TRIGONOMETRIC EQUATIONS & IDENTITIES INTRODUCTION

Question 1

Solve each of the following trigonometric equations in the range given.

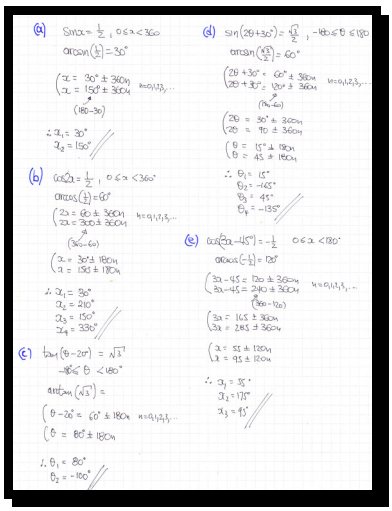
a) $\sin x = \frac{1}{2}$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 30°, 150°}$

b) $\cos 2x = \frac{1}{2}$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 30°, 150°, 210°, 330°}$

c) $\tan(\theta - 20°) = \sqrt{3}$ $\qquad -180° \le \theta < 180°$ $\qquad \boxed{\theta = -100°, 80°}$

d) $\sin(2\theta + 30°) = \frac{\sqrt{3}}{2}$ $\qquad -180° \le \theta < 180°$ $\qquad \boxed{\theta = -165°, -135°, 15°, 45°}$

e) $\cos(3x - 45°) = -\frac{1}{2}$ $\qquad 0 \le x < 180°$ $\qquad \boxed{x = 55°, 95°, 175°}$



Question 2

Solve each of the following trigonometric equations in the range given.

a)	$\sin x = -\frac{1}{2}$	$0 \le x < 360°$	$\boxed{x = 210°, 330°}$
b)	$\cos 2x = -0.1736$	$0 \le x < 360°$	$\boxed{x = 50°, 130°, 230°, 310°}$
c)	$\tan(x - 40°) = 0.7$	$-180° \le x < 180°$	$\boxed{x = -105°, 75°}$
d)	$\sin(2\theta + 40°) = 0.9848$	$-180° \le \theta < 180°$	$\boxed{\theta = -160°, -150°, 20°, 30°}$
e)	$\cos(4y - 60°) = \frac{\sqrt{6}-\sqrt{2}}{4}$	$0 \le y < 90°$	$\boxed{x = 33.8°, 86.3°}$



Question 3

Solve each of the following trigonometric equations in the range given.

a) $\tan\left(\frac{3}{2}x\right)=-\frac{\sqrt{3}}{3}$ $\quad 0\le x<360°$ $\quad x=100°,220°,340°$

b) $\sin(4y-45°)=\frac{\sqrt{6}-\sqrt{2}}{4}$ $\quad -90°\le y<90°$ $\quad y=-75°,-37.5°,15°,52.5°$

c) $5\cos 2x=2$ $\quad 0\le x<360°$ $\quad x=33.2°,146.8°,213.2°,326.8°$

d) $3-\tan(\theta-10°)=1$ $\quad 0\le \theta<360°$ $\quad \theta=73.4°,253.4°$

e) $3\sin(3\theta-30°)+4=5$ $\quad -90°\le \theta<90°$ $\quad \theta=-56.5°,16.5°,63.5°$



Question 4

Solve each of the following trigonometric equations in the range given.

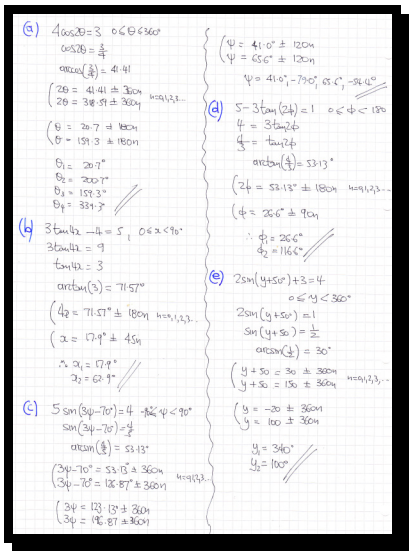
a) $4\cos 2\theta = 3$ $\quad 0 \le \theta < 360°$ $\quad \boxed{\theta \approx 20.7°, 159.3°, 200.7°, 339.3°}$

b) $3\tan 4x - 4 = 5$ $\quad 0 \le x < 90°$ $\quad \boxed{x \approx 17.9°, 62.9°}$

c) $5\sin(3\psi - 70°) = 4$ $\quad -90° \le \psi < 90°$ $\quad \boxed{\psi \approx -79.0°, -54.4°, 41.0°, 65.6°}$

d) $5 - 3\tan 2\varphi = 1$ $\quad 0 \le \varphi < 180°$ $\quad \boxed{\varphi \approx 26.6°, 116.6°}$

e) $2\sin(y + 50°) + 3 = 4$ $\quad 0 \le y < 360°$ $\quad \boxed{y = 100°, 340°}$



Question 5

Solve each of the following trigonometric equations in the range given.

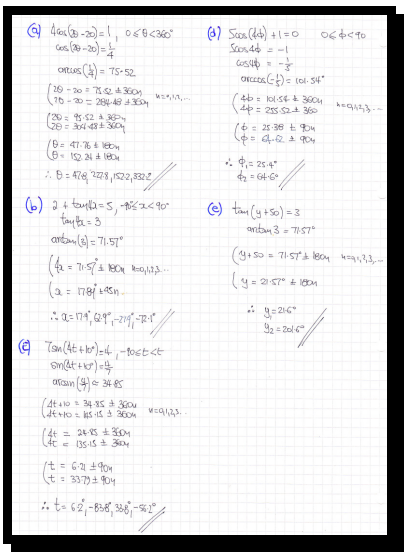
a) $4\cos(2\theta - 20°) = 1$ $\quad 0 \le \theta < 360°$ $\quad \boxed{\theta \approx 47.8°, 152.2°, 227.8°, 332.2°}$

b) $2 + \tan 4x = 5$ $\quad -90° \le x < 90°$ $\quad \boxed{x \approx -72.1°, -27.1°, 17.9°, 62.9°}$

c) $7\sin(4t + 10°) = 4$ $\quad -90° \le t < 90°$ $\quad \boxed{t \approx -83.8°, -56.2°, 6.2°, 33.8°}$

d) $5\cos(4\varphi) + 1 = 0$ $\quad 0 \le \varphi < 90°$ $\quad \boxed{\varphi \approx 25.4°, 64.6°}$

e) $\tan(y + 50°) = 3$ $\quad 0 \le y < 360°$ $\quad \boxed{y \approx 21.6°, 201.6°}$



Question 6

Solve each of the following trigonometric equations in the range given.

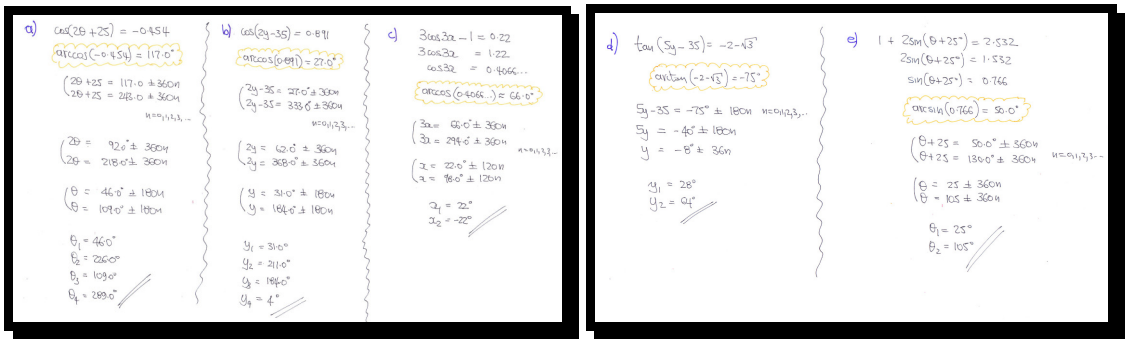
a) $\cos(2\theta+25)^\circ=-0.454$ $\quad 0\le\theta<360^\circ$ $\quad \boxed{\theta\approx 46^\circ,109^\circ,226^\circ,289^\circ}$

b) $\cos(2y-35)^\circ=0.891$ $\quad 0\le y<360^\circ$ $\quad \boxed{\theta\approx 4^\circ,31^\circ,184^\circ,211^\circ}$

c) $3\cos 3x-1=0.22$ $\quad -90^\circ\le x<90^\circ$ $\quad \boxed{x\approx -22^\circ,22^\circ}$

d) $\tan(5y-35)^\circ=-2-\sqrt{3}$ $\quad 0\le y<90^\circ$ $\quad \boxed{\theta\approx 28^\circ,64^\circ}$

e) $1+2\sin(\theta+25)^\circ=2.532$ $\quad 0\le\theta<360^\circ$ $\quad \boxed{\theta\approx 25^\circ,105^\circ}$



Question 7

Solve each of the following trigonometric equations in the range given.

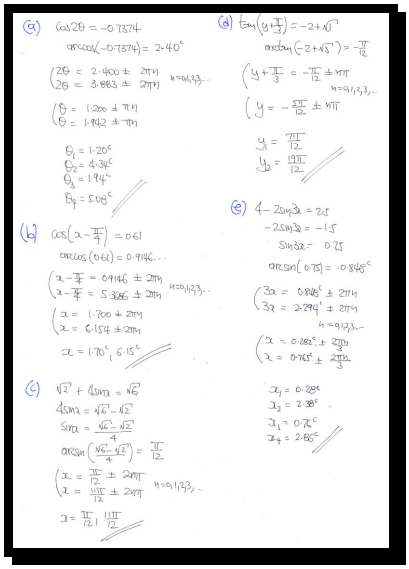
a) $\cos 2\theta = -0.7374$ $\qquad 0 \le \theta < 2\pi$ $\qquad \boxed{\theta \approx 1.20^{\text{c}}, 1.94^{\text{c}}, 4.34^{\text{c}}, 5.08^{\text{c}}}$

b) $\cos\left(x - \frac{\pi}{4}\right) = 0.61$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x \approx 1.70^{\text{c}}, 6.15^{\text{c}}}$

c) $\sqrt{2} + 4\sin x = \sqrt{6}$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{\pi}{12}, \frac{11\pi}{12}}$

d) $2 + \tan\left(y + \frac{\pi}{3}\right) = \sqrt{3}$ $\qquad 0 \le y < 2\pi$ $\qquad \boxed{y = \frac{7\pi}{12}, \frac{19\pi}{12}}$

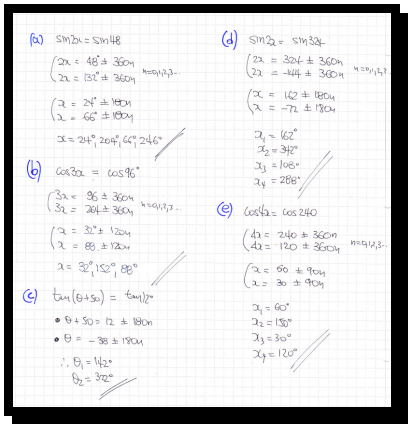
e) $4 - 2\sin 3x = 2.5$ $\qquad 0 \le x < \pi$ $\qquad \boxed{x \approx 0.28^{\text{c}}, 0.76^{\text{c}}, 2.38^{\text{c}}, 2.86^{\text{c}}}$



Question 8

Solve each of the following trigonometric equations in the range given.

a)	$\sin 2x = \sin 48°$	$0 \le x < 360°$	$\boxed{x = 24°, 66°, 204°, 246°}$
b)	$\cos 3x = \cos 96°$	$0 \le x < 180°$	$\boxed{x = 32°, 88°, 152°}$
c)	$\tan(\theta + 50°) = \tan 12°$	$0 \le \theta < 360°$	$\boxed{\theta = 142°, 322°}$
d)	$\sin 2x = \sin 324°$	$0 \le x < 360°$	$\boxed{x = 108°, 162°, 288°, 342°}$
e)	$\cos 4x = \cos 240°$	$0 \le x < 180°$	$\boxed{x = 30°, 60°, 120°, 150°}$



Question 9

Solve each of the following trigonometric equations in the range given.

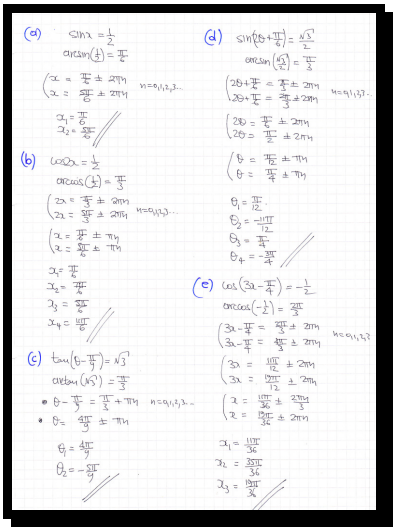
a) $\sin x = \frac{1}{2}$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

b) $\cos 2x = \frac{1}{2}$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

c) $\tan\left(\theta - \frac{\pi}{9}\right) = \sqrt{3}$ $\qquad -\pi \le \theta < \pi$ $\qquad \boxed{\theta = -\frac{5\pi}{9}, \frac{4\pi}{9}}$

d) $\sin\left(2\theta + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $\qquad -\pi \le \theta < \pi$ $\qquad \boxed{\theta = -\frac{11\pi}{12}, -\frac{3\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}}$

e) $\cos\left(3x - \frac{\pi}{4}\right) = -\frac{1}{2}$ $\qquad 0 \le x < \pi$ $\qquad \boxed{x = \frac{11\pi}{36}, \frac{19\pi}{36}, \frac{35\pi}{36}}$



Question 10

Solve each of the following trigonometric equations in the range given.

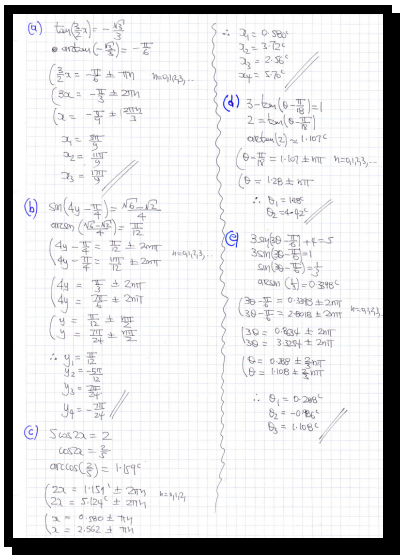
a) $\tan\left(\frac{3x}{2}\right)=-\frac{\sqrt{3}}{3}$ $\qquad 0\le x<2\pi$ $\qquad \boxed{x=\frac{5\pi}{9},\frac{11\pi}{9},\frac{17\pi}{9}}$

b) $\sin\left(4y-\frac{\pi}{4}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$ $\qquad -\frac{\pi}{2}\le y<\frac{\pi}{2}$ $\qquad \boxed{y=-\frac{5\pi}{12},-\frac{5\pi}{24},\frac{\pi}{12},\frac{7\pi}{24}}$

c) $5\cos 2x=2$ $\qquad 0\le x<2\pi$ $\qquad \boxed{x=0.58^{\text{c}},2.56^{\text{c}},3.72^{\text{c}},5.70^{\text{c}}}$

d) $3-\tan\left(\theta-\frac{\pi}{18}\right)=1$ $\qquad 0\le \theta<2\pi$ $\qquad \boxed{\theta=1.28^{\text{c}},4.42^{\text{c}}}$

e) $3\sin\left(3\theta-\frac{\pi}{6}\right)+4=5$ $\qquad -\frac{\pi}{2}\le \theta<\frac{\pi}{2}$ $\qquad \boxed{\theta=-0.99^{\text{c}},0.29^{\text{c}},1.11^{\text{c}}}$



Question 11

Solve each of the following trigonometric equations in the range given.

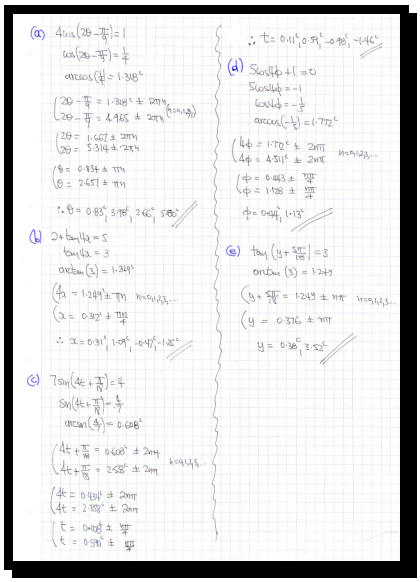
a) $4\cos\left(2\theta-\frac{\pi}{9}\right)=1$ $\qquad 0\le\theta<2\pi$ $\qquad \boxed{\theta\approx 0.83^{\text{c}},2.66^{\text{c}},3.98^{\text{c}},5.80^{\text{c}}}$

b) $2+\tan 4x=5$ $\qquad -\frac{\pi}{2}\le x<\frac{\pi}{2}$ $\qquad \boxed{x\approx -1.26^{\text{c}},-0.47^{\text{c}},0.31^{\text{c}},1.10^{\text{c}}}$

c) $7\sin\left(4t+\frac{\pi}{18}\right)=4$ $\qquad -\frac{\pi}{2}\le t<\frac{\pi}{2}$ $\qquad \boxed{t\approx -1.46^{\text{c}},-0.98^{\text{c}},0.11^{\text{c}},0.59^{\text{c}}}$

d) $5\cos 4\varphi+1=0$ $\qquad 0\le\varphi<\frac{\pi}{2}$ $\qquad \boxed{\varphi\approx 0.44^{\text{c}},1.13^{\text{c}}}$

e) $\tan\left(y+\frac{5\pi}{18}\right)=3$ $\qquad 0\le y<2\pi$ $\qquad \boxed{y\approx 0.38^{\text{c}},3.52^{\text{c}}}$



Question 12

Solve each of the following trigonometric equations in the range given.

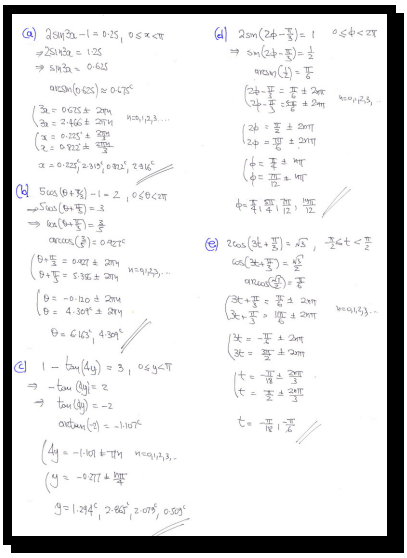
a) $2\sin 3x - 1 = 0.25$ $\qquad 0 \le \theta < \pi$ $\qquad \boxed{x \approx 0.225^{\text{c}}, 0.822^{\text{c}}, 2.32^{\text{c}}, 2.92^{\text{c}}}$

b) $5\cos\left(\theta + \frac{\pi}{3}\right) - 1 = 2$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{\theta = 4.31^{\text{c}}, 6.16^{\text{c}}}$

c) $1 - \tan 4y = 3$ $\qquad 0 \le y < \pi$ $\qquad \boxed{y \approx 0.509^{\text{c}}, 1.29^{\text{c}}, 2.08^{\text{c}}, 2.87^{\text{c}}}$

d) $2\sin\left(2\varphi - \frac{\pi}{3}\right) = 1$ $\qquad 0 \le \varphi < 2\pi$ $\qquad \boxed{\varphi = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}}$

e) $2\cos\left(3t + \frac{\pi}{3}\right) = \sqrt{3}$ $\qquad -\frac{\pi}{2} \le t < \frac{\pi}{2}$ $\qquad \boxed{t = -\frac{\pi}{6}, -\frac{\pi}{18}}$



Question 13

Solve each of the following trigonometric equations in the range given.

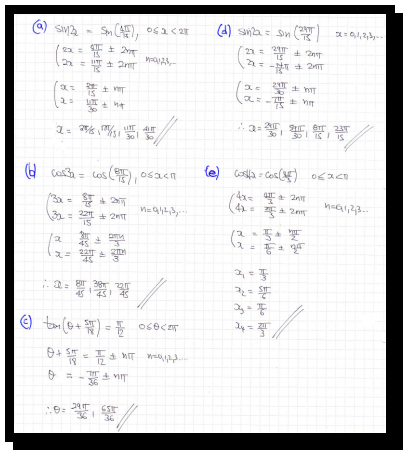
a) $\sin 2x = \sin\left(\frac{4\pi}{15}\right)$ $\qquad 0 \le \theta < 2\pi$ $\qquad \boxed{x = \frac{2\pi}{15}, \frac{11\pi}{30}, \frac{17\pi}{15}, \frac{41\pi}{30}}$

b) $\cos 3x = \cos\left(\frac{8\pi}{15}\right)$ $\qquad 0 \le x < \pi$ $\qquad \boxed{x = \frac{8\pi}{45}, \frac{22\pi}{45}, \frac{38\pi}{45}}$

c) $\tan\left(\theta + \frac{5\pi}{18}\right) = \tan\left(\frac{\pi}{12}\right)$ $\qquad 0 \le \theta < 2\pi$ $\qquad \boxed{\theta = \frac{29\pi}{36}, \frac{65\pi}{36}}$

d) $\sin 2x = \sin\left(\frac{29\pi}{15}\right)$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{8\pi}{15}, \frac{29\pi}{30}, \frac{23\pi}{15}, \frac{59\pi}{30}}$

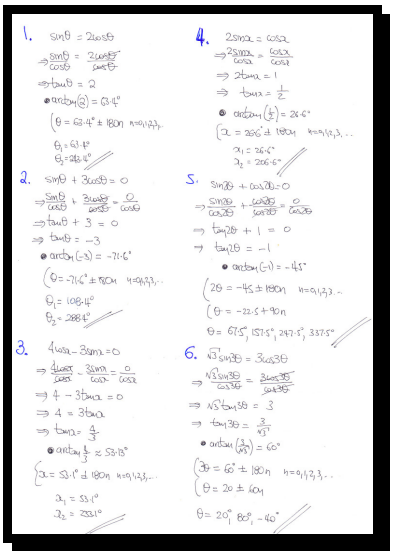
e) $\cos 4x = \cos\left(\frac{4\pi}{3}\right)$ $\qquad 0 \le x < \pi$ $\qquad \boxed{x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}}$



Question 14

Solve each of the following trigonometric equations in the range given.

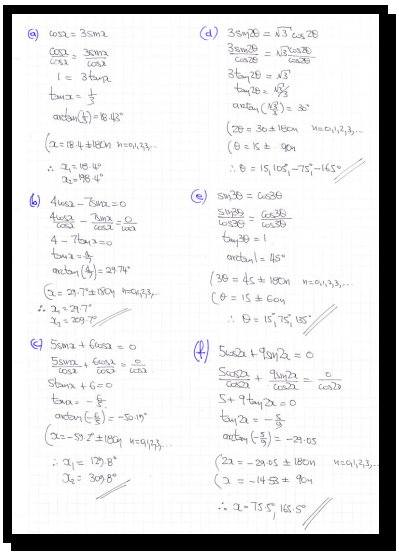
a)	$\sin\theta = 2\cos\theta$	$0 \le \theta < 360°$	$\theta \approx 63.4°, 243.4°$
b)	$\sin\theta + 3\cos\theta = 0$	$0 \le \theta < 360°$	$\theta \approx 108.4°, 288.4°$
c)	$4\cos x - 3\sin x = 0$	$0 \le x < 360°$	$x \approx 53.1°, 233.1°$
d)	$2\sin x = \cos x$	$0 \le x < 360°$	$x \approx 26.6°, 206.6°$
e)	$\sin 2\theta + \cos 2\theta = 0$	$0 \le \theta < 360°$	$\theta = 67.5°, 157.5°, 247.5°, 337.5°$
f)	$\sqrt{3}\sin 3\theta = 3\cos 3\theta$	$-90° \le \theta < 90°$	$\theta = -40°, 20°, 80°$



Question 15

Solve each of the following trigonometric equations in the range given.

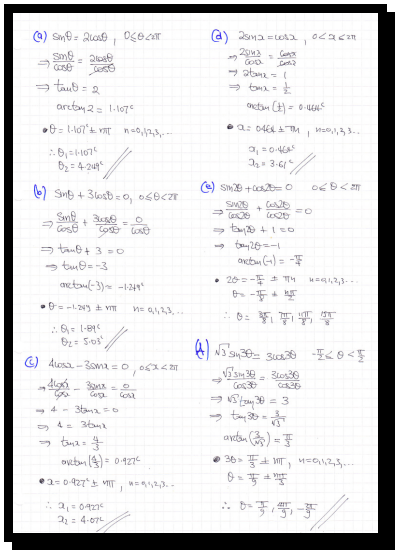
a)	$\cos x = 3\sin x$	$0 \le x < 360°$	$\boxed{x \approx 18.4°, 198.4°}$
b)	$4\cos x - 7\sin x = 0$	$0 \le x < 360°$	$\boxed{x \approx 29.7°, 209.7°}$
c)	$5\sin x + 6\cos x = 0$	$0 \le x < 360°$	$\boxed{x \approx 129.8°, 309.8°}$
d)	$3\sin 2\theta = \sqrt{3}\cos 2\theta$	$-180 \le \theta < 180°$	$\boxed{\theta = -165°, -75°, 15°, 105°}$
e)	$\sin 3\theta = \cos 3\theta$	$0 \le \theta < 180°$	$\boxed{\theta = 15°, 75°, 135°}$
f)	$5\cos 2x + 9\sin 2x = 0$	$0 \le x < 180°$	$\boxed{x \approx 75.5°, 165.5°}$



Question 16

Solve each of the following trigonometric equations in the range given.

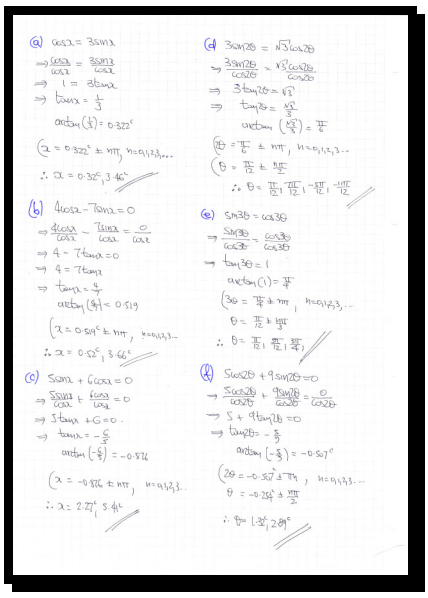
a)	$\sin\theta = 2\cos\theta$	$0 \le \theta < 2\pi$	$\theta \approx 1.11^{\text{c}}, 4.25^{\text{c}}$
b)	$\sin\theta + 3\cos\theta = 0$	$0 \le \theta < 2\pi$	$\theta \approx 1.89^{\text{c}}, 5.03^{\text{c}}$
c)	$4\cos x - 3\sin x = 0$	$0 \le x < 2\pi$	$x \approx 0.927^{\text{c}}, 4.07^{\text{c}}$
d)	$2\sin x = \cos x$	$0 \le x < 2\pi$	$x \approx 0.464^{\text{c}}, 3.61^{\text{c}}$
e)	$\sin 2\theta + \cos 2\theta = 0$	$0 \le \theta < 2\pi$	$\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$
f)	$\sqrt{3}\sin 3\theta = 3\cos 3\theta$	$-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$	$\theta = -\frac{2\pi}{9}, \frac{\pi}{9}, \frac{4\pi}{9}$



Question 17

Solve each of the following trigonometric equations in the range given.

a)	$\cos x = 3\sin x$	$0 \le x < 2\pi$	$\boxed{x \approx 0.32^{\text{c}}, 3.46^{\text{c}}}$
b)	$4\cos x - 7\sin x = 0$	$0 \le x < 2\pi$	$\boxed{x \approx 0.52^{\text{c}}, 3.66^{\text{c}}}$
c)	$5\sin x + 6\cos x = 0$	$0 \le x < 2\pi$	$\boxed{x \approx 2.27^{\text{c}}, 5.41^{\text{c}}}$
d)	$3\sin 2\theta = \sqrt{3}\cos 2\theta$	$-\pi \le \theta < \pi$	$\boxed{\theta = -\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}}$
e)	$\sin 3\theta = \cos 3\theta$	$0 \le \theta < \pi$	$\boxed{\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}}$
f)	$5\cos 2\theta + 9\sin 2\theta = 0$	$0 \le \theta < \pi$	$\boxed{\theta \approx 1.32^{\text{c}}, 2.89^{\text{c}}}$



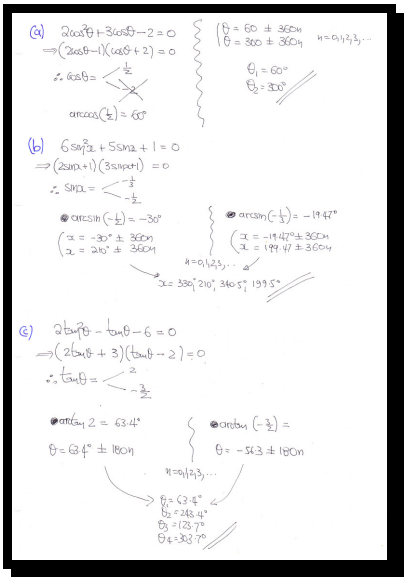
Question 18

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2\theta+3\cos\theta-2=0$ $\quad 0\le\theta<360°$ $\quad \boxed{\theta=60°,300°}$

b) $6\sin^2 x+5\sin x+1=0$ $\quad 0\le x<360°$ $\quad \boxed{x=210°,330°,x\approx 340.5°199.5°}$

c) $2\tan^2\theta-\tan\theta-6=0$ $\quad 0\le\theta<360°$ $\quad \boxed{\theta\approx 63.4°,123.7°,243.4°,303.7°}$



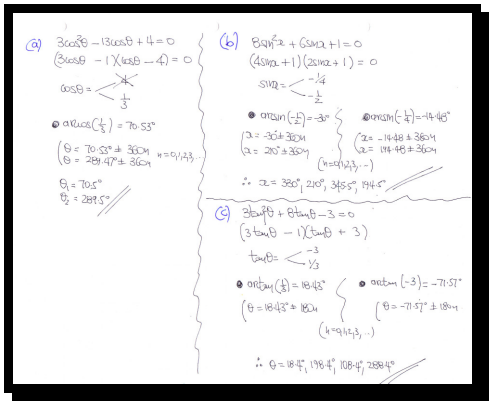
Question 19

Solve each of the following trigonometric equations in the range given.

a) $3\cos^2\theta - 13\cos\theta + 4 = 0 \qquad 0 \le \theta < 360° \qquad \boxed{\theta \approx 70.5°, 289.5°}$

b) $8\sin^2 x + 6\sin x + 1 = 0 \qquad 0 \le x < 360° \qquad \boxed{x = 210°, 330°, x \approx 194.5°, 345.5°}$

c) $3\tan^2\theta + 8\tan\theta - 3 = 0 \qquad 0 \le \theta < 360° \qquad \boxed{\theta \approx 18.4°, 108.4°, 198.4°, 288.4°}$



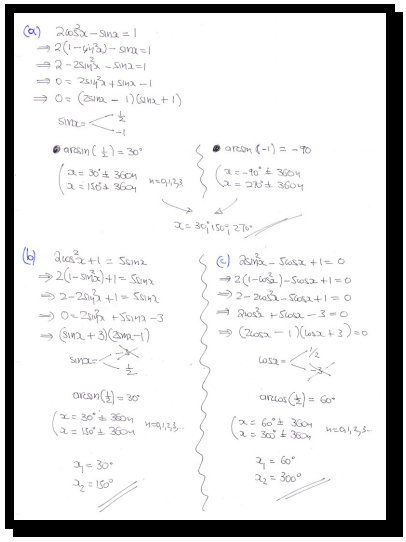
Question 20

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2 x - \sin x = 1$ $\quad 0 \le x < 360°$ $\quad \boxed{x = 30°, 150°, 270°}$

b) $2\cos^2 x + 1 = 5\sin x$ $\quad 0 \le x < 360°$ $\quad \boxed{x = 30°, 150°}$

c) $2\sin^2 x - 5\cos x + 1 = 0$ $\quad 0 \le x < 360°$ $\quad \boxed{x = 60°, 300°}$



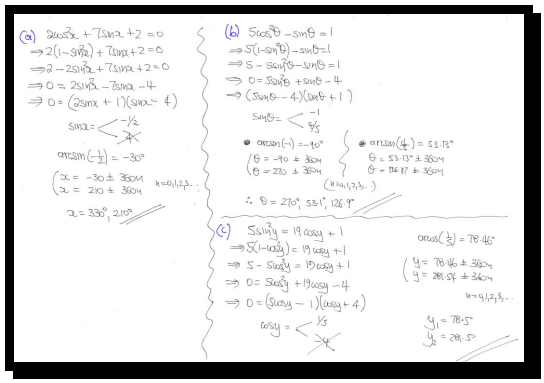
Question 21

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2 x + 7\sin x + 2 = 0$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 210°, 330°}$

b) $5\cos^2 \theta - \sin\theta = 1$ $\qquad 0 \le \theta < 360°$ $\qquad \boxed{\theta \approx 53.1°, 126.9°, \theta = 270°}$

c) $5\sin^2 y = 19\cos y + 1$ $\qquad 0 \le y < 360°$ $\qquad \boxed{y \approx 78.5°, 281.5°}$



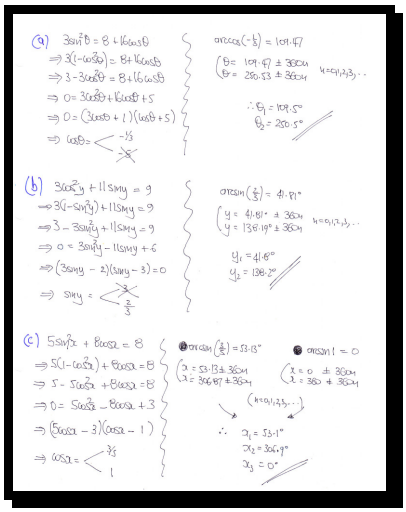
Question 22

Solve each of the following trigonometric equations in the range given.

a) $3\sin^2\theta = 8 + 16\cos\theta$ $\qquad 0 \le \theta < 360°$ $\qquad \boxed{\theta \approx 109.5°, 250.5°}$

b) $3\cos^2 y + 11\sin y = 9$ $\qquad 0 \le y < 360°$ $\qquad \boxed{y \approx 48.2°, 138.2°}$

c) $5\sin^2 x + 8\cos x = 8$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 0°, \quad x \approx 53.1°, 306.9°}$



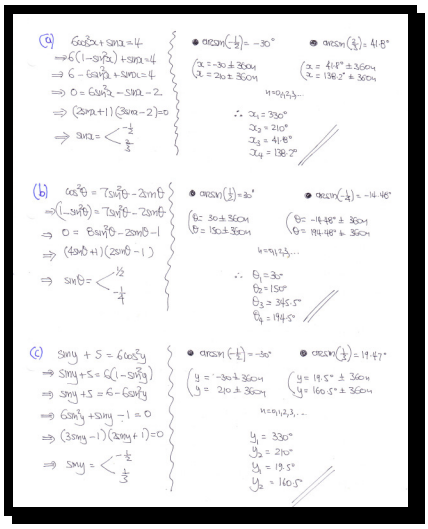
Question 23

Solve each of the following trigonometric equations in the range given.

a) $6\cos^2 x + \sin x = 4$ $\quad 0 \le x < 360°$ $\quad \boxed{x \approx 41.8°, 138.2°, x = 210°, 330°}$

b) $\cos^2 \theta = 7\sin^2 \theta - 2\sin\theta$ $\quad 0 \le \theta < 360°$ $\quad \boxed{\theta \approx 194.5°, 345.5°, \theta = 30°, 150°}$

c) $\sin y + 5 = 6\cos^2 y$ $\quad 0 \le \theta < 360°$ $\quad \boxed{y \approx 19.5°, 160.5°, y = 210°, 330°}$



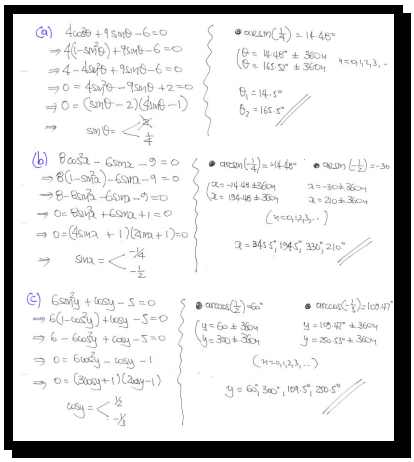
Question 24

Solve each of the following trigonometric equations in the range given.

a) $4\cos^2\theta+9\sin\theta-6=0$ $\quad 0\le\theta<360°$ $\quad \boxed{\theta\approx 14.5°, 165.5°}$

b) $8\cos^2 x-6\sin x-9=0$ $\quad 0\le x<360°$ $\quad \boxed{x\approx 194.5°, 345.5°, x=210°, 330°}$

c) $6\sin^2 y+\cos y-5=0$ $\quad 0\le y<360°$ $\quad \boxed{y\approx 109.5°, 250.5°, y=60°, 300°}$



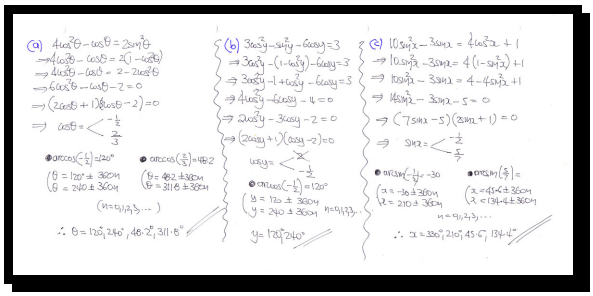
Question 25

Solve each of the following trigonometric equations in the range given.

a) $4\cos^2\theta-\cos\theta=2\sin^2\theta$ $\quad 0\le\theta<360°$ $\quad \boxed{\theta\approx 48.2°,311.8°,\theta=120°,240°}$

b) $3\cos^2 y-\sin^2 y-6\cos y=3$ $\quad 0\le y<360°$ $\quad \boxed{y=120°,240°}$

c) $10\sin^2 x-3\sin x=4\cos^2 x+1$ $\quad 0\le x<360°$ $\quad \boxed{x\approx 45.6°,134.4°,x=210°,330°}$



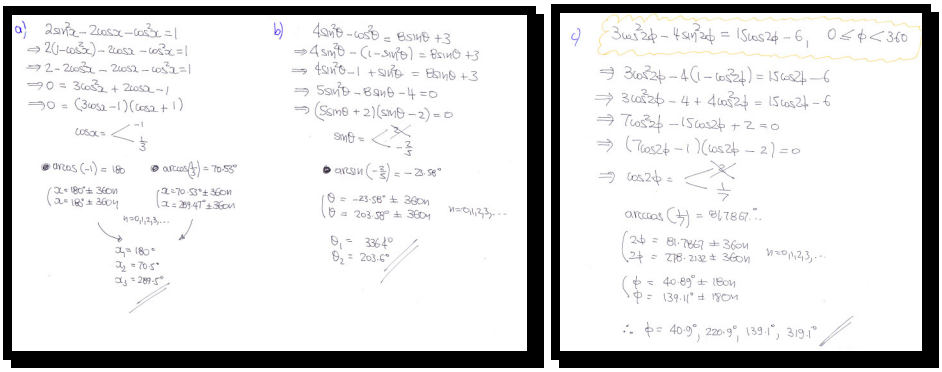
Question 26

Solve each of the following trigonometric equations in the range given.

a) $2\sin^2 x - 2\cos x - \cos^2 x = 1$ $\quad 0 \le x < 360°$ $\quad \boxed{x \approx 70.5°, 289.5°, \quad x = 180°}$

b) $4\sin^2 \theta - \cos^2 \theta = 8\sin\theta + 3$ $\quad 0 \le \theta < 360°$ $\quad \boxed{\theta \approx 203.6°, 336.4°}$

c) $3\cos^2 2\varphi - 4\sin^2 2\varphi = 15\cos 2\varphi - 6$ $\quad 0 \le \varphi < 360°$ $\quad \boxed{\varphi \approx 40.9°, 139.1°, 220.9°, 319.1°}$



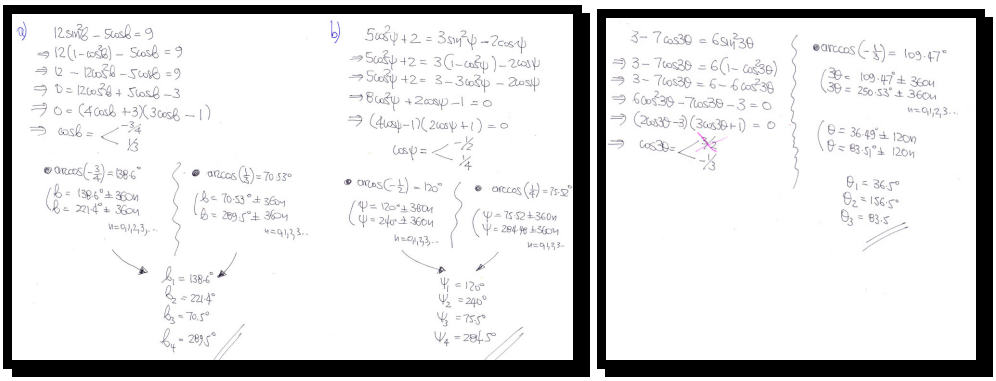
Question 27

Solve each of the following trigonometric equations in the range given.

a) $12\sin^2\beta - 5\cos\beta = 9$ $\quad 0 \le \beta < 360°$ $\quad \boxed{\beta \approx 70.5°, 138.6°, 221.4°, 289.5°}$

b) $5\cos^2\psi + 2 = 3\sin^2\psi - 2\cos\psi$ $\quad 0 \le \psi < 360°$ $\quad \boxed{\psi \approx 75.5°, 284.5°, \ \psi = 120°, 240°}$

c) $3 - 7\cos 3\theta = 6\sin^2 3\theta$ $\quad 0 \le \theta < 180°$ $\quad \boxed{\theta \approx 36.5°, 83.5°, 156.5°}$



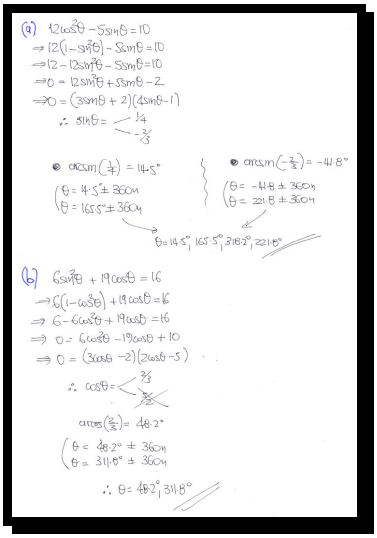
Question 28

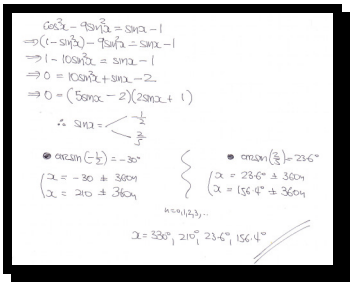
Solve each of the following trigonometric equations in the range given.

a) $12\cos^2\theta - 5\sin\theta = 10$ $\quad 0 \le \theta < 360°$ $\quad \boxed{\theta \approx 14.5°, 165.5°, 221.8°, 318.2°}$

b) $6\sin^2\theta + 19\cos\theta = 16$ $\quad 0 \le \theta < 360°$ $\quad \boxed{\theta \approx 48.2°, 311.8°}$

c) $\cos^2 x - 9\sin^2 x = \sin x - 1$ $\quad 0 \le x < 360°$ $\quad \boxed{x \approx 23.6°, 156.4°, x = 210°, 330°}$





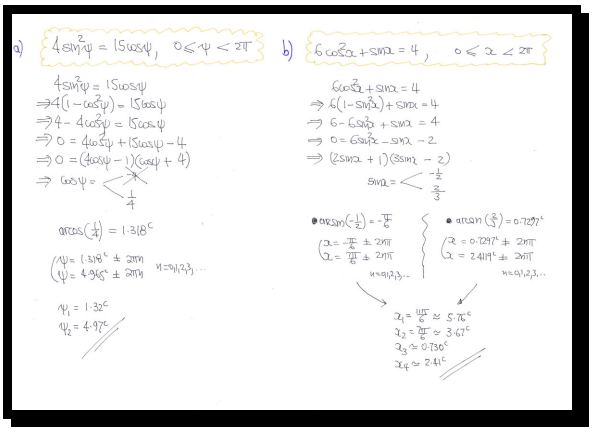
Question 29

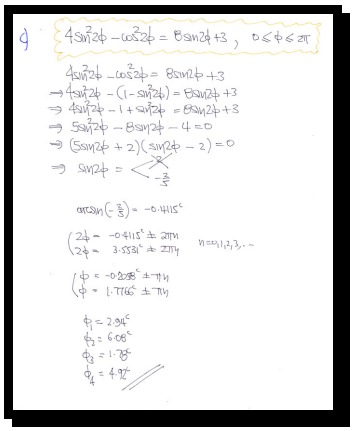
Solve each of the following trigonometric equations in the range given.

a) $4\sin^2\psi = 15\cos\psi$ $\quad 0 \le \psi < 2\pi$ $\quad \boxed{\psi \approx 1.32^{\text{c}},\ 4.97^{\text{c}}}$

b) $6\cos^2 x + \sin x = 4$ $\quad 0 \le x < 2\pi$ $\quad \boxed{x \approx 0.73^{\text{c}},\ 2.41^{\text{c}},\ 3.67^{\text{c}},\ 5.76^{\text{c}}}$

c) $4\sin^2 2\varphi - \cos^2 2\varphi = 3 + 8\sin 2\varphi$ $\quad 0 \le \varphi < 2\pi$ $\quad \boxed{\varphi \approx 1.78^{\text{c}},\ 2.94^{\text{c}},\ 4.92^{\text{c}},\ 6.08^{\text{c}}}$





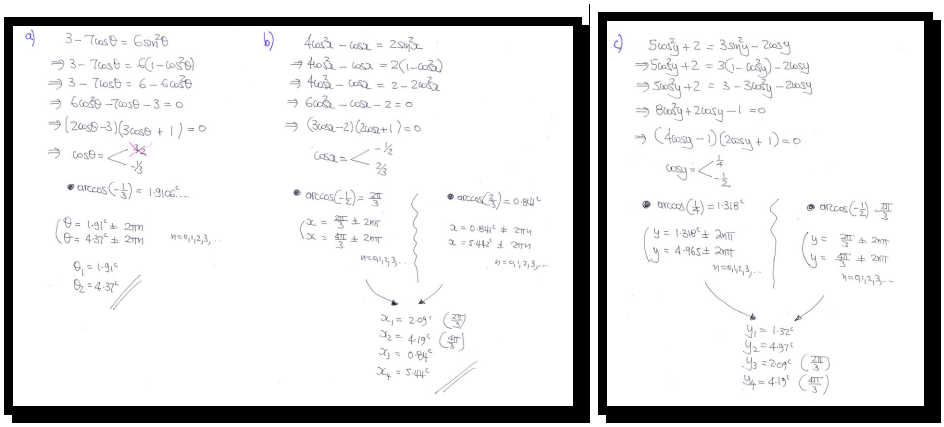
Question 30

Solve each of the following trigonometric equations in the range given.

a) $3-7\cos\theta=6\sin^2\theta$ $\qquad 0\le\theta<2\pi$ $\qquad \boxed{\theta\approx 1.91^{\text{c}},\ 4.37^{\text{c}}}$

b) $4\cos^2 x-\cos x=2\sin^2 x$ $\qquad 0\le x<2\pi$ $\qquad \boxed{x\approx 0.84^{\text{c}},\ 2.09^{\text{c}},\ 4.19^{\text{c}},\ 5.44^{\text{c}}}$

c) $5\cos^2 y+2=3\sin^2 y-2\cos y$ $\qquad 0\le y<2\pi$ $\qquad \boxed{y\approx 1.32^{\text{c}},\ 2.09^{\text{c}},\ 4.19^{\text{c}},\ 4.19^{\text{c}}}$



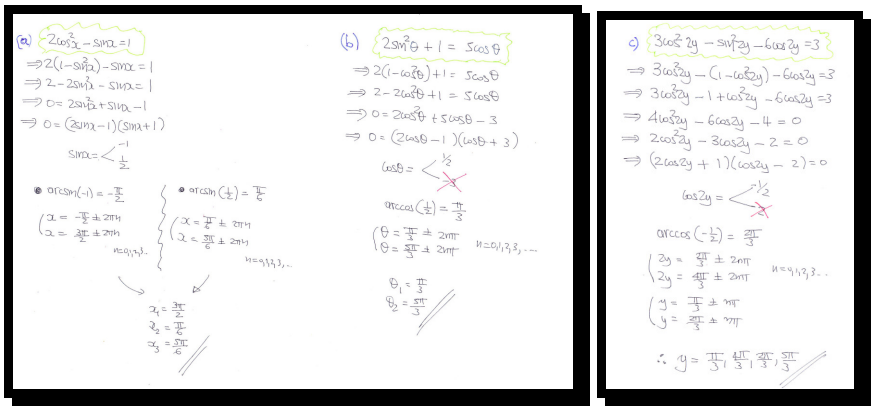
Question 31

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2 x - \sin x = 1$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

b) $2\sin^2 \theta + 1 = 5\cos\theta$ $\qquad 0 \le \theta < 2\pi$ $\qquad \boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

c) $3\cos^2 2y - \sin^2 2y - 6\cos 2y = 3$ $\qquad 0 \le y < 2\pi$ $\qquad \boxed{y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$



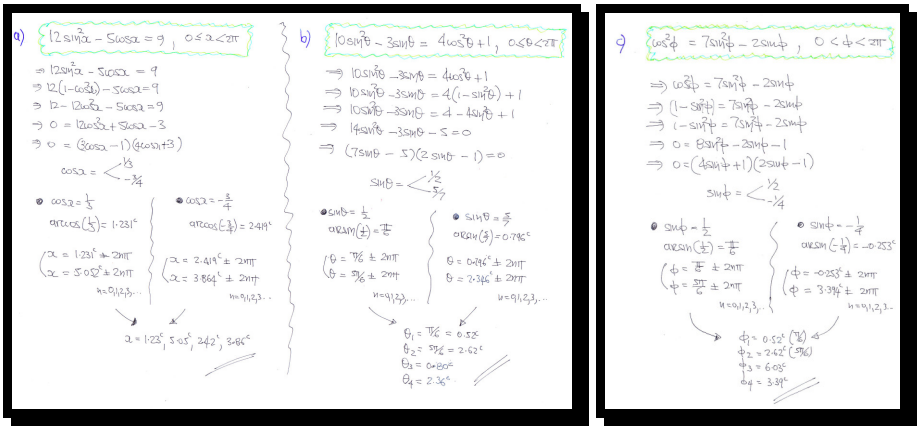
Question 32

Solve each of the following trigonometric equations in the range given.

a) $12\sin^2 x - 5\cos x = 9$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x \approx 1.23^{\text{c}},\ 2.42^{\text{c}},\ 3.86^{\text{c}},\ 5.05^{\text{c}}}$

b) $10\sin^2 \theta - 3\sin\theta = 4\cos^2 \theta + 1$ $\qquad 0 \le \theta < 2\pi$ $\qquad \boxed{\theta \approx 0.52^{\text{c}},\ 0.80^{\text{c}},\ 2.62^{\text{c}},\ 2.36^{\text{c}}}$

c) $\cos^2 \varphi = 7\sin^2 \varphi - 2\sin\varphi$ $\qquad 0 \le y < 2\pi$ $\qquad \boxed{\varphi \approx 0.52^{\text{c}}, 2.62^{\text{c}}, 3.39^{\text{c}}, 6.03^{\text{c}}}$



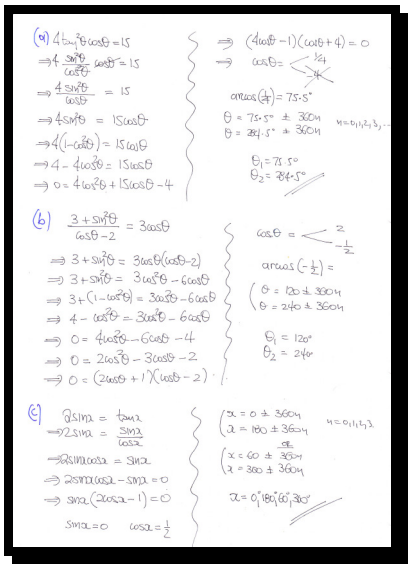
Question 33

Solve each of the following trigonometric equations in the range given.

a) $4\tan^2\theta\cos\theta=15$ $\qquad 0\le\theta<360°$ $\qquad \boxed{\theta\approx 75.5°,284.5°}$

b) $\dfrac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta$ $\qquad 0\le\theta<360°$ $\qquad \boxed{\theta=120°,240°}$

c) $2\sin x=\tan x$ $\qquad 0\le x<360°$ $\qquad \boxed{x=0°,60°,180°,300°}$



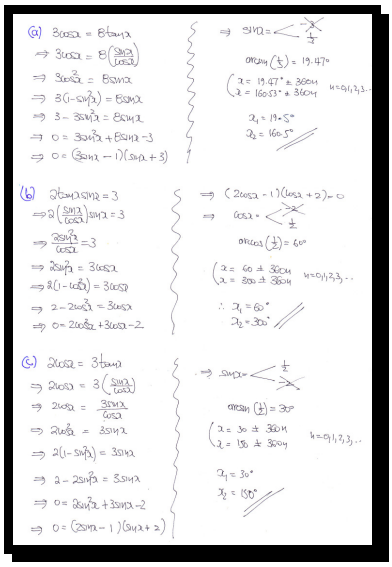
Question 34

Solve each of the following trigonometric equations in the range given.

a) $3\cos x = 8\tan x$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x \approx 19.5°, 160.5°}$

b) $2\tan x \sin x = 3$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 60°, 300°}$

c) $2\cos x = 3\tan x$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 30°, 150°}$



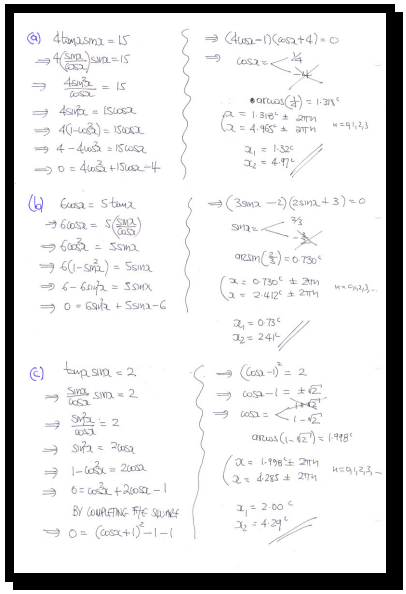
Question 35

Solve each of the following trigonometric equations in the range given.

a) $4\tan x \sin x = 15$ $\quad 0 \le x < 2\pi$ $\quad \boxed{x \approx 1.32^{\text{c}}, 4.97^{\text{c}}}$

b) $6\cos x = 5\tan x$ $\quad 0 \le x < 2\pi$ $\quad \boxed{x \approx 0.73^{\text{c}}, 2.41^{\text{c}}}$

c) $\tan x \sin x = 2$ $\quad 0 \le x < 2\pi$ $\quad \boxed{x \approx 2.00^{\text{c}}, 4.29^{\text{c}}}$



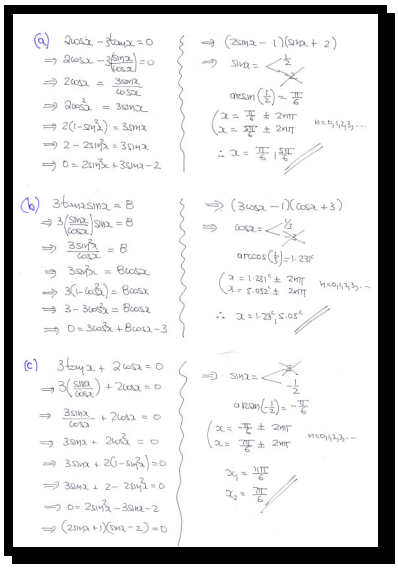
Question 36

Solve each of the following trigonometric equations in the range given.

a) $2\cos x - 3\tan x = 0$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

b) $3\tan x \sin x = 8$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x \approx 1.23^{\text{c}}, 5.05^{\text{c}}}$

c) $3\tan x + 2\cos x = 0$ $\qquad 0 \le x < 2\pi$ $\qquad \boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$



Question 37

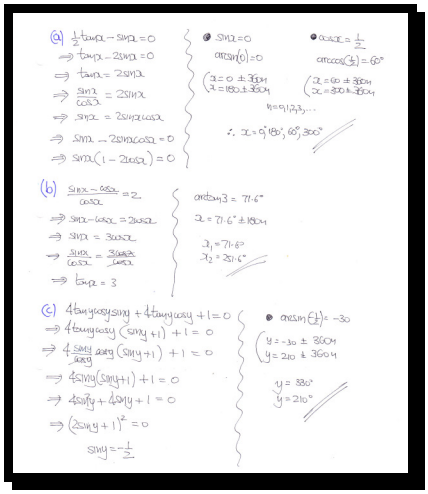
Solve each of the following trigonometric equations in the range given.

a) $\frac{1}{2}\tan x - \sin x = 0$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x = 0°, 60°, 180°, 300°}$

b) $\frac{\sin x - \cos x}{\cos x} = 2$ $\qquad 0 \le x < 360°$ $\qquad \boxed{x \approx 71.6°, 251.6°}$

c) $4\tan y \sin y \cos y + 4\tan y \cos y + 1 = 0$

$0 \le y < 360°$ $\qquad \boxed{y = 210°, 330°}$



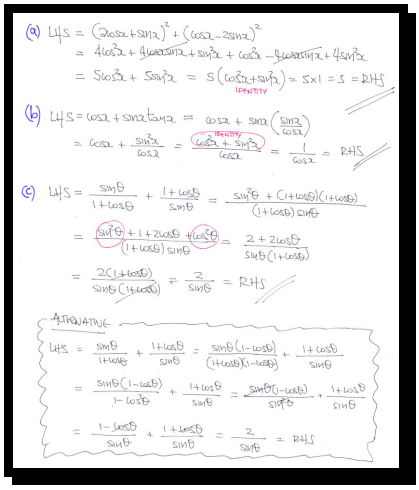
Question 38

Prove the validity of each of the following trigonometric identities.

a) $(2\cos x+\sin x)^2+(\cos x-2\sin x)^2\equiv 5$.

b) $\cos x+\sin x\tan x\equiv\dfrac{1}{\cos x}$.

c) $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}\equiv\dfrac{2}{\sin\theta}$.



Question 39

Prove the validity of each of the following trigonometric identities.

a) $\dfrac{\sin x}{1-\sin x}-\dfrac{\sin x}{1+\sin x}\equiv 2\tan^2 x$.

b) $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}\equiv \dfrac{2}{\cos x}$.

c) $\dfrac{1+\sin x}{\cos x}\equiv \dfrac{\cos x}{1-\sin x}$.

