TRIGONOMETRY

R-TRANSFORMATIONS

Question 1

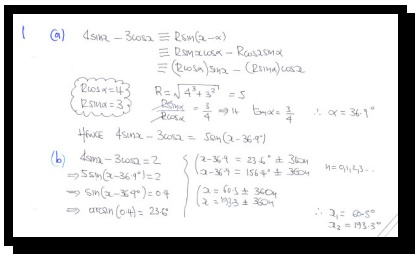
$$f(x) \equiv 4\sin x - 3\cos x .$$

a) Express $f(x)$ in the form $R\sin(x-\alpha)$, $R>0$, $0<\alpha<90°$.

b) Hence, solve the trigonometric equation

$$4\sin x - 3\cos x = 2, \quad 0<x<360° .$$

$$\boxed{f(x) \equiv 4\sin x - 3\cos x \equiv 5\sin(x-36.9°)}, \quad \boxed{x \approx 60.5°,\ 193.3°}$$



Question 2

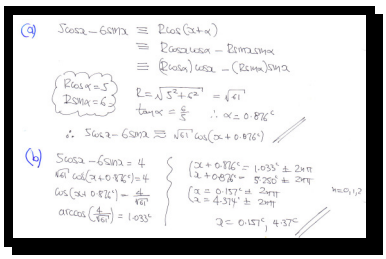
$$f(x) \equiv 5\cos x - 6\sin x, \quad x \in \mathbb{R} .$$

a) Express $f(x)$ in the form $R\cos(x+\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Hence, solve the trigonometric equation

$$5\cos x - 6\sin x = 4, \quad 0<x<2\pi .$$

$$\boxed{f(x) \equiv 5\cos x - 6\sin x \equiv \sqrt{61}\cos\left(x+0.876^{\text{c}}\right)}, \quad \boxed{x \approx 0.157^{\text{c}},\ 4.37^{\text{c}}}$$



Question 3

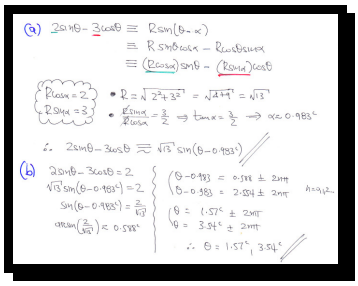
$$f(\theta) \equiv 2\sin\theta - 3\cos\theta\,,\ \theta \in \mathbb{R}\,.$$

a) Express $f(\theta)$ in the form $R\sin(\theta-\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Hence, solve the trigonometric equation

$$2\sin\theta - 3\cos\theta = 2\,,\ 0<\theta<2\pi\,.$$

$$\boxed{f(\theta) \equiv 2\sin\theta - 3\cos\theta \equiv \sqrt{13}\sin\left(\theta - 0.983^{\text{c}}\right)}\,,\quad \boxed{\theta \approx 1.57^{\text{c}},\ 3.54^{\text{c}}}$$



Question 4

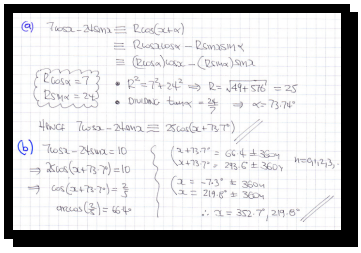
$$f(x) \equiv 7\cos x - 24\sin x\,,\ x \in \mathbb{R}\,.$$

a) Express $f(x)$ in the form $R\cos(x+\alpha)$, $R>0$, $0<\alpha<90°$.

b) Hence, solve the trigonometric equation

$$7\cos x - 24\sin x = 10\,,\ 0<x<360°\,.$$

$$\boxed{f(x) \equiv 7\cos x - 24\sin x \equiv 25\cos\left(x+73.7°\right)}\,,\quad \boxed{x \approx 219.9°,\ 352.7°}$$



Question 5

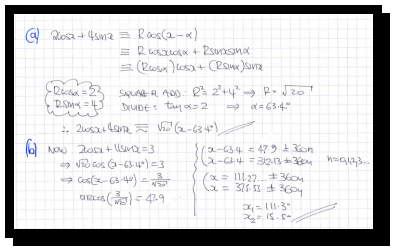
$$f(x) \equiv 2\cos x + 4\sin x,\ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, $R>0$, $0<\alpha<90°$.

b) Hence, solve the trigonometric equation

$$2\cos x + 4\sin x = 3,\ \ 0<x<360°.$$

$$\boxed{f(x) \equiv 2\cos x + 4\sin x \cong \sqrt{20}\cos(x-63.4°)},\ \boxed{x \approx 15.6°,\ 111.3°}$$



Question 6

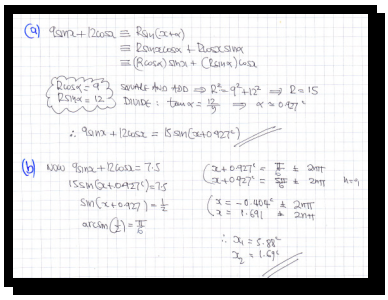
$$f(x) \equiv 9\sin x + 12\cos x\,,\ x \in \mathbb{R}\,.$$

a) Express $f(x)$ in the form $R\sin(x+\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Hence, solve the trigonometric equation

$$9\sin x + 12\cos x = 7.5\,,\ 0<x<2\pi\,.$$

$$\boxed{f(x) \equiv 9\sin x + 12\cos x \equiv 15\sin\left(x+0.927^{\text{c}}\right)}\,,\quad \boxed{x \approx 1.69^{\text{c}},\ 5.88^{\text{c}}}$$



Question 7

$$f(\theta) \equiv 9\cos 2\theta + 3\sin 2\theta,\ \theta \in \mathbb{R}.$$

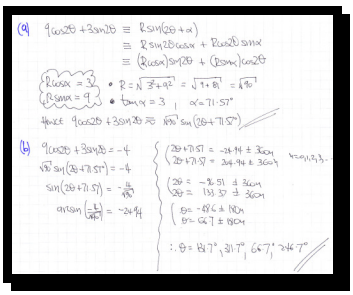
a) Express $f(\theta)$ in the form $R\sin(2\theta+\alpha)$, $R>0$, $0<\alpha<90°$.

b) Hence, solve the trigonometric equation

$$9\cos 2\theta + 3\sin 2\theta = -4,\ 0<\theta<360°.$$

$$\boxed{f(\theta) \equiv 9\cos 2\theta + 3\sin 2\theta \cong \sqrt{90}\sin(2\theta+71.6°)},$$

$$\boxed{\theta \approx 66.7°,\ 131.7°,\ 246.7°,\ 311.7°}$$



Question 8

$$f(\theta) \equiv 9\sin\theta + 12\cos\theta,\ \theta \in \mathbb{R}.$$

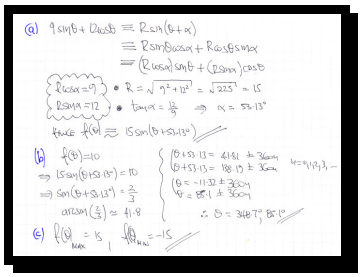
a) Express $f(\theta)$ the form $R\sin(\theta+\alpha)$, $R>0$, $0<\alpha<90°$.

b) Hence, solve the trigonometric equation

$$f(\theta) = 10,\ \ 0<\theta<360°.$$

c) Write down the minimum and the maximum value of $f(\theta)$.

$$\boxed{f(\theta) \cong 15\sin(\theta+53.1°)},\ \boxed{\theta \approx 85.1°,\ 348.7°},\ \boxed{f(\theta)_{\min} = -15,\ \ f(\theta)_{\max} = 15}$$



Question 9

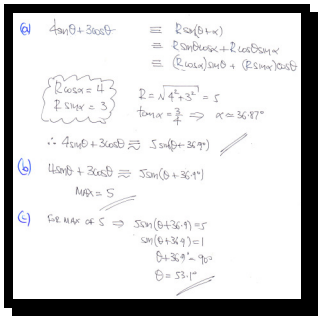
$$f(\theta) \equiv 4\sin\theta + 3\cos\theta,\ \theta \in \mathbb{R}.$$

a) Write the above expression in the form $R\sin(\theta+\alpha)$, $R>0$, $0<\alpha<90°$.

b) Write down the maximum value of $f(\theta)$.

c) Find the smallest positive value of θ for which this maximum value occurs.

$$\boxed{f(\theta) \equiv 4\sin\theta + 3\cos\theta \cong 5\sin(\theta + 36.9°)},\ \boxed{f(\theta)_{\max} = 5},\ \boxed{\theta \approx 53.1°}$$



Question 10

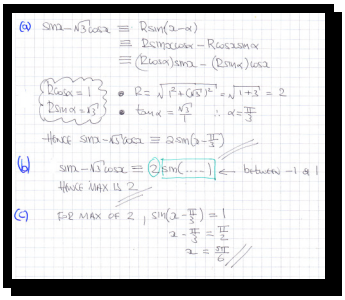
$$f(x) \equiv \sin x - \sqrt{3}\cos x, \ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\sin(x-\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Write down the maximum value of $f(x)$.

c) Find the smallest positive value of x for which this maximum value occurs.

$$\boxed{f(x) \equiv \sin x - \sqrt{3}\cos x \equiv 2\sin\left(x-\frac{\pi}{3}\right)}, \quad \boxed{f(x)_{\max} = 2}, \quad \boxed{x = \frac{5\pi}{6}}$$



Question 11

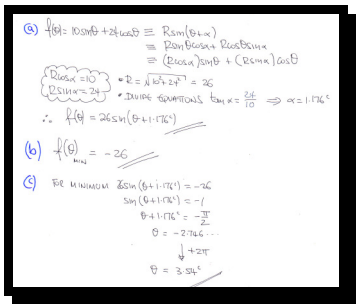
$$f(\theta) \equiv 10\sin\theta + 24\cos\theta\,,\ \theta \in \mathbb{R}\,.$$

a) Express $f(\theta)$ in the form $R\sin(\theta+\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Write down the minimum value of $f(\theta)$.

c) Find the smallest positive value of θ for which this minimum value occurs.

$$\boxed{f(\theta) \cong 26\sin\left(\theta+1.176^{\text{c}}\right)}\,,\ \boxed{f(\theta)_{\min} = -26}\,,\ \boxed{\theta \approx 3.54^{\text{c}}}$$



Question 12

$$f(x) \equiv 3\sin x + 2\cos x\,,\ x \in \mathbb{R}\,.$$

a) Express $f(x)$ in the form $R\sin(x+\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Hence solve the trigonometric equation

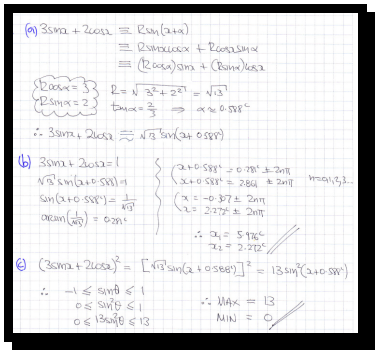
$$3\sin x + 2\cos x = 1,\ \ 0 < x < 2\pi\,.$$

c) Write down the minimum value and the maximum value of

$$(3\sin x + 2\cos x)^2\,.$$

$\boxed{f(x) \equiv 3\sin x + 2\cos x \cong \sqrt{13}\sin\left(x + 0.588^{\text{c}}\right)}$, $\boxed{x \approx 2.27^{\text{c}}, 5.98^{\text{c}}}$,

$\boxed{\min = 0,\ \max = 13}$



Question 13

$$y \equiv 5\sin x - 2\cos x,\ 0 < x < 2\pi .$$

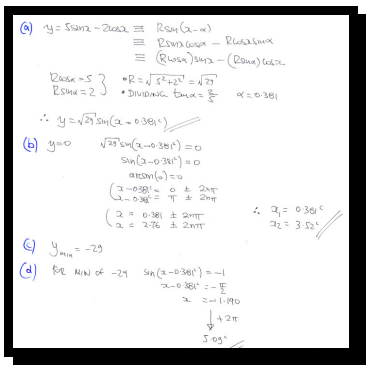
a) Express y in the form $R\sin(x-\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

b) Find the coordinates where the graph of y crosses the x axis.

c) Write down the minimum value of y.

d) Find the smallest positive value of x for which this minimum value of y occurs.

$$\boxed{y \equiv \sqrt{29}\sin\left(x-0.381^{\text{c}}\right)},\ \boxed{x \approx 0.381^{\text{c}}, 3.52^{\text{c}}},\ \boxed{y_{\min} = -\sqrt{29}},\ \boxed{x \approx 5.09^{\text{c}}}$$



Question 14

$$f(x) \equiv 2\sqrt{2}\cos x + 2\sqrt{2}\sin x,\ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\sin(x+\alpha)$, $R>0$, $0<\alpha<\dfrac{\pi}{2}$.

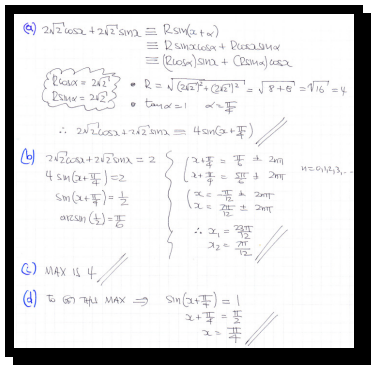
b) Solve the trigonometric equation

$$f(x) = 2 \text{ for } 0<x<2\pi.$$

c) Write down the maximum value of $f(x)$.

d) Find the smallest positive value of x for which this maximum value occurs.

$$\boxed{f(x) \equiv 4\sin\left(x+\frac{\pi}{4}\right)},\ \boxed{x=\frac{7\pi}{12},\frac{23\pi}{12}},\ \boxed{f(x)_{\max}=4},\ \boxed{x=\frac{\pi}{4}}$$



Question 15

$$f(x) \equiv 3\sin x + \cos x, \ x \in \mathbb{R}$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$.

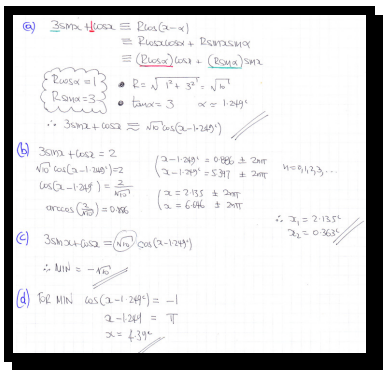
b) Solve the equation

$$f(x) = 2 \text{ for } 0<x<2\pi.$$

c) Write down the minimum value of $f(x)$.

d) Find the smallest positive value of x for which this minimum value occurs.

$$\boxed{f(x) \equiv \sqrt{10}\cos\left(x-1.249^{\text{c}}\right)}, \ \boxed{x = 0.363^{\text{c}}, 2.135^{\text{c}}}, \ \boxed{f(x)_{\min} = -\sqrt{10}}, \ \boxed{x = 4.391^{\text{c}}}$$



Question 16

$$y \equiv \sqrt{2}\cos\theta - \sqrt{6}\sin\theta\,,\ 0 < \theta < 360°\,.$$

a) Express y in the form $R\cos(\theta+\alpha)$, $R>0$, $0<\alpha<90°$.

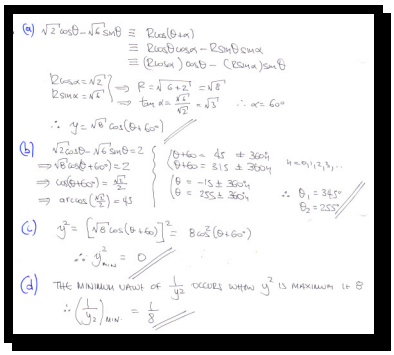
b) Solve the equation $y = 2$.

c) Write down the minimum values of …

i. … y^2.

ii. … $\dfrac{1}{y^2}$.

$\boxed{y \equiv \sqrt{8}\cos(\theta+60°)}$, $\boxed{\theta = 255°, 345°}$, $\boxed{\min = 0}$, $\boxed{\min = \frac{1}{8}}$



Question 17

$$f(x) \equiv 2\sin x + 2\cos x,\ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\sin(x+\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$.

b) State the minimum and the maximum value of …

i. … $y = f\left(x-\frac{\pi}{2}\right)$.

ii. … $y = 2f(x)+1$.

iii. … $y = \left[f(x)\right]^2$.

iv. … $y = \dfrac{10}{f(x)+3\sqrt{2}}$.

$\boxed{f(x) \equiv \sqrt{8}\sin\left(x+\frac{\pi}{4}\right)}$, $\boxed{\left[-\sqrt{8},\sqrt{8}\right]}$, $\boxed{\left[-2\sqrt{8}+1, 2\sqrt{8}+1\right]}$, $\boxed{[0,8]}$, $\boxed{\left[\sqrt{2},5\sqrt{2}\right]}$

