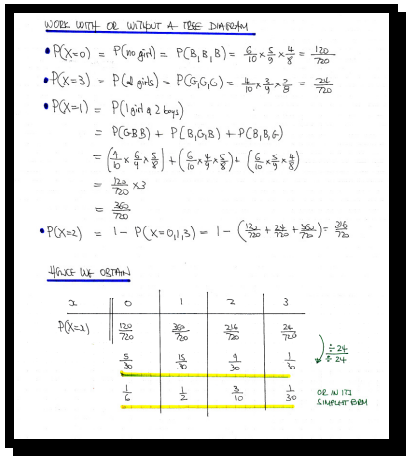
A sixth form class consists of 6 boys and 4 girls.

Three students are selected at random from this class and the variable X represents the number of girls selected.

Show that the probability distribution of X is given by

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

[ ], proof



Question 14 **(***+)**

The cumulative distribution of a discrete random variable X is given by

x	1	2	4	5
$\mathrm{F}(x)$	$\frac{3}{20}$	$\frac{2k+3}{20}$	$\frac{k+5}{10}$	$\frac{k+2}{4}$

where k is a positive constant.

a) Show clearly that $k=2$.

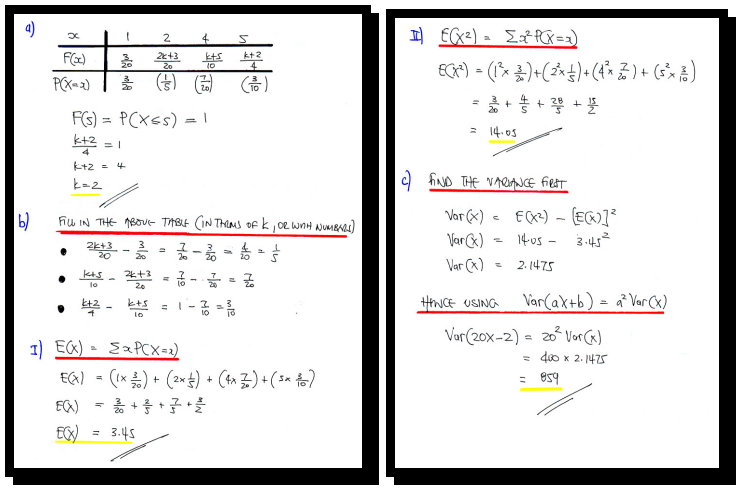
b) Find the value of …

i. … $\mathrm{E}(X)$.

ii. … $\mathrm{E}\left(X^2\right)$.

c) Calculate $\mathrm{Var}(20X-2)$.

$\boxed{}$, $\boxed{\mathrm{E}(X)=3.45}$, $\boxed{\mathrm{E}\left(X^2\right)=14.05}$, $\boxed{\mathrm{Var}(20X-2)=859}$



Question 15 (*+)**

A biased six sided die has the following probability distribution

x	1	2	3	4	5	6
$\mathrm{P}(X=x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{2}$

where the random variable X represents the number shown on its uppermost face when it comes to rest after it is rolled.

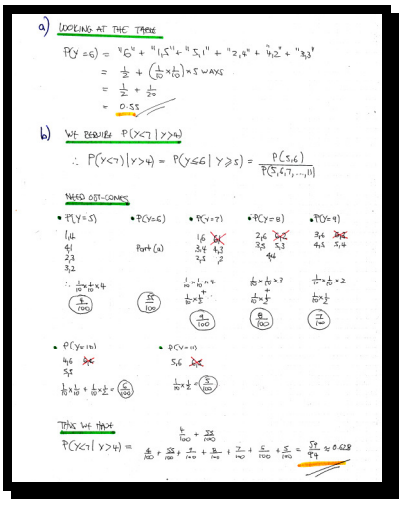
The die is rolled twice and the two independent observations of X, X_1 and X_2, produce the score Y defined as

$$Y=\begin{cases} 6 & \text{if } X_1=6 \\ X_1+X_2 & \text{if } X_1\neq 6 \end{cases}$$

a) Find the value of $\mathrm{P}(Y=6)$.

b) Find the value of $\mathrm{P}(Y<7|Y>4)$.

$\boxed{}$, $\boxed{\mathrm{P}(Y=6)=0.55}$, $\boxed{\mathrm{P}(Y<7|Y>4)=\frac{59}{94}}$



Question 16 **(***+)**

The probability distribution of a discrete random variable X is given by

x	0	1	2	3	4	5	6
$\mathrm{P}(X=x)$	0.05	0.1	0.15	0.2	0.25	0.2	0.05

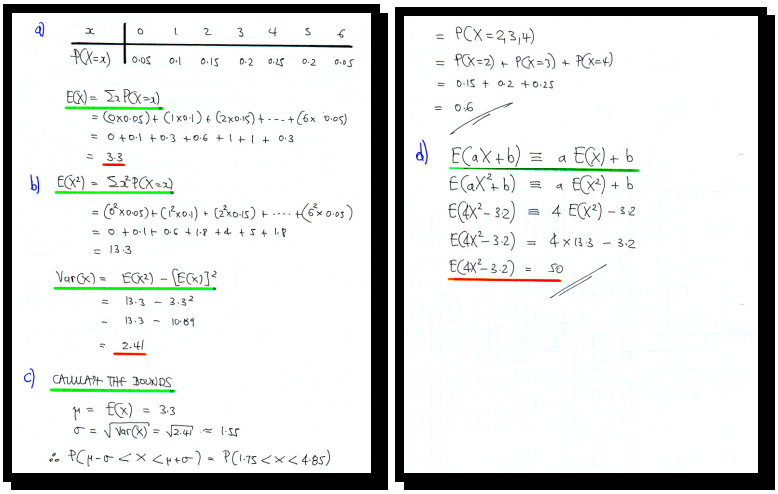
a) Find the value of $\mathrm{E}(X)$.

b) Calculate $\mathrm{Var}(X)$.

c) Determine $\mathrm{P}(\mu-\sigma\le X\le\mu+\sigma)$.

d) Find the value of $\mathrm{E}\left(4X^2-3.2\right)$.

$\boxed{}$, $\boxed{\mathrm{E}(X)=3.3}$, $\boxed{\mathrm{Var}(X)=2.41}$, $\boxed{\mathrm{P}(\mu-\sigma\le X\le\mu+\sigma)=0.6}$, $\boxed{\mathrm{E}\left(4X^2-3.2\right)=50}$



Question 17 (*+)**

A box contains three blue discs and two red discs.

Three discs are selected at random from the box without replacement.

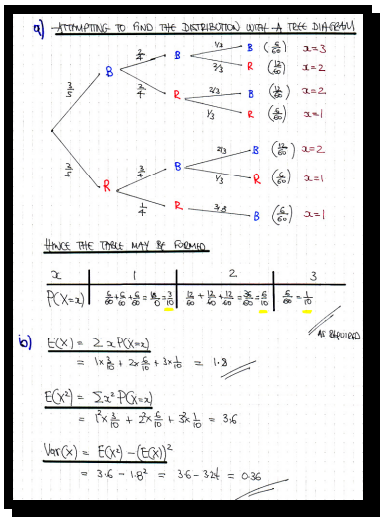
The variable X represents the number of blue discs selected.

a) Show that the probability distribution of X is given by

x	1	2	3
$\mathrm{P}(X=x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

b) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

$\boxed{}$, $\boxed{\mathrm{E}(X)=\frac{9}{5}=1.8}$, $\boxed{\mathrm{Var}(X)=\frac{9}{25}=0.36}$



Question 18 (*+)**

A sixth form class consists of 6 boys and 4 girls.

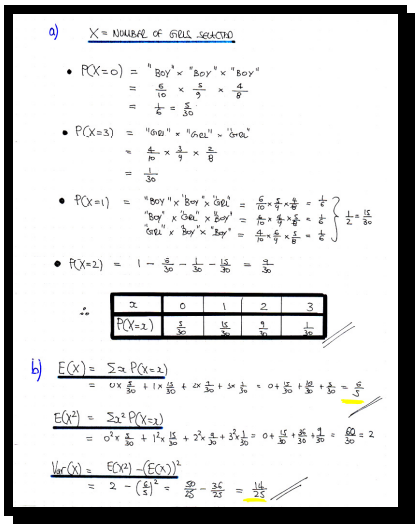
Three students are selected at random from this class and the variable X represents the number of girls selected.

a) Show that the probability distribution of X is given by

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

b) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

$\square$, $\boxed{\mathrm{E}(X)=\frac{6}{5}=1.2}$, $\boxed{\mathrm{Var}(X)=\frac{14}{25}=0.56}$



Question 19 **(***+)**

The probability distribution of the discrete random variable X is given by

x	2	3	4
$\mathrm{P}(X = x)$	$0.4 - a$	$2a$	$0.6 - a$

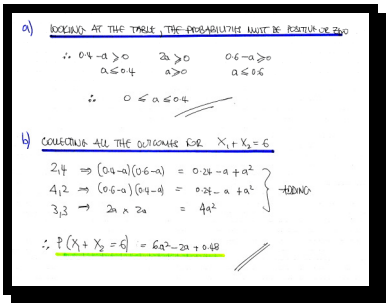
where a is a constant.

a) State the range of the possible values of a.

Two independent observations of X, denoted by X_1 and X_2, are considered.

b) Determine, in terms of a, a simplified expression for $\mathrm{P}(X_1 + X_2 = 6)$.

$\boxed{}$, $\boxed{0 \le a \le 0.4}$, $\boxed{P(X_1 + X_2 = 6) = 6a^2 - 2a + 0.48}$

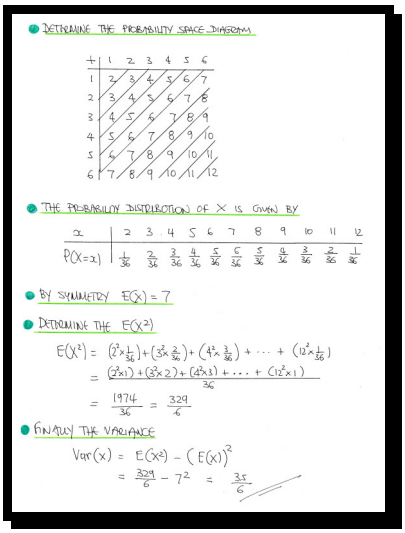


Question 20 (*+)**

Two standard fair cubical dice, numbered 1 to 6 are such rolled and the random variable X represents the sum of the scores of the two dice.

Determine the value of $\text{Var}(X)$.

$\boxed{}$, $\boxed{\text{Var}(X)=\frac{35}{6}}$



Question 21 (*+)**

The discrete random variable X has the following probability distribution

x	0	2	3
$\mathrm{P}(X=x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

a) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

A game in a fun fair consists of throwing 5 darts on a small target.

If a dart lands on the central portion of the target the dart scores 3 points.

If a dart lands on the outer portion of the target the dart scores 2 points, otherwise the dart scores no points.

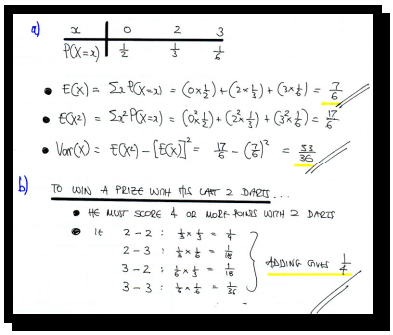
To win a prize, 10 or more points must be scored with 5 darts.

Paul has scored 6 points with his first 3 darts.

The likelihood of Paul scoring 0, 2 or 3 points is given by the probability distribution of part **(a)**.

b) Find the probability that Paul will win a prize after he throws his last 2 darts.

$\boxed{}$, $\boxed{\mathrm{E}(X)=\frac{7}{6}}$, $\boxed{\mathrm{Var}(X)=\frac{53}{36}}$, $\boxed{\frac{1}{4}}$



Question 22 (*+)**

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} k(4-x) & x=0,1,2,3 \\ \frac{1}{2} & x=4 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k=\frac{1}{20}$.

Two independent observations of X are made, denoted by X_1 and X_2.

b) Find the probability distribution of Y, where $Y=X_1+X_2$.

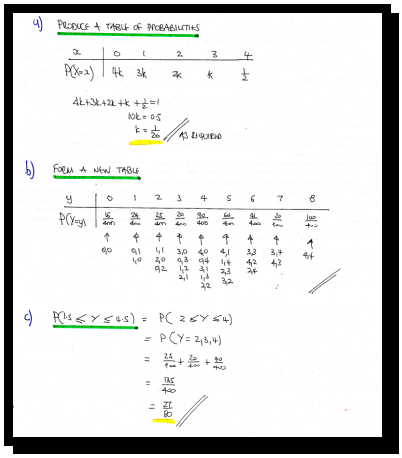
c) Calculate $P(1.5\le Y\le 4.5)$.

$\square$,

y	0	1	2	3	4	5	6	7	8
$\mathrm{P}(Y=y)$	$\frac{16}{400}$	$\frac{24}{400}$	$\frac{25}{400}$	$\frac{20}{400}$	$\frac{90}{400}$	$\frac{64}{400}$	$\frac{41}{400}$	$\frac{20}{400}$	$\frac{100}{400}$

,

$$P(1.5\le Y\le 4.5)=\frac{27}{80}$$



Question 23 **(***+)**

The probability distribution of a discrete random variable X is given by

x	1	2	3	4
$\mathrm{P}(X=x)$	a	b	b	c

where a, b and c are constants.

The cumulative distribution function of X is given by

x	1	2	3	4
$F(x)$	$\frac{1}{6}$	d	$\frac{2}{3}$	e

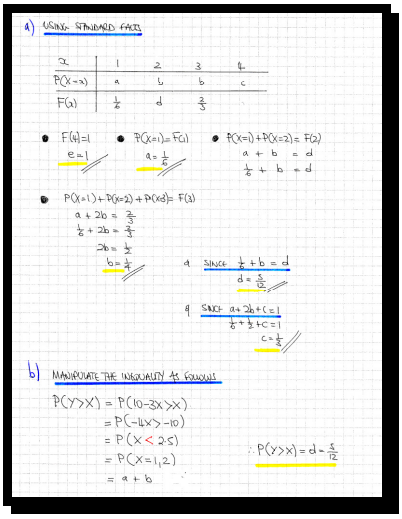
where d and e are constants.

a) Determine the value of each of the constants a, b, c, d and e.

The discrete random variable Y is defined as $Y=10-3X$.

b) Find the value of $P(Y>X)$.

$\boxed{}$, $\boxed{(a,b,c,d,e)=\left(\frac{1}{6},\frac{1}{4},\frac{1}{3},\frac{5}{12},1\right)}$, $\boxed{P(Y>X)=\frac{5}{12}}$



Question 24 (*+)**

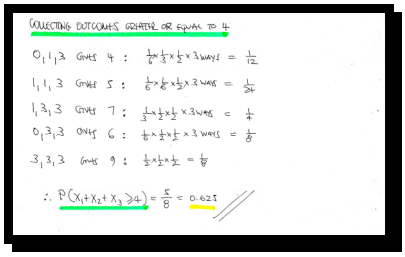
The discrete random variable X has the following probability distribution

x	0	1	3
$\mathrm{P}(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

Three independent observations of X are made, denoted by X_1, X_2 and X_3.

Calculate $P(X_1+X_2+X_3\geq 4)$.

$\boxed{}$, $\boxed{P(X_1+X_2+X_3\geq 4)=\frac{5}{8}}$



Question 25 (****)

The discrete random variable X has the following probability distribution

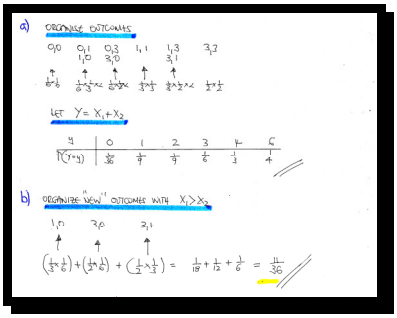
x	0	1	3
$\mathrm{P}(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

Two independent observations of X are made, denoted by X_1 and X_2.

a) Find the probability distribution of X_1+X_2.

b) Calculate $P(X_1>X_2)$.

$$\boxed{},\quad \boxed{\mathrm{P}(X_1+X_2=r)=\begin{cases}\frac{1}{36} & r=0\\ \frac{1}{9} & r=1,2\\ \frac{1}{4} & r=6\\ \frac{1}{3} & r=4\\ 0 & \text{otherwise}\end{cases}},\quad \boxed{P(X_1>X_2)=\frac{11}{36}}$$



Question 26 (****)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases} k(2-x) & x=0,1,2 \\ \frac{1}{4} & x=3 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k=\frac{1}{4}$.

b) Find the value of $\mathrm{E}(X)$ and $\mathrm{E}\left(X^2\right)$.

c) Determine $\mathrm{Var}(3-X)$.

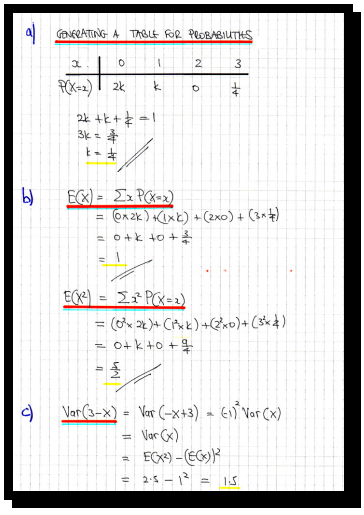
Two independent observations of X are made, denoted by X_1 and X_2.

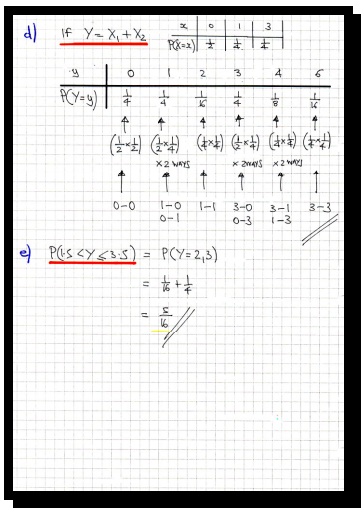
d) Find the probability distribution of Y, where $Y=X_1+X_2$.

e) Calculate $P(1.5\leq Y\leq 3.5)$.

$\square$, $\boxed{\mathrm{E}(X)=1}$, $\boxed{\mathrm{E}\left(X^2\right)=2.5}$, $\boxed{\mathrm{Var}(3-X)=1.5}$, $\boxed{\mathrm{P}(Y=y)=\begin{cases} \frac{1}{16} & y=2,6 \\ \frac{1}{8} & y=4 \\ \frac{1}{4} & y=0,1,3 \\ 0 & \text{otherwise} \end{cases}}$,

$\boxed{P(1.5\leq Y\leq 3.5)=\frac{5}{16}}$





Question 27 **(****)**

The probability distribution of a discrete random variable X is given by

x	1	2	3
$\mathrm{P}(X=x)$	$0.3-k$	$2k$	$0.7-k$

a) Find the range of possible values of the constant k.

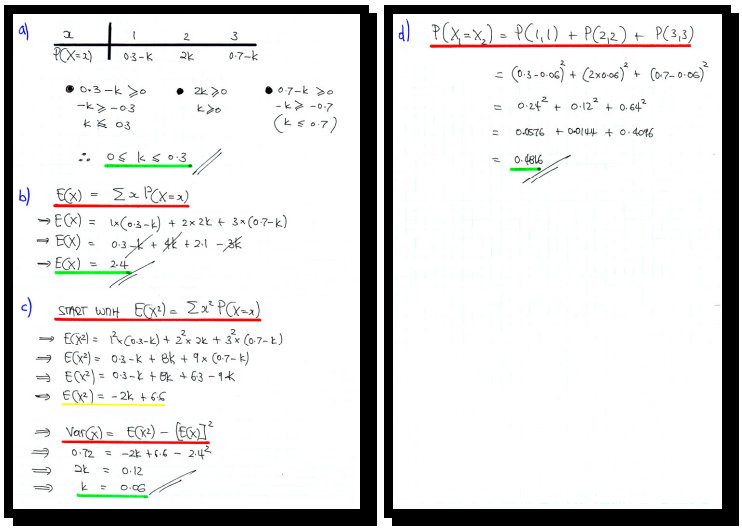
b) Determine $\mathrm{E}(X)$.

c) Given that $\mathrm{Var}(X)=0.72$, find the value of k.

X_1 and X_2 are two independent observations of X.

d) Find $P(X_1=X_2)$.

$\boxed{}$, $\boxed{0\le k\le 0.3}$, $\boxed{\mathrm{E}(X)=2.4}$, $\boxed{k=0.06}$, $\boxed{0.4816}$



Question 28 (****)

The probability distribution of the discrete random variable X is given by

x	2	3	4
$\mathrm{P}(X = x)$	$0.4 - a$	$2a$	$0.6 - a$

where a is a constant.

a) State the range of the possible values of a.

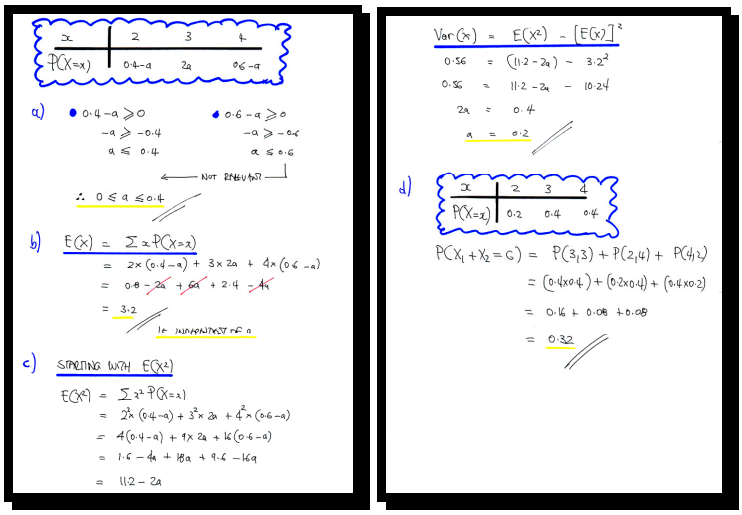
b) Show that $\mathrm{E}(X)$ is independent of a.

c) Given that $\mathrm{Var}(X) = 0.56$ show that $a = 0.2$.

Two independent observations of X, denoted by X_1 and X_2 are considered.

d) Calculate $P(X_1 + X_2 = 6)$.

$\boxed{}$, $\boxed{0 \le a \le 0.4}$, $\boxed{P(X_1 + X_2 = 6) = 0.32}$



Question 29 (****)

A biased spinner can show whole numbers from 1 to 8.

The probability of showing an 8 is 0.05 and the probability of showing a 7 is 0.11.

The probabilities of showing any of the other six whole numbers are all equal to one another.

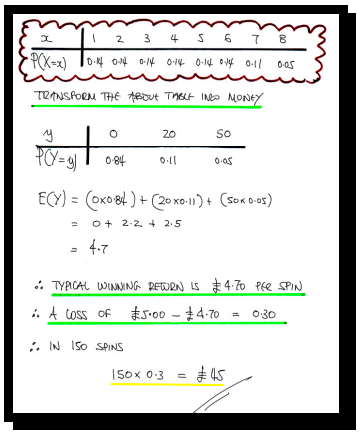
Players in a gambling parlour pay £5 for a single spin.

A score of 8 wins the player £50, a score of 7 wins the player £20, otherwise the player wins no money.

In a typical day, a gambling addict has 150 spins on this spinner.

Find the expected loss of the gambling addict in a typical day.

$\square$, £45



Question 30 **(****)**

The discrete random variable X has the following probability distribution

x	0	1	3
$\mathrm{P}(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

a) Determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

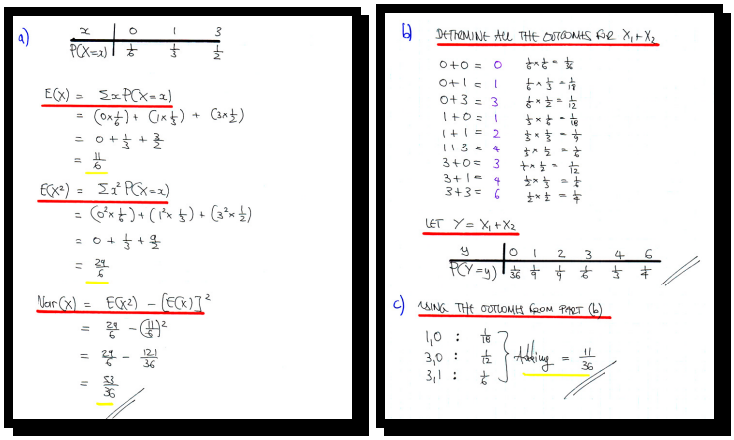
Two independent observations of X are made, denoted by X_1 and X_2.

b) Find the probability distribution of X_1+X_2.

c) Calculate $P(X_1>X_2)$.

$\square$, $\boxed{\mathrm{E}(X)=\frac{11}{6}}$, $\boxed{\mathrm{Var}(X)=\frac{53}{36}}$, $\boxed{\mathrm{P}(X_1+X_2=r)=\begin{cases}\frac{1}{36} & r=0\\ \frac{1}{9} & r=1,2\\ \frac{1}{4} & r=6\\ \frac{1}{3} & r=4\\ 0 & \text{otherwise}\end{cases}}$,

$\boxed{P(X_1>X_2)=\frac{11}{36}}$



Question 31 (****)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases}\frac{1}{20}x & x=1,2,3,4,5\\ \frac{1}{4} & x=6\\ 0 & \text{otherwise}\end{cases}$$

a) Find $\mathrm{P}(X>4)$.

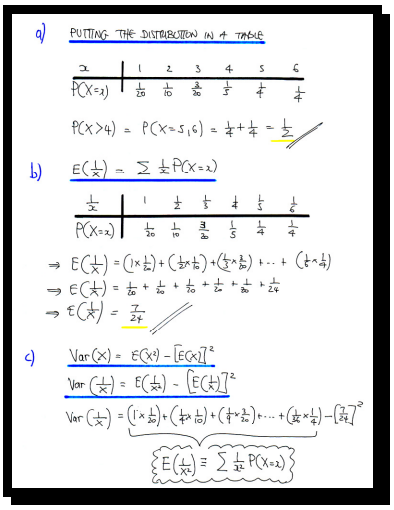
b) Calculate $\mathrm{E}\left(\frac{1}{X}\right)$.

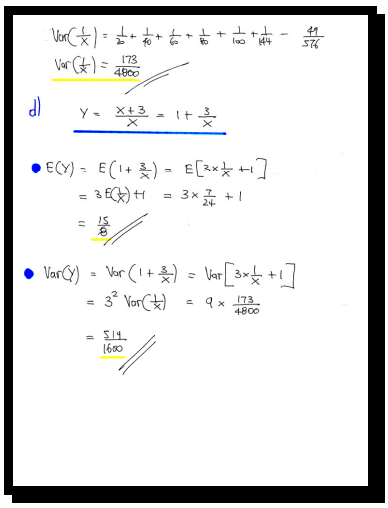
c) Show that $\mathrm{Var}\left(\frac{1}{X}\right)=\frac{173}{4800}$.

The discrete random variable Y is defined as $Y=\frac{X+3}{X}$.

d) Determine the value of $\mathrm{E}(Y)$ and the value of $\mathrm{Var}(Y)$.

$\square$, $\boxed{\mathrm{P}(X>4)=0.5}$, $\boxed{\mathrm{E}\left(\frac{1}{X}\right)=\frac{7}{24}}$, $\boxed{\mathrm{E}(Y)=\frac{15}{8}}$, $\boxed{\mathrm{Var}(Y)=\frac{519}{1600}}$





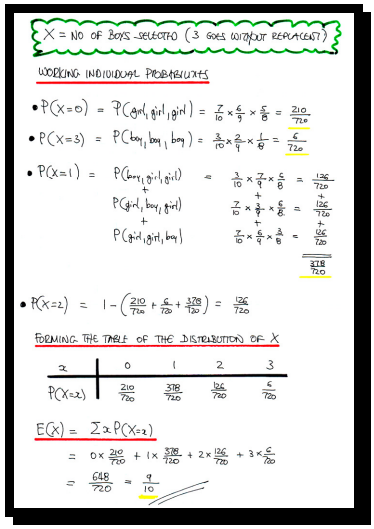
Question 32 **(****)**

A sixth form class consists of 3 boys and 7 girls.

Three students are selected at random from this class and the variable X represents the number of boys selected.

Show clearly that $\mathrm{E}(X)=0.9$.

$\boxed{}$, proof



Question 33 (****)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases}\frac{1}{10} & x=1,\,2,\,3,\,\ldots,\,10\\ 0 & \text{otherwise}\end{cases}$$

a) Find the value of $\mathrm{E}\left(X^2\right)$.

b) Determine $\mathrm{P}(X+2<3X-4<X+7)$.

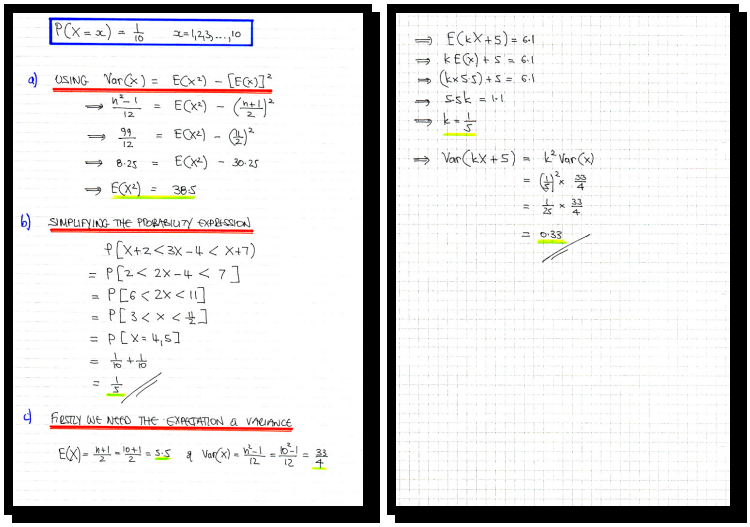
It is further given that

$$\mathrm{E}(kX+5)=6.1,$$

where k is a constant.

c) Find the value of $\mathrm{Var}(kX+5)$

$\square$, $\boxed{\mathrm{E}\left(X^2\right)=38.5}$, $\boxed{\mathrm{P}(X+2<3X-4<X+7)=\frac{1}{5}}$, $\boxed{\mathrm{Var}(kX+5)=0.33}$



Question 34 **(****+)**

The probability distribution of a discrete random variable X is given by

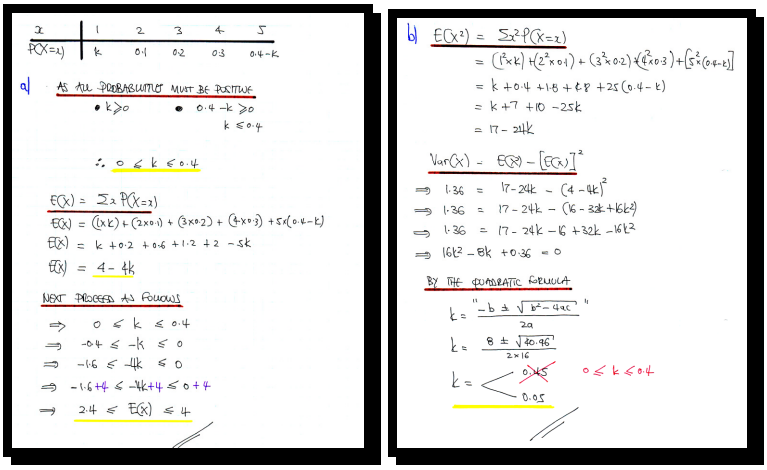
x	1	2	3	4	5
$\mathrm{P}(X=x)$	k	0.1	0.2	0.3	$0.4-k$

where k is a positive constant.

a) Determine the range of values of $\mathrm{E}(X)$.

b) Given that $\mathrm{Var}(X)=1.36$, find the value of k.

$\boxed{}$, $\boxed{2.4\le \mathrm{E}(X)\le 4}$, $\boxed{k=0.05}$



Question 35 (**+)**

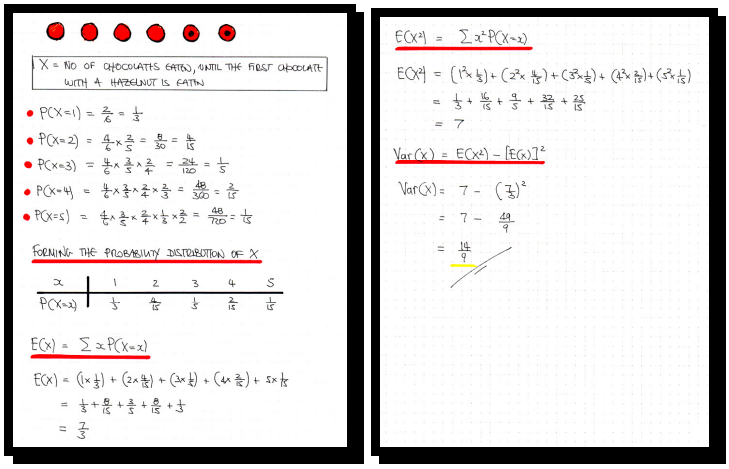
Luke has 6 chocolates of which 2 have a hazelnut at their centre.

Luke eats his chocolates one after the other.

The random variable X represents the number of chocolates Luke eats, up and including the first chocolate with a hazelnut at its centre.

Show, with detailed workings, that $\text{Var}(X)=\frac{14}{9}$.

[ ], proof



Question 36 (**+)**

The probability of a biased coin landing on "*tails*" is 0.3.

An experiment consists of tossing the coin until "*tails*" is shown for the first time, up to a maximum of 4 tosses.

If "*tails*" is shown before the 4^{th} toss the experiment stops.

If after the 4^{th} toss no "*tails*" has been obtained, then no more tosses are made.

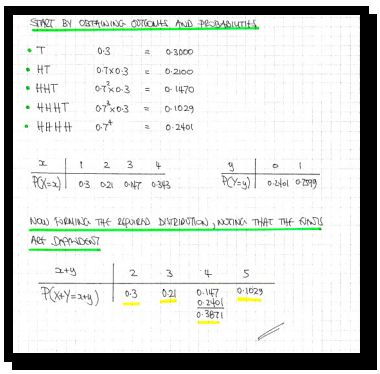
- The discrete random variable X is defined as the number of tosses in this experiment.

- The discrete random variable Y is defined as the number of "*tails*" in this experiment.

Determine the probability distribution of $X+Y$.

$\square$,

$x+y$	2	3	4	5
probability	0.3	0.21	0.3871	0.1029



Question 37 (**+)**

A box contains three blue discs and two red discs.

An experiment is conducted where three discs are selected at random from the box **without** replacement.

The variable X represents the number of blue discs selected.

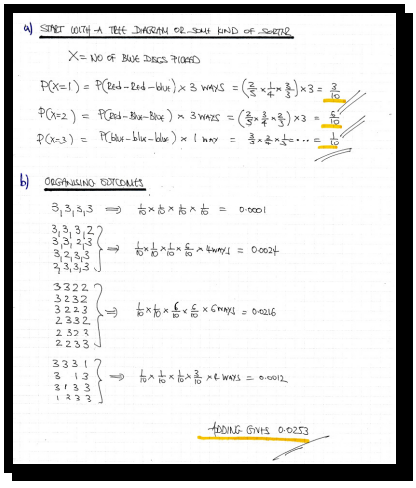
a) Show that the probability distribution of X is given by

x	1	2	3
$\mathrm{P}(X=x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

Four independent observations of X are recorded, labelled as X_1, X_2, X_3 and X_4.

b) Determine $\mathrm{P}(X_1+X_2+X_3+X_4\geq 10)$ and $\mathrm{Var}(X)$.

$\boxed{}$, $\boxed{\mathrm{P}(X_1+X_2+X_3+X_4\geq 10)=0.0253}$



Question 38 (*****)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=\begin{cases}\frac{1}{7} & x=1,\,2,\,3,\,\ldots\,7\\ 0 & \text{otherwise}\end{cases}$$

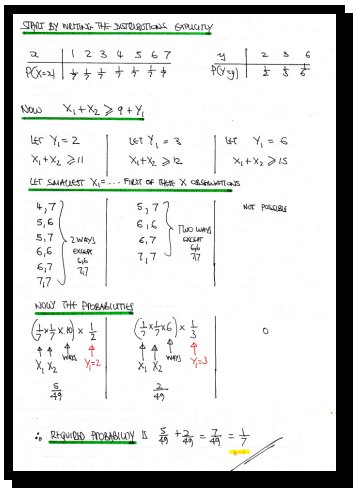
The probability distribution of another discrete random variable Y is given by

$$\mathrm{P}(Y=y)=\begin{cases}\frac{1}{y} & x=2,\,3,\,6\\ 0 & \text{otherwise}\end{cases}$$

Two observations of X are made, denoted by X_1 and X_2, and one observation of Y, denoted by Y_1 are considered.

Assuming these three observations are independent, calculate $P(X_1+X_2\geq 9+Y_1)$.

$\boxed{}$, $\boxed{P(X_1+X_2\geq 9+Y_1)=\frac{1}{7}}$



Question 39 **(*****)**

A biased six sided die has the following probability distribution

x	1	2	3	4	5	6
$\mathrm{P}(X=x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{2}$

where the random variable X represents the number shown on its uppermost face when it comes to rest after it is rolled.

The die is rolled twice and the two independent observations of X, X_1 and X_2, produce the score Y defined as

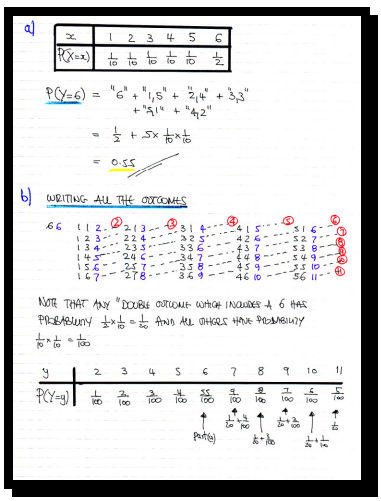
$$Y=\begin{cases} 6 & \text{if } X_1=6 \\ X_1+X_2 & \text{if } X_1\neq 6 \end{cases}$$

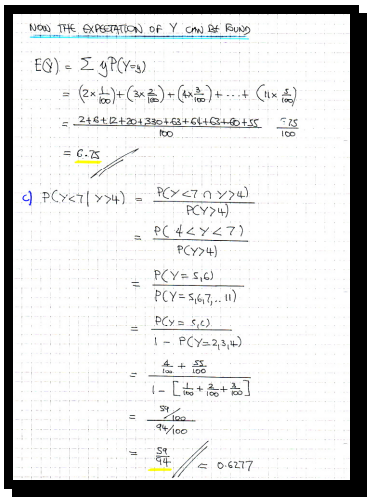
a) Find the value of $\mathrm{P}(Y=6)$.

b) Determine the probability distribution of Y and hence calculate the $\mathrm{E}(Y)$.

c) Find the value of $\mathrm{P}(Y<7|Y>4)$.

$\boxed{}$, $\boxed{\mathrm{P}(Y=6)=0.55}$, $\boxed{\mathrm{E}(Y)=6.75}$, $\boxed{\mathrm{P}(Y<7|Y>4)=\frac{59}{94}}$





Question 40 (*****)

The probability distribution of a discrete random variable X is given by

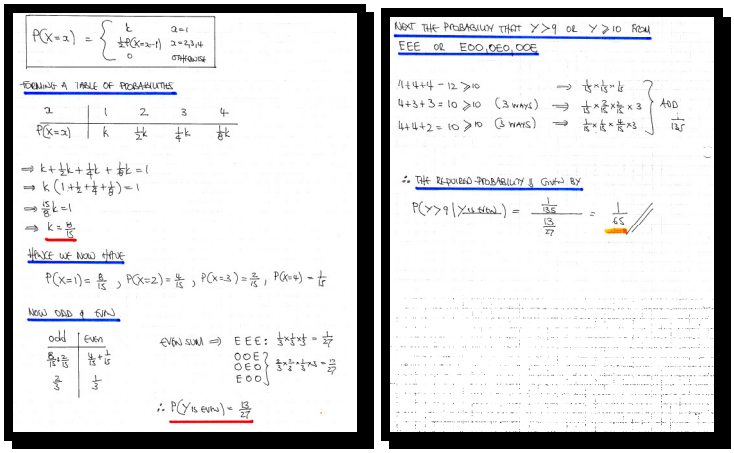
$$\mathrm{P}(X=x)=\begin{cases} k & x=1 \\ \frac{1}{2}\mathrm{P}(X=x-1) & x=2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

Three independent observations of X are made, denoted by X_1, X_2 and X_3, and the variable Y is defined as $Y=X_1+X_2+X_3$.

If Y is an even number, determine the probability that Y is greater than 9.

$\square$, $\boxed{\frac{1}{65}}$



Question 41 (*****)

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=r+1)=\begin{cases}\frac{2}{3}\mathrm{P}(X=r) & r=1,2,3,4,5,\ldots \\ 0 & \text{otherwise}\end{cases}$$

Determine $\mathrm{P}(2\le X\le 4)$.

$\boxed{}$, $\boxed{\mathrm{P}(2\le X\le 4)=\frac{38}{31}}$

