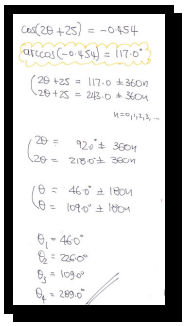
# TRIGONOMETRY EXAM QUESTIONS INTRODUCTION

**Question 1 (**+)**

Solve the following trigonometric equation in the range given.

$$\cos(2\theta+25)^\circ=-0.454\,,\quad 0\le\theta<360\,.$$

$\boxed{\phantom{xx}}$, $\boxed{\theta\approx 46,\,109,\,226,\,289}$

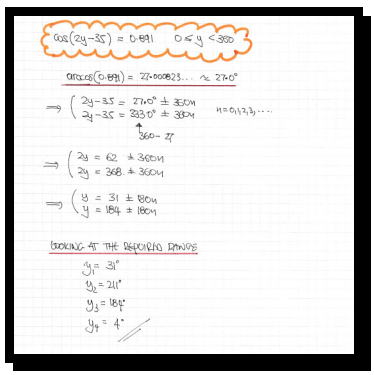



**Question 2 (**+)**

Solve the following trigonometric equation in the range given.

$$\cos(2y-35)^\circ=0.891\,,\quad 0\le y<360\,.$$

$\boxed{\phantom{xx}}$, $\boxed{y\approx 4,\;31,\;184,\;211}$

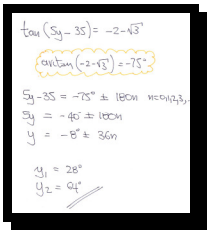


**Question 3 (**+)**

Solve the following trigonometric equation in the range given.

$$\tan(5y-35)^\circ = -2-\sqrt{3}, \quad 0 \le y < 90.$$

$\square$, $\boxed{y \approx 28,\ 64}$



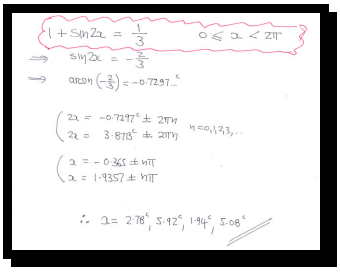
**Question 4 (**+)**

Solve, in **radians**, the following trigonometric equation

$$1+\sin 2x = \frac{1}{3}, \quad 0 \le x < 2\pi,$$

giving the answers correct to three significant figures.

$$\boxed{x = 1.94^{\text{c}},\ 2.78^{\text{c}},\ 5.08^{\text{c}},\ 5.92^{\text{c}}}$$

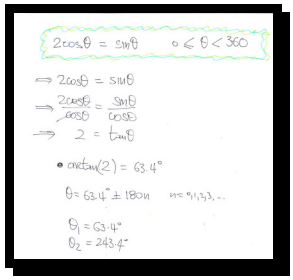


**Question 5 (**+)**

Solve the following trigonometric equation in the range given.

$$2\cos\theta = \sin\theta, \quad 0° \le \theta < 360° .$$

$$\boxed{\theta = 63.4°,\ 243.4°}$$

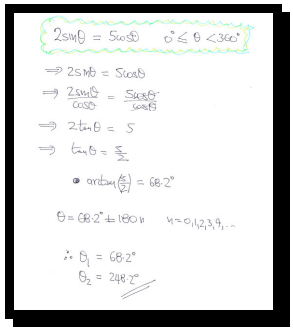



**Question 6 (**+)**

Solve the following trigonometric equation in the range given.

$$2\sin\theta = 5\cos\theta, \quad 0° \le \theta < 360° .$$

$$\boxed{\ \ }, \ \boxed{x \approx 68.2°,\ 248.2°}$$

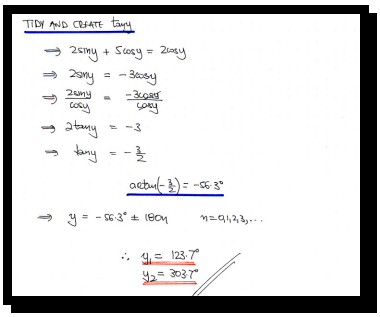


**Question 7 (**+)**

Solve the following trigonometric equation in the range given.

$$2\sin y+5\cos y=2\cos y\,,\quad 0^\circ\le y<360^\circ\,.$$

$\boxed{\phantom{xxxx}}$, $\boxed{y\approx 123.7^\circ,\ 303.7^\circ}$

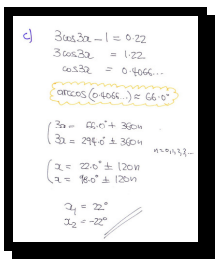



**Question 8 (***)**

Solve the following trigonometric equation in the range given.

$$3\cos 3x-1=0.22\,,\quad -90^\circ\le x<90^\circ\,.$$

$\boxed{x\approx -22^\circ, 22^\circ}$

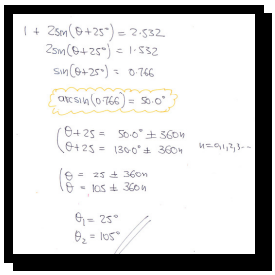


**Question 9 (***)**

Solve the following trigonometric equation in the range given.

$$1+2\sin(\theta+25)^\circ=2.532\,,\quad 0\le\theta<360\,.$$

$$\boxed{\theta\approx 25,\ 105}$$



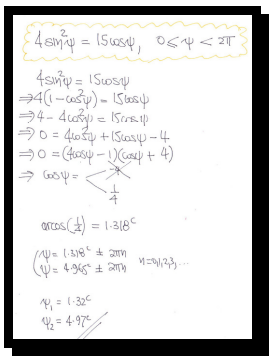

**Question 10 (***)**

Solve, in **radians**, the following trigonometric equation

$$4\sin^2\psi=15\cos\psi\,,\quad 0\le\psi<2\pi\,,$$

giving the answers correct to three significant figures.

$$\boxed{\psi\approx 1.32^{\text{c}},4.97^{\text{c}}}$$

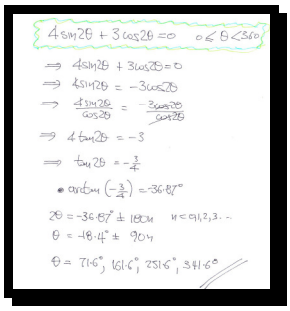


**Question 11 (***)**

Solve the following trigonometric equation in the range given.

$$4\sin 2\theta + 3\cos 2\theta = 0\,, \quad 0° \le \theta < 360°\,.$$

$$\boxed{\theta = 71.6°, 161.6°, 251.6°, 341.6°}$$

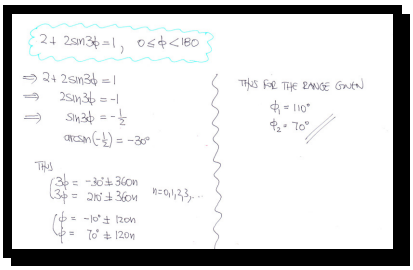



**Question 12 (***)**

Solve the following trigonometric equation in the range given.

$$2 + 2\sin 3\varphi = 1\,, \ 0° \le \varphi < 180°\,.$$

$$\boxed{\varphi = 70°,\ 110°}$$

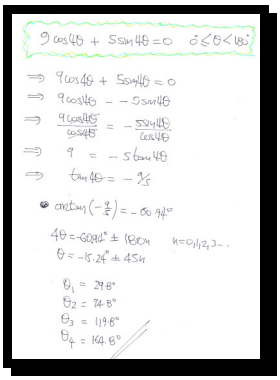


**Question 13 (***)**

Solve the following trigonometric equation in the range given.

$$9\cos 4\theta + 5\sin 4\theta = 0\,, \quad 0° \le \theta < 180°\,.$$

$$\boxed{\theta \approx 29.8°,\ 74.8°,\ 119.8°,\ 164.8°}$$

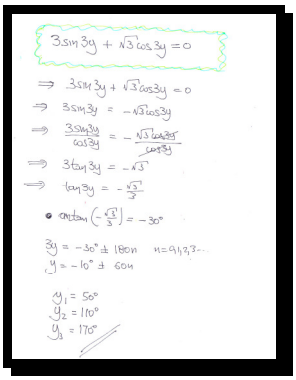



**Question 14 (***)**

Solve the following trigonometric equation in the range given.

$$3\sin 3y + \sqrt{3}\cos 3y = 0\,, \quad 0° \le y < 180°\,.$$

$$\boxed{y = 50°,\ 110°, 170°}$$



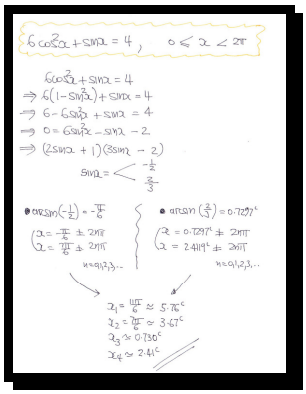
**Question 15 (***)**

Solve, in **radians**, the following trigonometric equation

$$6\cos^2 x+\sin x=4\,,\quad 0\le x<2\pi\,,$$

giving the answers correct to three significant figures.

$$\boxed{x\approx 0.73^{\text{c}},\,2.41^{\text{c}},\,3.67^{\text{c}},\,5.76^{\text{c}}}$$



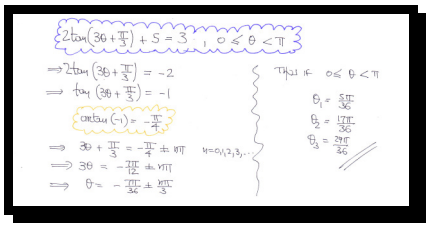
**Question 16 (***)**

Solve, in **radians**, the following trigonometric equation

$$5+2\tan\left(3\theta+\frac{\pi}{3}\right)=3\,,\quad 0\le\theta<\pi\,,$$

giving the answers in terms of $\pi$.

$$\boxed{\theta=\frac{5\pi}{36},\,\frac{17\pi}{36},\,\frac{29\pi}{36}}$$

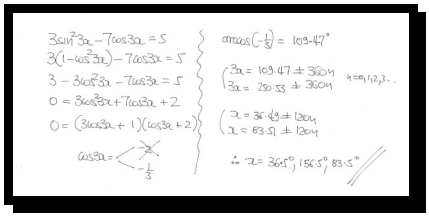


**Question 17 (***)**

Solve, in **degrees**, the following trigonometric equation

$$3\sin^2 3x - 7\cos 3x = 5, \quad 0° \le x < 180°.$$

$\square$, $\boxed{x \approx 36.5°,\ 83.5°,\ 156.5°}$



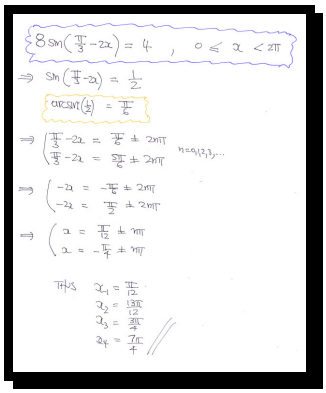
**Question 18 (***)**

Solve, in **radians**, the following trigonometric equation

$$8\sin\left(\frac{\pi}{3} - 2x\right) = 4, \quad 0 \le \theta < 2\pi,$$

giving the answers in terms of $\pi$.

$$\boxed{x = \frac{\pi}{12},\ \frac{3\pi}{4},\ \frac{13\pi}{12},\ \frac{7\pi}{4}}$$

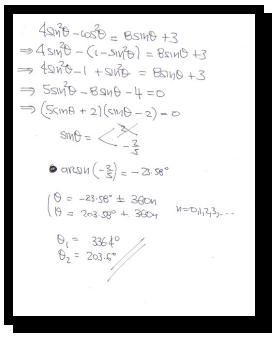


**Question 19 (***)**

Solve the following trigonometric equation in the range given.

$$4\sin^2\theta-\cos^2\theta=8\sin\theta+3,\quad 0^\circ\le\theta<360^\circ.$$

$$\boxed{\theta\approx 203.6^\circ,336.4^\circ}$$

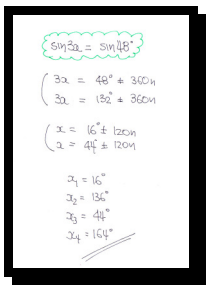



**Question 20 (***)**

Solve, in **degrees**, the following trigonometric equation

$$\sin 3x=\sin 48^\circ,\quad 0^\circ\le x<180^\circ.$$

$$\boxed{\phantom{x}},\ \boxed{x=16^\circ,44^\circ,136^\circ,164^\circ}$$



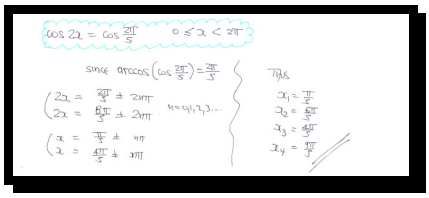
**Question 21 (***)**

Solve, in **radians**, the following trigonometric equation

$$\cos 2x = \cos\frac{2\pi}{5}, \quad 0 \le x < 2\pi,$$

giving the answers in terms of $\pi$.

$$\boxed{x = \frac{\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{9\pi}{5}}$$

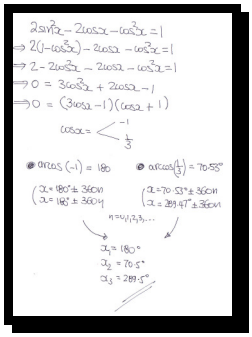


**Question 22 (***)**

Solve the following trigonometric equation in the range given.

$$2\sin^2 x - 2\cos x - \cos^2 x = 1, \quad 0° \le x < 360°.$$

$$\boxed{x \approx 70.5°, 289.5°, \quad x = 180°}$$

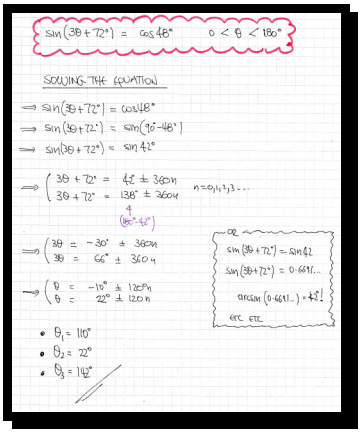


**Question 23 (***)**

Solve the following trigonometric equation.

$$\sin(3\theta+72)^\circ=\cos 48^\circ, \quad 0\le\theta<180.$$

$\boxed{\phantom{xxxx}}$, $\boxed{\theta=\{22,110,142\}}$

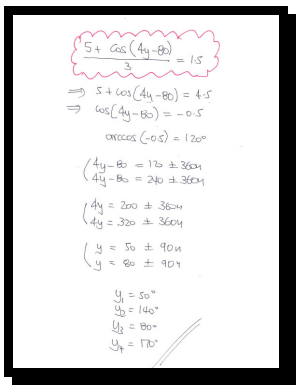


**Question 24 (***+)**

Solve the following trigonometric equation in the range given.

$$\frac{5+\cos(4y-80)^\circ}{3}=1.5, \quad 0\le y<180.$$

$$\boxed{y=50,\ 80,\ 140,\ 170}$$

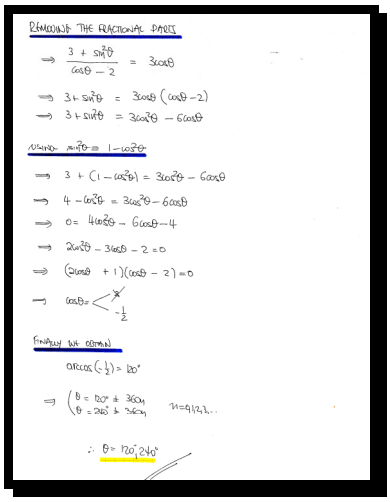


**Question 25 (***+)**

Solve the following trigonometric equation in the range given.

$$\frac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta, \quad 0^\circ\le\theta<360^\circ.$$

$$\boxed{\ \ \ \ \ }, \quad \boxed{\theta=120^\circ,\ 240^\circ}$$



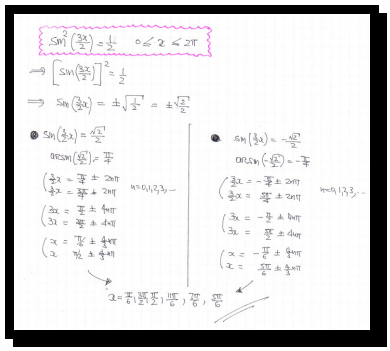
**Question 26 (***+)**

Solve, in **radians**, the following trigonometric equation

$$\sin^2\left(\frac{3x}{2}\right)=\frac{1}{2}, \quad 0\le x<2\pi,$$

giving the answers in terms of $\pi$.

$$\boxed{x=\frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{3\pi}{2},\ \frac{11\pi}{6}}$$

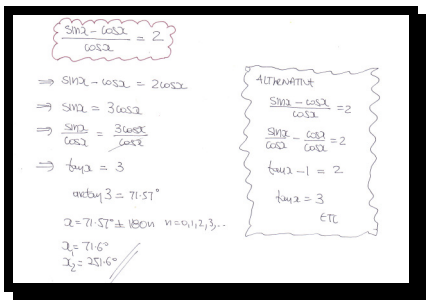


**Question 27 (***+)**

Solve the following trigonometric equation in the range given.

$$\frac{\sin x-\cos x}{\cos x}=2, \quad 0^\circ\le x<360^\circ.$$

$\boxed{\phantom{x}}$, $\boxed{x\approx 71.6^\circ,\ 251.6^\circ}$



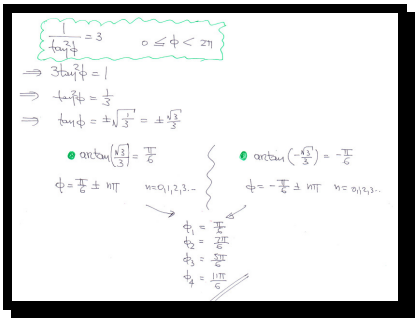
**Question 28 (***+)**

Solve, in **radians**, the following trigonometric equation

$$\frac{1}{\tan^2\varphi}=3\,,\quad 0\le\varphi<2\pi\,,$$

giving the answers in terms of $\pi$.

$\square$, $\boxed{\varphi=\dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}}$



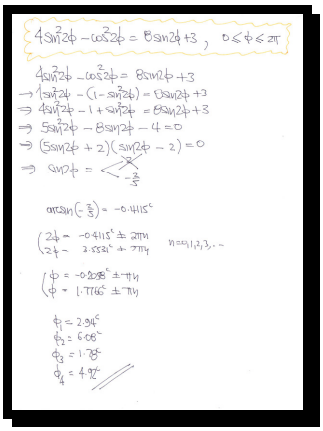
**Question 29 (***+)**

Solve, in **radians**, the following trigonometric equation

$$4\sin^2 2\varphi-\cos^2 2\varphi=3+8\sin 2\varphi\,,\quad 0\le\varphi<2\pi\,,$$

giving the answers correct to three significant figures.

$\boxed{\varphi\approx 1.78^{\text{c}},2.94^{\text{c}},4.92^{\text{c}},6.08^{\text{c}}}$

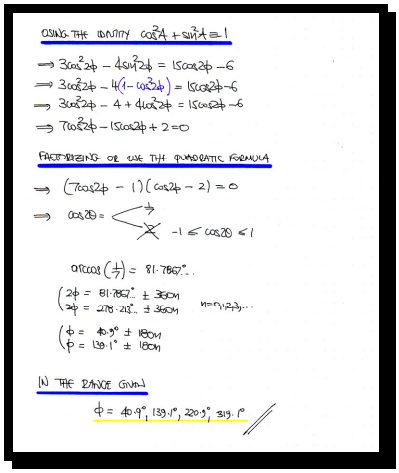


**Question 30 (***+)**

Solve the following trigonometric equation in the range given.

$$3\cos^2 2\varphi - 4\sin^2 2\varphi = 15\cos 2\varphi - 6, \quad 0° \le \varphi < 360°.$$

$\boxed{\quad}$, $\boxed{\varphi \approx 40.9°,\ 139.1°,\ 220.9°,\ 319.1°}$



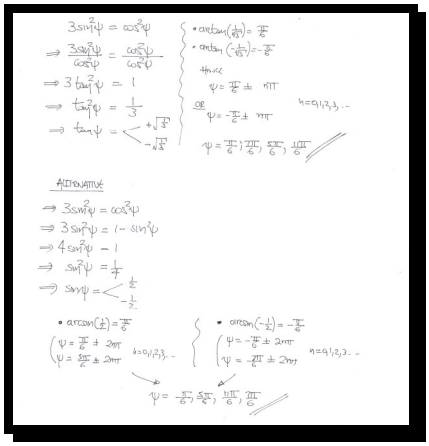
**Question 31 (***+)**

Solve, in **radians**, the following trigonometric equation

$$3\sin^2\psi=\cos^2\psi\,,\quad 0\le\psi<2\pi\,,$$

giving the answers in terms of $\pi$.

$$\boxed{\psi=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}}$$

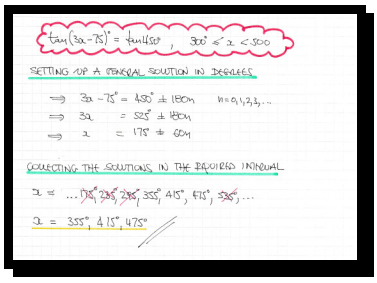


**Question 32 (***+)**

Solve, in **degrees**, the following trigonometric equation

$$\tan(3x-75)^\circ=\tan 450^\circ\,,\quad 300^\circ\le x<500^\circ\,.$$

$$\boxed{\phantom{x}}\,,\quad \boxed{x=355^\circ,\ x=415^\circ,\ x=475^\circ}$$

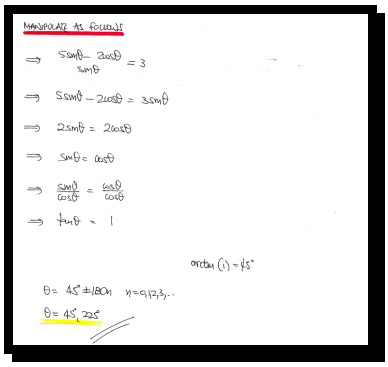


**Question 33 (***+)**

Solve the following trigonometric equation in the range given.

$$\frac{5\sin\theta-2\cos\theta}{\sin\theta}=3, \quad 0^\circ\le\theta<360^\circ.$$

$\boxed{\phantom{x}}$, $\boxed{x=45^\circ,\ 225^\circ}$




**Question 34 (***+)**

$$2CT-2C+T-1$$

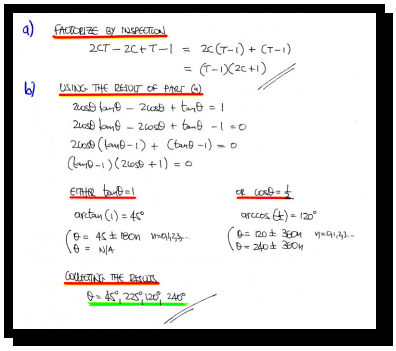
**a)** Write the above expression as a product of two linear factors.

**b)** Hence solve the trigonometric equation

$$2\cos\theta\tan\theta-2\cos\theta+\tan\theta=1,$$

for $0^\circ\le\theta<360^\circ$.

$\boxed{\phantom{x}}$, $\boxed{(2C+1)(T-1)}$, $\boxed{\theta=45^\circ,\ 120^\circ,\ 135^\circ,\ 240^\circ}$

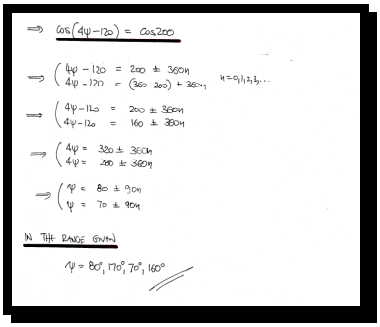


**Question 35 (***+)**

Solve the following trigonometric equation in the range given.

$$\cos(4\psi-120)^\circ=\cos 200^\circ,\ 0\le\psi<180.$$

$\boxed{\phantom{xxxx}}$, $\boxed{\psi=70,\ 80,\ 160,\ 170}$



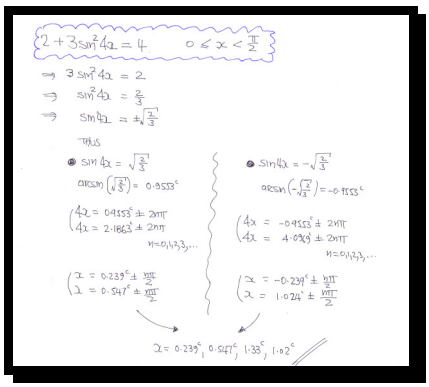

**Question 36 (***+)**

Solve, in **radians**, the following trigonometric equation

$$2+3\sin^2 4x=4,\ \ 0\le x<\frac{\pi}{2},$$

giving the answers correct to three significant figures.

$\boxed{x=0.239^{\text{c}},\ 0.547^{\text{c}},1.02^{\text{c}},\ 1.33^{\text{c}}}$

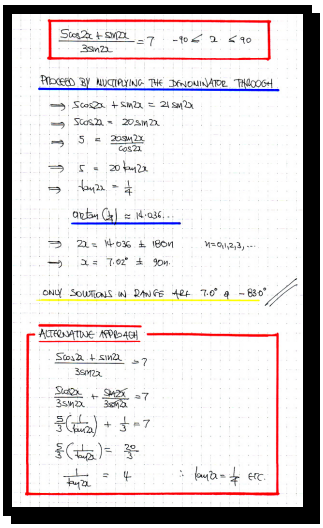


**Question 37 (***+)**

Solve the following trigonometric equation in the range given.

$$\frac{5\cos 2x+\sin 2x}{3\sin 2x}=7\,,\quad -90^\circ \le x<90^\circ\,.$$

$\boxed{\phantom{x}}$, $\boxed{x\approx -83.0^\circ,\ 7.0^\circ}$



**Question 38 (***+)**

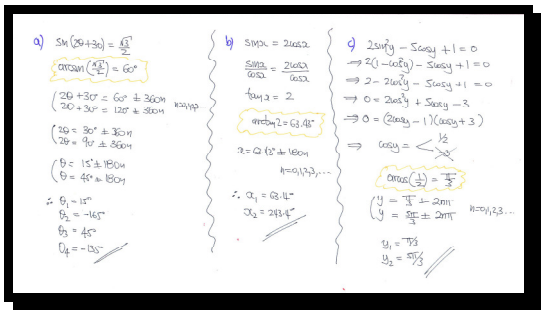
Solve each of the following trigonometric equations, in the range given.

**a)** $\sin(2\theta+30^\circ)=\frac{\sqrt{3}}{2}$, $-180^\circ \le \theta < 180^\circ$

**b)** $\sin x = 2\cos x$, $0 \le x < 360^\circ$

**c)** $2\sin^2 y - 5\cos y + 1 = 0$, $0 \le y < 2\pi$

$\boxed{\quad}$, $\boxed{\theta = -165^\circ, -135^\circ, 15^\circ, 45^\circ}$, $\boxed{x \approx 63.4^\circ, \ 243.4^\circ}$, $\boxed{y = \frac{\pi}{3}, \ \frac{5\pi}{3}}$



**Question 39 (***+)**

A cubic curve is given by

$$f(x) \equiv 4x^3 - 8x^2 - x + k ,$$

where $k$ is a non zero constant.

**a)** Given that $(x-2)$ is a factor of $f(x)$, show that $(2x-1)$ is also a factor of $f(x)$.

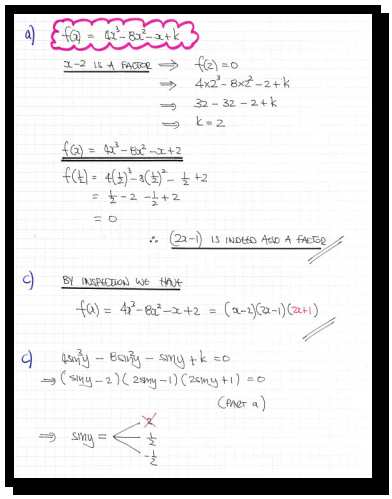
**b)** Express $f(x)$ as the product of three linear factors.

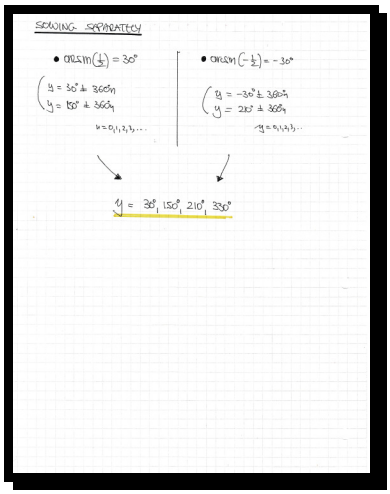
**c)** Hence solve the following trigonometric equation

$$4\sin^3 y - 8\sin^2 y - \sin y + k = 0 ,$$

for $0° \le y < 360°$.

$\boxed{\phantom{xxxx}}$, $\boxed{y = 30°, \ 150°, \ 210°, \ 330°}$



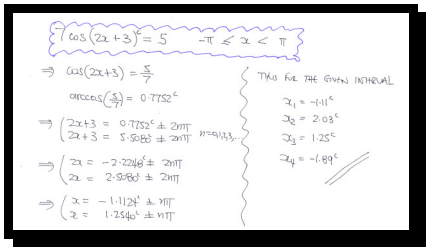
**Question 40 (***+)**

Solve, in **radians**, the following trigonometric equation

$$7\cos\left(2x+3\right)^{\text{c}}=5,\ \ -\pi\le x<\pi,$$

giving the answers correct to three significant figures.

$$\boxed{x=-1.89^{\text{c}},\ -1.11^{\text{c}},1.25^{\text{c}},\ 2.08^{\text{c}}}$$



**Question 41 (***+)**

The graph of the curve with equation

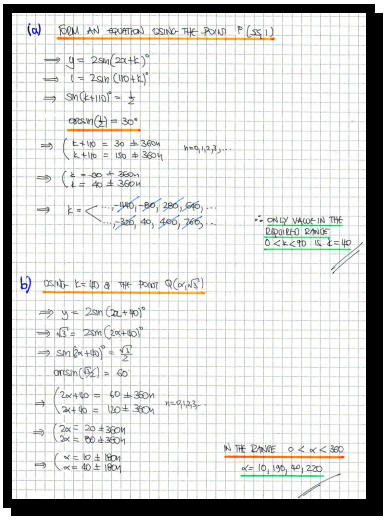
$$y = 2\sin(2x+k)^\circ, \quad 0 \le x < 360,$$

where $k$ is a constant so that $0 < k < 90$, passes through the points with coordinates $P(55,1)$ and $Q(\alpha,\sqrt{3})$.

**a)** Show, without verification, that $k = 40$.

**b)** Determine the possible values of $\alpha$.

$\boxed{\phantom{x}}$, $\boxed{\alpha = 10, 40, 190, 220}$



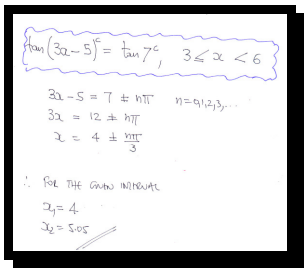
**Question 42 (***+)**

Solve, in **radians**, the following trigonometric equation

$$\tan\left(3x-5\right)^{\text{c}} = \tan 7^{\text{c}}, \quad 3 \le x < 6,$$

giving the answers correct to three significant figures, where appropriate.

$$\boxed{x = 4, \ x \approx 5.05}$$



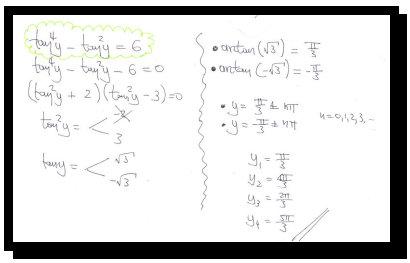

**Question 43 (***+)**

Solve, in **radians**, the following trigonometric equation

$$\tan^4 y - \tan^2 y = 6, \quad 0 \le y < 2\pi,$$

giving the answers in terms of $\pi$.

$$\boxed{y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

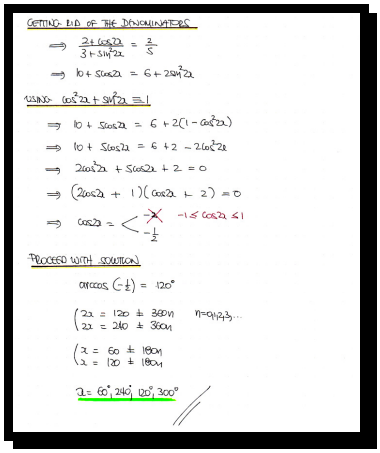


**Question 44 (***+)**

Solve the following trigonometric equation

$$\frac{2+\cos 2x}{3+\sin^2 2x}=\frac{2}{5}, \quad \text{for } 0^\circ \le x < 360^\circ .$$

$\boxed{\phantom{x}}$, $\boxed{x=60^\circ,120^\circ,240^\circ,300^\circ}$

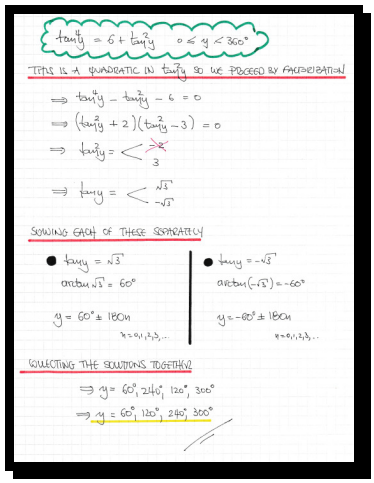



**Question 45 (***+)**

Solve, in **degrees**, the following trigonometric equation

$$\tan^4 y = 6+\tan^2 y , \quad 0^\circ \le y < 360^\circ .$$

$\boxed{\phantom{x}}$, $\boxed{y=60^\circ,\ 120^\circ,\ 240^\circ,\ 300^\circ}$



**Question 46 (***+)**

A trigonometric curve is defined by the equation

$$f(x)=3-4\sin(2x+k)^\circ,\ 0\le x\le 360$$

where $k$ is a constant such that $-90<k<90$.

The curve passes through the point with coordinates $(15,5)$ and further satisfies

$$A\le f(x)\le B,$$

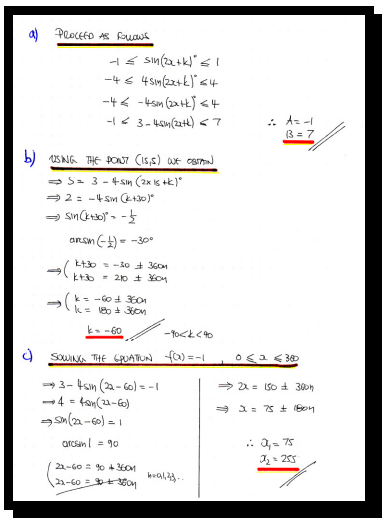
for some constants $A$ and $B$.

**a)** State the value of $A$ and the value of $B$.

**b)** Show that $k=-60$.

**c)** Solve the equation $f(x)=-1$.

$\boxed{\phantom{x}}$, $\boxed{A=-1}$, $\boxed{B=7}$, $\boxed{x=75,\ \ 255}$

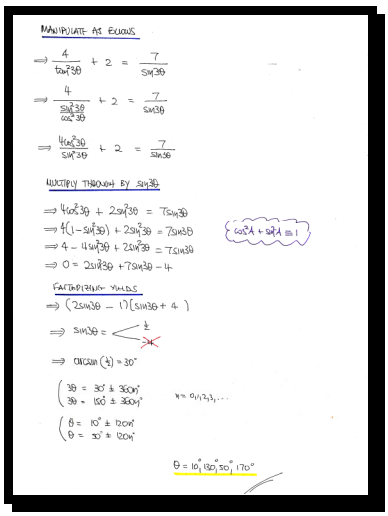


**Question 47 (****)**

Given that $\theta$ is measured in degrees, solve the following trigonometric equation

$$\frac{4}{\tan^2 3\theta}+2=\frac{7}{\sin 3\theta}, \quad 0\leq\theta\leq 180.$$

$\boxed{\phantom{xxxx}}$, $\boxed{\theta = 10°,\ 50°,\ 130°,\ 170°}$



**Question 48 (****)**

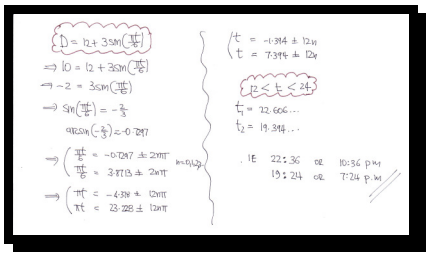
The depth of water in a harbour on a particular day can be modelled by the equation

$$D = 12 + 3\sin\left(\frac{\pi t}{6}\right),$$

where $D$ is the depth of the water in metres, $t$ hours after midnight.

Determine the times after noon, when the depth of water in the harbour is $10$ metres.

$\boxed{\phantom{00}}$, $\boxed{19:24}$, $\boxed{22:36}$



**Question 49 (****)**

The height of tides in a harbour on a particular day can be modelled by the equation

$$h = a + b\sin(30t)^\circ,$$

where $h$ is the height of the water in metres, $t$ hours after midnight, and $a$ and $b$ are constants.

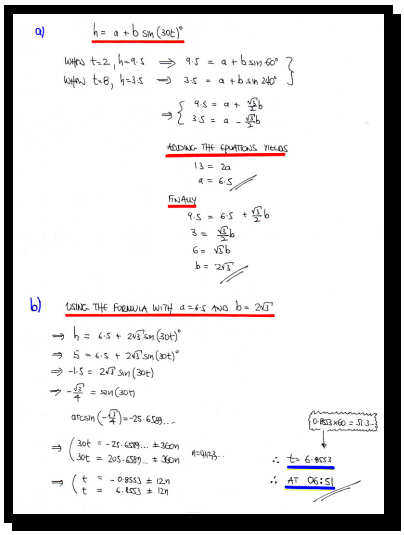
At $02.00$, $h = 9.5$ m and at $08.00$, $h = 3.5$ m.

Determine …

**a)** … the value of $a$ and the exact value of $b$.

**b)** … the first time after midnight when the height of the tide is $5$ metres.

$\boxed{\phantom{00}}$, $\boxed{a = 6.5}$, $\boxed{b = 2\sqrt{3}}$, $\boxed{06:51}$



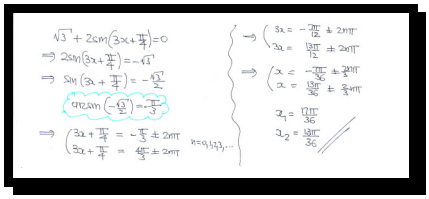
**Question 50 (****)**

Solve the following trigonometric equation, in the range given.

$$\sqrt{3}+2\sin\left(3x+\frac{\pi}{4}\right)=0\,,\quad 0\le x<\frac{\pi}{2}\,.$$

Give the answers in terms of $\pi$.

$\square$, $\boxed{x=\frac{13\pi}{36},\ \frac{17\pi}{36}}$

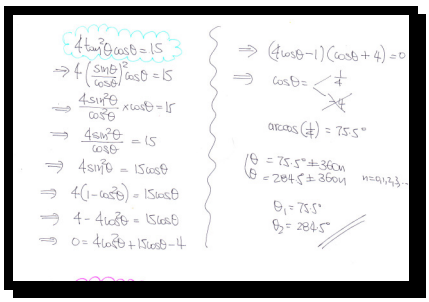


**Question 51 (****)**

Solve the following trigonometric equation in the range given.

$$4\tan^2\theta\cos\theta=15\,,\ 0\le\theta<360^\circ\,.$$

$\square$, $\boxed{\theta\approx 75.5^\circ,\ 284.5^\circ}$



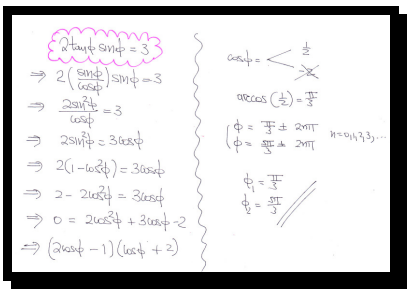
**Question 52 (****)**

Solve the following trigonometric equation in the range given.

$$2\tan\varphi\sin\varphi=3,\quad 0\le\varphi<2\pi.$$

Give the answers in terms of $\pi$.

$$\boxed{\varphi=\frac{\pi}{3},\frac{5\pi}{3}}$$

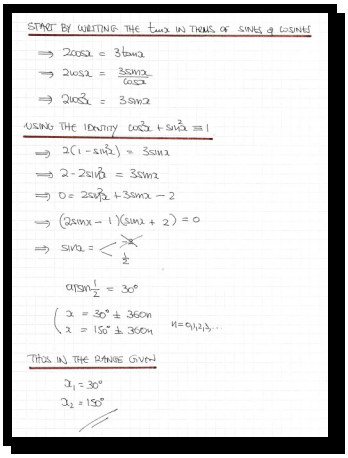


**Question 53 (****)**

Solve the following trigonometric equation in the range given.

$$2\cos x=3\tan x,\ 0^\circ\le x<360^\circ.$$

$$\boxed{\phantom{x}},\ \boxed{x=30^\circ,150^\circ}$$



**Question 54 (****)**

$$f(x)=x^3-4x^2-\frac{1}{2}x+2\,,\ x\in\mathbb{R}\,.$$

**a)** Show that $(x-4)$ is a factor of $f(x)$.

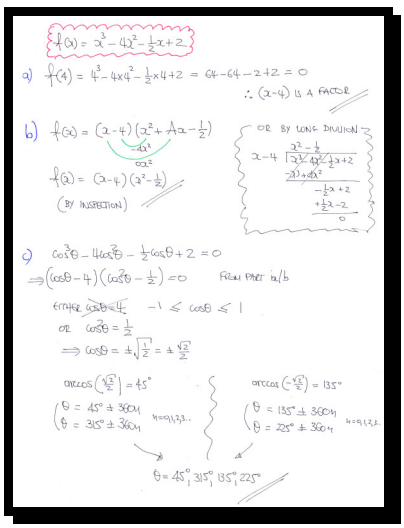
**b)** Express $f(x)$ as the product of a linear and one quadratic factor.

**c)** Hence solve the trigonometric equation

$$\cos^3\theta-4\cos^2\theta-\frac{1}{2}\cos\theta+2=0\,,$$

for $0^\circ\le\theta<360^\circ$.

$\square$, $\boxed{f(x)\equiv(x-4)\left(x^2-\frac{1}{2}\right)}$, $\boxed{\theta=45^\circ,135^\circ,225^\circ,315^\circ}$



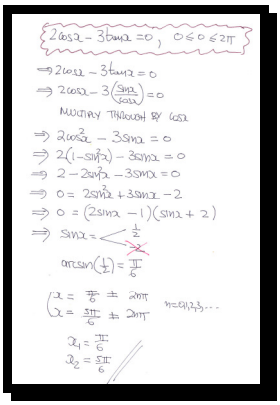
**Question 55 (****)**

Solve the following trigonometric equation in the range given.

$$2\cos x - 3\tan x = 0, \quad 0 \le x < 2\pi .$$

Give the answers in terms of $\pi$.

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$



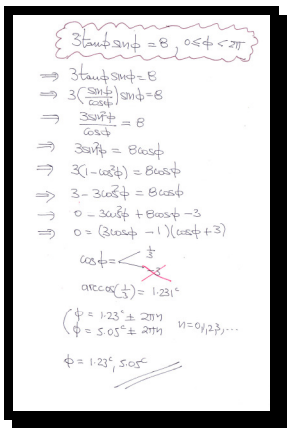

**Question 56 (****)**

Solve the following trigonometric equation in the range given.

$$3\tan\varphi\sin\varphi = 8, \quad 0 \le \varphi < 2\pi .$$

Give the answers in radians correct to two decimal places.

$$\boxed{\varphi \approx 1.23^{\text{c}}, 5.05^{\text{c}}}$$

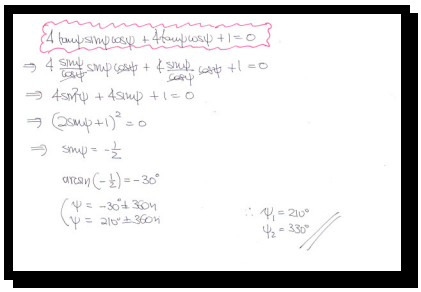


**Question 57 (****)**

Solve the following trigonometric equation in the range given.

$$4\tan\psi\sin\psi\cos\psi+4\tan\psi\cos\psi+1=0\,,\quad 0°\le\psi<360°\,.$$

$\boxed{\phantom{x}}$, $\boxed{\psi=210°,\,330°}$

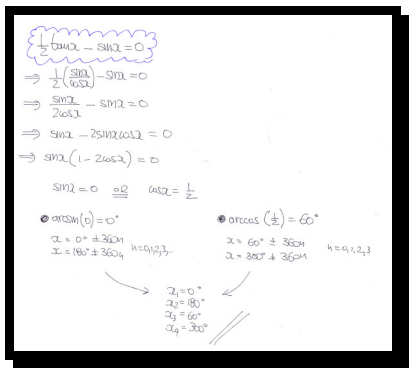



**Question 58 (****)**

Solve the following trigonometric equation in the range given.

$$\frac{1}{2}\tan x-\sin x=0\,,\quad 0°\le x<360°\,.$$

$\boxed{\phantom{x}}$, $\boxed{x=0°,\;60°,\;180°,\;300°}$



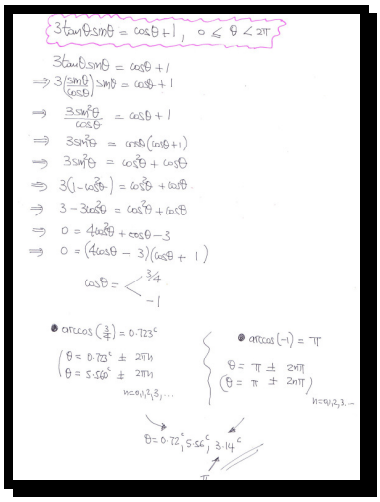
**Question 59 (****)**

Solve the following trigonometric equation in the range given.

$$3\tan\theta\sin\theta=\cos\theta+1,\quad 0\le\theta<2\pi.$$

Give the answers in radians correct to two decimal places.

$\square$, $\boxed{\theta\approx 0.72^{\text{c}},\ 3.14^{\text{c}},\ 5.56^{\text{c}}}$



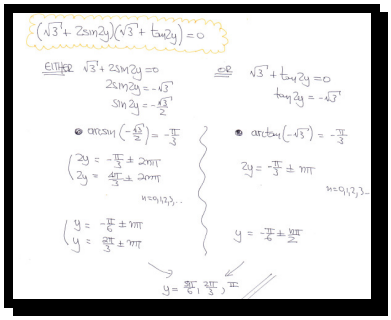

**Question 60 (****)**

Solve the following trigonometric equation in the range given.

$$\left(\sqrt{3}+2\sin 2y\right)\left(\sqrt{3}+\tan 2y\right)=0,\quad 0\le y<\pi.$$

Give the answers in terms of $\pi$.

$\square$, $\boxed{y=\frac{\pi}{3},\frac{2\pi}{3},\frac{5\pi}{6}}$



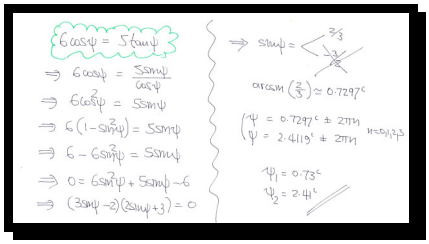
**Question 61 (****)**

Solve the following trigonometric equation in the range given.

$$6\cos\psi = 5\tan\psi \ , \ 0 \le \psi < 2\pi \ .$$

Give the answers in radians, correct to two decimal places.

$\boxed{\phantom{xx}}$, $\boxed{\psi \approx 0.73^{\text{c}}, 2.41^{\text{c}}}$



**Question 62 (****)**

$$f(x)=x^3-x^2-3x+3\,.$$

**a)** Show that $(x-1)$ is a factor of $f(x)$.

**b)** Express $f(x)$ as the product of **three linear factors**.

**c)** Hence solve the trigonometric equation

$$\tan^3\theta-\tan^2\theta-3\tan\theta+3=0\,,$$

for $0^\circ\le\theta<360^\circ$.

$\boxed{\phantom{x}}$, $\boxed{\theta=45^\circ,60^\circ,120^\circ,225^\circ,240^\circ,300^\circ}$



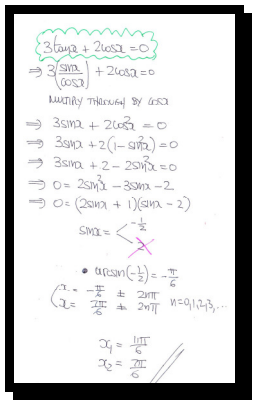
**Question 63 (****)**

Solve the following trigonometric equation in the range given.

$$3\tan x+2\cos x=0, \quad 0\le x<2\pi .$$

Give the answers in terms of $\pi$.

$$\boxed{x=\frac{7\pi}{6},\frac{11\pi}{6}}$$

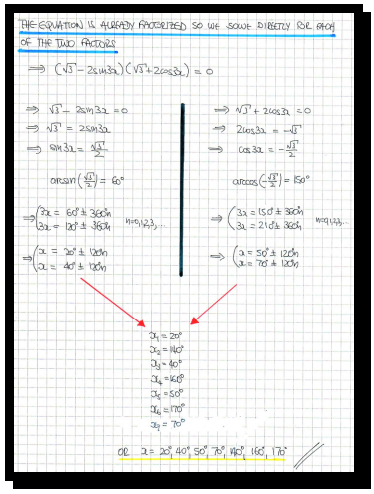



**Question 64 (****)**

Solve the following trigonometric equation in the range given.

$$\left(\sqrt{3}-2\sin 3x\right)\left(\sqrt{3}+2\cos 3x\right)=0, \quad 0^\circ\le x<180^\circ .$$

$$\boxed{\phantom{x=20^\circ}}, \quad \boxed{x=20^\circ,\ 50^\circ,\ 70^\circ,\ 140^\circ,\ 160^\circ,\ 170^\circ}$$



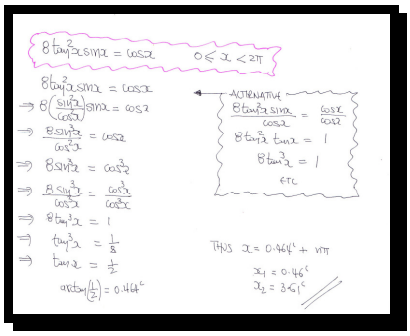
**Question 65 (****)**

Solve the following trigonometric equation in the range given.

$$8\tan^2 x \sin x = \cos x, \quad 0 \le x < 2\pi .$$

Give the answers in radians correct to two decimal places.

$\boxed{\phantom{x}}$, $\boxed{x \approx 0.46^{\text{c}},\ 3.61^{\text{c}}}$

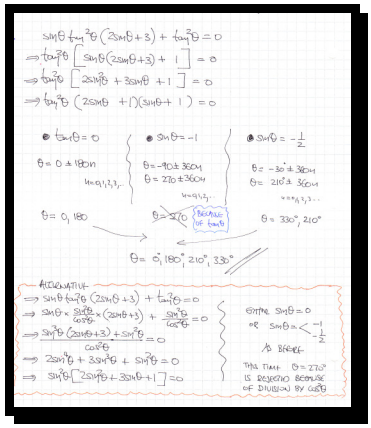


**Question 66 (****+)**

Solve the following trigonometric equation for $0 \le \theta < 360°$

$$\sin\theta \tan^2\theta\left(2\sin\theta + 3\right) + \tan^2\theta = 0 .$$

$\boxed{\phantom{x}}$, $\boxed{\theta = 0°,\ 180°,\ 210°,\ 330°}$

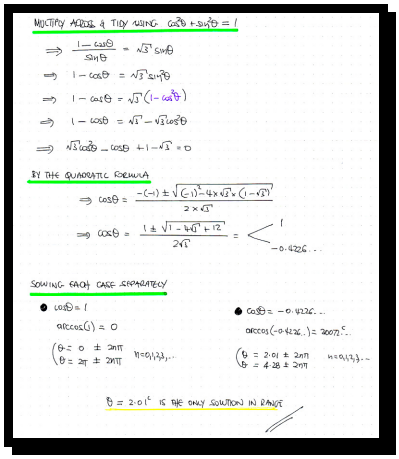


**Question 67 (****+)**

Calculate in **degrees**, correct to one decimal place, the solution of the following trigonometric equation

$$\frac{1-\cos\theta}{\sin\theta}=\sqrt{3}\sin\theta\,,\quad 0<\theta<\pi\,.$$

$\boxed{\phantom{xxxx}}$, $\boxed{\theta\approx 2.01^{\text{c}}}$



**Question 68 (****+)**

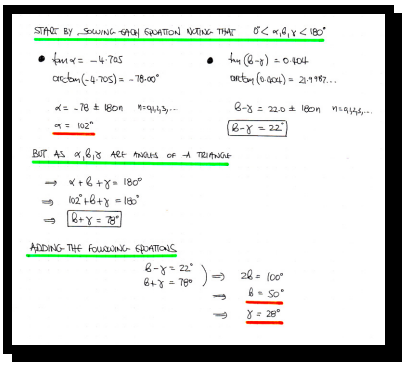
The three angles in a triangle are denoted as $\alpha$, $\beta$ and $\gamma$.

It is further given that

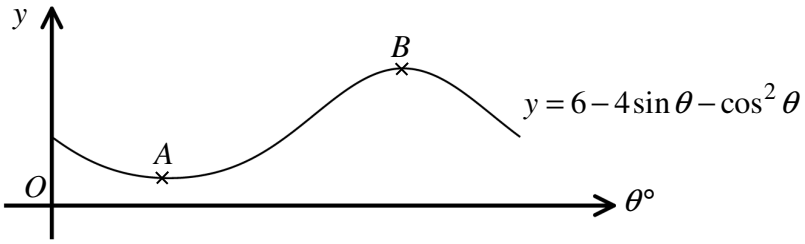
$$\tan\alpha = -4.705 \qquad \text{and} \qquad \tan(\beta-\gamma) = 0.404$$

Determine, in degrees, the size of each of the angles $\alpha$, $\beta$ and $\gamma$.

$\boxed{\phantom{xx}}$, $\boxed{\alpha \approx 102^\circ}$, $\boxed{\beta \approx 50^\circ}$, $\boxed{\gamma \approx 28^\circ}$



**Question 69** **(****+)**




The figure above shows the graph of the curve with equation

$$y = 6 - 4\sin\theta - \cos^2\theta, \quad 0° \le \theta \le 360°.$$

The curve has a minimum at the point $A$ and a maximum at the point $B$.

Determine the coordinates of $A$ and $B$.

$\boxed{\phantom{A(90°,2)}}$, $\boxed{A(90°,2)}$, $\boxed{B(270°,10)}$

FIRST WRITE THE EQUATION IN $\sin\theta$

$y = 6 - 4\sin\theta - \cos^2\theta$

$y = 6 - 4\sin\theta - (1 - \sin^2\theta)$

$y = 5 - 4\sin\theta + \sin^2\theta$

BY INSPECTION, LOOKING AT THE SINE GRAPH, AND NOTING THAT $\sin 270° = -1$

$y_{MAX} = 5 - 4(-1) + (-1)^2 = 10$

$\therefore B(270, 10)$

COMPLETING THE SQUARE IN $\sin\theta$

$y = \sin^2\theta - 4\sin\theta + 5$

$y = (\sin\theta - 2)^2 + 1$

BUT $\sin\theta \neq 2$, SO MINIMUM WILL BE ACHIEVED WHEN $\sin\theta = +1$

$y_{MIN} = (+1-2)^2 + 1 = 1 + 1 = 2$

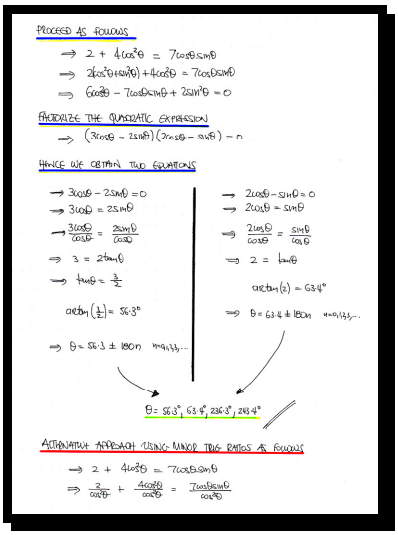
$\therefore A(90, 2)$

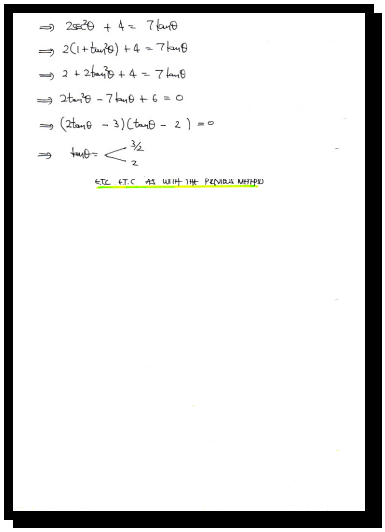
**Question 70 (*****)**

Solve the following trigonometric equation for $0 \leq \theta < 360°$

$$2+4\cos^2\theta = 7\cos\theta\sin\theta .$$

$\boxed{\phantom{xx}}$, $\boxed{\theta \approx 56.3°,\ \theta \approx 63.4°,\ \theta \approx 236.3°,\ \theta \approx 243.4°}$



**Question 71 (*****)**

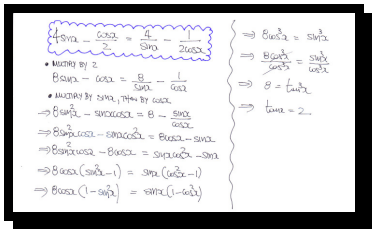
It is given that

$$4\sin x-\frac{\cos x}{2}=\frac{4}{\sin x}-\frac{1}{2\cos x}.$$

Show clearly that the above equation is equivalent to

$$\tan x=2\,.$$

$\boxed{\phantom{xx}}$ , $\boxed{\text{proof}}$

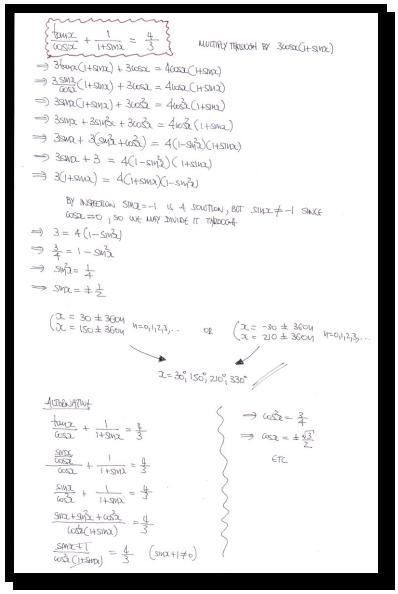


**Question 72 (*****)**

Solve the following trigonometric equation for $0 \leq x < 360°$

$$\frac{\tan x}{\cos x}+\frac{1}{1+\sin x}=\frac{4}{3}.$$

$\square$, $\boxed{x = 30°,\ 150°,\ 210°,\ 330°}$

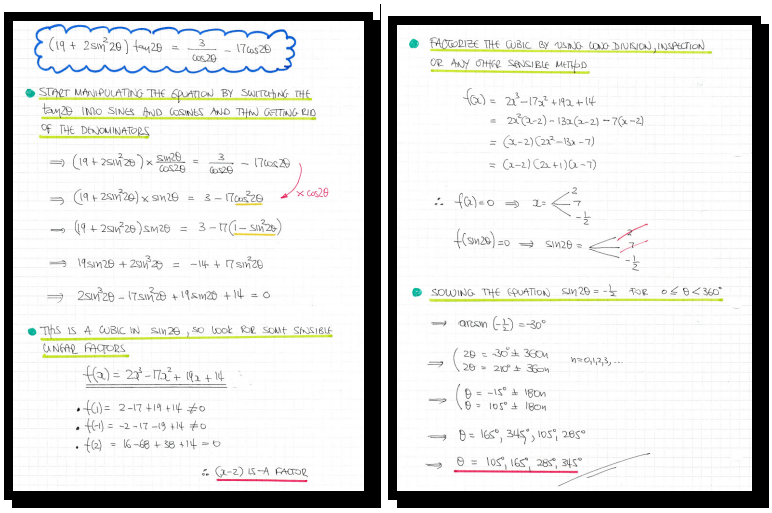


**Question 73 (*****)**

Solve the following trigonometric equation

$$\left(19+2\sin^2 2\theta\right)\tan 2\theta=\frac{3}{\cos 2\theta}-17\cos 2\theta\,,\qquad 0^\circ\le\theta<360^\circ\,.$$

$\boxed{\phantom{xxxx}}$, $\boxed{\theta=105^\circ,\quad \theta=165^\circ,\quad \theta=285^\circ,\quad \theta=345^\circ}$



**Question 74** **(*****)**

$$4\cos^2\theta + \tan^4\theta = 10, \quad 0 \le \theta < 2\pi .$$

Show that $\theta = \frac{1}{3}\pi$ is a solution of the above trigonometric equation and use a non verification method to find the other solutions.

$\square$, $\boxed{\theta = \frac{2}{3}\pi,\ \frac{4}{3}\pi,\ \frac{5}{3}\pi}$

