TRIGONOMETRIC EQUATIONS

Question 1

Solve each of the following trigonometric equations.

a) $\sec\theta = 4\,, \qquad 0 \le \theta < 360°$

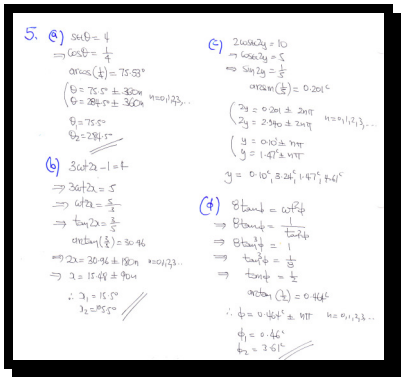
b) $3\cot 2x - 1 = 4\,, \qquad 0 \le x < 180°$

c) $2\operatorname{cosec} 2y = 10\,, \qquad 0 \le y < 2\pi$

d) $8\tan\varphi = \cot^2\varphi\,, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 75.5°,\ 284.5°}$, $\boxed{x \approx 15.5°,\ 105.5°}$, $\boxed{y \approx 0.10^{\text{c}},\ 1.47^{\text{c}},\ 3.24^{\text{c}},\ 4.61^{\text{c}}}$,

$\boxed{\varphi \approx 0.46^{\text{c}},\ 3.61^{\text{c}}}$



Question 2

Solve each of the following trigonometric equations.

a) $2\sec\theta = 3, \qquad 0 \le \theta < 360°$

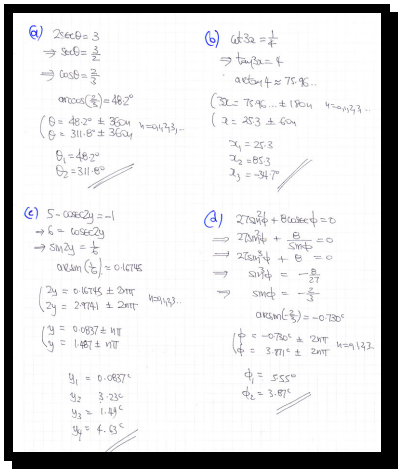
b) $\cot 3x = \frac{1}{4}, \qquad -90° \le x < 90°$

c) $5 - \operatorname{cosec} 2y = -1, \qquad 0 \le y < 2\pi$

d) $27\sin^2\varphi + 8\operatorname{cosec}\varphi = 0, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 48.2°,\ 311.8°}$, $\boxed{x \approx -34.7°,\ 25.3°,\ 85.3°}$, $\boxed{y \approx 0.0837^{\text{c}},\ 1.49^{\text{c}},\ 3.23^{\text{c}},\ 4.63^{\text{c}}}$,

$\boxed{\varphi \approx 3.87^{\text{c}},\ 5.55^{\text{c}}}$



Question 3

Solve each of the following trigonometric equations.

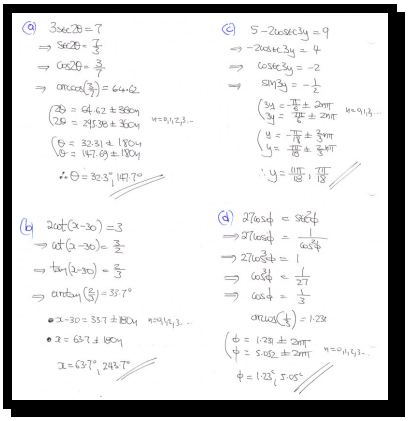
a) $3\sec 2\theta = 7\,, \qquad 0 \le \theta < 180°$

b) $2\cot\left(x - 30°\right) = 3\,, \qquad 0 \le x < 360°$

c) $5 - 2\operatorname{cosec} 3y = 9\,, \qquad 0 \le y < \pi$

d) $27\cos\varphi = \sec^2\varphi\,, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 32.3°,\ 147.7°}$, $\boxed{x \approx 63.7°,\ 243.7°}$, $\boxed{y = \frac{7\pi}{18},\ \frac{11\pi}{18}}$, $\boxed{\varphi \approx 1.23^{\text{c}},\ 5.05^{\text{c}}}$



Question 4

Solve each of the following trigonometric equations.

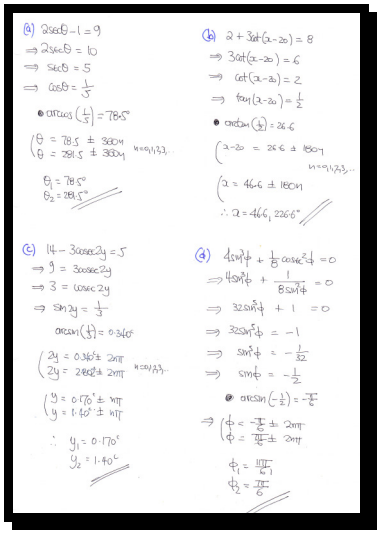
a) $2\sec\theta-1=9\,,\qquad 0\leq\theta<360°$

b) $2+3\cot\left(x-20°\right)=8\,,\qquad 0\leq x<360°$

c) $14-3\operatorname{cosec}2y=5\,,\qquad 0\leq y<\pi$

d) $4\sin^3\varphi+\frac{1}{8}\operatorname{cosec}^2\varphi=0\,,\quad 0\leq\varphi<2\pi$

$\boxed{\theta\approx 78.5°,\ 281.5°}$, $\boxed{x\approx 46.6°,\ 226.6°}$, $\boxed{y\approx 0.170^{\text{c}},\ 1.40^{\text{c}}}$, $\boxed{\varphi=\frac{7\pi}{6},\ \frac{11\pi}{6}}$



Question 5

Solve each of the following trigonometric equations.

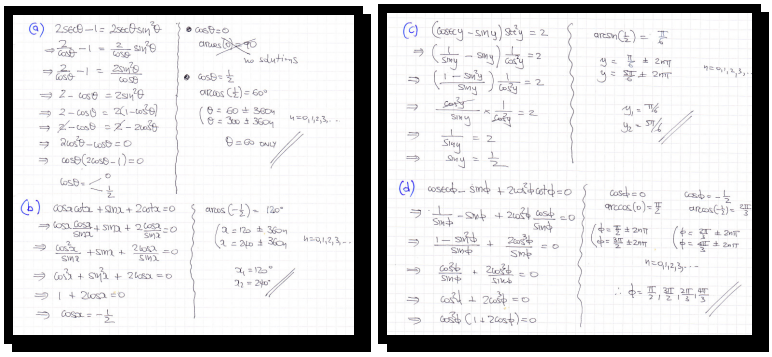
a) $2\sec\theta - 1 = 2\sec\theta\sin^2\theta\,, \qquad 0 \le \theta < 180°\,, \quad \theta \neq 90°$

b) $\cos x\cot x + \sin x + 2\cot x = 0\,, \qquad 0 < x < 360°, \quad x \neq 180°$

c) $(\operatorname{cosec} y - \sin y)\sec^2 y = 2\,, \qquad 0 \le y < \pi, \quad y \neq \dfrac{\pi}{2}$

d) $\operatorname{cosec}\varphi - \sin\varphi + 2\cos^2\varphi\cot\varphi = 0\,, \quad 0 < \varphi < 2\pi, \quad \varphi \neq \pi$

$\boxed{\theta = 60°}$, $\boxed{x = 120°, 240°}$, $\boxed{y = \dfrac{\pi}{6}, \dfrac{5\pi}{6}}$, $\boxed{\varphi = \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}}$



Question 6

Solve each of the following trigonometric equations.

a) $\sec\theta+\cos\theta=\frac{5}{2}, \quad 0\le\theta<360°, \quad \theta\neq 90°,270°$

b) $\sec x-\cos x=8\left(\operatorname{cosec} x-\sin x\right), \quad 0\le x<360°, \quad x\neq 90°$

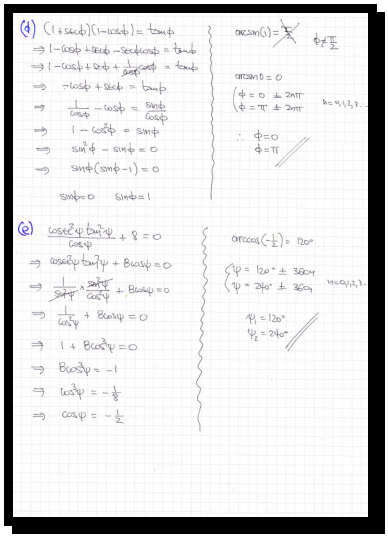
c) $2\cot y-3\operatorname{cosec} y=2\sec y\operatorname{cosec} y, \quad 0<y<2\pi, \quad y\neq\frac{k\pi}{2}, k\in\mathbb{Z}$

d) $\left(1+\sec\varphi\right)\left(1-\cos\varphi\right)=\tan\varphi, \quad 0\le\varphi<2\pi, \quad \varphi\neq\frac{\pi}{2},\frac{3\pi}{2}$

e) $\frac{\operatorname{cosec}^2\psi\tan^2\psi}{\cos\psi}+8=0, \quad 0\le\psi<360°, \quad \psi\neq 90°,270°$

$\boxed{\theta=60°,300°}$, $\boxed{x=63.4°,243.6°}$, $\boxed{y=\frac{2\pi}{3},\frac{4\pi}{3}}$, $\boxed{\varphi=0,\pi}$, $\boxed{\psi=120°,240°}$





Question 7 (hard questions)

Solve each of the following trigonometric equations.

a) $2\sin\theta+3\sec\theta=6+\tan\theta\,,\quad 0\le\theta<2\pi\,,\quad \theta\neq\dfrac{\pi}{2},\dfrac{3\pi}{2}$

b) $\sin^2 x\tan x+\cos^2 x\cot x+2\sin x\cos x=2\,,\quad 0<x<360°,\quad x\neq 90°,180°,270°$

you may use in this part the fact that $\boxed{2\sin x\cos x\equiv\sin 2x}$

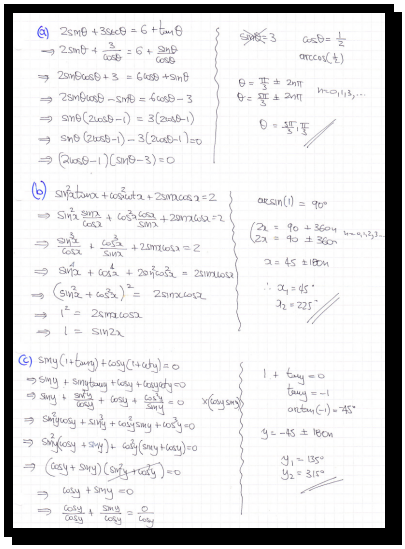
c) $\sin y\left(1+\tan y\right)+\cos y\left(1+\cot y\right)=0\,,\quad 0<y<360°,\quad y\neq 90°,180°,270°$

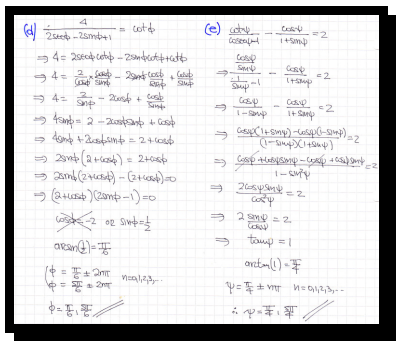
d) $\dfrac{4}{2\sec\varphi-2\sin\varphi+1}=\cot\varphi\,,\quad 0<\varphi<2\pi,\quad \varphi\neq\pi$

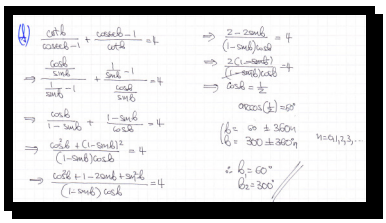
e) $\dfrac{\cot\psi}{\operatorname{cosec}\psi-1}-\dfrac{\cos\psi}{1+\sin\psi}=2\,,\quad 0<\psi<2\pi,\quad \psi\neq\dfrac{\pi}{2},\pi,\dfrac{3\pi}{2}$

f) $\dfrac{\cot\beta}{\operatorname{cosec}\beta-1}+\dfrac{\operatorname{cosec}\beta-1}{\cot\beta}=4\,,\qquad 0\le x<360°,\quad x\neq 90°,180°,270°$

$\boxed{\varphi=\dfrac{\pi}{3},\dfrac{5\pi}{3}}$, $\boxed{x=45°,225°}$, $\boxed{y=135°,315°}$, $\boxed{\varphi=\dfrac{\pi}{6},\dfrac{5\pi}{6}}$, $\boxed{\psi=\dfrac{\pi}{4},\dfrac{5\pi}{4}}$, $\boxed{\beta=60°,300°}$







Question 8

Solve each of the following equations.

a) $2\tan^2\theta = 11\sec\theta - 7, \qquad 0 \le \theta < 360°$

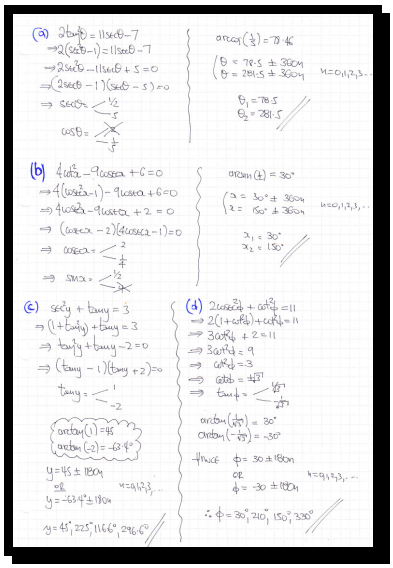
b) $4\cot^2 x - 9\operatorname{cosec} x + 6 = 0, \qquad 0 \le x < 360°$

c) $\sec^2 y + \tan y = 3, \qquad 0 \le y < 360°$

d) $2\operatorname{cosec}^2\varphi + \cot^2\varphi = 11, \quad 0 \le \varphi < 360°$

$\boxed{\theta = 78.5°,\ 281.5°}$, $\boxed{x = 30°,\ 150°}$, $\boxed{y = 45°,\ 225° \quad y \approx 116.6°,\ 296.6°}$,

$\boxed{\varphi = 30°,\ 150°,\ 210°,\ 330°}$



Question 9

Solve each of the following equations.

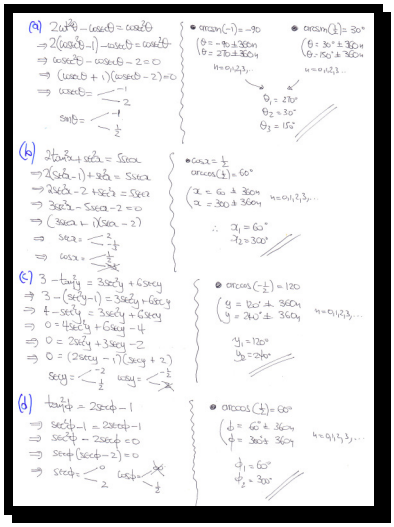
a) $2\cot^2\theta - \operatorname{cosec}\theta = \operatorname{cosec}^2\theta, \qquad 0 \le \theta < 360°$

b) $2\tan^2 x + \sec^2 x = 5\sec x, \qquad 0 \le x < 360°$

c) $3 - \tan^2 y = 3\sec^2 y + 6\sec y, \qquad 0 \le y < 360°$

d) $\tan^2\varphi = 2\sec\varphi - 1, \quad 0 \le \varphi < 360°$

$\boxed{\theta = 30°,\ 150°,\ 270°}$, $\boxed{x = 60°,\ 300°}$, $\boxed{y = 120°,\ 240°}$, $\boxed{\varphi = 60°,\ 300°}$



Question 10

Solve each of the following equations.

a) $2\cot^2\theta+6=9\operatorname{cosec}\theta\,, \qquad 0\le\theta<360°$

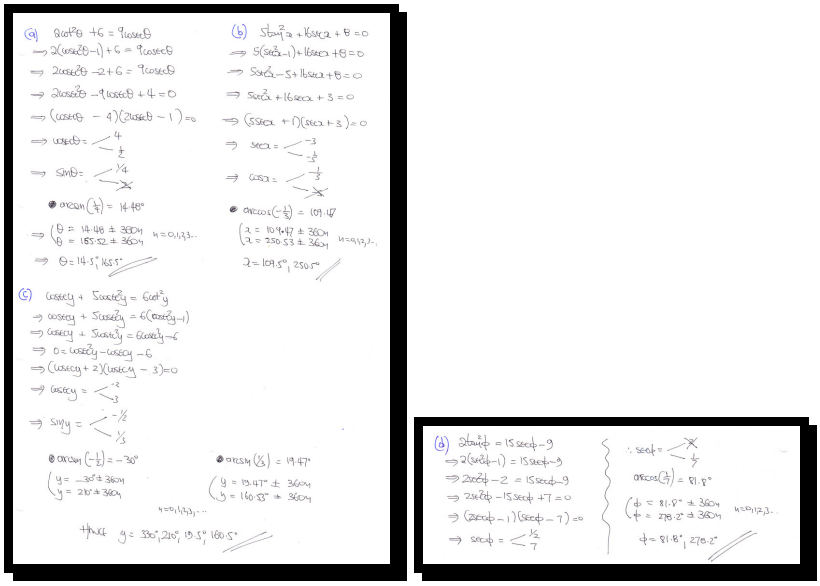
b) $5\tan^2 x+16\sec x+8=0\,, \qquad 0\le x<360°$

c) $\operatorname{cosec} y+5\operatorname{cosec}^2 y=6\cot^2 y\,, \qquad 0\le y<360°$

d) $2\tan^2\varphi=15\sec\varphi-9\,, \quad 0\le\varphi<360°$

$\boxed{\theta\approx 14.5°,\ 165.5°}$, $\boxed{x\approx 109.5°,\ 250.5°}$, $\boxed{y\approx 19.5°,\ 160.5°\quad y=210°,\ 330°}$,

$\boxed{\varphi\approx 81.8°,\ 278.2°}$



Question 11

Solve each of the following equations.

a) $4\cot^2\theta = 1 + \operatorname{cosec}\theta, \qquad 0 \le \theta < 360°$

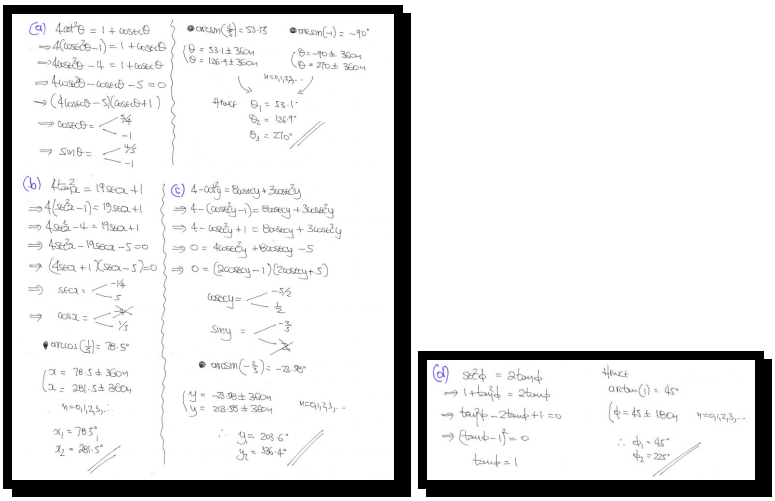
b) $4\tan^2 x = 19\sec x + 1, \qquad 0 \le x < 360°$

c) $4 - \cot^2 y = 8\operatorname{cosec} y + 3\operatorname{cosec}^2 y, \qquad 0 \le y < 360°$

d) $\sec^2\varphi = 2\tan\varphi, \quad 0 \le \varphi < 360°$

$\boxed{\theta \approx 53.1°,\ 126.9° \quad \theta = 270°}$, $\boxed{x \approx 78.5°,\ 281.5°}$, $\boxed{y \approx 203.6°,\ 336.4°}$,

$\boxed{\varphi = 45°,\ 225°}$



Question 12

Solve each of the following equations.

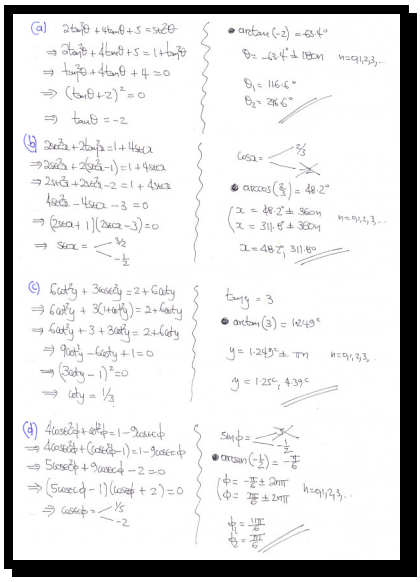
a) $2\tan^2\theta+4\tan\theta+5=\sec^2\theta\,,\qquad 0\le\theta<360^\circ$

b) $2\sec^2 x+2\tan^2 x=1+4\sec x\,,\qquad 0\le x<360^\circ$

c) $6\cot^2 y+3\operatorname{cosec}^2 y=2+6\cot y\,,\qquad 0\le y<2\pi$

d) $4\operatorname{cosec}^2\varphi+\cot^2\varphi=1-9\operatorname{cosec}\varphi\,,\quad 0\le\varphi<2\pi$

$\boxed{\theta\approx 116.6^\circ,\ 296.6^\circ}$, $\boxed{x\approx 48.2^\circ,\ 311.8^\circ}$, $\boxed{y\approx 1.25^{\text{c}},\ 4.39^{\text{c}}}$, $\boxed{\varphi=\dfrac{7\pi}{6},\ \dfrac{11\pi}{6}}$



Question 13

Solve each of the following equations.

a) $10\sec^2\theta = 11\tan\theta + 16, \qquad 0 \le \theta < 360°$

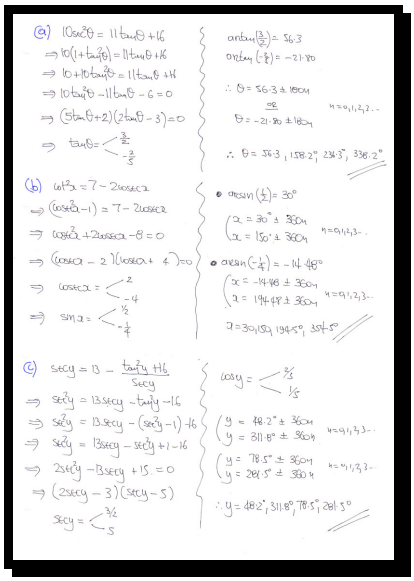
b) $\cot^2 x = 7 - 2\operatorname{cosec} x, \qquad 0 \le x < 360°$

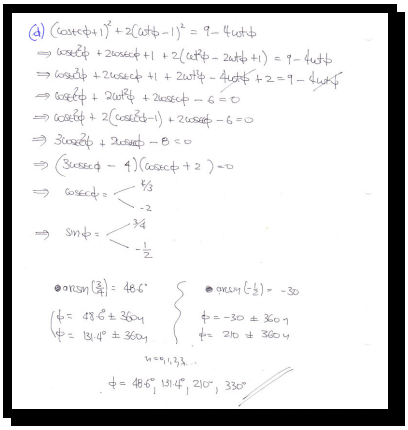
c) $\sec y = 13 - \dfrac{\tan^2 y + 16}{\sec y}, \qquad 0 \le y < 360°$

d) $(\operatorname{cosec}\varphi + 1)^2 + 2(\cot\varphi - 1)^2 = 9 - 4\cot\varphi, \quad 0 \le \varphi < 360°$

$\boxed{\theta \approx 56.3°,\ 158.2°,\ 236.3°,\ 338.2°}$, $\boxed{x = 30°,\ 150° \quad x \approx 194.5°,\ 344.5°}$,

$\boxed{y \approx 48.2°,\ 78.5°,\ 281.5°,\ 311.8°}$, $\boxed{\varphi \approx 48.6°,\ 131.4° \quad \varphi = 210°,\ 330°}$





Question 14

Solve each of the following equations.

a) $3\tan^2\theta = 8\sec\theta, \qquad 0 \le \theta < 2\pi$

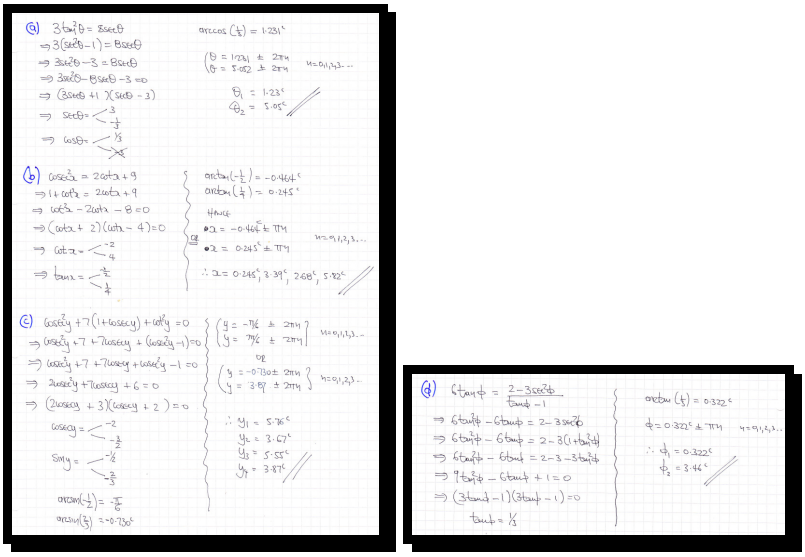
b) $\operatorname{cosec}^2 x = 2\cot x + 9, \qquad 0 \le x < 2\pi$

c) $\operatorname{cosec}^2 y + 7\left(1 + \operatorname{cosec} y\right) + \cot^2 y = 0, \qquad 0 \le y < 2\pi$

d) $6\tan\varphi = \dfrac{2 - 3\sec^2\varphi}{\tan\varphi - 1}, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 1.23^{\text{c}},\ 5.05^{\text{c}}}$, $\boxed{x \approx 0.245^{\text{c}},\ 2.68^{\text{c}},\ 3.39^{\text{c}},\ 5.82^{\text{c}}}$, $\boxed{y \approx 3.67^{\text{c}},\ 3.87^{\text{c}},\ 5.55^{\text{c}},\ 5.76^{\text{c}}}$,

$\boxed{\varphi \approx 0.322^{\text{c}},\ 3.46^{\text{c}}}$



Question 15

Solve each of the following equations.

a) $5\tan^2\theta - 12\sec\theta + 9 = 0, \qquad 0 \le \theta < 360°$

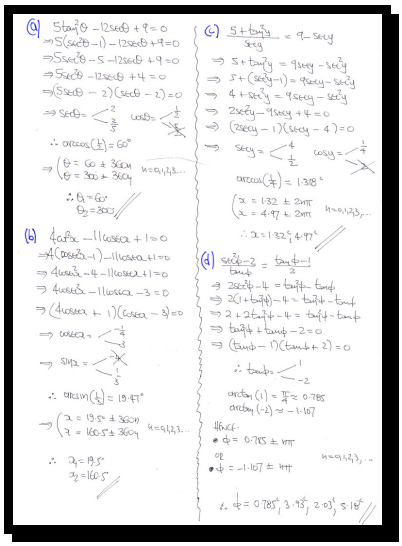
b) $4\cot^2 x - 11\operatorname{cosec} x + 1 = 0, \qquad 0 \le x < 360°$

c) $\dfrac{5+\tan^2 y}{\sec y} = 9 - \sec y, \qquad 0 \le y < 2\pi$

d) $\dfrac{\sec^2\varphi - 2}{\tan\varphi} = \dfrac{\tan\varphi - 1}{2}, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta = 60°,\ 300°}$, $\boxed{x \approx 19.5°,\ 160.5°}$, $\boxed{y \approx 1.32^{\text{c}},\ 4.97^{\text{c}}}$,

$\boxed{\varphi \approx 0.785^{\text{c}},\ 2.03^{\text{c}},\ 3.93^{\text{c}},\ 5.18^{\text{c}}}$



Question 16

Solve each of the following equations.

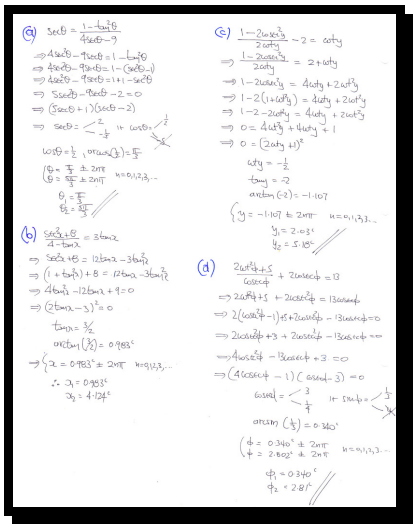
a) $\sec\theta = \dfrac{1-\tan^2\theta}{4\sec\theta - 9}, \qquad 0 \le \theta < 2\pi$

b) $\dfrac{\sec^2 x + 8}{4 - \tan x} = 3\tan x, \qquad 0 \le x < 2\pi$

c) $\dfrac{1 - 2\operatorname{cosec}^2 y}{2\cot y} - 2 = \cot y, \qquad 0 \le y < 2\pi$

d) $\dfrac{2\cot^2\varphi + 5}{\operatorname{cosec}\varphi} + 2\operatorname{cosec}\varphi = 13, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta = \dfrac{\pi}{3},\ \dfrac{5\pi}{3}}$, $\boxed{x \approx 0.983^{\text{c}},\ 4.12^{\text{c}}}$, $\boxed{y \approx 2.03^{\text{c}},\ 5.18^{\text{c}}}$, $\boxed{\varphi \approx 0.340^{\text{c}},\ 2.80^{\text{c}}}$



Question 17

Solve each of the following trigonometric equations.

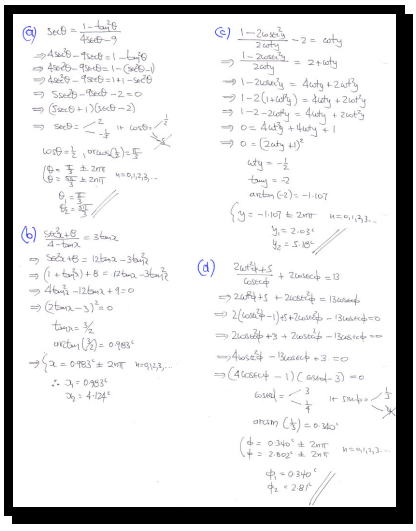
a) $\sec\theta = \dfrac{1-\tan^2\theta}{4\sec\theta - 9}, \qquad 0 \le \theta < 2\pi$

b) $\dfrac{\sec^2 x + 8}{4 - \tan x} = 3\tan x, \qquad 0 \le x < 2\pi$

c) $\dfrac{1 - 2\operatorname{cosec}^2 y}{2\cot y} - 2 = \cot y, \qquad 0 \le y < 2\pi$

d) $\dfrac{2\cot^2\varphi + 5}{\operatorname{cosec}\varphi} + 2\operatorname{cosec}\varphi = 13, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta = \dfrac{\pi}{3},\ \dfrac{5\pi}{3}}$, $\boxed{x = 0.983^{\text{c}},\ 4.12^{\text{c}}}$, $\boxed{y = 2.03^{\text{c}},\ 5.18^{\text{c}}}$, $\boxed{\varphi = 0.340^{\text{c}},\ 2.80^{\text{c}}}$



Question 17

Solve each of the following trigonometric equations.

a) $\cos(\theta+30^\circ)=\sin\theta\,, \qquad 0\le\theta<360^\circ$

b) $3\cos(x+30^\circ)=\sin(x-60^\circ)\,, \qquad 0\le x<360^\circ$

c) $\sin(y-30^\circ)=\sin(y+45^\circ)\,, \qquad 0\le y<360^\circ$

d) $\sin(\varphi+30^\circ)=\cos(\varphi-45^\circ)\,, \qquad 0\le\varphi<360^\circ$

e) $\cos(\alpha-60^\circ)=\cos(\alpha-45^\circ)\,, \qquad 0\le\alpha<360^\circ$

$\boxed{\theta=30^\circ,\ 210^\circ}$, $\boxed{x=60^\circ,\ 240^\circ}$, $\boxed{y=82.5^\circ,\ 262.5^\circ}$, $\boxed{\varphi=52.5^\circ,\ 232.5^\circ}$,
$\boxed{\alpha=52.5^\circ,\ 232.5^\circ}$



Question 18

Solve each of the following trigonometric equations.

a) $\sin(\theta-45^\circ)=\sin\theta, \qquad 0\le\theta<360^\circ$

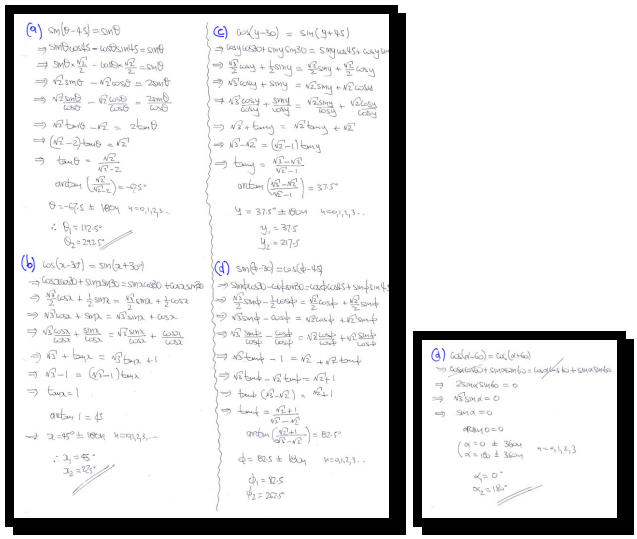
b) $\cos(x-30^\circ)=\sin(x+30^\circ), \qquad 0\le x<360^\circ$

c) $\cos(y-30^\circ)=\sin(y+45^\circ), \qquad 0\le y<360^\circ$

d) $\sin(\varphi-30^\circ)=\cos(\varphi-45^\circ), \qquad 0\le\varphi<360^\circ$

e) $\cos(\alpha-60^\circ)=\cos(\alpha+60^\circ), \qquad 0\le\alpha<360^\circ$

$\boxed{\theta=112.5^\circ,\ 292.5^\circ}$, $\boxed{x=45^\circ,\ 225^\circ}$, $\boxed{y=37.5^\circ,\ 217.5^\circ}$, $\boxed{\varphi=82.5^\circ,\ 262.5^\circ}$,
$\boxed{\alpha=0^\circ,\ 180^\circ}$



Question 19

Solve each of the following trigonometric equations.

a) $\sin\left(\theta+\frac{\pi}{4}\right)=\sin\theta\,, \qquad 0\le\theta<2\pi$

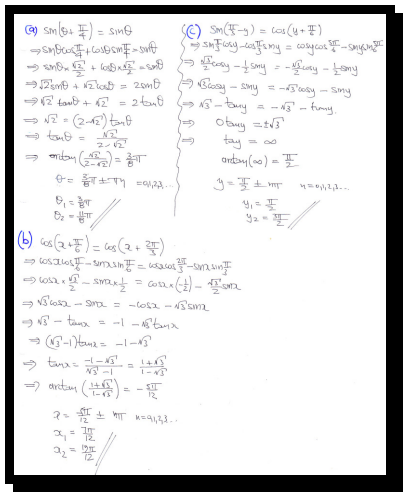
b) $\cos\left(x+\frac{\pi}{6}\right)=\cos\left(x+\frac{2\pi}{3}\right), \qquad 0\le\theta<2\pi$

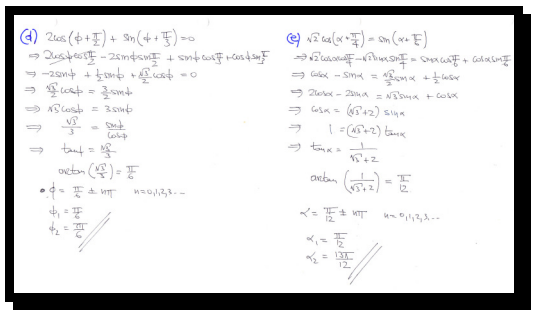
c) $\sin\left(\frac{\pi}{3}-y\right)=\cos\left(y+\frac{5\pi}{6}\right), \qquad 0\le y<2\pi$ (very hard)

d) $2\cos\left(\varphi+\frac{\pi}{2}\right)+\sin\left(\varphi+\frac{\pi}{3}\right)=0\,, \qquad 0\le\varphi<2\pi$

e) $\sqrt{2}\cos\left(\alpha+\frac{\pi}{4}\right)=\sin\left(\alpha+\frac{\pi}{6}\right), \qquad 0\le\alpha<2\pi$

$$\boxed{\theta=\frac{3\pi}{8},\ \frac{11\pi}{8}}\,,\quad \boxed{x=\frac{7\pi}{12},\ \frac{19\pi}{12}}\,,\quad \boxed{y=\frac{\pi}{2},\ \frac{3\pi}{2}}\,,\quad \boxed{\varphi=\frac{\pi}{6},\ \frac{7\pi}{6}}\,,\quad \boxed{\alpha=\frac{\pi}{12},\ \frac{13\pi}{12}}$$





Question 20

Solve each of the following trigonometric equations.

a) $\sin(\theta-20°)=\sin(\theta+60°), \qquad 0\le\theta<360°$

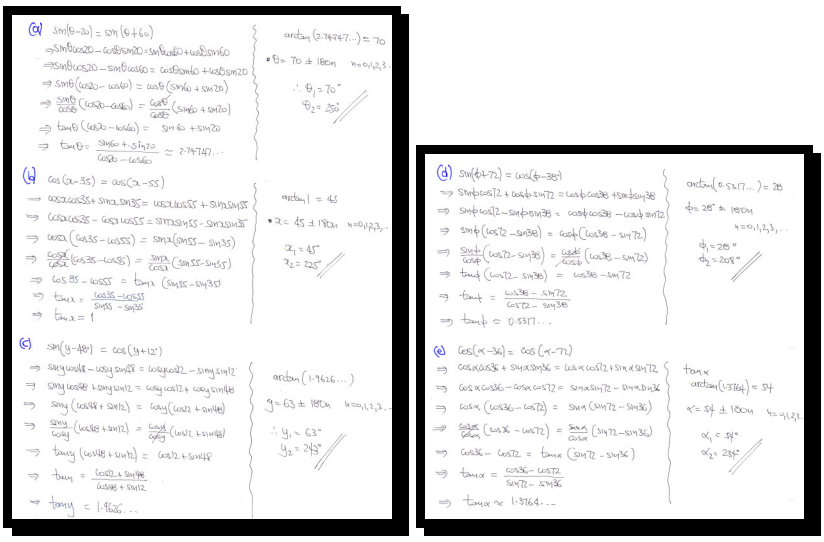
b) $\cos(x-35°)=\cos(x-55°), \qquad 0\le x<360°$

c) $\sin(y-48°)=\cos(y+12°), \qquad 0\le y<360°$

d) $\sin(\varphi+72°)=\cos(\varphi-38°), \qquad 0\le\varphi<360°$

e) $\cos(\alpha-36°)=\cos(\alpha-72°), \qquad 0\le\alpha<360°$

$\boxed{\theta=70°,\ 250°}$, $\boxed{x=45°,\ 225°}$, $\boxed{y=63°,\ 243°}$, $\boxed{\varphi=28°,\ 208°}$, $\boxed{\alpha=54°,\ 234°}$



Question 21

Solve each of the following trigonometric equations.

a) $\sin 2\theta = \tan\theta, \qquad 0 \le \theta \le 180°$

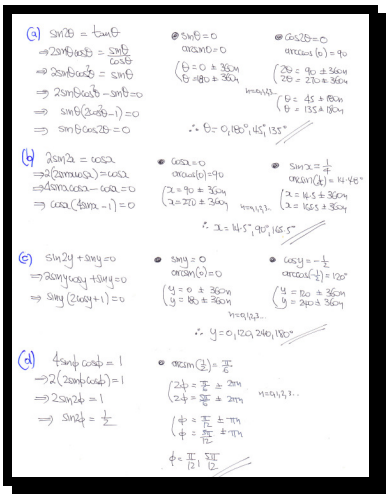
b) $2\sin 2x = \cos x, \qquad 0 \le x < 180°$

c) $\sin 2y + \sin y = 0, \qquad 0 \le y < 360°$

d) $4\sin\varphi\cos\varphi = 1, \qquad 0 \le \varphi < \pi$

$\boxed{\theta = 0°,\ 45°,\ 135°,\ 180°}$, $\boxed{x = 90°,\ x \approx 14.5°,\ 165.5°}$, $\boxed{y = 0°,\ 120°,\ 180°,\ 240°}$,

$\boxed{\varphi = \frac{\pi}{12},\ \frac{5\pi}{12}}$



Question 22

Solve each of the following trigonometric equations.

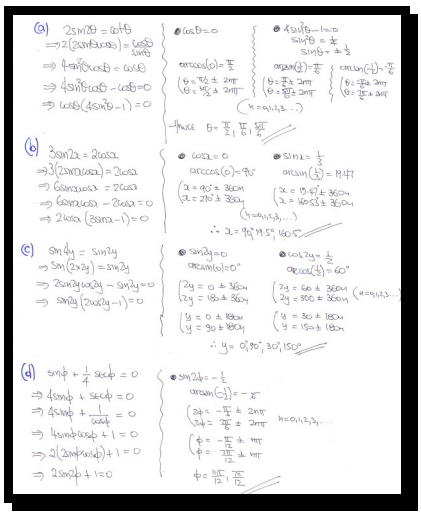
a) $2\sin 2\theta = \cot\theta\,, \qquad 0 \le \theta \le \pi$

b) $3\sin 2x = 2\cos x\,, \qquad 0 \le x < 180°$

c) $\sin 4y = \sin 2y\,, \qquad 0 \le y < 180°$

d) $\sin\varphi + \frac{1}{4}\sec\varphi = 0\,, \qquad 0 \le \varphi < \pi$

$\boxed{\theta = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}}$, $\boxed{x = 90°,\ x \approx 19.5°,\ 160.5°}$, $\boxed{y = 0°,\ 30°,\ 90°,\ 150°}$, $\boxed{\varphi = \frac{7\pi}{12},\ \frac{11\pi}{12}}$



Question 23

Solve each of the following trigonometric equations.

a) $\cos\theta - \sin 2\theta = 0\,, \qquad 0 \le \theta \le 360°$

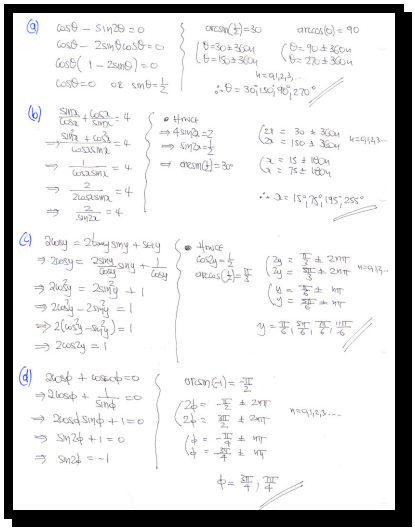
b) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = 4\,, \qquad 0 \le x < 360°$

c) $2\cos y = 2\tan y \sin y + \sec y\,, \qquad 0 \le y < 2\pi$

d) $2\cos\varphi + \operatorname{cosec}\varphi = 0\,, \qquad 0 \le \varphi < 2\pi$

$\boxed{\theta = 30°,\ 90°,\ 150°,\ 270°}$, $\boxed{x = 15°,\ 75°,\ 195°,\ 255°}$, $\boxed{y = \dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}}$,

$\boxed{\varphi = \dfrac{3\pi}{4},\ \dfrac{7\pi}{4}}$



Question 24

Solve each of the following trigonometric equations.

a) $2\cos 2\theta = 1+\cos\theta, \qquad 0 \le \theta < 360°$

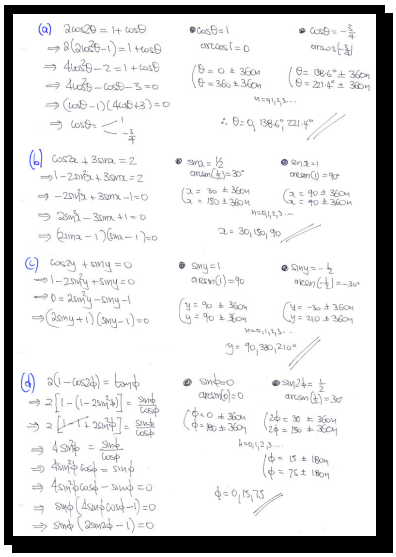
b) $\cos 2x + 3\sin x = 2, \qquad 0 \le x < 360°$

c) $\cos 2y + \sin y = 0, \qquad 0 \le y < 360°$

d) $2(1-\cos 2\varphi) = \tan\varphi, \qquad 0 \le \varphi < 180°$

$\boxed{\theta = 0°,\ \theta \approx 138.6°,\ 221.4°}$, $\boxed{x = 30°,\ 90°,\ 150°}$, $\boxed{y = 90°,\ 210°,\ 330°}$,

$\boxed{\varphi = 0°,\ 15°,\ 75°}$



Question 25

Solve each of the following trigonometric equations.

a) $\cos 2\theta = 1 + \sin\theta\,, \qquad 0 \le \theta < 360°$

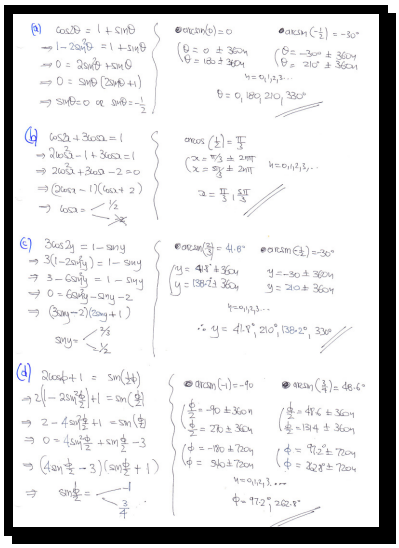
b) $\cos 2x + 3\cos x = 1\,, \qquad 0 \le x < 2\pi$

c) $3\cos 2y = 1 - \sin y\,, \qquad 0 \le y < 360°$

d) $2\cos\varphi + 1 = \sin\left(\frac{1}{2}\varphi\right), \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 0°,\ 180°,\ 210°,\ 330°}$, $\boxed{x = \frac{\pi}{3},\ \frac{5\pi}{3}}$, $\boxed{y \approx 41.8°,\ 138.2° \quad y = 210°,\ 330°}$,

$\boxed{\varphi = 97.2°,\ 262.8°}$



Question 26

a) $\cos 2\theta - 7\sin\theta - 4 = 0, \qquad 0 \le \theta < 360°$

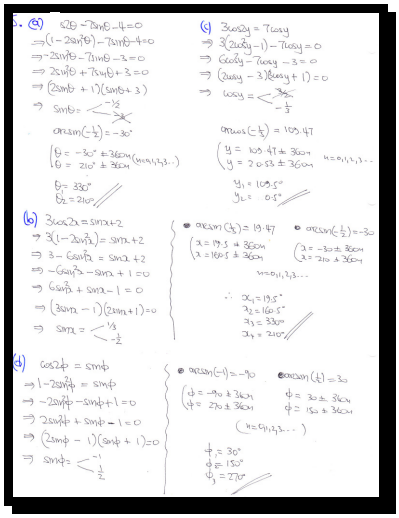
b) $3\cos 2x = \sin x + 2, \qquad 0 \le x < 360°$

c) $3\cos 2y = 7\cos y, \qquad 0 \le y < 360°$

d) $\cos 2\varphi = \sin\varphi, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330°}$, $\boxed{x \approx 19.5°,\ 160.5° \quad x = 210°,\ 330°}$, $\boxed{y \approx 109.5°,\ 250.5°}$,

$\boxed{\varphi = 30°,\ 150°,\ 270°}$



Question 27

Solve each of the following trigonometric equations.

a) $3\cos 2\theta - 5\sin\theta = 4\,, \qquad 0 \le \theta < 360°$

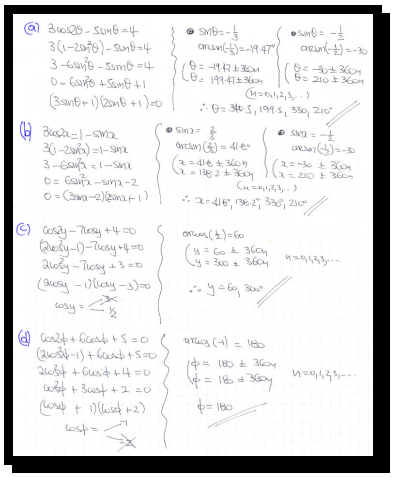
b) $3\cos 2x = 1 - \sin x\,, \qquad 0 \le x < 360°$

c) $\cos 2y - 7\cos y + 4 = 0\,, \qquad 0 \le y < 360°$

d) $\cos 2\varphi + 6\cos\varphi + 5 = 0\,, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330° \quad \theta \approx 199.5°,\ 340.5°}$, $\boxed{x \approx 41.8°,\ 138.2° \quad x = 210°,\ 330°}$,

$\boxed{y = 60°,\ 300°}$, $\boxed{\varphi = 180°}$



Question 28

Solve each of the following trigonometric equations.

a) $\cos 2\theta = 7\cos\theta + 3\,, \qquad 0 \le \theta < 360°$

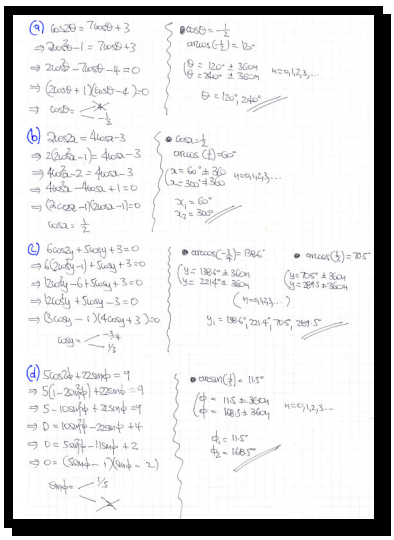
b) $2\cos 2x = 4\cos x - 3\,, \qquad 0 \le x < 360°$

c) $6\cos 2y + 5\cos y + 3 = 0\,, \qquad 0 \le y < 360°$

d) $5\cos 2\varphi + 22\sin\varphi = 9\,, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 120°,\ 240°}$, $\boxed{x = 60°,\ 300°}$, $\boxed{y \approx 70.5°,\ 138.6°,\ 221.4°,\ 289.5°}$,

$\boxed{\varphi \approx 11.5°,\ 168.5°}$



Question 29

Solve each of the following trigonometric equations.

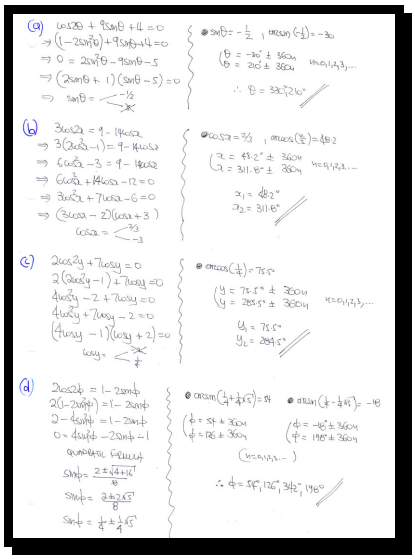
a) $\cos 2\theta + 9\sin\theta + 4 = 0\,, \qquad 0 \le \theta < 360°$

b) $3\cos 2x = 9 - 14\cos x\,, \qquad 0 \le x < 360°$

c) $2\cos 2y + 7\cos y = 0\,, \qquad 0 \le y < 360°$

d) $2\cos 2\varphi = 1 - 2\sin\varphi\,, \qquad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330°}$, $\boxed{x \approx 48.2°,\ 311.8°}$, $\boxed{y \approx 75.5°,\ 284.5°}$, $\boxed{\varphi = 54°,\ 126°,\ 198°,\ 342°}$



Question 30

Solve each of the following trigonometric equations.

a) $\tan\theta\left(1+\cos 2\theta\right)=2\sin^2 2\theta\,,\qquad 0\le\theta\le 90°$

b) $4\tan 2\varphi+3\cot\varphi\sec^2\varphi=0\,,\qquad 0\le\varphi<2\pi$ (hard)

$\boxed{\theta=0°,15°,75°,90°}$, $\boxed{\varphi=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}}$