# TRANSFORMATIONS OF GRAPHS

**Question 1**

Describe geometrically each of the following transformations.

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|---|---|---|
| **a)** | $f(x)+2$ | Translation "upwards" by $2$ units |
| **b)** | $f(x+2)$ | Translation "to the left" by $2$ units |
| **c)** | $2f(x)$ | Stretch in the $y$ direction by a factor of 2 |
| **d)** | $f(2x)$ | Stretch in the $x$ direction by a factor of $\frac{1}{2}$ |
| **e)** | $-f(x)$ | Reflection about the $x$ axis |
| **f)** | $f(x)-4$ | Translation "downwards" by $4$ units |
| **g)** | $f(x-3)$ | Translation "to the right" by $3$ units |
| **h)** | $3f(x)$ | Stretch in the $y$ direction by a factor of $3$ |
| **i)** | $f(-x)$ | Reflection about the $y$ axis |
| **j)** | $f\left(\frac{1}{4}x\right)$ | Stretch in the $x$ direction by a factor of $4$ |

**Question 2**

Describe geometrically each of the following transformations.

**a)** $f(x+1)-2$ — Translation, "to the left" by $1$ unit, followed by translation, "downwards" by $2$ units.

**b)** $4f(x-1)$ — Translation "to the right" by $1$ unit, followed by stretch in the $y$ direction by a factor of $4$.

**c)** $3f(x)-1$ — Stretch in the $y$ direction by a factor of $3$, followed by translation "downwards" by $1$ unit.

**d)** $f(2x+3)$ — Translation "to the left" by $3$ units, followed by stretch in the $x$ direction by a factor of $\frac{1}{2}$.

**e)** $4-f(x)$ — Reflection about the $x$ axis, followed by translation "upwards" by $4$ units.

**f)** $f(-3x)$ — Reflection about the $y$ axis, followed by stretch in the $x$ direction by a factor of $\frac{1}{3}$.

**g)** $-f\left(\frac{1}{2}x\right)$ — Reflection about the $x$ axis, followed by stretch in the $x$ direction by a factor of $2$.

**h)** $2\left[f(x)+3\right]$ — Translation "upwards" by $3$ units, followed by stretch in the $y$ direction by a factor of $2$.

**i)** $-4f(x)$ — Reflection about the $x$ axis, followed by stretch in the $y$ direction by a factor of $4$.

**j)** $f(2-x)$ — Translation "to the left" by 2 units, followed by reflection in the $y$ axis.

**Question 3**

Describe geometrically each of the following transformations.

**a)** $2f(x-1)$ — Translation "to the right" by $1$ unit, followed by stretch in the $y$ direction by factor of $2$.

**b)** $f(3x+1)$ — Translation "to the left" by $1$ unit, followed by stretch in the $x$ direction by a factor of $\frac{1}{3}$.

**c)** $f(4-x)$ — Translation "to the left" by $4$ units, followed by reflection in the $y$ axis.

**d)** $\frac{1}{2}f\left(\frac{1}{2}x+1\right)$ — Translation "to the left" by $1$ unit, followed by stretch in the $x$ direction by a factor of $2$, followed by stretch in the $y$ direction by a factor of $\frac{1}{2}$

**e)** $f\left(\frac{1}{2}x-2\right)$ — Translation "to the right" by $2$ units, followed by stretch in the $x$ direction by a factor of $2$.

**f)** $f(5-x)$ — Translation "to the left" by $5$ units, followed by reflection in the $y$ axis.

**g)** $f(4-2x)$ — Translation "left" by $4$ units, followed by stretch in the $x$ direction by a factor of $\frac{1}{2}$ and reflection in the $y$ axis in either order

**h)** $-4f(x)$ — Reflection about the $x$ axis, followed by stretch in the $y$ direction by a factor of $4$.

**i)** $f(2-x)$ — Translation "to the left" by 2 units, followed by reflection in the $y$ axis.

**Question 4**

Write each of the transformations described below in $f$ notation.

**a)** Translation "upwards" by $5$ units $\boxed{f(x)+5}$

**b)** Translation "to the left" by $4$ units $\boxed{f(x+4)}$

**c)** Stretch in the $y$ direction by a factor of $3$ $\boxed{3f(x)}$

**d)** Stretch in the $x$ direction by a factor of $\frac{1}{2}$ $\boxed{f(2x)}$

**e)** Reflection about the $x$ axis $\boxed{-f(x)}$

**f)** Translation "downwards" by $1$ unit $\boxed{f(x)-1}$

**g)** Translation "to the right" by $5$ units $\boxed{f(x-5)}$

**h)** Stretch in the $y$ direction by a factor of $4$ $\boxed{4f(x)}$

**i)** Reflection about the $y$ axis $\boxed{f(-x)}$

**j)** Stretch in the $x$ direction by a factor of $3$ $\boxed{f\left(\frac{1}{3}x\right)}$

**Question 5**

Write each of the transformations described below in $f$ notation.

**a)** Translation "to the left" by $2$ units, followed by translation "downwards" by $4$ units

**b)** Translation "to the right" by $3$ units, followed by stretch in the $y$ direction by a factor of $2$

**c)** Stretch in the $y$ direction by a factor of $6$, followed by translation "downwards" by $5$ units

**d)** Translation "to the left" by $5$ units, followed by stretch in the $x$ direction by a factor of $\frac{1}{2}$

**e)** Reflection about the $x$ axis, followed by translation "upwards" by $2$ units

**f)** Reflection about the $y$ axis, followed by stretch in the $x$ direction by a factor of $3$

**g)** Reflection about the $x$ axis, followed by stretch in the $x$ direction by a factor of $2$

**h)** Translation "upwards" by $1$ unit, followed by stretch in the $y$ direction by a factor of $3$

**i)** Reflection about the $x$ axis, followed by stretch in the $y$ direction by a factor of $3$

**j)** Translation "to the left" by $4$ units, followed by reflection about the $y$ axis

**a)** $\boxed{f(x+2)-4}$

**b)** $\boxed{2f(x-3)}$

**c)** $\boxed{6f(x)-5}$

**d)** $\boxed{f(2x+5)}$

**e)** $\boxed{2-f(x)}$

**f)** $\boxed{f\left(-\frac{1}{3}x\right)}$

**g)** $\boxed{-f\left(\frac{1}{2}x\right)}$

**h)** $\boxed{3\left[f(x)+1\right]}$

**i)** $\boxed{-3f(x)}$

**j)** $\boxed{f(4-x)}$

**Question 6**

Describe the transformation or set of transformation for each of the following mappings.

**a)** $\ln x \mapsto 2\ln(x-3)$

**b)** $\sin x \mapsto \sin\left(2x+\frac{\pi}{6}\right)$

**c)** $\sqrt{x} \mapsto \sqrt{3x-6}$

**d)** $\mathrm{e}^{x} \mapsto 4\mathrm{e}^{3x}-1$

**e)** $\frac{1}{x} \mapsto \frac{1}{2-x}$

**f)** $\ln\left(x^2+1\right) \mapsto \ln\left(4x^2+1\right)$

**g)** $\frac{1}{x^2+1} \mapsto \frac{4}{x^2+4}$ (hint: it is a single stretch)

**a)** stretch in the $y$ direction, followed by translation "to the right" by 3 units. (either order)

**b)** translation "to the left" by $\frac{\pi}{6}$ units, followed by stretch in the $x$ direction by factor of $\frac{1}{2}$.

**c)** translation "to the right" by 6 units, followed by stretch in the $x$ direction by factor of $\frac{1}{3}$.

**d)** stretch in the $y$ direction by factor of $4$, followed by translation "downwards" by 1 unit, followed by stretch in the $x$ direction by factor of $\frac{1}{3}$.

**e)** translation "to the left" by $2$ units, followed by reflection in the $y$ axis.

**f)** stretch in the $y$ direction by factor of $\frac{1}{3}$.

**g)** stretch in the $x$ direction by factor of $2$.