TRIGONOMETRY

THE DOUBLE ANGLE IDENTITIES

Question 1

Prove the validity of each of the following trigonometric identities.

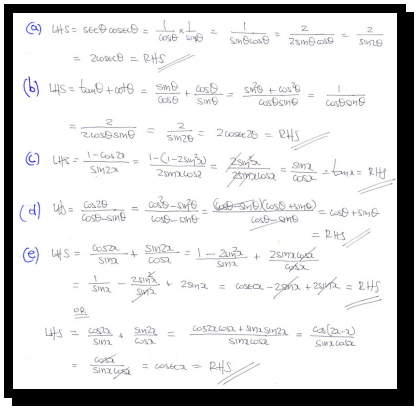
a) $\sec\theta\operatorname{cosec}\theta \equiv 2\operatorname{cosec}2\theta$

b) $\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta$

c) $\dfrac{1-\cos 2x}{\sin 2x} \equiv \tan x$

d) $\dfrac{\cos 2\theta}{\cos\theta - \sin\theta} \equiv \cos\theta + \sin\theta$

e) $\dfrac{\cos 2x}{\sin x} + \dfrac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x$



Question 2

Prove the validity of each of the following trigonometric identities.

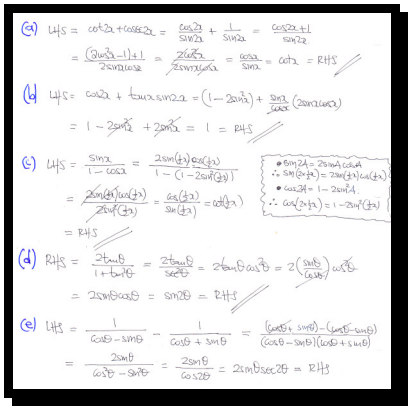
a) $\cot 2x + \operatorname{cosec} 2x \equiv \cot x$

b) $\cos 2x + \tan x \sin 2x \equiv 1$

c) $\dfrac{\sin x}{1-\cos x} \equiv \cot \dfrac{1}{2} x$

d) $\sin 2\theta \equiv \dfrac{2\tan\theta}{1+\tan^2\theta}$

e) $\dfrac{1}{\cos\theta - \sin\theta} - \dfrac{1}{\cos\theta + \sin\theta} \equiv 2\sin\theta \sec 2\theta$



Question 3

Prove the validity of each of the following trigonometric identities.

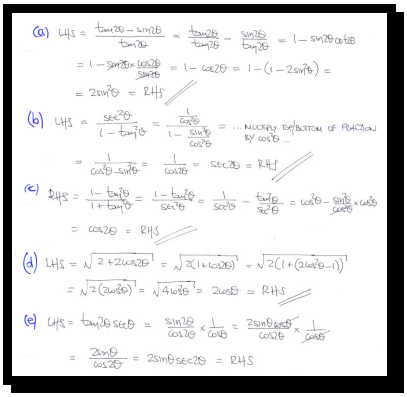
a) $\dfrac{\tan 2\theta - \sin 2\theta}{\tan 2\theta} \equiv 2\sin^2\theta$

b) $\dfrac{\sec^2\theta}{1-\tan^2\theta} \equiv \sec 2\theta$

c) $\cos 2\theta \equiv \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

d) $\sqrt{2+2\cos 2\theta} \equiv 2\cos\theta$

e) $\tan 2\theta \sec\theta \equiv 2\sin\theta \sec 2\theta$



Question 4

Prove the validity of each of the following trigonometric identities.

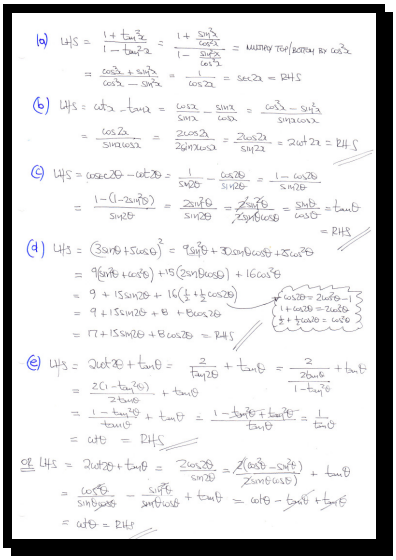
a) $\dfrac{1+\tan^2 x}{1-\tan^2 x} \equiv \sec 2x$

b) $\cot x - \tan x \equiv 2\cot 2x$

c) $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan\theta$

d) $(3\sin\theta + 5\cos\theta)^2 \equiv 17 + 8\cos 2\theta + 15\sin 2\theta$

e) $2\cot 2\theta + \tan\theta \equiv \cot\theta$



Question 5

Prove the validity of each of the following trigonometric identities.

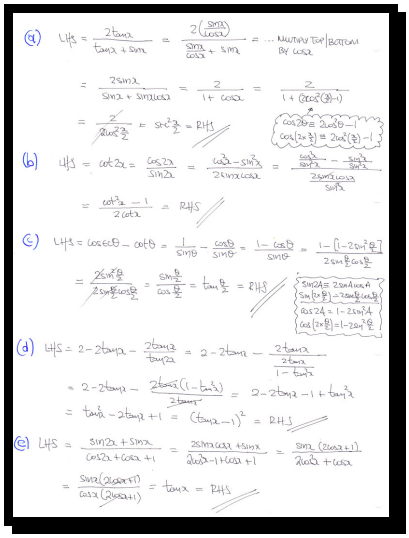
a) $\dfrac{2\tan x}{\tan x+\sin x}\equiv \sec^2\left(\dfrac{x}{2}\right)$

b) $\cot 2x\equiv \dfrac{\cot^2 x-1}{2\cot x}$

c) $\operatorname{cosec}\theta-\cot\theta\equiv \tan\dfrac{1}{2}\theta$

d) $2-2\tan x-\dfrac{2\tan x}{\tan 2x}\equiv (1-\tan x)^2$

e) $\dfrac{\sin 2x+\sin x}{\cos 2x+\cos x+1}\equiv \tan x$



Question 6

Prove the validity of each of the following trigonometric identities.

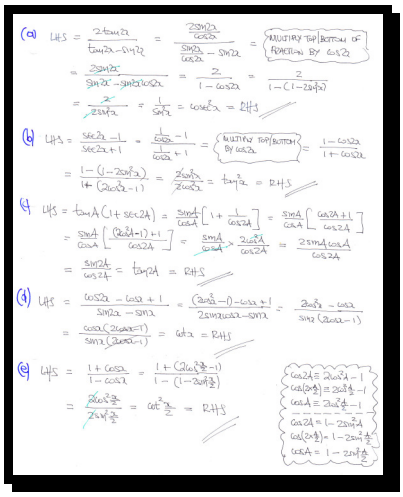
a) $\dfrac{2\tan 2x}{\tan 2x - \sin 2x} \equiv \operatorname{cosec}^2 x$

b) $\dfrac{\sec 2x - 1}{\sec 2x + 1} \equiv \tan^2 x$

c) $\tan A\left(1 + \sec 2A\right) \equiv \tan 2A$

d) $\dfrac{\cos 2x - \cos x + 1}{\sin 2x - \sin x} \equiv \cot x$

e) $\dfrac{1 + \cos x}{1 - \cos x} \equiv \cot^2 \dfrac{1}{2} x$



Question 7

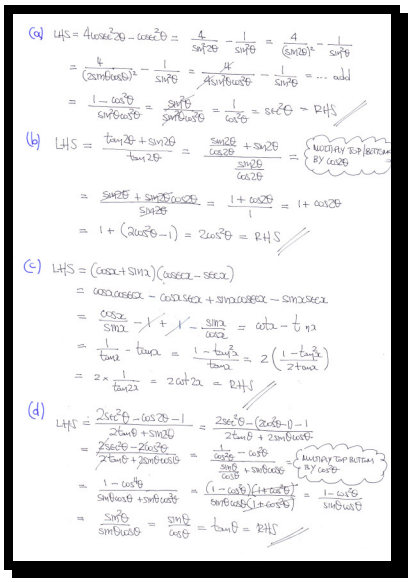
Prove the validity of each of the following trigonometric identities.

a) $4\text{cosec}^2 2\theta - \text{cosec}^2\theta \equiv \sec^2\theta$

b) $\dfrac{\tan 2\theta + \sin 2\theta}{\tan 2\theta} \equiv 2\cos^2\theta$

c) $(\cos x + \sin x)(\text{cosec}\, x - \sec x) \equiv 2\cot 2x$

d) $\dfrac{2\sec^2\theta - \cos 2\theta - 1}{2\tan\theta + \sin 2\theta} \equiv \tan\theta$



Question 8

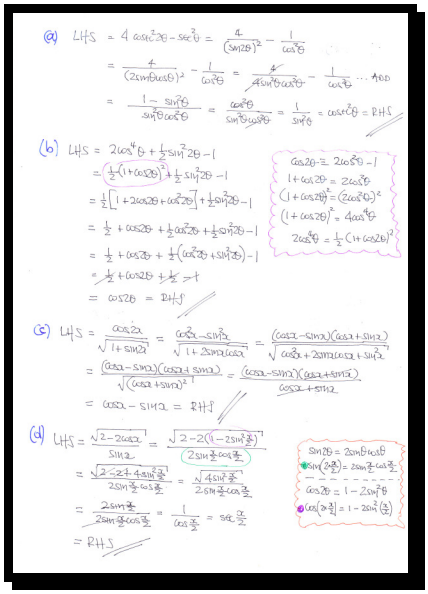
Prove the validity of each of the following trigonometric identities.

a) $4\text{cosec}^2 2\theta - \sec^2\theta \equiv \text{cosec}^2\,\theta$

b) $2\cos^4\theta + \frac{1}{2}\sin^2 2\theta - 1 \equiv \cos 2\theta$

c) $\frac{\cos 2x}{\sqrt{1+\sin 2x}} \equiv \cos x - \sin x$

d) $\frac{\sqrt{2-2\cos x}}{\sin x} \equiv \sec\frac{x}{2}$



Question 9

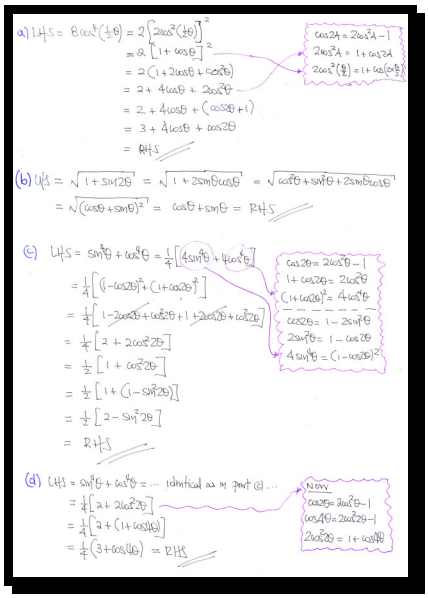
Prove the validity of each of the following trigonometric identities.

a) $8\cos^4\left(\frac{1}{2}\theta\right) \equiv \cos 2\theta + 4\cos\theta + 3$

b) $\sqrt{1+\sin 2\theta} \equiv \sin\theta + \cos\theta$

c) $\sin^4\theta + \cos^4\theta \equiv \frac{1}{2}\left(2 - \sin^2 2\theta\right)$

d) $\sin^4\theta + \cos^4\theta \equiv \frac{1}{4}(3 + \cos 4\theta)$



Question 10

Solve each of the following trigonometric equations.

a) $\sin 2\theta = \tan\theta, \qquad 0 \le \theta \le 180°$

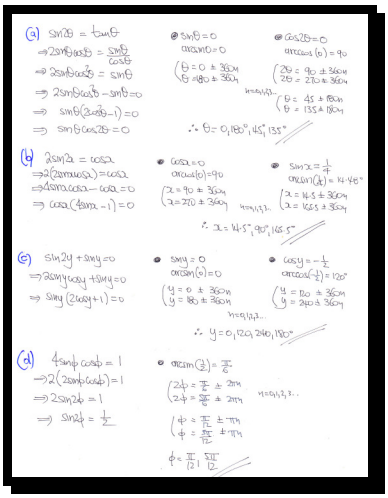
b) $2\sin 2x = \cos x, \qquad 0 \le x < 180°$

c) $\sin 2y + \sin y = 0, \qquad 0 \le y < 360°$

d) $4\sin\varphi\cos\varphi = 1, \qquad 0 \le \varphi < \pi$

$\boxed{\theta = 0°,\ 45°,\ 135°,\ 180°}$, $\boxed{x = 90°,\ x \approx 14.5°,\ 165.5°}$, $\boxed{y = 0°,\ 120°,\ 180°,\ 240°}$,

$$\boxed{\varphi = \frac{\pi}{12},\ \frac{5\pi}{12}}$$



Question 11

Solve each of the following trigonometric equations.

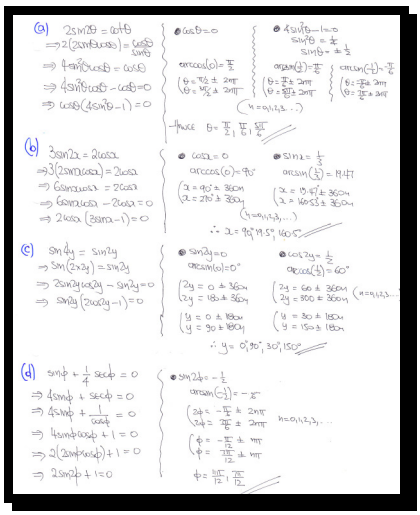
a) $2\sin 2\theta = \cot\theta\,, \qquad 0 \le \theta \le \pi$

b) $3\sin 2x = 2\cos x\,, \qquad 0 \le x < 180°$

c) $\sin 4y = \sin 2y\,, \qquad 0 \le y < 180°$

d) $\sin\varphi + \frac{1}{4}\sec\varphi = 0\,, \qquad 0 \le \varphi < \pi$

$\boxed{\theta = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}}$, $\boxed{x = 90°,\ x \approx 19.5°,\ 160.5°}$, $\boxed{y = 0°,\ 30°,\ 90°,\ 150°}$, $\boxed{\varphi = \frac{7\pi}{12},\ \frac{11\pi}{12}}$



Question 12

Solve each of the following trigonometric equations.

a) $\cos\theta - \sin 2\theta = 0\,, \qquad 0 \le \theta \le 360°$

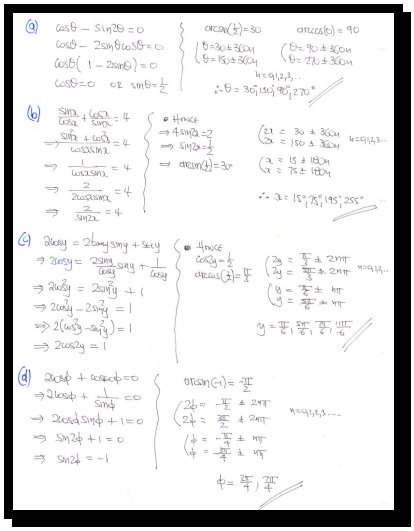
b) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = 4\,, \qquad 0 \le x < 360°$

c) $2\cos y = 2\tan y \sin y + \sec y\,, \qquad 0 \le y < 2\pi$

d) $2\cos\varphi + \operatorname{cosec}\varphi = 0\,, \qquad 0 \le \varphi < 2\pi$

$\boxed{\theta = 30°,\ 90°,\ 150°,\ 270°}$, $\boxed{x = 15°,\ 75°,\ 195°,\ 255°}$, $\boxed{y = \dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}}$,

$\boxed{\varphi = \dfrac{3\pi}{4},\ \dfrac{7\pi}{4}}$



Question 13

Solve each of the following trigonometric equations.

a) $2\cos 2\theta = 1+\cos\theta, \qquad 0 \le \theta < 360°$

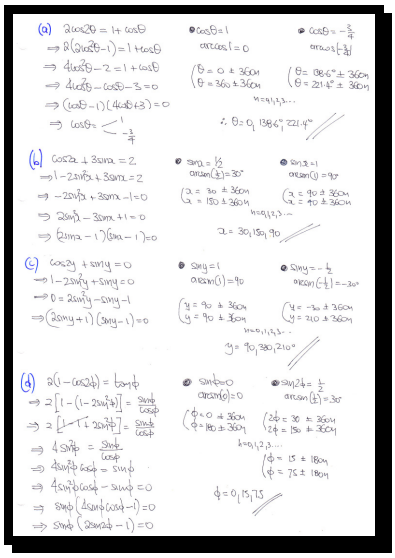
b) $\cos 2x + 3\sin x = 2, \qquad 0 \le x < 360°$

c) $\cos 2y + \sin y = 0, \qquad 0 \le y < 360°$

d) $2(1-\cos 2\varphi) = \tan\varphi, \qquad 0 \le \varphi < 180°$

$\boxed{\theta = 0°,\ \theta \approx 138.6°,\ 221.4°}$, $\boxed{x = 30°,\ 90°,\ 150°}$, $\boxed{y = 90°,\ 210°,\ 330°}$,

$\boxed{\varphi = 0°,\ 15°,\ 75°}$



Question 14

Solve each of the following trigonometric equations.

a) $\cos 2\theta - 7\sin\theta - 4 = 0, \qquad 0 \le \theta < 360°$

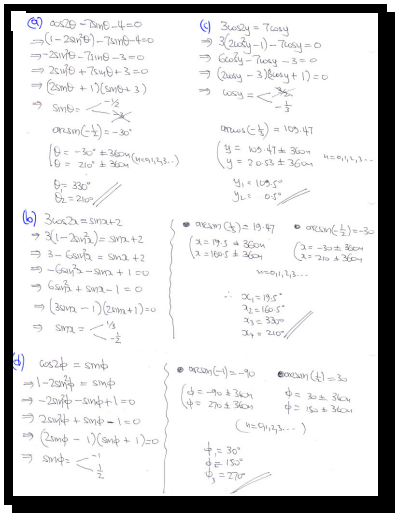
b) $3\cos 2x = \sin x + 2, \qquad 0 \le x < 360°$

c) $3\cos 2y = 7\cos y, \qquad 0 \le y < 360°$

d) $\cos 2\varphi = \sin\varphi, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330°}$, $\boxed{x \approx 19.5°,\ 160.5° \quad x = 210°,\ 330°}$, $\boxed{y \approx 109.5°,\ 250.5°}$,

$\boxed{\varphi = 30°,\ 150°,\ 270°}$



Question 15

Solve each of the following trigonometric equations.

a) $3\cos 2\theta - 5\sin\theta = 4\,, \qquad 0 \le \theta < 360°$

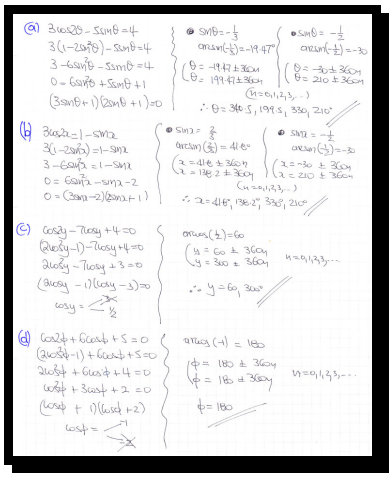
b) $3\cos 2x = 1 - \sin x\,, \qquad 0 \le x < 360°$

c) $\cos 2y - 7\cos y + 4 = 0\,, \qquad 0 \le y < 360°$

d) $\cos 2\varphi + 6\cos\varphi + 5 = 0\,, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330° \quad \theta \approx 199.5°,\ 340.5°}$, $\boxed{x \approx 41.8°,\ 138.2° \quad x = 210°,\ 330°}$,

$\boxed{y = 60°,\ 300°}$, $\boxed{\varphi = 180°}$



Question 16

Solve each of the following trigonometric equations.

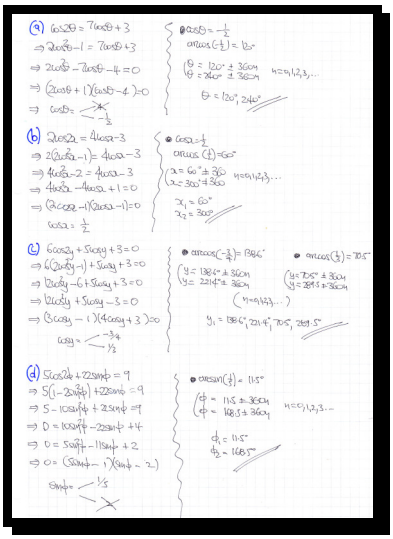
a) $\cos 2\theta = 7\cos\theta + 3, \qquad 0 \le \theta < 360°$

b) $2\cos 2x = 4\cos x - 3, \qquad 0 \le x < 360°$

c) $6\cos 2y + 5\cos y + 3 = 0, \qquad 0 \le y < 360°$

d) $5\cos 2\varphi + 22\sin\varphi = 9, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 120°,\ 240°}$, $\boxed{x = 60°,\ 300°}$, $\boxed{y \approx 70.5°,\ 138.6°,\ 221.4°,\ 289.5°}$, $\boxed{\varphi \approx 11.5°,\ 168.5°}$



Question 17

Solve each of the following trigonometric equations.

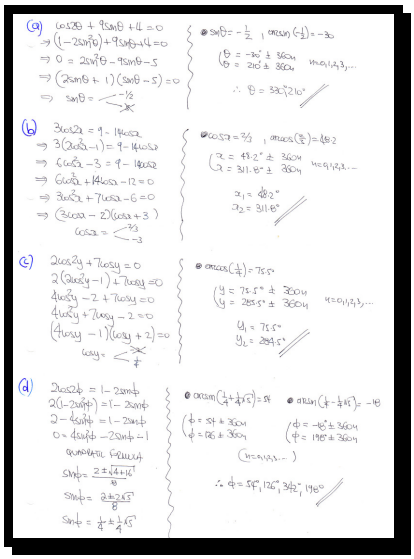
a) $\cos 2\theta + 9\sin\theta + 4 = 0, \qquad 0 \le \theta < 360°$

b) $3\cos 2x = 9 - 14\cos x, \qquad 0 \le x < 360°$

c) $2\cos 2y + 7\cos y = 0, \qquad 0 \le y < 360°$

d) $2\cos 2\varphi = 1 - 2\sin\varphi, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330°}$, $\boxed{x \approx 48.2°,\ 311.8°}$, $\boxed{y \approx 75.5°,\ 284.5°}$, $\boxed{\varphi = 54°,\ 126°,\ 198°,\ 342°}$



Question 18

Solve each of the following trigonometric equations.

a) $\cos 2\theta = 1 + \sin\theta, \qquad 0 \le \theta < 360°$

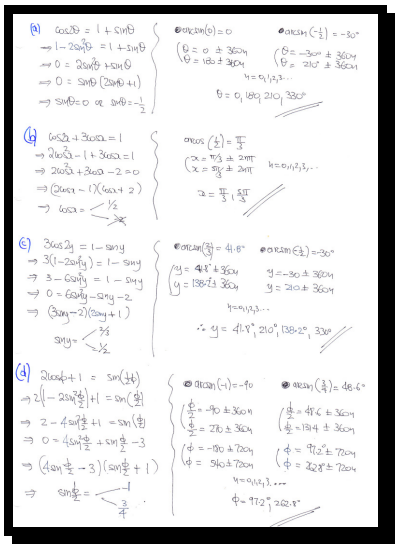
b) $\cos 2x + 3\cos x = 1, \qquad 0 \le x < 2\pi$

c) $3\cos 2y = 1 - \sin y, \qquad 0 \le y < 360°$

d) $2\cos\varphi + 1 = \sin\left(\frac{1}{2}\varphi\right), \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 0°,\ 180°,\ 210°,\ 330°}$, $\boxed{x = \frac{\pi}{3},\ \frac{5\pi}{3}}$, $\boxed{y \approx 41.8°,\ 138.2° \quad y = 210°,\ 330°}$,

$\boxed{\varphi = 97.2°,\ 262.8°}$



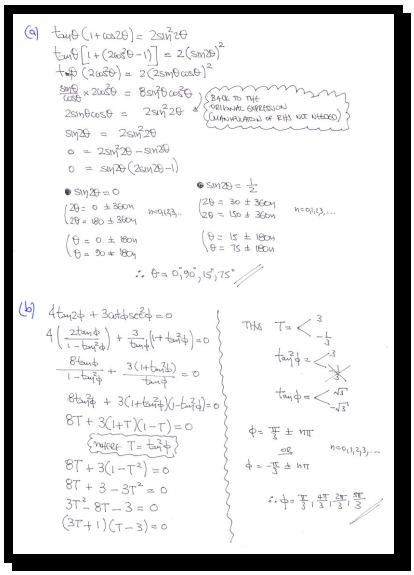
Question 19

Solve each of the following trigonometric equations.

a) $\tan\theta\left(1+\cos 2\theta\right)=2\sin^2 2\theta\,, \qquad 0\le\theta\le 90°$

b) $4\tan 2\varphi+3\cot\varphi\sec^2\varphi=0\,, \qquad 0\le\varphi<2\pi$ **(hard)**

$\boxed{\theta=0°,\ 15°,\ 75°,\ 90°}$, $\boxed{\varphi=\dfrac{\pi}{3},\ \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ \dfrac{5\pi}{3}}$



Question 20

Show clearly that each of the following trigonometric equations has no real roots, regardless of the solution interval.

a) $\cos 2\theta = 23 + 14\cos\theta$

b) $\cos 2x + \cos x + 2 = 0$

proof

