PARAMETRIC EQUATIONS

ALGEBRAIC ELIMINATIONS

Question 1

Find a Cartesian equation for each of the following parametric relationships.

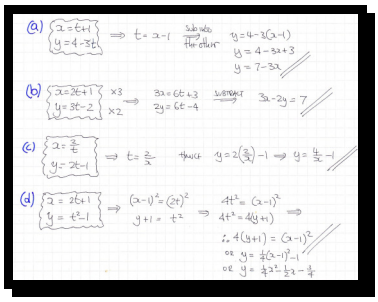
a) $x = t + 1, \quad y = 4 - 3t, \qquad t \in \mathbb{R}$

b) $x = 2t + 1, \quad y = 3t - 2, \qquad t \in \mathbb{R}$

c) $x = \dfrac{2}{t}, \quad y = 2t - 1, \qquad t \in \mathbb{R}, \quad t \neq 0$

d) $x = 2t + 1, \quad y = t^2 - 1, \qquad t \in \mathbb{R}$

$\boxed{y = 7 - 3x}$, $\boxed{3x - 2y = 7}$, $\boxed{y = \dfrac{4}{x} - 1}$, $\boxed{(x-1)^2 = 4(y+1)}$



Question 2

Find a Cartesian equation for each of the following parametric relationships.

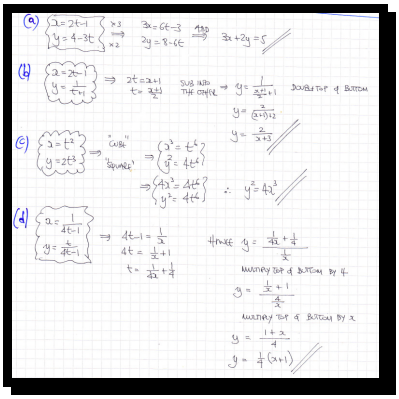
a) $x = 2t - 1, \quad y = 4 - 3t, \qquad t \in \mathbb{R}$

b) $x = 2t - 1, \quad y = \dfrac{1}{t+1}, \qquad t \in \mathbb{R}, \quad t \neq -1$

c) $x = t^2, \quad y = 2t^3, \qquad t \in \mathbb{R}$

d) $x = \dfrac{1}{4t-1}, \quad y = \dfrac{t}{4t-1}, \qquad t \in \mathbb{R}, \quad t \neq \dfrac{1}{4}$

$\boxed{3x + 2y = 5}$, $\boxed{y = \dfrac{2}{x+3}}$, $\boxed{y^2 = 4x^3}$, $\boxed{y = \dfrac{1}{4}(x+1)}$



Question 3

Find a Cartesian equation for each of the following parametric relationships.

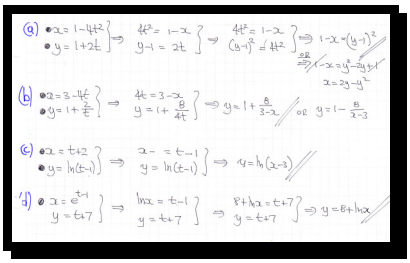
a) $x=1-4t^2, \quad y=1+2t, \qquad t\in\mathbb{R}$

b) $x=3-4t, \quad y=1+\dfrac{2}{t}, \qquad t\in\mathbb{R} \quad t\neq 0$

c) $x=t+2, \quad y=\ln\left(t-1\right), \qquad t\in\mathbb{R} \quad t>1$

d) $x=\mathrm{e}^{t-1}, \quad y=t+7, \qquad t\in\mathbb{R}$

$\boxed{x=2y-y^2}$, $\boxed{y=1-\dfrac{8}{x-3}}$, $\boxed{y=\ln\left(x-3\right)}$, $\boxed{y=8+\ln x}$



Question 4

Find a Cartesian equation for each of the following parametric expressions.

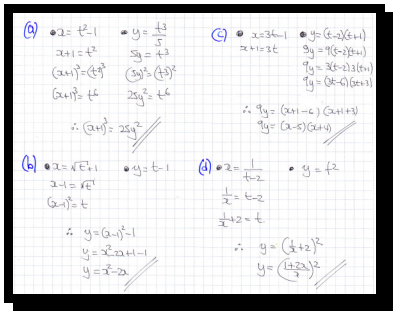
a) $x = t^2 - 1, \quad y = \dfrac{t^3}{5}, \qquad t \in \mathbb{R}$

b) $x = \sqrt{t} + 1, \quad y = t - 1, \qquad t \in \mathbb{R}, t \geq 0$

c) $x = 3t - 1, \quad y = (t-2)(t+1), \qquad t \in \mathbb{R}$

d) $x = \dfrac{1}{t-2}, \quad y = t^2, \qquad t \in \mathbb{R}, t \neq 2$

$\boxed{25y^2 = (x+1)^3}$, $\boxed{y = x^2 - 2x}$, $\boxed{9y = (x-5)(x+4)}$, $\boxed{y = \left(\dfrac{2x+1}{x}\right)^2}$



Question 5

Find a Cartesian equation for each of the following parametric equations.

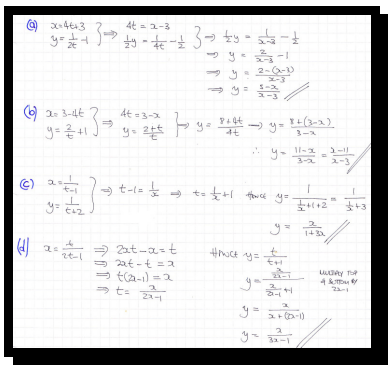
a) $x = 4t + 3, \quad y = \dfrac{1}{2t} - 1, \qquad t \in \mathbb{R}, t \neq 0$

b) $x = 3 - 4t, \quad y = \dfrac{2}{t} + 1, \qquad t \in \mathbb{R}, \ t \neq 0$

c) $x = \dfrac{1}{t-1}, \quad y = \dfrac{1}{t+2}, \qquad t \in \mathbb{R}, t \neq 1, -2$

d) $x = \dfrac{t}{2t-1}, \quad y = \dfrac{t}{t+1}, \qquad t \in \mathbb{R}, \ t \neq -1, \dfrac{1}{2}$

$$\boxed{y = \frac{2}{x-3} - 1}, \quad \boxed{y = \frac{x-11}{x-3}}, \quad \boxed{y = \frac{x}{1+3x}}, \quad \boxed{y = \frac{x}{3x-1}}$$



Question 6

Find a Cartesian equation for each of the following parametric expressions.

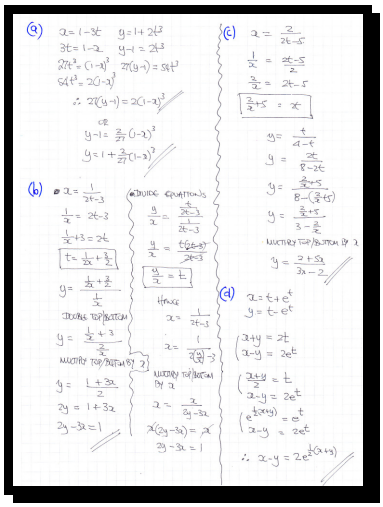
a) $x=1-3t,\quad y=1+2t^3,\qquad t\in\mathbb{R}$

b) $x=\dfrac{1}{2t-3},\quad y=\dfrac{t}{2t-3},\qquad t\in\mathbb{R},\ t\neq\dfrac{3}{2}$

c) $x=\dfrac{2}{2t-5},\quad y=\dfrac{t}{4-t},\qquad t\in\mathbb{R},\ t\neq 4,\ t\neq\dfrac{5}{2}$

d) $x=t+\mathrm{e}^t,\quad y=t-\mathrm{e}^t,\qquad t\in\mathbb{R}$

$\boxed{y=1+\dfrac{2}{27}(1-x)^3}$, $\boxed{2y-3x=1}$, $\boxed{y=\dfrac{5x+2}{3x-2}}$, $\boxed{x-y=2\mathrm{e}^{\frac{1}{2}(x+y)}}$



Question 7

Find a Cartesian equation for each of the following parametric expressions.

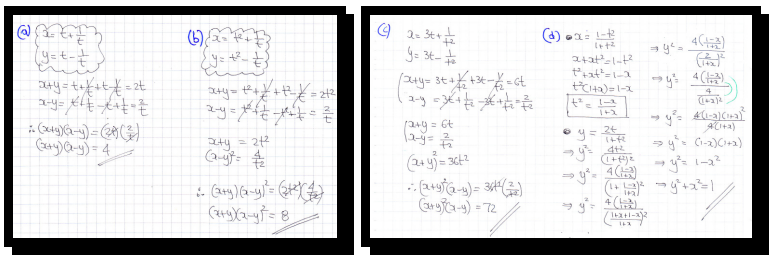
a) $x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \qquad t \in \mathbb{R}, \ t \neq 0$

b) $x = t^2 + \frac{1}{t}, \quad y = t^2 - \frac{1}{t}, \qquad t \in \mathbb{R}, \quad t \neq 0$

c) $x = 3t + \frac{1}{t^2}, \quad y = 3t - \frac{1}{t^2}, \qquad t \in \mathbb{R}, \ t \neq 0$

d) $x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2}, \qquad t \in \mathbb{R}$

$\boxed{x^2 - y^2 = 4}$, $\boxed{(x+y)(x-y)^2 = 8}$, $\boxed{(x-y)(x+y)^2 = 72}$, $\boxed{x^2 + y^2 = 1}$



Question 8

Find a Cartesian equation for each of the following parametric equations.

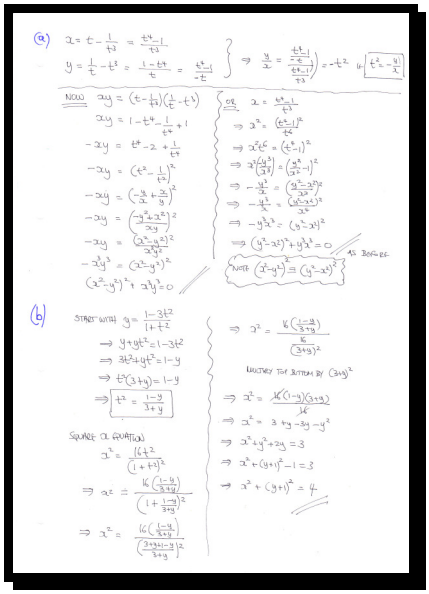
a) $x = t - \dfrac{1}{t^3}$, $y = \dfrac{1}{t} - t^3$, $t \in \mathbb{R},\ t \neq 0$

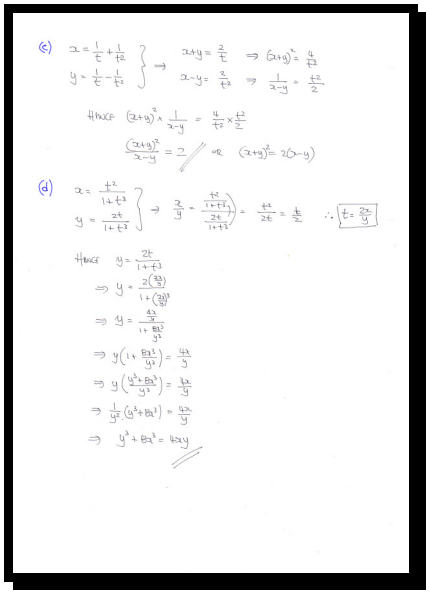
b) $x = \dfrac{4t}{1+t^2}$, $y = \dfrac{1-3t^2}{1+t^2}$, $t \in \mathbb{R}$

c) $x = \dfrac{1}{t} + \dfrac{1}{t^2}$, $y = \dfrac{1}{t} - \dfrac{1}{t^2}$, $t \in \mathbb{R}, t \neq 0$

d) $x = \dfrac{t^2}{1+t^3}$, $y = \dfrac{2t}{1+t^3}$, $t \in \mathbb{R}, t \neq -1$

$\boxed{\left(y^2 - x^2\right)^2 + x^3 y^3 = 0}$, $\boxed{x^2 + (y+1)^2 = 4}$, $\boxed{(x+y)^2 = 2(x-y)}$, $\boxed{y^3 + 8x^3 = 4xy}$





TRIGONOMETRIC ELIMINATIONS

Question 1

Find a Cartesian equation for each of the following parametric relationships.

a) $x=5\cos t, \quad y=5\sin t, \quad 0\le t<2\pi$

b) $x=1+2\cos\theta, \quad y=2+\sin\theta, \quad 0\le\theta<2\pi$

c) $x=2\tan\theta, \quad y=\cos\theta, \quad 0\le\theta<2\pi$

d) $x=\cos t, \quad y=\operatorname{cosec} t, \quad 0\le t<2\pi$

e) $x=\tan\theta, \quad y=\sin\theta, \quad 0\le\theta<2\pi$

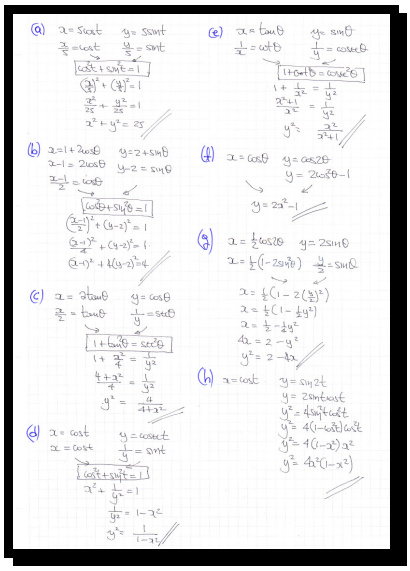
f) $x=\cos\theta, \quad y=\cos 2\theta, \quad 0\le\theta<2\pi$

g) $x=\frac{1}{2}\cos 2\theta, \quad y=2\sin\theta, \quad 0\le\theta<2\pi$

h) $x=\cos t, \quad y=\sin 2t, \quad 0\le t<2\pi$

$\boxed{x^2+y^2=25}$, $\boxed{(x-1)^2+4(y-2)^2=4}$, $\boxed{y^2=\frac{4}{4+x^2}}$, $\boxed{y^2=\frac{1}{1-x^2}}$, $\boxed{y^2=\frac{x^2}{1+x^2}}$,

$\boxed{y=2x^2-1}$, $\boxed{y^2=2-4x}$, $\boxed{y^2=4x^2\left(1-x^2\right)}$



Question 2

Find a Cartesian equation for each of the following parametric relationships.

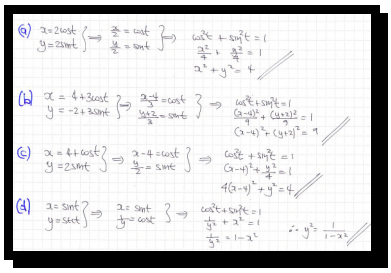
a) $x = 2\cos t, \quad y = 2\sin t, \quad 0 \le t < 2\pi$

b) $x = 4 + 3\cos t, \quad y = -2 + 3\sin t, \quad 0 \le t < 2\pi$

c) $x = 4 + \cos t, \quad y = 2\sin t, \quad 0 \le t < 2\pi$

d) $x = \sin t, \quad y = \sec t, \quad 0 \le t < 2\pi$

$\boxed{x^2 + y^2 = 4}$, $\boxed{(x-4)^2 + (y+2)^2 = 9}$, $\boxed{4(x-4)^2 + y^2 = 4}$, $\boxed{y^2 = \dfrac{1}{1-x^2}}$



Question 3

Eliminate θ to obtain a Cartesian equation for the following parametric equations.

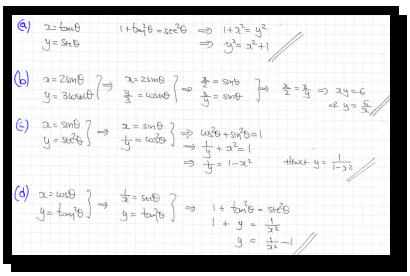
a) $x = \tan\theta$, $y = \sec\theta$, $0 \le \theta < 2\pi$

b) $x = 2\sin\theta$, $y = 3\operatorname{cosec}\theta$, $0 \le \theta < 2\pi$

c) $x = \sin\theta$, $y = \sec^2\theta$, $0 \le \theta < 2\pi$

d) $x = \cos\theta$, $y = \tan^2\theta$, $0 \le \theta < 2\pi$

$\boxed{y^2 = x^2 + 1}$, $\boxed{y = \dfrac{6}{x}}$, $\boxed{y = \dfrac{1}{1-x^2}}$, $\boxed{y = \dfrac{1}{x^2} - 1}$



Question 4

Eliminate the parameter θ to obtain a Cartesian equation for each of the following parametric equations.

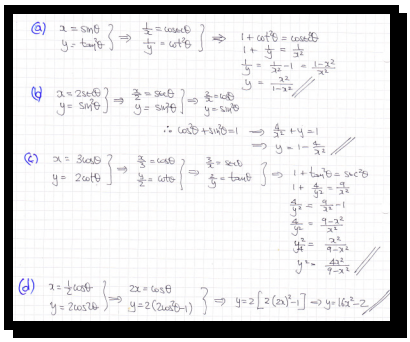
a) $x = \sin\theta, \quad y = \tan^2\theta, \quad 0 \le \theta < 2\pi$

b) $x = 2\sec\theta, \quad y = \sin^2\theta, \quad 0 \le \theta < 2\pi$

c) $x = 3\cos\theta, \quad y = 2\cot\theta, \quad 0 \le \theta < 2\pi$

d) $x = \frac{1}{2}\cos\theta, \quad y = 2\cos 2\theta, \quad 0 \le \theta < 2\pi$

$$\boxed{y = \frac{x^2}{1-x^2}}, \quad \boxed{y = 1 - \frac{4}{x^2}}, \quad \boxed{y^2 = \frac{4x^2}{9-x^2}}, \quad \boxed{y = 2\left(8x^2 - 1\right)}$$



Question 5

Find a Cartesian equation for each of the following parametric relationships.

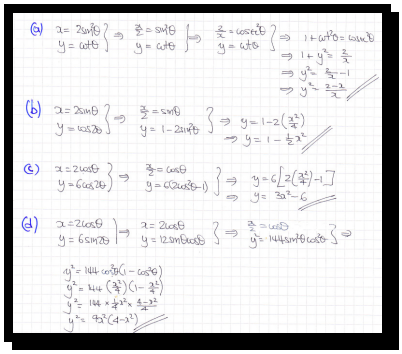
a) $x = 2\sin^2\theta, \quad y = \cot\theta, \qquad 0 \le \theta < 2\pi$

b) $x = 2\sin\theta, \quad y = \cos 2\theta, \qquad 0 \le \theta < 2\pi$

c) $x = 2\cos\theta, \quad y = 6\cos 2\theta, \qquad 0 \le \theta < 2\pi$

d) $x = 2\cos\theta, \quad y = 6\sin 2\theta, \qquad 0 \le \theta < 2\pi$

$\boxed{y^2 = \dfrac{2-x}{x}}$, $\boxed{y = 1 - \frac{1}{2}x^2}$, $\boxed{y = 3x^2 - 6}$, $\boxed{y^2 = 9x^2\left(4 - x^2\right)}$



Question 6

Eliminate the parameter θ to obtain a Cartesian equation for each of the following parametric equations.

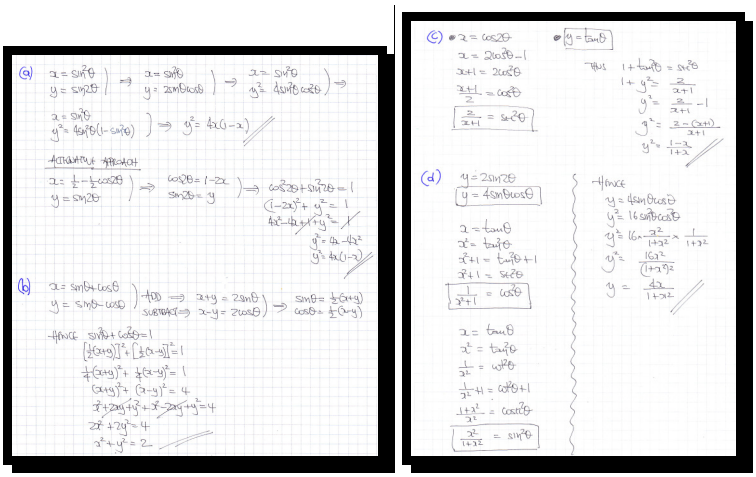
a) $x=\sin^2\theta,\quad y=\sin 2\theta,\qquad 0\le\theta<2\pi$

b) $x=\sin\theta+\cos\theta,\quad y=\sin\theta-\cos\theta,\qquad 0\le\theta<2\pi$

c) $x=\cos 2\theta,\quad y=\tan\theta,\qquad 0\le\theta<2\pi$

d) $x=\tan\theta,\quad y=2\sin 2\theta,\qquad 0\le\theta<2\pi$

$\boxed{y^2=4x(1-x)}$, $\boxed{x^2+y^2=2}$, $\boxed{y^2=\dfrac{1-x}{1+x}}$, $\boxed{y=\dfrac{4x}{1+x^2}}$



Question 7

Eliminate the parameter θ to obtain a Cartesian equation for each of the following parametric expressions.

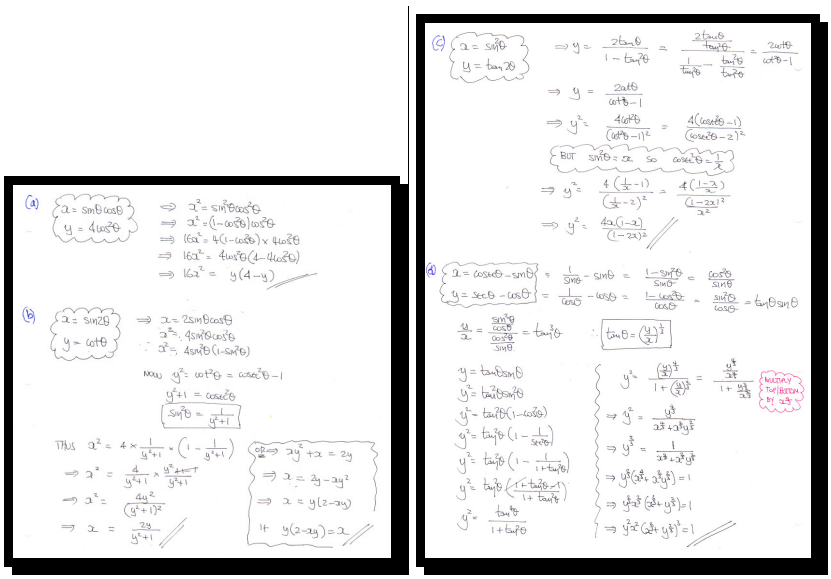
a) $x = \sin\theta\cos\theta, \quad y = 4\cos^2\theta, \qquad 0 \le \theta < 2\pi$

b) $x = \sin 2\theta, \quad y = \cot\theta, \qquad 0 \le \theta < 2\pi$

c) $x = \sin^2\theta, \quad y = \tan 2\theta, \qquad 0 \le \theta < 2\pi$

d) $x = \operatorname{cosec}\theta - \sin\theta, \quad y = \sec\theta - \cos\theta, \qquad 0 \le \theta < 2\pi$

$\boxed{16x^2 = y(4-y)}$, $\boxed{y(2-xy) = x}$, $\boxed{y^2 = \dfrac{4x(1-x)}{(1-2x)^2}}$, $\boxed{y^2x^2\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right)^3 = 1}$



PARAMETRIC ALGEBRA

Question 1

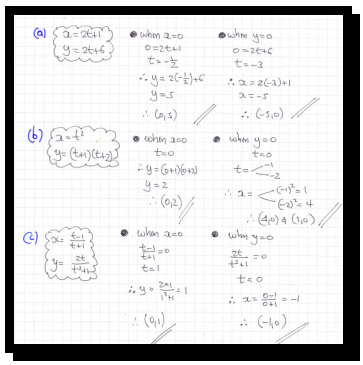
Find the x and y intercepts for each pair of parametric equations.

a) $x=2t+1, \quad y=2t+6, \quad t\in\mathbb{R}$

b) $x=t^2, \quad y=(t+1)(t+2), \quad t\in\mathbb{R}$

c) $x=\dfrac{t-1}{t+1}, \quad y=\dfrac{2t}{t^2+1}, \quad t\in\mathbb{R}, \quad t\neq -1$

$\boxed{(0,5)\,\&\,(-5,0)}$, $\boxed{(0,2)\,\&\,(4,0),(1,0)}$, $\boxed{(0,1)\,\&\,(-1,0)}$



Question 2

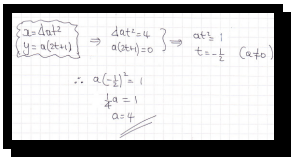
A curve is defined by the following parametric equations

$$x=4at^2, \quad y=a(2t+1), \ t\in\mathbb{R}.$$

where a is non zero constant.

Given the curves passes through the point $A(4,0)$, find the value of a.

$\boxed{a=4}$



Question 3

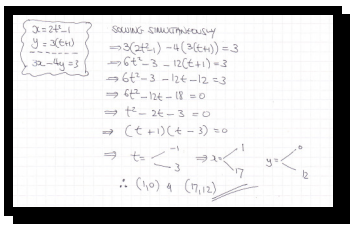
A curve is given by the parametric equations

$$x = 2t^2 - 1, \quad y = 3(t+1), \quad t \in \mathbb{R}.$$

Find the coordinates of the points of intersection of this curve and the line with equation

$$3x - 4y = 3.$$

$$\boxed{(17,12) \ \& \ (1,0)}$$



Question 4

The curve C_1 has Cartesian equation

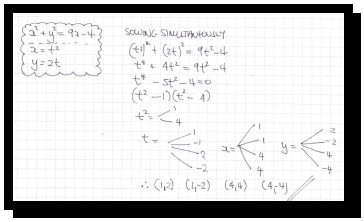
$$x^2 + y^2 = 9x - 4.$$

The curve C_2 has parametric equations

$$x = t^2, \quad y = 2t \quad , \quad t \in \mathbb{R}.$$

Find the coordinates of the points of intersection of C_1 and C_2.

$$\boxed{(4,4), \ (4,-4), (1,2), \ (1,-2)}$$



Question 5

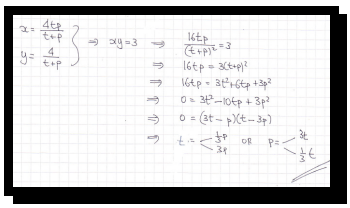
The curve with Cartesian equation $xy = 3$ is also traced by the following parametric equations

$$x = \frac{4tp}{t+p}, \quad y = \frac{4}{t+p}, \quad t, p \in \mathbb{R}, \ t \neq p$$

where t and p are parameters.

Find the relationship between the two parameters t and p in the form $p = f(t)$.

$$\boxed{p = 3t \ \text{ or } \ p = \frac{1}{3}t}$$



PARAMETRIC DIFFERENTIATION

Question 1

A curve is given parametrically by the equations

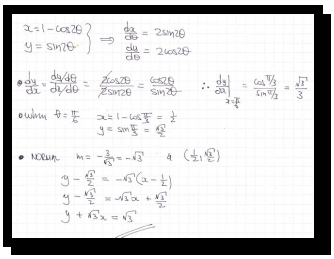
$$x = 1 - \cos 2\theta, \quad y = \sin 2\theta, \quad 0 \le \theta < 2\pi .$$

The point P lies on this curve, and the value of θ at that point is $\dfrac{\pi}{6}$.

Show that an equation of the normal at the point P is given by

$$y + \sqrt{3}x = \sqrt{3} .$$

proof



Question 2

A curve is given parametrically by the equations

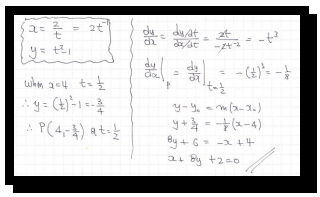
$$x=\frac{2}{t},\ y=t^2-1,\ t\in\mathbb{R},\ t\neq 0.$$

The point $P(4, y)$ lies on this curve.

Show that an equation of the tangent at the point P is given by

$$x+8y+2=0.$$

proof



Question 3

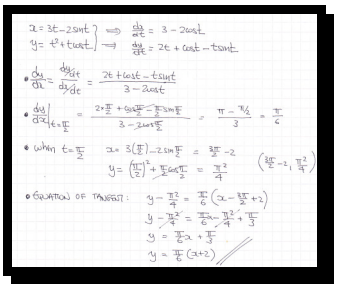
A curve is given parametrically by the equations

$$x = 3t - 2\sin t\,,\ \ y = t^2 + t\cos t\,,\ \ 0 \le t < 2\pi\,.$$

Show that an equation of the tangent at the point on the curve where $t = \dfrac{\pi}{2}$ is given by

$$y = \frac{\pi}{6}\left(x + 2\right).$$

proof



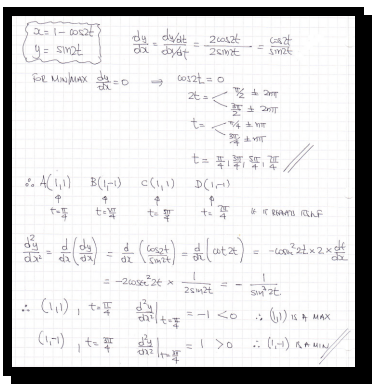
Question 4

Find the turning points of the curve given parametrically by the equations

$$x = 1 - \cos 2t\,,\ \ y = \sin 2t\,,\ \ 0 \le t < 2\pi\,.$$

Determine the nature of these turning points.

$\max\left(1,1\right),\ \ \min\left(1,-1\right)$



Question 5

A curve is given parametrically by the equations

$$x = 3\sin 2\theta, \quad y = 4\cos 2\theta, \quad 0 \le \theta \le 2\pi .$$

The point P is such so that $\cos\theta = \frac{3}{5}$ with $0 \le \theta \le \frac{\pi}{2}$.

Show that an equation of the tangent at the point P is

$$32x - 7y = 100 .$$

proof



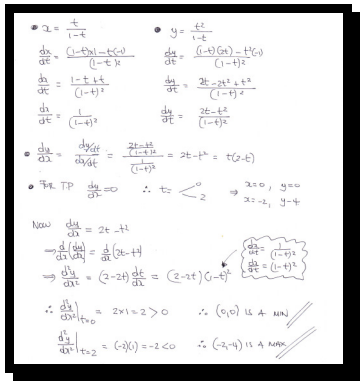
Question 6

For the curve given parametrically by

$$x=\frac{t}{1-t}\,, \quad y=\frac{t^2}{1-t} \quad , \quad t\in\mathbb{R},\ t\neq 1,$$

find the coordinates of the turning points and determine their nature.

$$\boxed{\max\left(-2,-4\right),\ \ \min\left(0,0\right)}$$



Question 7

A curve is given parametrically by the equations

$$x=\frac{2t}{1+t^2}\,,\quad y=\frac{1-t^2}{1+t^2}\,,\quad t\in\mathbb{R}\,.$$

The point $P\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ lies on this curve.

Show that an equation of the tangent at the point P is given by

$$x+y=\sqrt{2}\,.$$

proof

