TRIGONOMETRY

MINOR TRIGONOMETRIC RATIOS

Question 1

Simplify the following trigonometric expressions.

The final answer must not contain trigonometric fractions.

a) $\dfrac{\operatorname{cosec} x}{\sin^3 x}$

b) $\dfrac{\cos\theta}{\sec^2\theta}$

c) $\dfrac{2\sin^2 x}{\operatorname{cosec} x}$

d) $\dfrac{\sec^2\theta}{2\cos^2\theta}$

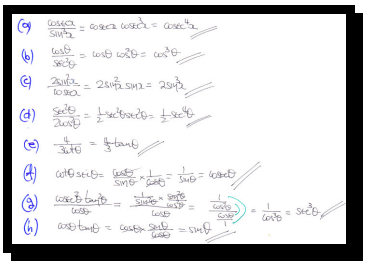
e) $\dfrac{4}{3\cot x}$

f) $\cot\theta\sec\theta$

g) $\dfrac{\operatorname{cosec}^2\theta\tan^2\theta}{\cos\theta}$

h) $\cos\theta\tan\theta$

$\boxed{\operatorname{cosec}^4 x}$, $\boxed{\cos^3\theta}$, $\boxed{2\sin^3 x}$, $\boxed{\frac{1}{2}\sec^4\theta}$, $\boxed{\frac{4}{3}\tan x}$, $\boxed{\operatorname{cosec}\theta}$, $\boxed{\sec^3\theta}$, $\boxed{\sin\theta}$



Question 2

Simplify the following trigonometric expressions.

The final answer must not contain trigonometric fractions.

a) $\dfrac{1-\sin^2 x}{\sin^2 x}$

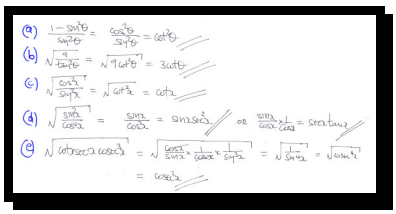
b) $\sqrt{\dfrac{9}{\tan^2\theta}}$

c) $\sqrt{\dfrac{\cos^2 x}{\sin^2 x}}$

d) $\sqrt{\dfrac{\sin^2 x}{\cos^4 x}}$

e) $\sqrt{\cot x \sec x \operatorname{cosec}^3 x}$

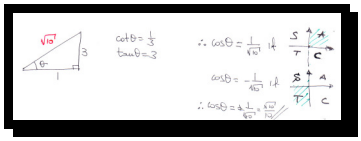
$\boxed{\cot^2 x}$, $\boxed{3\cot\theta}$, $\boxed{\cot x}$, $\boxed{\sec x \tan x}$, $\boxed{\operatorname{cosec}^2 x}$



Question 3

If $\cot\theta = \dfrac{1}{3}$, show that $\cos\theta = \pm\dfrac{\sqrt{10}}{10}$.

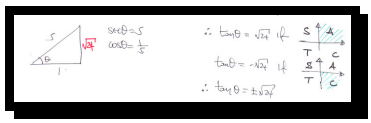
$\boxed{\text{proof}}$



Question 4

If $\sec\theta = 5$, show that $\tan\theta = \pm\sqrt{24}$.

proof



Question 5

Solve each of the following trigonometric equations.

a) $\sec\theta = 4$, $\quad 0 \le \theta < 360°$

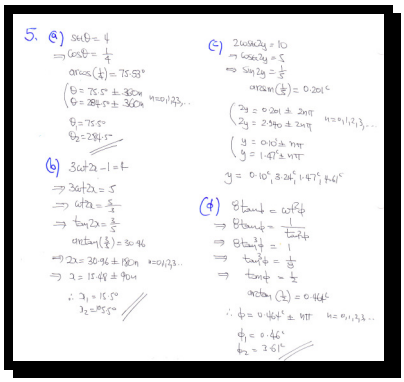
b) $3\cot 2x - 1 = 4$, $\quad 0 \le x < 180°$

c) $2\operatorname{cosec} 2y = 10$, $\quad 0 \le y < 2\pi$

d) $8\tan\varphi = \cot^2\varphi$, $\quad 0 \le \varphi < 2\pi$

$\theta \approx 75.5°,\ 284.5°$, $x \approx 15.5°,\ 105.5°$, $y \approx 0.10^c,\ 1.47^c,\ 3.24^c,\ 4.61^c$,

$\varphi \approx 0.46^c,\ 3.61^c$



Question 6

Solve each of the following trigonometric equations.

a) $2\sec\theta = 3, \qquad 0 \le \theta < 360°$

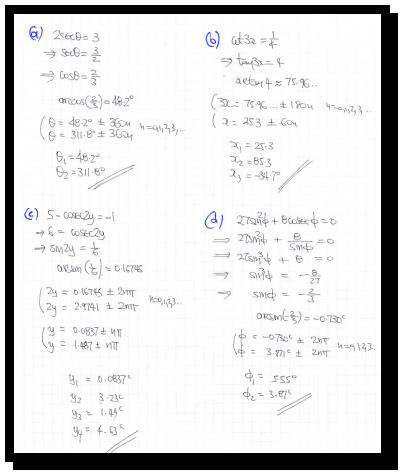
b) $\cot 3x = \dfrac{1}{4}, \qquad -90° \le x < 90°$

c) $5 - \operatorname{cosec} 2y = -1, \qquad 0 \le y < 2\pi$

d) $27\sin^2\varphi + 8\operatorname{cosec}\varphi = 0, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 48.2°,\ 311.8°}$, $\boxed{x \approx -34.7°,\ 25.3°,\ 85.3°}$,

$\boxed{y \approx 0.0837^{\text{c}},\ 1.49^{\text{c}},\ 3.23^{\text{c}},\ 4.63^{\text{c}}}$, $\boxed{\varphi \approx 3.87^{\text{c}},\ 5.55^{\text{c}}}$



Question 7

Solve each of the following trigonometric equations.

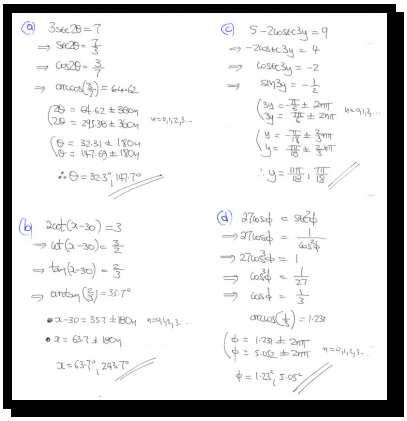
a) $3\sec 2\theta = 7\,, \qquad 0 \le \theta < 180°$

b) $2\cot\left(x - 30°\right) = 3\,, \qquad 0 \le x < 360°$

c) $5 - 2\operatorname{cosec} 3y = 9\,, \qquad 0 \le y < \pi$

d) $27\cos\varphi = \sec^2\varphi\,, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 32.3°,\ 147.7°}$, $\boxed{x \approx 63.7°,\ 243.7°}$, $\boxed{y = \frac{7\pi}{18},\ \frac{11\pi}{18}}$, $\boxed{\varphi \approx 1.23^{\text{c}},\ 5.05^{\text{c}}}$



Question 8

Solve each of the following trigonometric equations.

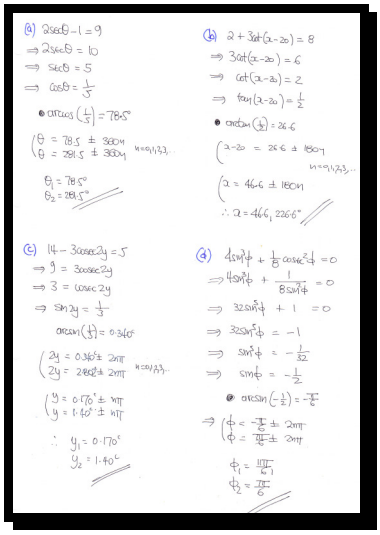
a) $2\sec\theta - 1 = 9, \qquad 0 \le \theta < 360°$

b) $2 + 3\cot(x - 20°) = 8, \qquad 0 \le x < 360°$

c) $14 - 3\operatorname{cosec} 2y = 5, \qquad 0 \le y < \pi$

d) $4\sin^3\varphi + \frac{1}{8}\operatorname{cosec}^2\varphi = 0, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 78.5°,\ 281.5°}$, $\boxed{x \approx 46.6°,\ 226.6°}$, $\boxed{y \approx 0.170^{\text{c}},\ 1.40^{\text{c}}}$, $\boxed{\varphi = \frac{7\pi}{6},\ \frac{11\pi}{6}}$



Question 9

Solve each of the following trigonometric equations.

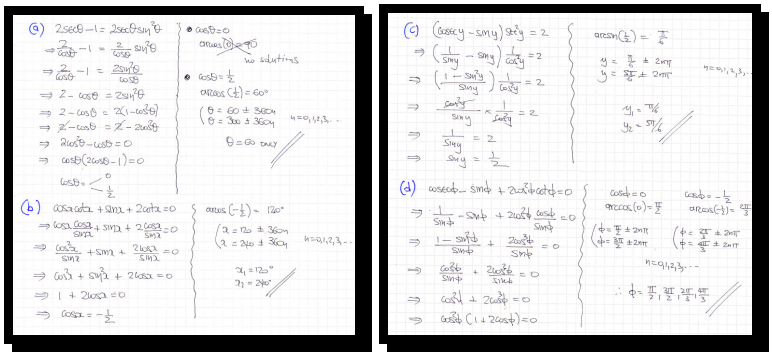
a) $2\sec\theta - 1 = 2\sec\theta\sin^2\theta, \quad 0 \le \theta < 180°, \quad \theta \neq 90°$

b) $\cos x\cot x + \sin x + 2\cot x = 0, \quad 0 < x < 360°, \quad x \neq 180°$

c) $(\operatorname{cosec} y - \sin y)\sec^2 y = 2, \quad 0 \le y < \pi, \quad y \neq \dfrac{\pi}{2}$

d) $\operatorname{cosec}\varphi - \sin\varphi + 2\cos^2\varphi\cot\varphi = 0, \quad 0 < \varphi < 2\pi, \quad \varphi \neq \pi$

$\boxed{\theta = 60°}$, $\boxed{x = 120°, 240°}$, $\boxed{y = \dfrac{\pi}{6}, \dfrac{5\pi}{6}}$, $\boxed{\varphi = \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}}$



Question 10

Solve each of the following trigonometric equations.

a) $\sec\theta+\cos\theta=\frac{5}{2}, \quad 0\le\theta<360°, \quad \theta\neq 90°,270°$

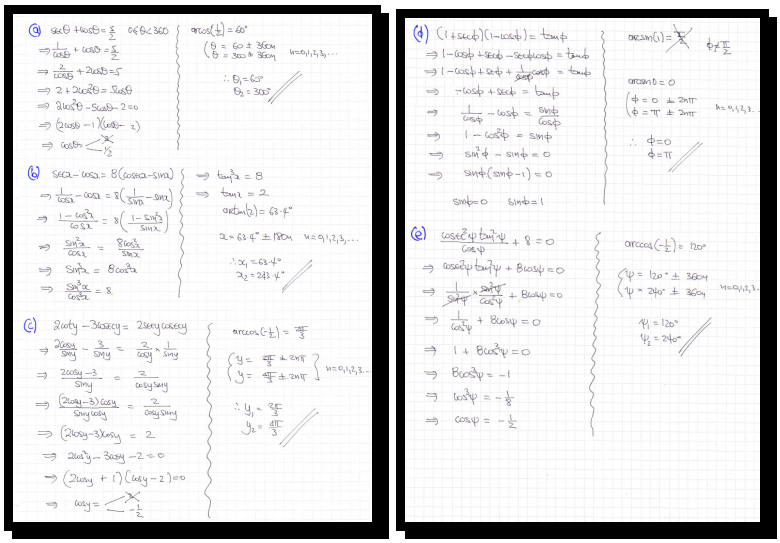
b) $\sec x-\cos x=8\left(\operatorname{cosec} x-\sin x\right), \quad 0\le x<360°, \quad x\neq 90°$

c) $2\cot y-3\operatorname{cosec} y=2\sec y\operatorname{cosec} y, \quad 0<y<2\pi, \quad y\neq\frac{k\pi}{2}, k\in\mathbb{Z}$

d) $(1+\sec\varphi)(1-\cos\varphi)=\tan\varphi, \quad 0\le\varphi<2\pi, \quad \varphi\neq\frac{\pi}{2},\frac{3\pi}{2}$

e) $\frac{\operatorname{cosec}^2\psi\tan^2\psi}{\cos\psi}+8=0, \quad 0\le\psi<360°, \quad \psi\neq 90°,270°$

$\boxed{\theta=60°,300°}$, $\boxed{x=63.4°,243.6°}$, $\boxed{y=\frac{2\pi}{3},\frac{4\pi}{3}}$, $\boxed{\varphi=0,\pi}$, $\boxed{\psi=120°,240°}$



Question 11 (hard questions)

Solve each of the following trigonometric equations.

a) $2\sin\theta+3\sec\theta=6+\tan\theta\,,\quad 0\le\theta<2\pi\,,\quad \theta\neq\dfrac{\pi}{2},\dfrac{3\pi}{2}$

b) $\sin^2 x\tan x+\cos^2 x\cot x+2\sin x\cos x=2\,,\quad 0<x<360°,\quad x\neq 90°,180°,270°$

you may use in this part the fact that $\boxed{2\sin x\cos x\equiv\sin 2x}$

c) $\sin y\left(1+\tan y\right)+\cos y\left(1+\cot y\right)=0\,,\quad 0<y<360°,\quad y\neq 90°,180°,270°$

d) $\dfrac{4}{2\sec\varphi-2\sin\varphi+1}=\cot\varphi\,,\quad 0<\varphi<2\pi,\quad \varphi\neq\pi$

e) $\dfrac{\cot\psi}{\operatorname{cosec}\psi-1}-\dfrac{\cos\psi}{1+\sin\psi}=2\,,\quad 0<\psi<2\pi,\quad \psi\neq\dfrac{\pi}{2},\pi,\dfrac{3\pi}{2}$

f) $\dfrac{\cot\beta}{\operatorname{cosec}\beta-1}+\dfrac{\operatorname{cosec}\beta-1}{\cot\beta}=4\,,\quad 0\le x<360°,\quad x\neq 90°,180°,270°$

$\boxed{\varphi=\dfrac{\pi}{3},\dfrac{5\pi}{3}}$, $\boxed{x=45°,225°}$, $\boxed{y=135°,315°}$, $\boxed{\varphi=\dfrac{\pi}{6},\dfrac{5\pi}{6}}$, $\boxed{\psi=\dfrac{\pi}{4},\dfrac{5\pi}{4}}$, $\boxed{\beta=60°,300°}$



Question 12

Prove the validity of each of the following trigonometric identities.

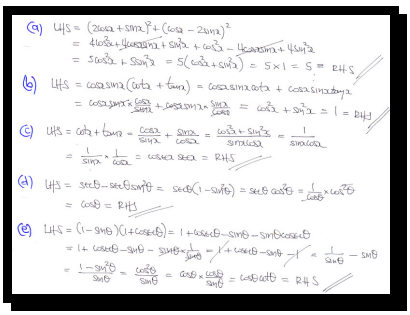
a) $(2\cos x+\sin x)^2+(\cos x-2\sin x)^2\equiv 5$

b) $\cos x\sin x(\cot x+\tan x)\equiv 1$

c) $\cot x+\tan x\equiv \sec x\operatorname{cosec} x$

d) $\sec\theta-\sec\theta\sin^2\theta\equiv\cos\theta$

e) $(1-\sin\theta)(1+\operatorname{cosec}\theta)\equiv\cos\theta\cot\theta$



Question 13

Prove the validity of each of the following trigonometric identities.

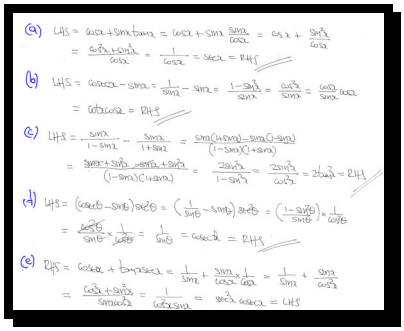
a) $\cos x + \sin x \tan x \equiv \sec x$

b) $\operatorname{cosec} x - \sin x \equiv \cos x \cot x$

c) $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x} \equiv 2\tan^2 x$

d) $(\operatorname{cosec}\theta - \sin\theta)\sec^2\theta \equiv \operatorname{cosec}\theta$

e) $\operatorname{cosec} x \sec^2 x \equiv \operatorname{cosec} x + \tan x \sec x$



Question 14

Prove the validity of each of the following trigonometric identities.

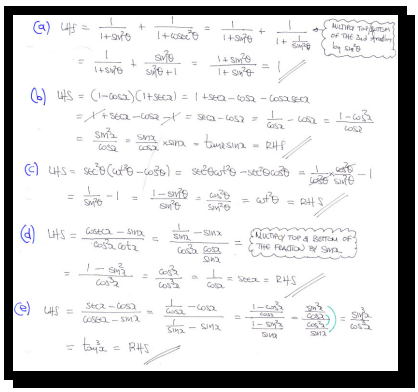
a) $\dfrac{1}{1+\sin^2\theta}+\dfrac{1}{1+\operatorname{cosec}^2\theta}\equiv 1$

b) $(1-\cos x)(1+\sec x)\equiv \sin x\tan x$

c) $\sec^2\theta\left(\cot^2\theta-\cos^2\theta\right)\equiv \cot^2\theta$

d) $\dfrac{\operatorname{cosec} x-\sin x}{\cos^2 x\cot x}\equiv \sec x$

e) $\dfrac{\sec x-\cos x}{\operatorname{cosec} x-\sin x}\equiv \tan^3 x$



Question 15

Prove the validity of each of the following trigonometric identities.

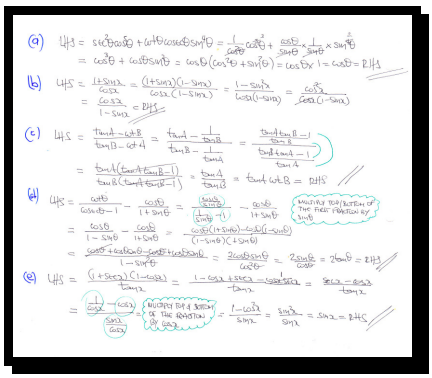
a) $\sec^2\theta\cos^5\theta+\cot\theta\operatorname{cosec}\theta\sin^4\theta\equiv\cos\theta$

b) $\dfrac{1+\sin x}{\cos x}\equiv\dfrac{\cos x}{1-\sin x}$

c) $\dfrac{\tan A-\cot B}{\tan B-\cot A}\equiv\tan A\cot B$

d) $\dfrac{\cot\theta}{\operatorname{cosec}\theta-1}-\dfrac{\cos\theta}{1+\sin\theta}\equiv 2\tan\theta$

e) $\dfrac{(1+\sec x)(1-\cos x)}{\tan x}\equiv\sin x$



Question 16

Prove the validity of each of the following trigonometric identities.

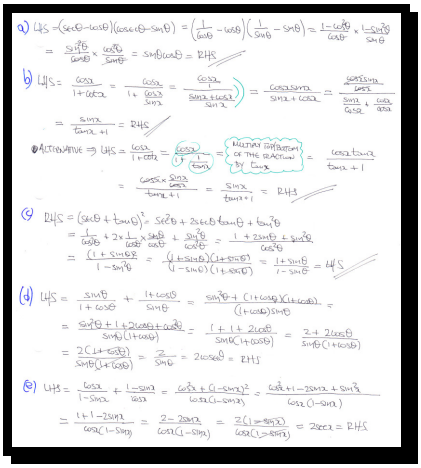
a) $(\sec\theta-\cos\theta)(\operatorname{cosec}\theta-\sin\theta)\equiv\sin\theta\cos\theta$

b) $\dfrac{\cos x}{1+\cot x}\equiv\dfrac{\sin x}{1+\tan x}$

c) $\dfrac{1+\sin\theta}{1-\sin\theta}\equiv(\sec\theta+\tan\theta)^2$

d) $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}\equiv 2\operatorname{cosec}\theta$

e) $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}\equiv 2\sec x$



Question 17

Prove the validity of each of the following trigonometric identities.

a) $(\tan\theta+\cot\theta)(\sin\theta+\cos\theta)\equiv\sec\theta+\operatorname{cosec}\theta$

b) $\cos^3\theta+\sin^3\theta\equiv(\cos\theta+\sin\theta)(1-\sin\theta\cos\theta)$

c) $\dfrac{1}{\cos\theta-\sin\theta}+\dfrac{1}{\cos\theta+\sin\theta}\equiv\dfrac{2\sec\theta}{1-\tan^2\theta}$

d) $\dfrac{2\sin x\cos x-\cos x}{1-\sin x+\sin^2 x-\cos^2 x}\equiv\cot x$

e) $\sin^2\theta\tan\theta+\cos^2\theta\cot\theta+2\sin\theta\cos\theta\equiv\tan\theta+\cot\theta$

f) $\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)\equiv\sec\theta+\operatorname{cosec}\theta$

