INTEGRATION STRUCTURED EXAM QUESTIONS PART I

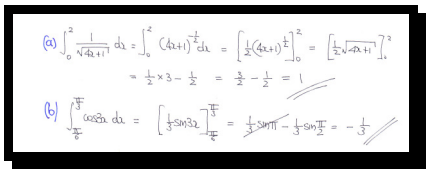
Question 1 (**)

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^2 \frac{1}{\sqrt{4x+1}}\,dx$.

b) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos 3x\,dx$.

$\square$, $\boxed{1}$, $\boxed{-\frac{1}{3}}$

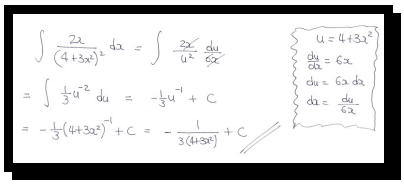


Question 2 (**)

By using the substitution $u = 4+3x^2$, or otherwise, find

$$\int \frac{2x}{\left(4+3x^2\right)^2}\,dx.$$

$\square$, $\boxed{-\frac{1}{3\left(4+3x^2\right)}+C}$

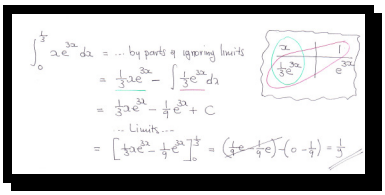


Question 3 (**)

Show clearly that

$$\int_0^{\frac{1}{3}} x\mathrm{e}^{3x}\ dx=\frac{1}{9}.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 4 (**)

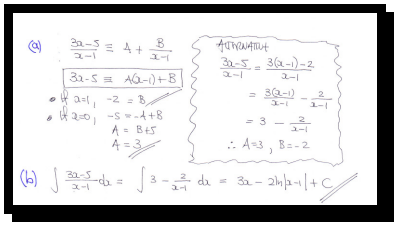
$$\frac{3x-5}{x-1}\equiv A+\frac{B}{x-1}.$$

a) Determine the value of each of the constants A and B.

b) Hence find

$$\int\frac{3x-5}{x-1}\ dx.$$

$\boxed{A=3}$, $\boxed{B=-2}$, $\boxed{3x-2\ln|x-1|+C}$



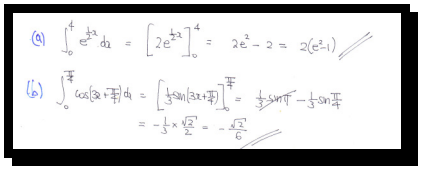
Question 5 (**)

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^4 \mathrm{e}^{\frac{1}{2}x}\,dx$.

b) $\displaystyle\int_0^{\frac{\pi}{4}} \cos\left(3x+\frac{\pi}{4}\right)\,dx$.

$\square$, $\boxed{2\left(\mathrm{e}^2-1\right)}$, $\boxed{-\frac{\sqrt{2}}{6}}$



Question 6 **(**)**

$$\frac{5x+13}{(2x+1)(x+4)} \equiv \frac{A}{2x+1}+\frac{B}{x+4}.$$

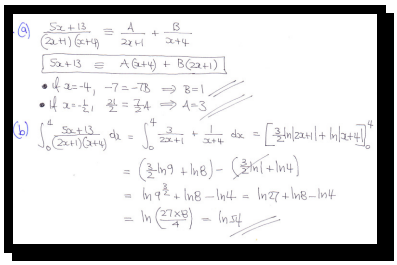
a) Determine the value of each of the constants A and B.

b) Evaluate

$$\int_0^4 \frac{5x+13}{(2x+1)(x+4)}\,dx,$$

giving the answer as a single simplified natural logarithm.

$\boxed{}$, $\boxed{A=3}$, $\boxed{B=1}$, $\boxed{\ln 54}$

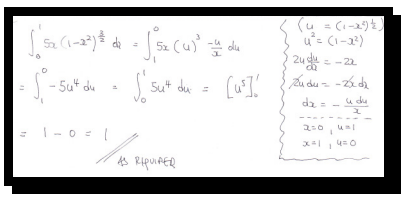


Question 7 **(**)**

By using the substitution $u^2=1-x^2$, or otherwise, show that

$$\int_0^1 5x\left(1-x^2\right)^{\frac{3}{2}}\,dx=1.$$

$\boxed{}$, $\boxed{\text{proof}}$

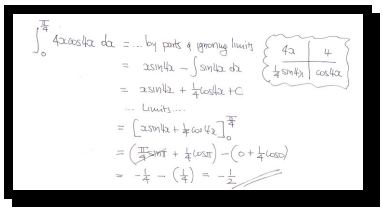


Question 8 (**)

Use integration by parts to find the value of

$$\int_0^{\frac{\pi}{4}} 4x\cos 4x \ dx .$$

$\square$, $\boxed{-\frac{1}{2}}$

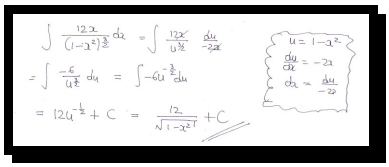


Question 9 (**)

By using the substitution $u = 1 - x^2$, or otherwise, find

$$\int \frac{12x}{\left(1-x^2\right)^{\frac{3}{2}}} dx .$$

$\square$, $\boxed{\frac{12}{\sqrt{1-x^2}} + C}$



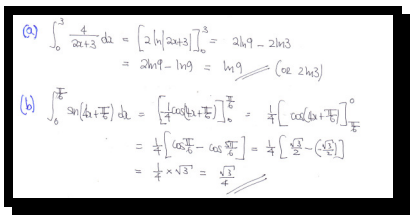
Question 10 (**)

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^3 \frac{4}{2x+3}\,dx$.

b) $\displaystyle\int_0^{\frac{\pi}{6}} \sin\left(4x+\frac{\pi}{6}\right)\,dx$.

$\boxed{}$, $\boxed{\ln 9}$, $\boxed{\dfrac{\sqrt{3}}{4}}$

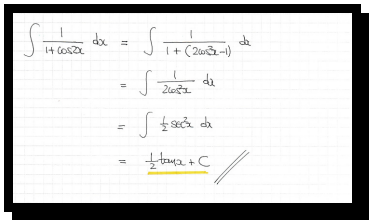


Question 11 (**)

Use a trigonometric identity to integrate

$$\int \frac{1}{1+\cos 2x}\,dx\,.$$

$\boxed{}$, $\boxed{\dfrac{1}{2}\tan x+C}$



Question 12 (**)

$$\frac{30}{(x+3)(9-2x)} \equiv \frac{A}{x+3} + \frac{B}{9-2x}.$$

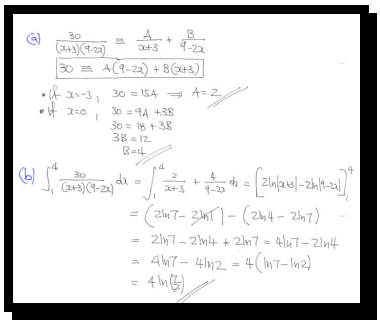
a) Determine the value of each of the constants A and B.

b) Evaluate

$$\int_1^4 \frac{30}{(x+3)(9-2x)}\, dx,$$

giving the answer as a single simplified natural logarithm.

$\boxed{}$, $\boxed{A=2}$, $\boxed{B=4}$, $\boxed{4\ln\left(\frac{7}{2}\right)=\ln\left(\frac{2401}{16}\right)}$



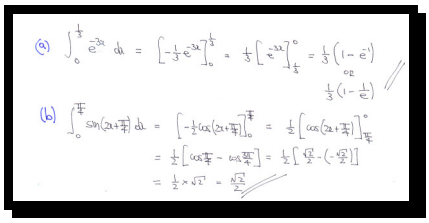
Question 13 (**)

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^{\frac{1}{3}} \mathrm{e}^{-3x}\ dx$.

b) $\displaystyle\int_0^{\frac{\pi}{4}} \sin\left(2x+\frac{\pi}{4}\right)\ dx$.

$\boxed{\frac{1}{3}\left(1-\mathrm{e}^{-1}\right)}$, $\boxed{\frac{\sqrt{2}}{2}}$

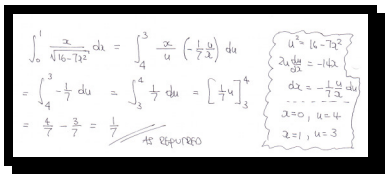


Question 14 (+)**

By using the substitution $u^2=16-7x^2$, or otherwise, show that

$$\int_0^1 \frac{x}{\sqrt{16-7x^2}}\,dx=\frac{1}{7}.$$

$\boxed{}$, $\boxed{\text{proof}}$



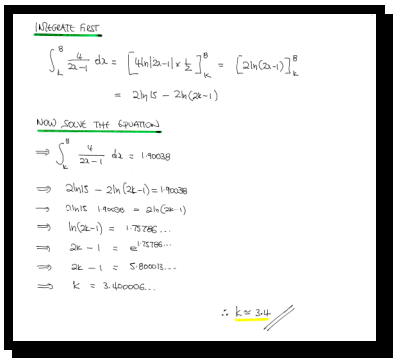
Question 15 (+)**

Determine the value of the positive constant k given further that

$$\int_k^8 \frac{4}{2x-1}\,dx = 1.90038\,.$$

Give the value of k to an appropriate degree of accuracy.

$\boxed{}$, $\boxed{k \approx 3.4}$

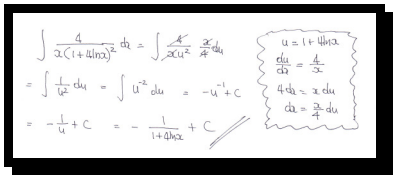


Question 16 (+)**

By using the substitution $u = 1 + 4\ln x$, or otherwise, find

$$\int \frac{4}{x(1+4\ln x)^2}\,dx\,.$$

$\boxed{\phantom{-\frac{1}{1+4\ln x}+C}}$, $\boxed{-\frac{1}{1+4\ln x}+C}$



Question 17 (+)**

$$\frac{8x}{4x-3} \equiv A + \frac{B}{4x-3}.$$

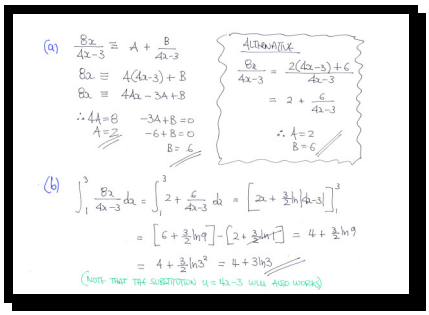
a) Determine the value of each of the constants A and B.

b) Hence, or otherwise, evaluate

$$\int_1^3 \frac{8x}{4x-3}\, dx,$$

giving the answer in terms of natural logarithms.

$\boxed{A=2}$, $\boxed{B=6}$, $\boxed{4+3\ln 3}$

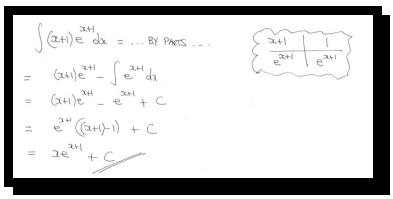


Question 18 (+)**

Use an appropriate integration method to find

$$\int (x+1)\mathrm{e}^{x+1}\, dx.$$

$\boxed{}$, $\boxed{x\mathrm{e}^{x+1}+C}$



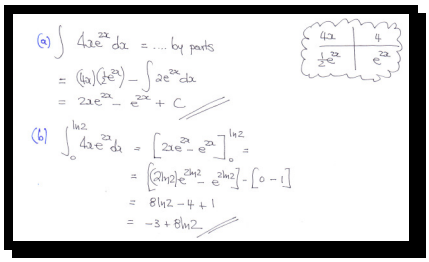
Question 19 (+)**

$$f(x)=4x\mathrm{e}^{2x}.$$

a) Use integration by parts to find $\int f(x)\,dx$.

b) Find an exact vale for $\int_0^{\ln 2} f(x)\,dx$.

$\boxed{}$, $\boxed{2x\mathrm{e}^{2x}-\mathrm{e}^{2x}+C}$, $\boxed{-3+8\ln 2}$



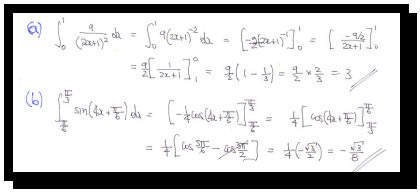
Question 20 (+)**

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^1 \frac{9}{(2x+1)^2}\,dx$.

b) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin\left(4x+\frac{\pi}{6}\right)\,dx$.

$\boxed{3}$, $\boxed{-\dfrac{\sqrt{3}}{8}}$

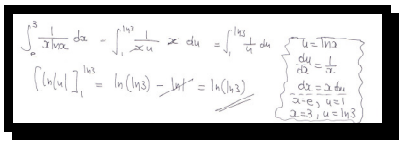


Question 21 (+)**

By using the substitution $u = \ln x$, or otherwise, find an exact value for

$$\int_{\mathrm{e}}^{3} \frac{1}{x \ln x} \, dx .$$

$\boxed{\ln\left(\ln 3\right)}$



Question 22 **(**+)**

$$f(x)\equiv\frac{x-5}{x^2+5x+4}.$$

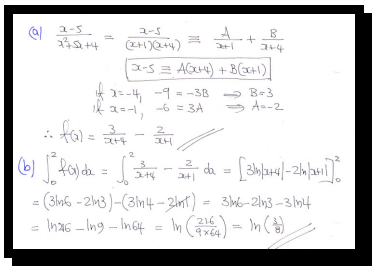
a) Express $f(x)$ in partial fractions.

b) Find the value of

$$\int_0^2 f(x)\ dx\ ,$$

giving the answer as a single simplified logarithm.

$\square$, $\boxed{f(x)\equiv\frac{3}{x+4}-\frac{2}{x+1}}$, $\boxed{\int_0^2 f(x)\ dx=\ln\left(\frac{3}{8}\right)}$

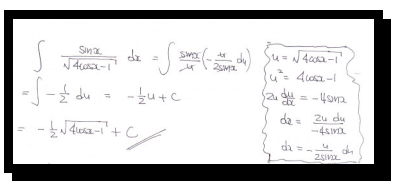


Question 23 **(**+)**

By using the substitution $u^2=4\cos x-1$, or otherwise, find

$$\int\frac{\sin x}{\sqrt{4\cos x-1}}\ dx\ .$$

$\boxed{-\frac{1}{2}\sqrt{4\cos x-1}+C}$

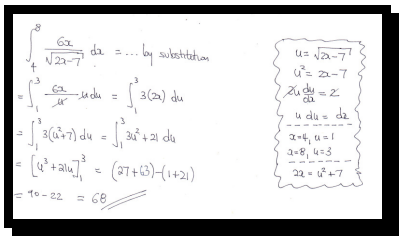


Question 24 (+)**

Use the substitution $u=\sqrt{2x-7}$ to find

$$\int_4^8 \frac{6x}{\sqrt{2x-7}}\,dx\,.$$

$\square$, $\boxed{68}$



Question 25 (+)**

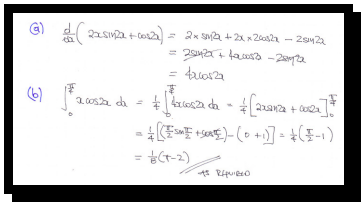
$$y=2x\sin 2x+\cos 2x\,,\ x\in\mathbb{R}$$

a) Find an expression for $\dfrac{dy}{dx}$.

b) Hence show that

$$\int_0^{\frac{\pi}{4}} x\cos 2x\ dx=\frac{1}{8}(\pi-2)\,.$$

$$\boxed{\frac{dy}{dx}=4x\cos 2x}$$



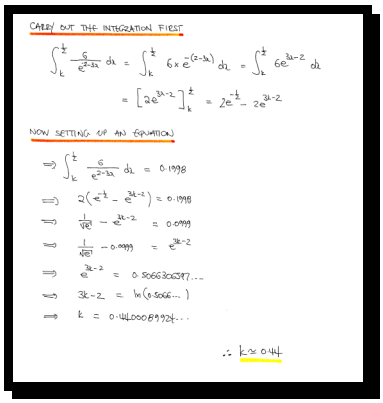
Question 26 (+)**

Determine the value of the positive constant k given further that

$$\int_k^{\frac{1}{2}} \frac{6}{\mathrm{e}^{2-3x}}\,dx = 0.1998\,.$$

Give the value of k to an appropriate degree of accuracy.

$\boxed{}$, $\boxed{k \approx 0.44}$



Question 27 (***)

$$y=\frac{3x}{2+x-x^2}$$

a) Calculate the three missing values of y in the following table.

x	0	0.25	0.5	0.75	1
y	0				1.5

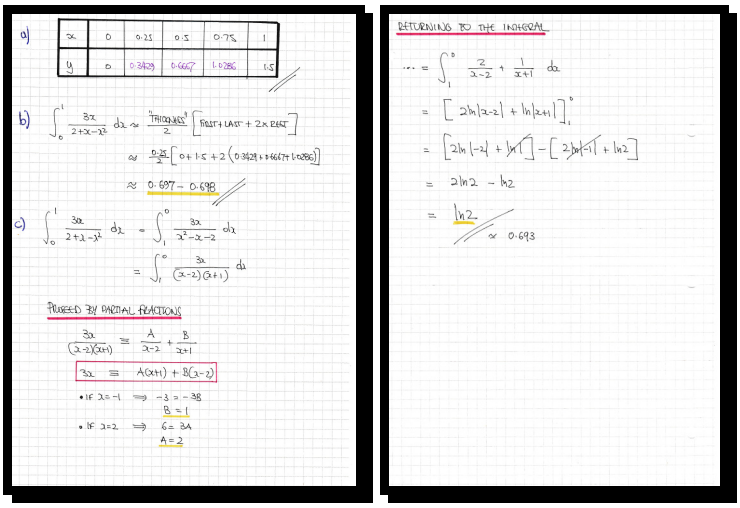
b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

$$\int_0^1 \frac{3x}{2+x-x^2}\,dx\,.$$

c) Use a suitable method to find the exact value of

$$\int_0^1 \frac{3x}{2+x-x^2}\,dx\,.$$

$\boxed{}$, $\boxed{0.3429}$, $\boxed{0.6667}$, $\boxed{1.0286}$, $\boxed{\ln 2}$

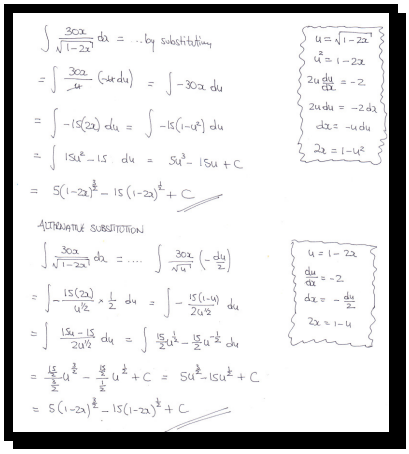


Question 28 (***)

Use a suitable substitution to find

$$\int \frac{30x}{\sqrt{1-2x}}\,dx\,.$$

$$\boxed{5(1-2x)^{\frac{3}{2}}-15(1-2x)^{\frac{1}{2}}+C}$$



Question 29 **(***)**

$$\frac{x^2+3}{x-1} \equiv Ax+B+\frac{C}{x-1}.$$

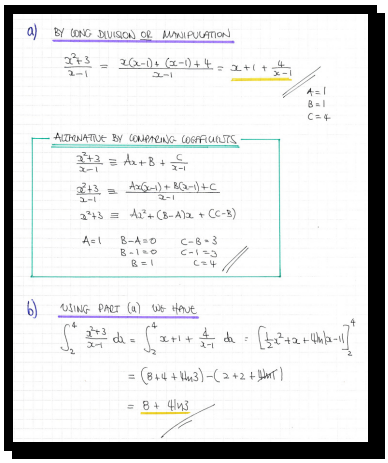
a) Determine the value of each of the constants A, B and C.

b) Hence, or otherwise, evaluate

$$\int_2^4 \frac{x^2+3}{x-1}\,dx,$$

giving the answer in terms of natural logarithms.

$\boxed{}$, $\boxed{A=1}$, $\boxed{B=1}$, $\boxed{C=4}$, $\boxed{8+4\ln 3}$

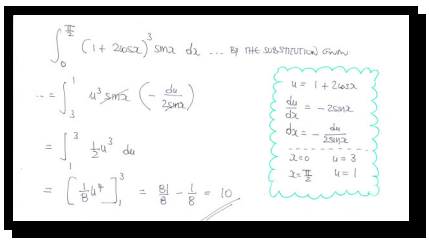


Question 30 (***)

Use the substitution $u = 1 + 2\cos x$ to find

$$\int_0^{\frac{\pi}{2}} (1+2\cos x)^3 \sin x \ dx .$$

$\boxed{10}$

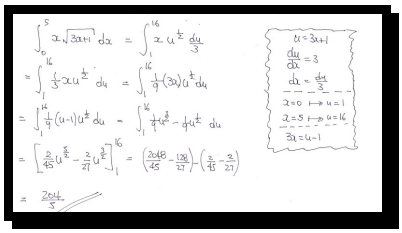


Question 31 (***)

By using the substitution $u = 3x+1$, or otherwise, find

$$\int_0^5 x\sqrt{3x+1} \ dx .$$

$\boxed{\dfrac{204}{5} = 40.8}$

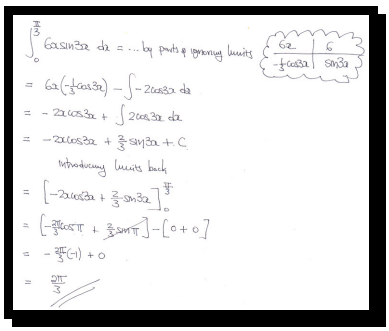


Question 32 (***)

Use an appropriate integration method to find an exact value for

$$\int_0^{\frac{\pi}{3}} 6x\sin 3x \ \ dx .$$

$\square$, $\boxed{\dfrac{2\pi}{3}}$

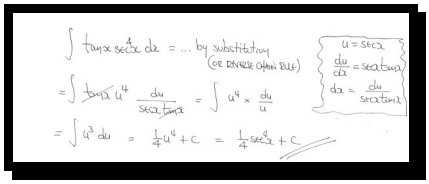


Question 33 (***)

By using the substitution $u = \sec x$, or otherwise, find

$$\int \tan x \sec^4 x \ dx .$$

$$\boxed{\frac{1}{4}\sec^4 x + C}$$



Question 34 (***)

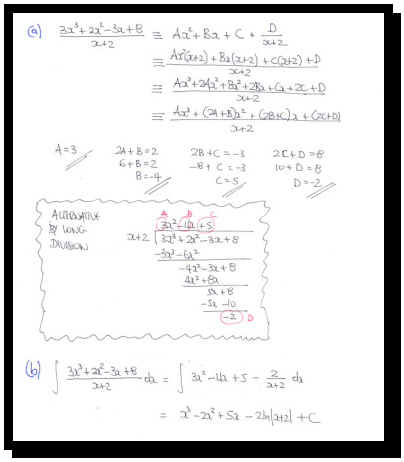
$$\frac{3x^3+2x^2-3x+8}{x+2} \equiv Ax^2+Bx+C+\frac{D}{x+2}.$$

a) Find the value of each of the constants A, B, C and D.

b) Hence find

$$\int \frac{3x^3+2x^2-3x+8}{x+2}\,dx.$$

$\boxed{A=3}$, $\boxed{B=-4}$, $\boxed{C=5}$, $\boxed{D=-2}$, $\boxed{x^3-2x^2+5x-2\ln|x+2|+C}$



Question 35 (***)

$$f(x)\equiv\frac{5}{3x^2-5x}.$$

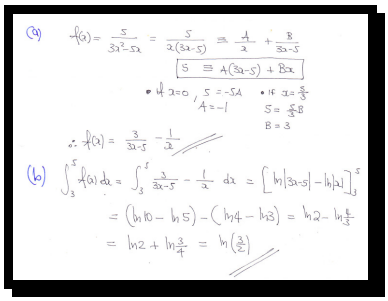
a) Express $f(x)$ in partial fractions.

b) Find the value of

$$\int_3^5 f(x)\ dx\ ,$$

giving the answer as a single simplified logarithm.

$$\boxed{},\quad \boxed{f(x)\equiv\frac{3}{3x-5}-\frac{1}{x}},\quad \boxed{\ln\left(\frac{3}{2}\right)}$$



(a) $f(x)=\frac{5}{3x^2-5x}=\frac{5}{x(3x-5)}\equiv\frac{A}{x}+\frac{B}{3x-5}$

$5\equiv A(3x-5)+Bx$

• If $x=0$, $5=-5A$, $A=-1$

• If $x=\frac{5}{3}$, $5=\frac{5}{3}B$, $B=3$

$\therefore f(x)=\frac{3}{3x-5}-\frac{1}{x}$

(b) $\int_3^5 f(x)\,dx=\int_3^5\frac{3}{3x-5}-\frac{1}{x}\,dx=\left[\ln|3x-5|-\ln|x|\right]_3^5$

$=(\ln 10-\ln 5)-(\ln 4-\ln 3)=\ln 2-\ln\frac{4}{3}$

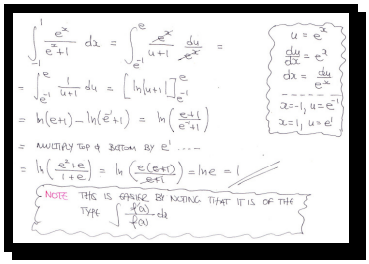
$=\ln 2+\ln\frac{3}{4}=\ln\left(\frac{3}{2}\right)$

Question 36 (***)

By using the substitution $u = \mathrm{e}^x$, or otherwise, show clearly that

$$\int_{-1}^{1} \frac{\mathrm{e}^x}{\mathrm{e}^x + 1}\, dx = 1 .$$

proof

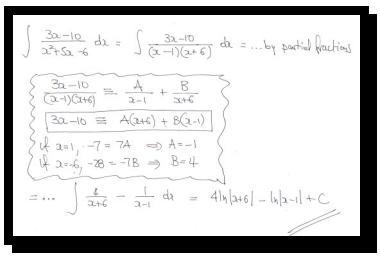


Question 37 (***)

Find an expression for the integral

$$\int \frac{3x - 10}{x^2 + 5x - 6}\, dx .$$

$$\boxed{4\ln|x+6| - \ln|x-1| + C}$$

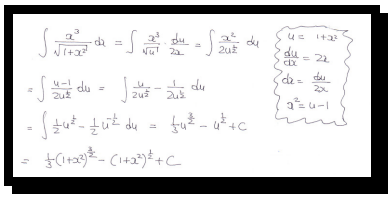


Question 38 (***)

By using the substitution $u = 1 + x^2$, or otherwise, find

$$\int \frac{x^3}{\sqrt{1+x^2}}\, dx .$$

$$\boxed{\frac{1}{3}\left(1+x^2\right)^{\frac{3}{2}} - \left(1+x^2\right)^{\frac{1}{2}} + C}$$

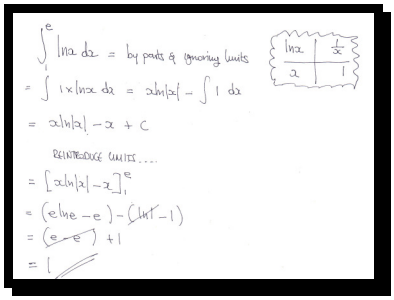


Question 39 (***)

Use integration by parts to find the value of

$$\int_1^{\mathrm{e}} \ln x \ \ dx .$$

$$\boxed{}, \ \boxed{1}$$

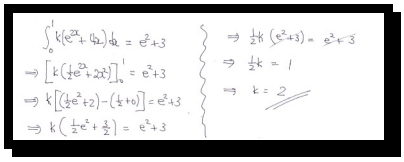


Question 40 (***)

Find the value of the constant k given that

$$\int_0^1 k\left(\mathrm{e}^{2x}+4x\right)\,dx=\mathrm{e}^2+3\,.$$

$$\boxed{k=2}$$



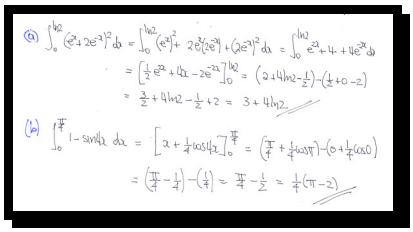
Question 41 (***)

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^{\ln 2}\left(\mathrm{e}^{x}+2\mathrm{e}^{-x}\right)^2\,dx\,.$

b) $\displaystyle\int_0^{\frac{\pi}{4}} 1-\sin 4x\,\,dx\,.$

$$\boxed{3+4\ln 2}\,,\quad \boxed{\frac{1}{4}(\pi-2)}$$

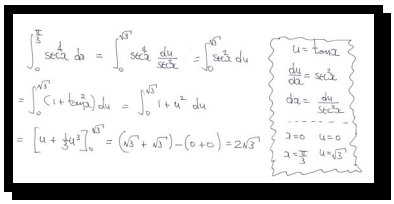


Question 42 (***)

By using the substitution $u=\tan x$ and the trigonometric identity $1+\tan^2 x=\sec^2 x$, show clearly that

$$\int_0^{\frac{\pi}{3}} \sec^4 x \ dx = 2\sqrt{3}\,.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 43 (***)

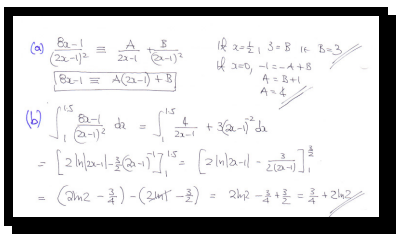
$$\frac{8x-1}{(2x-1)^2} \equiv \frac{A}{2x-1}+\frac{B}{(2x-1)^2}\,.$$

a) Determine the value of each of the constants A and B.

b) Hence find the exact value of

$$\int_1^{1.5} \frac{8x-1}{(2x-1)^2} \ dx\,.$$

$\boxed{A=4}$, $\boxed{B=3}$, $\boxed{\frac{3}{4}+2\ln 2}$

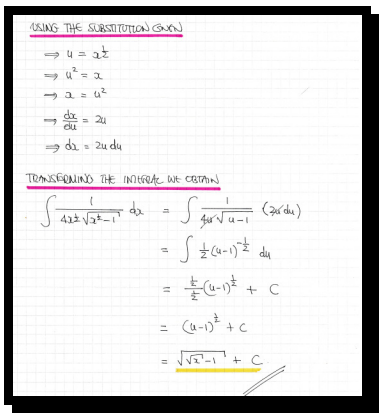


Question 44 (***)

By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find

$$\int \frac{1}{4x^{\frac{1}{2}}\sqrt{x^{\frac{1}{2}}-1}}\,dx\,.$$

$\square$, $\boxed{\left(x^{\frac{1}{2}}-1\right)^{\frac{1}{2}}+C}$



Question 44 **(***)**

a) Use integration by parts to find

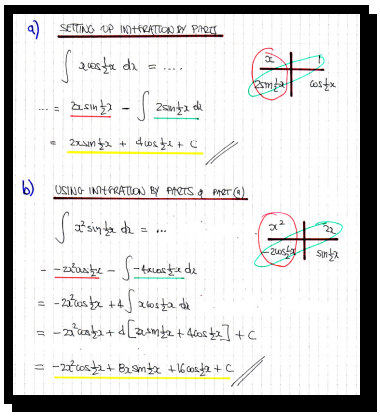
$$\int x\cos\left(\frac{1}{2}x\right)dx\,.$$

b) Hence determine

$$\int x^2\sin\left(\frac{1}{2}x\right)dx\,.$$

$\square$, $\boxed{2x\sin\left(\frac{1}{2}x\right)+4\cos\left(\frac{1}{2}x\right)+C}$,

$\boxed{-2x^2\cos\left(\frac{1}{2}x\right)+8x\sin\left(\frac{1}{2}x\right)+16\cos\left(\frac{1}{2}x\right)+C}$

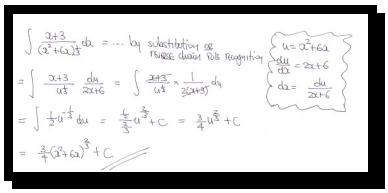


Question 45 (***)

By using the substitution $u = x^2 + 6x$, or otherwise find

$$\int \frac{x+3}{\left(x^2+6x\right)^{\frac{1}{3}}}\,dx\,.$$

$$\boxed{\frac{1}{2}\left(x^2+6x\right)^{\frac{2}{3}}+C}$$

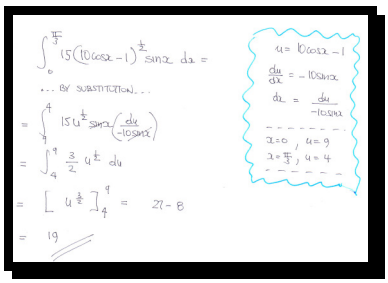


Question 46 (***)

Use the substitution $u = 10\cos x - 1$ to find

$$\int_0^{\frac{\pi}{3}} 15\left(10\cos x-1\right)^{\frac{1}{2}}\sin x\;\;dx\,.$$

$$\boxed{19}$$



Question 47 (***)

$$\frac{2x^2-x+6}{x^2(3-2x)} \equiv \frac{A}{x}+\frac{B}{x^2}+\frac{C}{3-2x}.$$

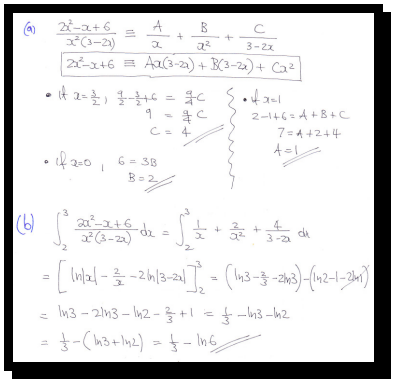
a) Determine the value of each of the constants A, B and C.

b) Evaluate

$$\int_2^3 \frac{2x^2-x+6}{x^2(3-2x)}\,dx,$$

giving the answer in the form $p-\ln q$, where p and q are constants.

$$\boxed{A=1},\ \boxed{B=2},\ \boxed{C=4},\ \boxed{\frac{1}{3}-\ln 6}$$



Question 48 **(***+)**

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$
y	0	0.1309	0.4534	0.7854		0.6545

The table above shows tabulated values for the equation

$$y = x\sin 2x\,,\ 0 \le x \le \frac{5\pi}{12}\,.$$

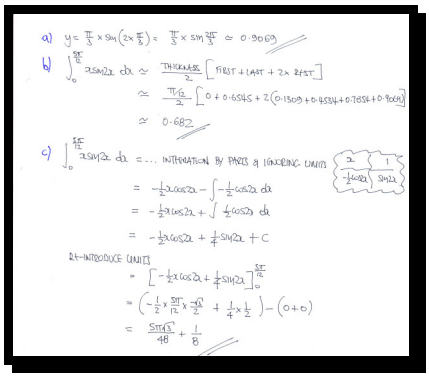
a) Complete the missing value in the table.

b) Use the trapezium rule with all the values from the table to find an approximate value for

$$\int_0^{\frac{5\pi}{12}} x\sin 2x\ \ dx\,.$$

c) Use integration by parts to find an exact value for $\displaystyle\int_0^{\frac{5\pi}{12}} x\sin 2x\ \ dx\,.$

$\boxed{}$, $\boxed{0.9069}$, $\boxed{0.682}$, $\boxed{\frac{5\pi\sqrt{3}}{48}+\frac{1}{8}}$

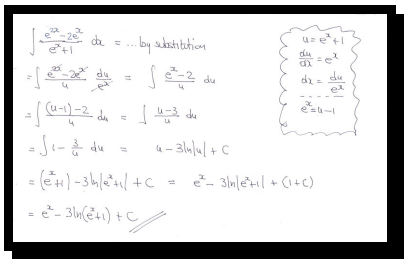


Question 49 (*+)**

By using the substitution $u = \mathrm{e}^x + 1$, or otherwise, find

$$\int \frac{\mathrm{e}^{2x} - 2\mathrm{e}^x}{\mathrm{e}^x + 1}\, dx .$$

$\square$, $\boxed{\mathrm{e}^x - 3\ln\left(\mathrm{e}^x + 1\right) + C}$

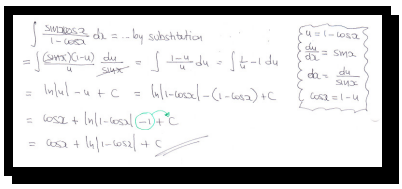


Question 50 (*+)**

By using the substitution $u = 1 - \cos x$, or otherwise, find

$$\int \frac{\sin x \cos x}{1 - \cos x}\, dx .$$

$\boxed{\cos x + \ln\left(1 - \cos x\right) + C}$

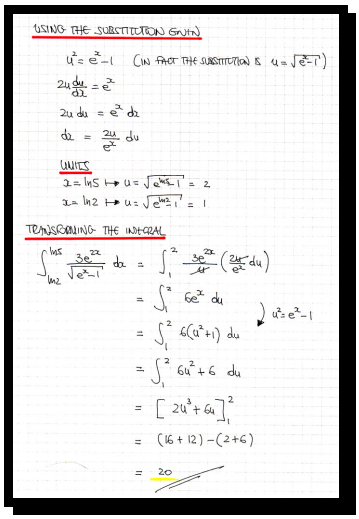


Question 51 (*+)**

By using the substitution $u^2 = \mathrm{e}^x - 1$, or otherwise, find

$$\int_{\ln 2}^{\ln 5} \frac{3\mathrm{e}^{2x}}{\sqrt{\mathrm{e}^x - 1}}\, dx .$$

$\boxed{}$, $\boxed{20}$

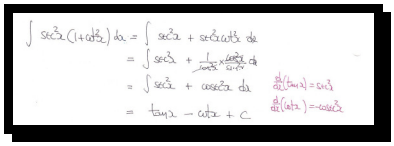


Question 52 (*+)**

Use trigonometric identities to find

$$\int \sec^2 x\left(1+\cot^2 x\right) dx .$$

$\boxed{\tan x - \cot x + C}$

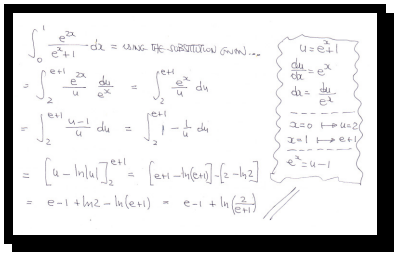


Question 53 (*+)**

By using the substitution $u = \mathrm{e}^x + 1$, or otherwise, find the exact value of

$$\int_0^1 \frac{\mathrm{e}^{2x}}{\mathrm{e}^x + 1}\, dx .$$

$$\boxed{\mathrm{e} - 1 + \ln\left(\frac{2}{1+\mathrm{e}}\right)}$$

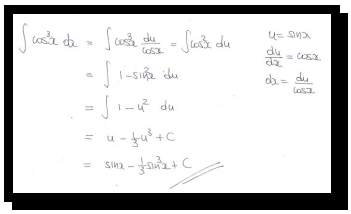


Question 54 (*+)**

By using the substitution $u = \sin x$, or otherwise, find

$$\int \cos^3 x\, dx .$$

$$\boxed{}, \quad \boxed{\sin x - \frac{1}{3}\sin^3 x + C}$$



Question 55 **(***+)**

$$I=\int (x-1)(4-x)^{\frac{1}{2}}\,dx\,,\ x\in\mathbb{R}\,,\ x\leq 4\,.$$

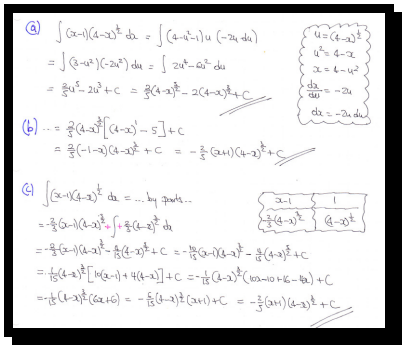
a) Use the substitution $u=(4-x)^{\frac{1}{2}}$ to find an expression for I.

b) Show that the answer of part **(a)** can be written as

$$I=-\frac{2}{5}(x+1)(4-x)^{\frac{3}{2}}+C\,.$$

c) Use integration by parts to verify the answer of part **(b)**.

$\square$, $\boxed{I=\frac{2}{5}(4-x)^{\frac{5}{2}}-2(4-x)^{\frac{3}{2}}+C}$

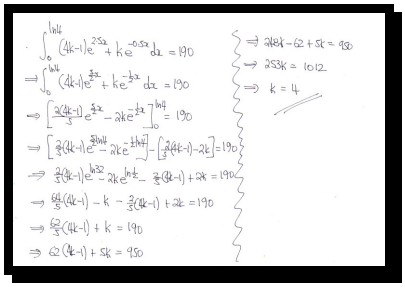


Question 56 (*+)**

Find the value of the constant k given that

$$\int_0^{\ln 4} (4k-1)\mathrm{e}^{2.5x} + k\,\mathrm{e}^{-0.5x}\ dx = 190 .$$

$\boxed{k=4}$

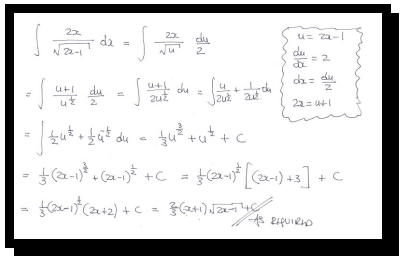


Question 57 (*+)**

By using the substitution $u = 2x-1$, or otherwise, show that

$$\int \frac{2x}{\sqrt{2x-1}}\ dx = \frac{2}{3}(x+1)\sqrt{2x-1} + C .$$

proof



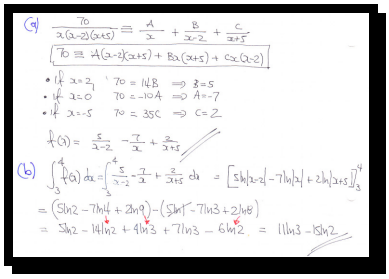
Question 58 (*+)**

$$f(x)\equiv\frac{70}{x(x-2)(x+5)}.$$

a) Express $f(x)$ in partial fractions.

b) Show that $\displaystyle\int_3^4 f(x)\,dx$ can be written in the form $p\ln 3+q\ln 2$, where p and q are integers to be found.

$$\boxed{f(x)\equiv\frac{2}{x+5}+\frac{5}{x-2}-\frac{7}{x}},\qquad \boxed{\int_3^4 f(x)\,dx=11\ln 3-15\ln 2}$$

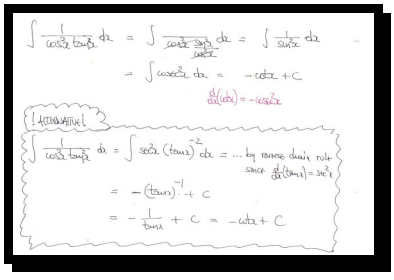


Question 59 (*+)**

Use trigonometric identities to find

$$\int\frac{1}{\cos^2 x\tan^2 x}\,dx.$$

$$\boxed{-\cot x+C}$$



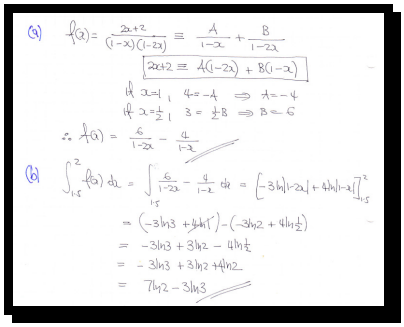
Question 60 (*+)**

$$f(x)\equiv\frac{2x+2}{(1-x)(1-2x)}.$$

a) Express $f(x)$ in partial fractions.

b) Show that $\displaystyle\int_{1.5}^{2} f(x)\,dx$ can be written in the form $p\ln 2+q\ln 3$, where p and q are integers to be found.

$$\boxed{f(x)\equiv\frac{6}{1-2x}-\frac{4}{1-x}},\qquad \boxed{\int_{1.5}^{2} f(x)\,dx=7\ln 2-3\ln 3}$$

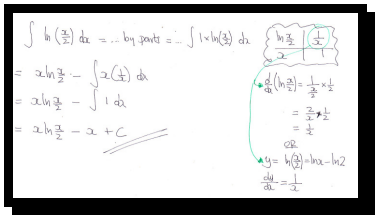


Question 61 (*+)**

Use a suitable method to find

$$\int \ln\left(\frac{x}{2}\right)dx.$$

$$\boxed{x\ln\left(\frac{x}{2}\right)-x+C}$$



Question 62 **(***+)**

$$f(x)\equiv\frac{32-17x}{(x+1)(3x-4)^2}.$$

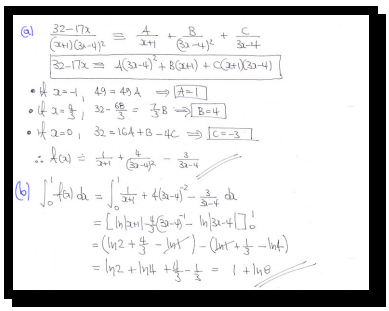
a) Express $f(x)$ in partial fractions.

b) Show that

$$\int_0^1 f(x)\ dx,$$

can be evaluated in the form $p+\ln q$, where p and q are integers to be found.

$$\boxed{f(x)\equiv\frac{4}{(3x-4)^2}-\frac{3}{(3x-4)}+\frac{1}{(x+1)}},\qquad \boxed{\int_0^1 f(x)\ dx=1+\ln 8}$$

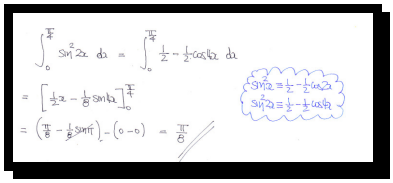


Question 63 (*+)**

Use a trigonometric identity to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sin^2 2x \ dx .$$

$$\boxed{\frac{\pi}{8}}$$

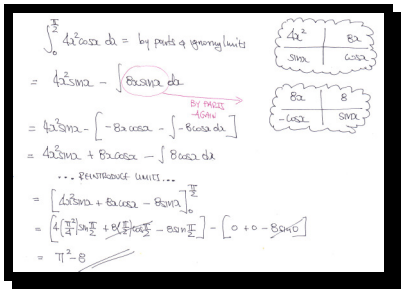


Question 64 (*+)**

Use integration by parts **twice** to find an exact value for

$$\int_0^{\frac{\pi}{2}} 4x^2 \cos x \ dx .$$

$$\boxed{\ \ }, \ \boxed{\pi^2 - 8}$$



Question 65 (*+)**

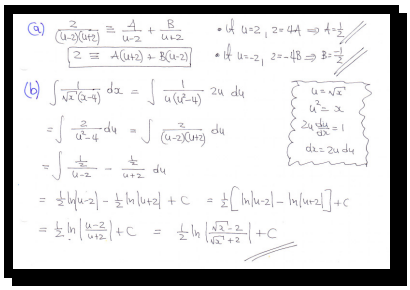
$$\frac{2}{(u-2)(u+2)} \equiv \frac{A}{u-2} + \frac{B}{u+2}.$$

a) Find the value of A and B in the above identity.

b) By using the substitution $u = \sqrt{x}$, or otherwise, find

$$\int \frac{1}{\sqrt{x}(x-4)}\, dx.$$

$$\boxed{A = \frac{1}{2}}, \quad \boxed{B = -\frac{1}{2}}, \quad \boxed{\frac{1}{2}\ln\left|\frac{\sqrt{x}-2}{\sqrt{x}-2}\right| + C}$$

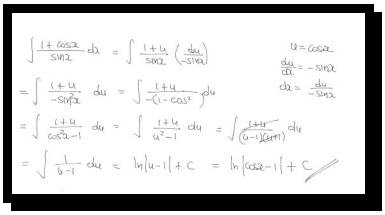


Question 66 (*+)**

By using the substitution $u = \cos x$, or otherwise, find

$$\int \frac{1+\cos x}{\sin x}\, dx.$$

$$\boxed{\ln|\cos x - 1| + C}$$

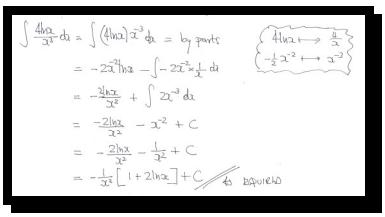


Question 67 (*+)**

Use integration by parts to show that

$$\int \frac{4\ln x}{x^3}\,dx = -\frac{1+2\ln x}{x^2}+C\,.$$

$$\boxed{\sin x-\tfrac{1}{3}\sin^3 x+C}$$

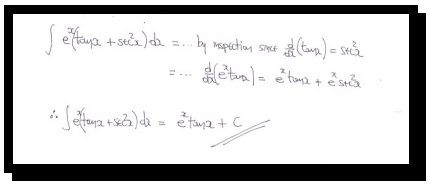


Question 68 (*+)**

By considering the differentiation of a product of two appropriate functions, find

$$\int \mathrm{e}^x\left(\tan x+\sec^2 x\right)dx\,.$$

$$\boxed{\mathrm{e}^x\tan x+C}$$



Question 69 (*+)**

$$\frac{2x^2-3}{(x-1)^2}\equiv A+\frac{B}{x-1}+\frac{C}{(x-1)^2}.$$

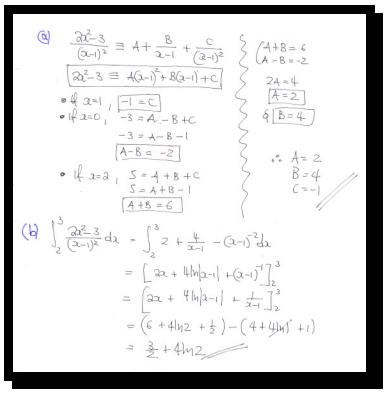
a) Determine the value of each of the constants A, B and C.

b) Evaluate

$$\int_2^3 \frac{2x^2-3}{(x-1)^2}\,dx,$$

giving the answer in the form $p+\ln q$, where p and q are constants.

$\boxed{}$, $\boxed{A=2}$, $\boxed{B=4}$, $\boxed{C=-1}$, $\boxed{\frac{3}{2}+\ln 16}$

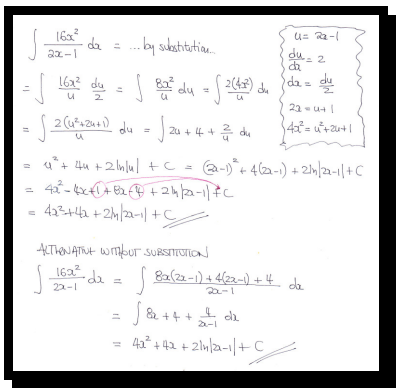


Question 70 (*+)**

By using the substitution $u = 2x-1$, or otherwise, find

$$\int \frac{16x^2}{2x-1}\,dx\,.$$

$$\boxed{(2x-1)^2+4(2x-1)+2\ln|2x-1|+C \;=\; 4x^2+4x+2\ln|2x-1|+C}$$

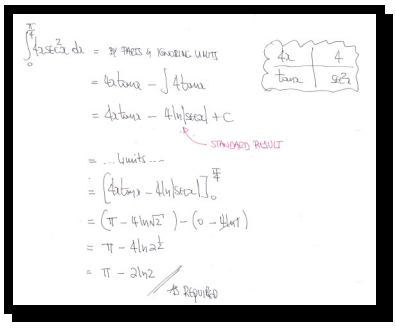


Question 71 (*+)**

Use integration by parts to show that

$$\int_0^{\frac{\pi}{4}} 4x\sec^2 x\;\;dx = \pi - 2\ln 2\,.$$

proof



Question 72 (*+)**

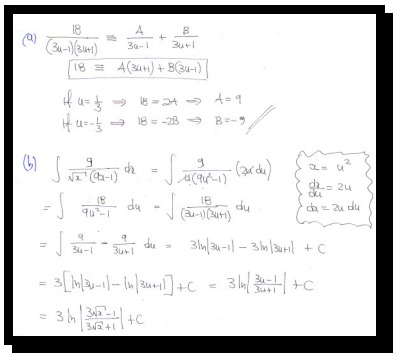
$$\frac{18}{(3u-1)(3u+1)} \equiv \frac{A}{3u-1} + \frac{B}{3u+1}.$$

a) Find the value of A and B in the above identity.

b) By using the substitution $x = u^2$, or otherwise, find

$$\int \frac{9}{\sqrt{x}(9x-1)}\,dx.$$

$\boxed{A=9}$, $\boxed{B=-9}$, $\boxed{3\ln\left|\frac{3\sqrt{x}-1}{3\sqrt{x}+1}\right|+C}$



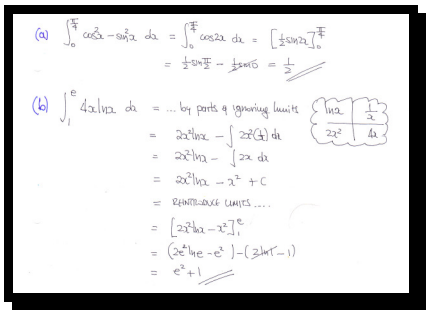
Question 73 (*+)**

Use an appropriate integration method to find an exact value for each of the following integrals

a) $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 x - \sin^2 x \ dx$.

b) $\displaystyle\int_1^{\mathrm{e}} 4x\ln x \ dx$.

$\boxed{\frac{1}{2}}$, $\boxed{\mathrm{e}^2+1}$

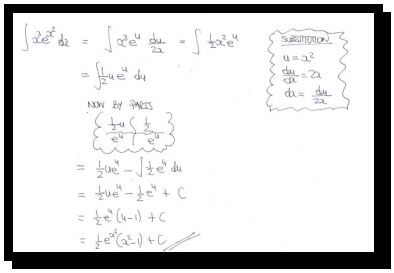


Question 74 (*+)**

Use the substitution $u = x^2$, followed by integration by parts to find

$$\int x^3 \mathrm{e}^{x^2} \ dx .$$

$\boxed{}$, $\boxed{\frac{1}{2}\mathrm{e}^{x^2}\left(x^2-1\right)+C}$

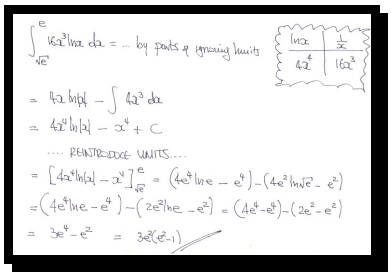


Question 75 (*+)**

Use integration by parts to find the exact value of

$$\int_{\sqrt{\mathrm{e}}}^{\mathrm{e}} 16x^3 \ln x \ dx .$$

$$\boxed{3\mathrm{e}^2\left(\mathrm{e}^2-1\right)}$$

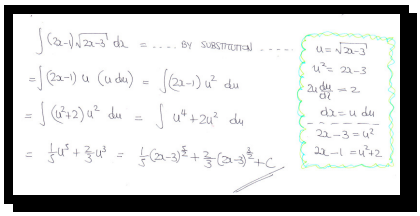


Question 76 (*+)**

By using the substitution $u=\sqrt{2x-3}$, or otherwise, find an expression for

$$\int (2x-1)\sqrt{2x-3}\ dx .$$

$$\boxed{\frac{1}{5}(2x-3)^{\frac{5}{2}}+\frac{2}{3}(2x-3)^{\frac{3}{2}}+C}$$



Question 77 (*+)**

$$\frac{3x^2-2x+1}{2x(x-1)^2}\equiv\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}.$$

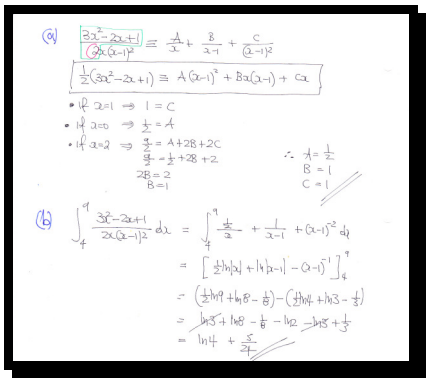
a) Determine the value of each of the constants A, B and C.

b) Evaluate

$$\int_4^9 \frac{3x^2-2x+1}{2x(x-1)^2}\,dx,$$

giving the answer in the form $p+q\ln 4$, where p and q are constants.

$$\boxed{A=\frac{1}{2}},\ \boxed{B=1},\ \boxed{C=1},\ \boxed{\frac{5}{24}+\ln 4}$$



Question 78 (*+)**

It is given that

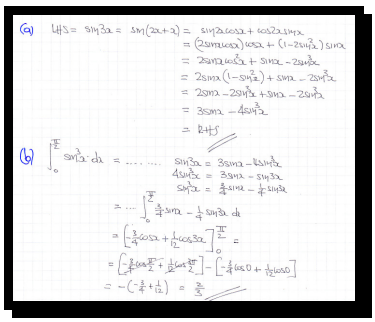
$$\sin 3x \equiv 3\sin x - 4\sin^3 x .$$

a) Prove the above trigonometric identity, by writing $\sin 3x$ as $\sin(2x+x)$.

b) Hence, or otherwise, find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin^3 x \ dx .$$

$\boxed{\frac{2}{3}}$

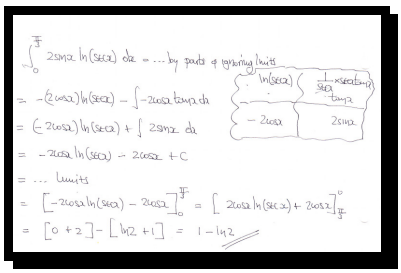


Question 79 (*+)**

Use integration by parts to find an exact value for

$$\int_{1}^{\frac{\pi}{3}} 2\sin x \ln\left(\sec x\right) \, dx .$$

$\boxed{1-\ln 2}$



Question 80 **(***+)**

x	0	$\frac{\pi}{18}$	$\frac{\pi}{9}$	$\frac{\pi}{6}$
y	0	0.1632		0.2500

The table above shows tabulated values for the equation

$$y=\sin x\cos 2x,\ 0\le x\le\frac{\pi}{6}.$$

a) Complete the missing value in the table. **(1)**

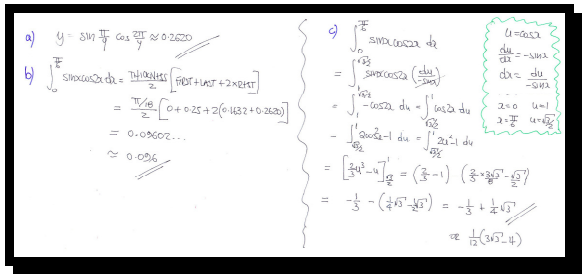
b) Use the trapezium rule with all the values from the table to find an approximate value for

$$\int_0^{\frac{\pi}{6}}\sin x\cos 2x\ \ dx.$$ **(3)**

c) By using the substitution $u=\cos x$, or otherwise, find an exact value for **(6)**

$$\int_0^{\frac{\pi}{6}}\sin x\cos 2x\ \ dx.$$

$\boxed{}$, $\boxed{0.2620}$, $\boxed{0.096}$, $\boxed{\frac{1}{12}\left(3\sqrt{3}-4\right)}$



Question 81 **(***+)**

$$\frac{12x^2+x+3}{(6x+1)\left(2x^2+1\right)}\equiv\frac{A}{6x+1}+\frac{Bx+C}{2x^2+1}.$$

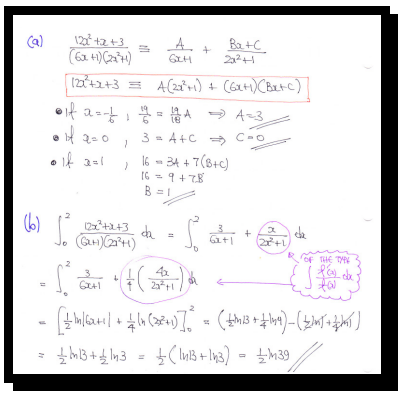
a) Determine the value of each of the constants A, B and C.

b) Evaluate

$$\int_0^2\frac{12x^2+x+3}{(6x+1)\left(2x^2+1\right)}\,dx,$$

giving the answer in the form $p\ln q$, where p and q are constants.

$\boxed{A=3}$, $\boxed{B=1}$, $\boxed{C=0}$, $\boxed{\frac{1}{2}\ln 39}$



Question 82 (*+)**

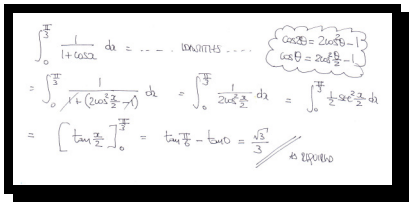
By using the trigonometric identity

$$\cos 2\theta \equiv 2\cos^2\theta - 1$$

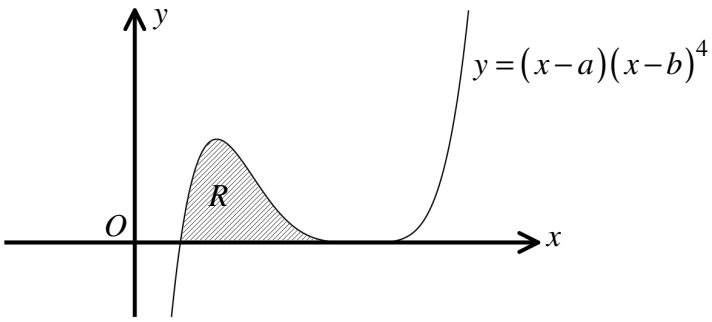
and the fact that $\dfrac{d}{dx}\left(\tan x\right) = \sec^2 x$, show clearly that

$$\int_0^{\frac{\pi}{3}} \frac{1}{1+\cos x}\, dx = \frac{\sqrt{3}}{3}.$$

proof



Question 83 (*+)**



The figure above shows the graph of the curve with equation

$$y=(x-a)(x-b)^4,$$

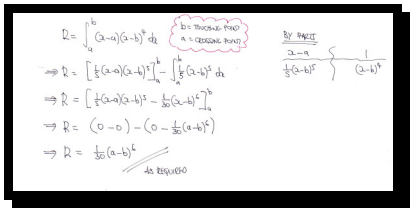
where a and b are positive constants.

The shaded region R is bounded by the curve and the x axis.

By using integration by parts, or otherwise, show that the area of the shaded region is

$$\frac{1}{30}(a-b)^6.$$

proof

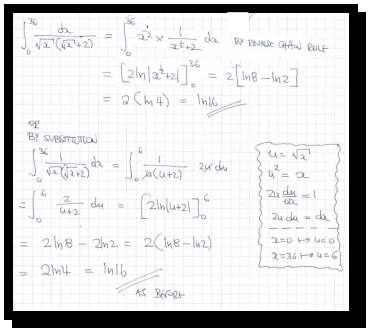


Question 84 (*+)**

By using the substitution $u=\sqrt{x}$, or otherwise, show that

$$\int_0^{36}\frac{1}{\sqrt{x}\left(\sqrt{x}+2\right)}\,dx=\ln 16\,.$$

proof

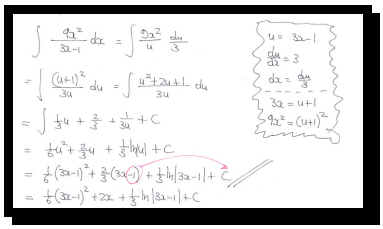


Question 85 (*+)**

By using the substitution $u=3x-1$, or otherwise, find

$$\int\frac{9x^2}{3x-1}\,dx\,.$$

$$\boxed{\frac{1}{6}(3x-1)^2+\frac{2}{3}(3x-1)+\frac{1}{3}\ln|3x-1|+C}$$

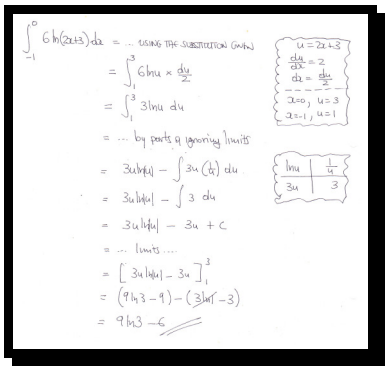


Question 86 (*+)**

By using the substitution $u=2x+3$, or otherwise, show clearly that

$$\int_{-1}^{0} 6\ln\left(2x+3\right)\ dx=9\ln 3-6\,.$$

proof

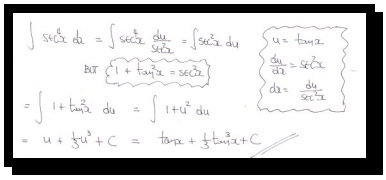


Question 87 (*+)**

By using the substitution $u=\tan x$, or otherwise, find

$$\int \sec^4 x\ dx\,.$$

$$\tan x+\frac{1}{3}\tan^3 x+C$$



Question 88 **(***+)**

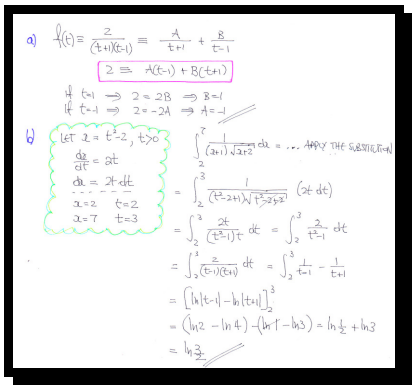
$$f(t)\equiv\frac{2}{(t-1)(t+1)}\equiv\frac{A}{(t-1)}+\frac{B}{(t+1)}.$$

a) Find the value of each of the constants A and B in the above identity.

b) Use the substitution $x=t^2-2$, $t>0$ to show that

$$\int_4^9\frac{1}{(x+1)\sqrt{x+2}}\,dx=\ln\frac{3}{2}.$$

$\boxed{A=2}$, $\boxed{B=2}$



Question 89 **(***+)**

x	0	$\frac{2\pi}{5}$	$\frac{4\pi}{5}$	$\frac{6\pi}{5}$	$\frac{8\pi}{5}$	2π
y	0	0.2031	0.8602			0

The table above shows some tabulated values for the equation

$$y=\sin^3\left(\frac{1}{2}x\right),\ 0\le x\le 2\pi\,.$$

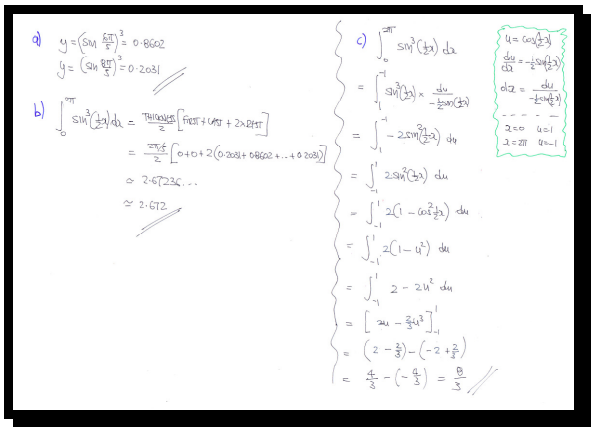
a) Complete the missing values in the table.

b) Use the trapezium rule with all the values from the table to find an approximate value for

$$\int_0^{2\pi}\sin^3\left(\frac{1}{2}x\right)\,dx\,.$$

c) By using the substitution $u=\cos\left(\frac{1}{2}x\right)$, or otherwise, find the value of the integral of part (b).

$\boxed{}$, $\boxed{0.8602,\ 0.2031}$, $\boxed{2.672}$, $\boxed{\frac{8}{3}}$

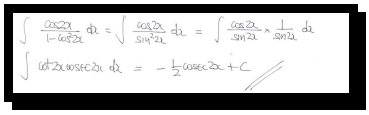


Question 90 (*+)**

Use trigonometric identities to integrate

$$\int \frac{\cos 2x}{1-\cos^2 2x}\,dx\,.$$

$\boxed{}$, $\boxed{-\frac{1}{2}\operatorname{cosec} 2x + C}$

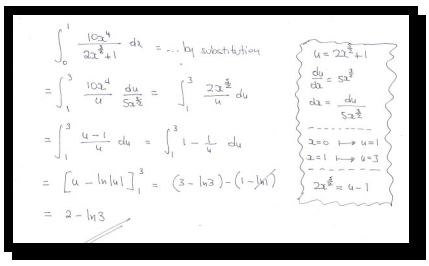


Question 91 (****)

By using the substitution $u = 2x^{\frac{5}{2}} + 1$, or otherwise, find an exact simplified value for

$$\int_0^1 \frac{10x^4}{2x^{\frac{5}{2}}+1}\,dx\,.$$

$\boxed{}$, $\boxed{2-\ln 3}$



Question 92 (****)

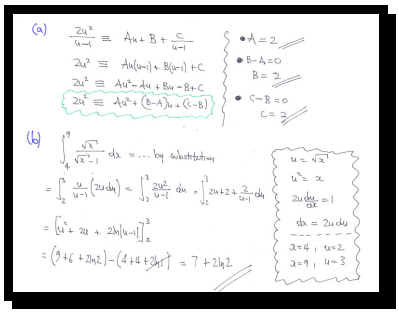
$$\frac{2u^2}{u-1} \equiv Au + B + \frac{C}{u-1}.$$

a) Find the value of each of the constants A, B and C in the above identity.

b) Use the substitution $u = \sqrt{x}$ to show

$$\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1}\, dx = 7 + 2\ln 2.$$

$\boxed{A=2}$, $\boxed{B=2}$, $\boxed{C=2}$



Question 93 (****)

$$\int_1^9 \frac{1}{2x\left(1+\sqrt{x}\right)}\,dx\,.$$

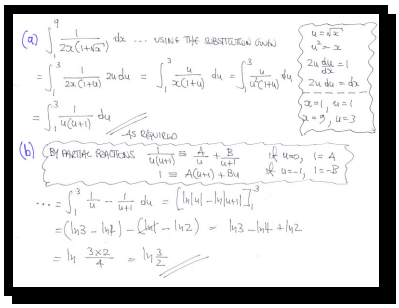
a) Show that the substitution $u=\sqrt{x}$ transforms the above integral to

$$\int_{x_1}^{x_2} \frac{1}{u\left(u+1\right)}\,du\,,$$

where x_1 and x_2 are constants to be found.

b) Hence find an exact value for the original integral.

$$\boxed{\ln\left(\frac{3}{2}\right)}$$



Question 94 (****)

$$f(x)\equiv\frac{x^3}{x^2-4},\ x\neq\pm2\,.$$

a) Use a suitable substitution to show that

$$\int_{\sqrt{6}}^{\sqrt{8}} f(x)\ dx=1+\ln 4\,.$$

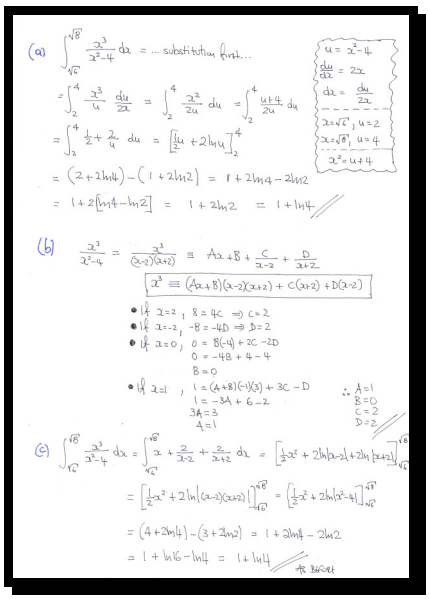
b) Express $f(x)$ in the form

$$Ax+B+\frac{C}{x-2}+\frac{D}{x+2},$$

where A, B, C and D are constants to be found.

c) Use the result part (b) to verify the result of part (a).

$\boxed{A=1}$, $\boxed{B=0}$, $\boxed{C=2}$, $\boxed{D=2}$

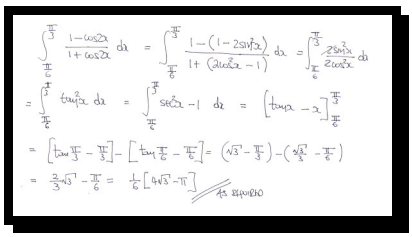


Question 95 (****)

By using the cosine double angle identities and the fact that $\dfrac{d}{dx}(\tan x)=\sec^2 x$, show clearly that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1-\cos 2x}{1+\cos 2x}\,dx=\frac{1}{6}\left(4\sqrt{3}-\pi\right).$$

proof

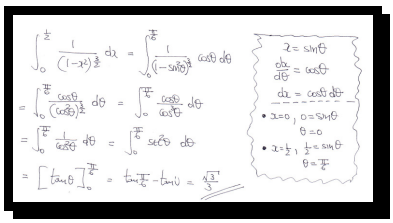


Question 96 (****)

Use the substitution $x=\sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{\left(1-x^2\right)^{\frac{3}{2}}}\,dx.$$

$\dfrac{\sqrt{3}}{3}$



Question 97 (****)

$$f(x)\equiv\frac{1}{x\left(x^2+1\right)},\ x\neq 0.$$

a) Use the substitution $x=\tan\theta$ to find

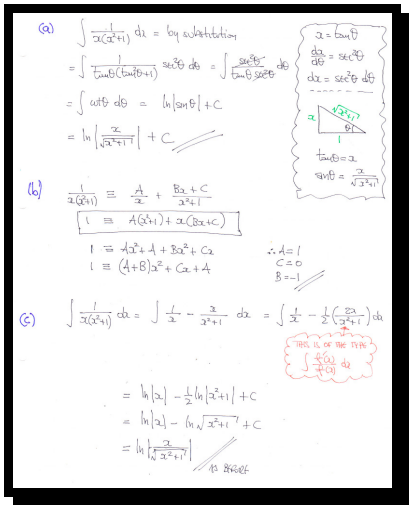
$$\int f(x)\,dx.$$

b) Find the value of each of the constants A, B and C, so that

$$f(x)\equiv\frac{A}{x}+\frac{Bx+C}{x^2+1}.$$

c) Use the result of part (b) to independently verify the answer of part (a).

$$\boxed{\ln\left(\frac{x}{\sqrt{x^2+1}}\right)},\ \boxed{A=1},\ \boxed{B=-1},\ \boxed{C=0}$$



Question 98 **(****)**

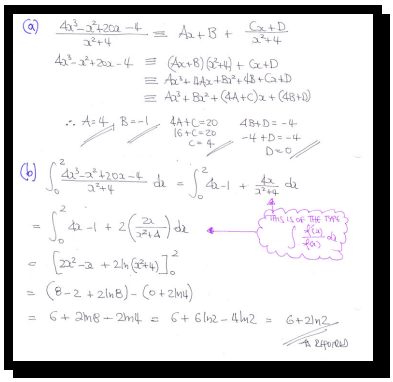
$$\frac{4x^3-x^2+20x-4}{x^2+4}\equiv Ax+B+\frac{Cx+D}{x^2+4}.$$

a) Determine the value of each of the constants A, B, C and D.

b) Hence show that

$$\int_0^2 \frac{4x^3-x^2+20x-4}{x^2+4}\,dx=6+2\ln 2,$$

$\boxed{A=4}$, $\boxed{B=-1}$, $\boxed{C=4}$, $\boxed{D=0}$

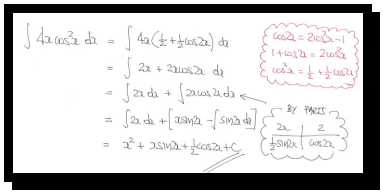


Question 99 **(****)**

Use a cosine double identity and integration by parts to find

$$\int 4x\cos^2 x\,dx.$$

$\boxed{x^2+x\sin 2x+\frac{1}{2}\cos 2x+C}$

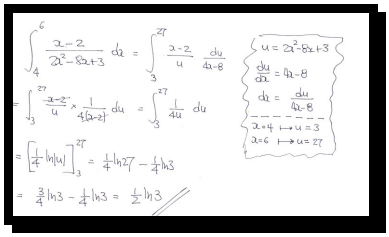


Question 100 (****)

By using the substitution $u = 2x^2 - 8x + 3$, or otherwise, find the exact value

$$\int_4^6 \frac{x-2}{2x^2-8x+3}\, dx .$$

$\boxed{\frac{1}{2}\ln 3}$



Question 101 (****)

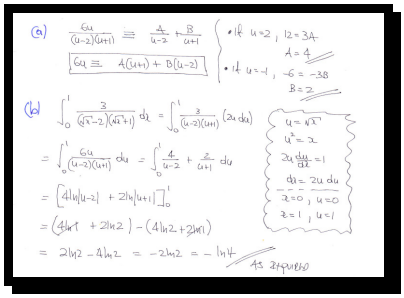
$$\frac{6u}{(u-2)(u+1)} \equiv \frac{A}{u-2} + \frac{B}{u+1}.$$

a) Find the value of each of the constants A and B in the above identity.

b) By using the substitution $u = \sqrt{x}$, or otherwise, show that

$$\int_0^1 \frac{3}{\left(\sqrt{x}-2\right)\left(\sqrt{x}+1\right)}\, dx = -\ln 4 .$$

$\boxed{A=4}$, $\boxed{B=2}$



Question 102 (****)

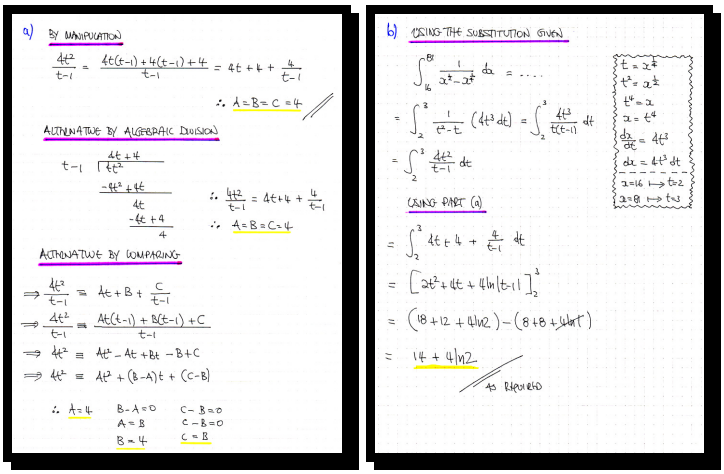
$$\frac{4t^2}{t-1} \equiv At + B + \frac{C}{t-1}.$$

a) Determine the value of each of the constants A, B and C.

b) Use the substitution $t = x^{\frac{1}{4}}$ to show

$$\int_{16}^{81} \frac{1}{x^{\frac{1}{2}} - x^{\frac{1}{4}}}\, dx = 14 + 4\ln 2.$$

$\boxed{}$, $\boxed{A=4}$, $\boxed{B=4}$, $\boxed{C=4}$



Question 103 (****)

It is given that

$$\int_{k}^{2k} \frac{3x-5}{x(x-1)}\,dx = \ln 72,$$

determine the value of k, $0<k<1$.

$\square$, $\boxed{k=\frac{1}{4}}$

START BY PARTIAL FRACTIONS

$$\frac{3x-5}{x(x-1)} \equiv \frac{A}{x}+\frac{B}{x-1}$$

$$\boxed{3x-5 \equiv A(x-1)+Bx}$$

IF $x=1 \Rightarrow -2=B$

$\Rightarrow B=-2$

IF $x=0 \Rightarrow -5=-A$

$\Rightarrow A=5$

HENCE THE INTEGRAL BECOMES

$$\Rightarrow \int_{k}^{2k} \frac{3x-5}{x(x-1)}\,dx = \ln 72$$

$$\Rightarrow \int_{k}^{2k} \frac{5}{x}-\frac{2}{x-1}\,dx = \ln 72$$

$$\Rightarrow \left[5\ln|x|-2\ln|x-1|\right]_{k}^{2k} = \ln 72$$

$$\Rightarrow \left[5\ln|2k|-2\ln|2k-1|\right]-\left[5\ln|k|-2\ln|k-1|\right]=\ln 72$$

$$\Rightarrow 5\ln|2k|-2\ln|2k-1|-5\ln|k|+2\ln|k-1| = \ln 72$$

$$\Rightarrow 5\ln\left|\frac{2k}{k}\right|+2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72$$

$$\Rightarrow 5\ln 2+2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72$$

$$\Rightarrow 2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72-5\ln 2$$

$$\Rightarrow 2\ln\left|\frac{k-1}{2k-1}\right| = \ln 72-\ln 32$$

$$\Rightarrow 2\ln\left|\frac{k-1}{2k-1}\right| = \ln\frac{9}{4}$$

$$\Rightarrow 2\ln\left|\frac{k-1}{2k-1}\right| = \ln\left(\frac{3}{2}\right)^2$$

$$\Rightarrow 2\ln\left|\frac{k-1}{2k-1}\right| = 2\ln\frac{3}{2}$$

$$\Rightarrow \frac{k-1}{2k-1}=\frac{3}{2}$$

$$\Rightarrow 6k-3=2k-2$$

$$\Rightarrow 4k=1$$

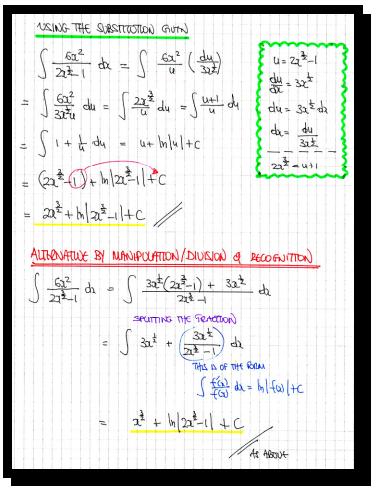
$$\Rightarrow k=\frac{1}{4}$$

Question 104 (****)

By using the substitution $u = 2x^{\frac{3}{2}} - 1$, or otherwise, find an expression for the integral

$$\int \frac{6x^2}{2x^{\frac{3}{2}} - 1} \, dx .$$

$\square$, $\boxed{2x^{\frac{1}{2}} + \ln\left|2x^{\frac{3}{2}} - 1\right| + C}$

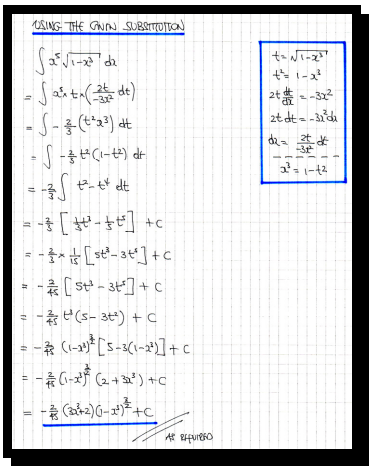


Question 105 (****)

Use the substitution $t=\sqrt{1-x^3}$ to show that

$$\int x^5\sqrt{1-x^3}\,dx=-\frac{2}{45}\left(3x^3+2\right)\left(1-x^3\right)^{\frac{3}{2}}+C\,.$$

[ ], [proof]



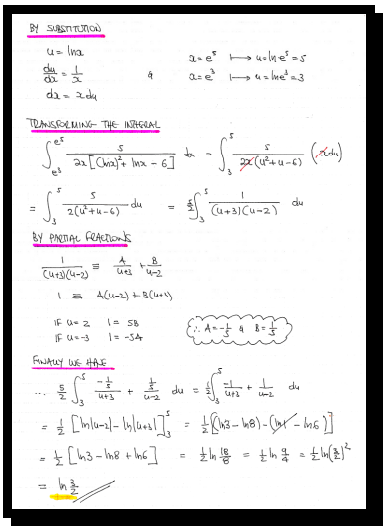
Question 106 (****)

Use an appropriate substitution, followed by partial fractions, to show that

$$\int_{e^3}^{e^5} \frac{5}{2x\left[(\ln x)^2+\ln x-6\right]}\,dx = \ln\left(\frac{3}{2}\right).$$

You may assume that the integral converges.

[ ], proof



Question 107 (****)

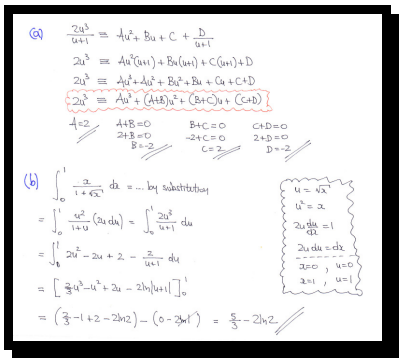
$$\frac{2u^3}{u+1} \equiv Au^2 + Bu + C + \frac{D}{u+1}.$$

a) Find the value of each of the constants A, B and C in the above identity.

b) Use the substitution $u = \sqrt{x}$ to show

$$\int_0^1 \frac{x}{1+\sqrt{x}}\, dx = \frac{5}{3} - 2\ln 2.$$

$\boxed{A=2}$, $\boxed{B=-2}$, $\boxed{C=2}$, $\boxed{D=-2}$



Question 109 (****)

$$\int \frac{\cos x}{1-\cos x}\,dx\,.$$

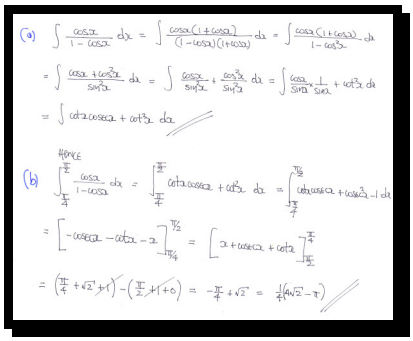
a) Show by multiplying the numerator and denominator of the integrand by $(1+\cos x)$, that the above integral can eventually be written as

$$\int \cot x \operatorname{cosec} x + \cot^2 x\,dx\,.$$

b) Hence show further that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{1-\cos x}\,dx = \frac{1}{4}\left(4\sqrt{2}-\pi\right).$$

$\boxed{}$, $\boxed{\text{proof}}$

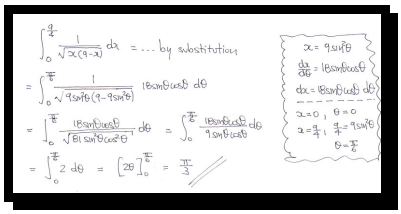


Question 109 **(****)**

By using the substitution $x=9\sin^2\theta$, or otherwise, find the exact value of

$$\int_0^{\frac{9}{4}} \frac{1}{\sqrt{x(9-x)}}\,dx .$$

$$\frac{\pi}{3}$$



Question 110 **(****)**

It is given that

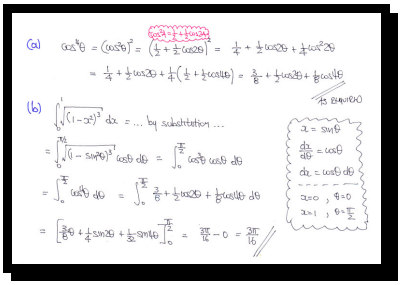
$$\cos^4\theta \equiv \frac{3}{8}+\frac{1}{2}\cos 2\theta+\frac{1}{8}\cos 4\theta .$$

a) Prove the validity of the above trigonometric identity.

b) Use the substitution $u=\sin\theta$ to show

$$\int_0^1 \sqrt{\left(1-x^2\right)^3}\;dx=\frac{3\pi}{16} .$$

proof



Question 111 (****)

$$y=\frac{x^2+2x-2}{x^2-2x+2}.$$

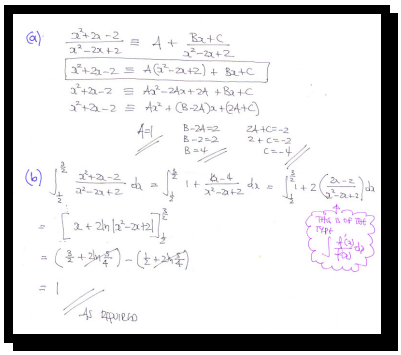
a) Find the value of each of the constants A, B and C, so that

$$y\equiv A+\frac{Bx+C}{x^2-2x+2}.$$

b) Hence, or otherwise, show that

$$\int_{\frac{1}{2}}^{\frac{3}{2}} y\ dx=1.$$

$\boxed{A=1}$, $\boxed{B=4}$, $\boxed{C=-4}$



Question 112 (****)

$$f(x)\equiv\frac{2}{x+\sqrt{2x-1}},\ x\geq\frac{1}{2}.$$

a) Use the substitution $u=\sqrt{2x-1}$ transforms to show

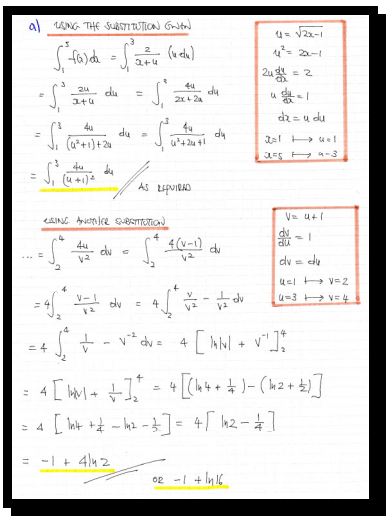
$$\int_1^5 f(x)\ dx\equiv\int_{u_1}^{u_2}\frac{4u}{(u+1)^2}\ du,$$

where u_1 and u_2 are constants to be found.

b) By using another suitable substitution, or otherwise, show that

$$\int_1^5 f(x)\ dx=-1+\ln 16.$$

$[\ \ \ \ \ ]$, proof

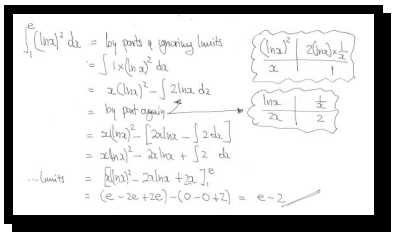


Question 113 (****)

Use integration by parts find an exact value in terms of e, for the integral

$$\int_1^{\mathrm{e}} (\ln x)^2 \, dx .$$

$\boxed{\mathrm{e}-2}$

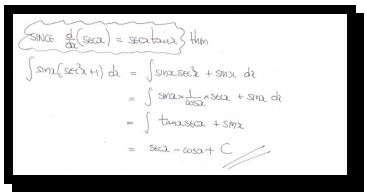


Question 114 (****)

Use the fact that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$, to find

$$\int \sin x \left(1+\sec^2 x\right) dx .$$

$\boxed{\sec x - \cos x + C}$



Question 115 (****)

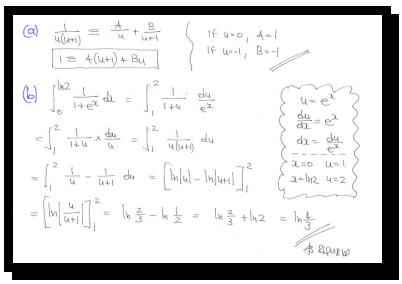
$$\frac{1}{u(u+1)} \equiv \frac{A}{u} + \frac{B}{u+1}.$$

a) Find the value of A and B in the above identity.

b) By using the substitution $u = \mathrm{e}^x$, or otherwise, show that

$$\int_0^{\ln 2} \frac{1}{1+\mathrm{e}^x}\, dx = \ln\left(\frac{4}{3}\right).$$

$\boxed{A=1}$, $\boxed{B=-1}$



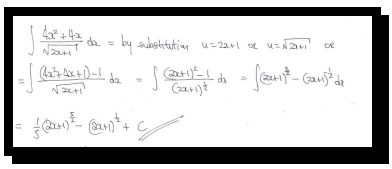
Question 116 (****)

By completing the square in the expression $4x^2+4x$, or otherwise, show that

$$\int \frac{4x^2+4x}{\sqrt{2x+1}}\, dx = A(2x+1)^{\frac{5}{2}} + B(2x+1)^{\frac{1}{2}} + C,$$

where A and B are constants to be found and C is the arbitrary constant of the integration.

$\boxed{A=\frac{1}{5}}$, $\boxed{B=-1}$



Question 117 (****)

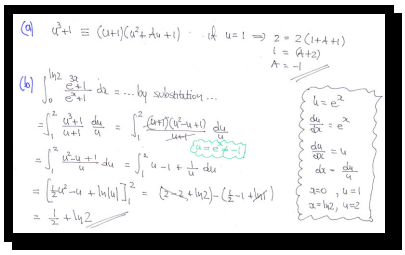
$$u^3+1\equiv(u+1)\left(u^2+Au+1\right).$$

a) Determine the value of A in the above identity.

b) Use the substitution $u=\mathrm{e}^x$ to show

$$\int_0^{\ln 2}\frac{\mathrm{e}^{3x}+1}{\mathrm{e}^x+1}\,dx=\frac{1}{2}+\ln 2\,.$$

$\boxed{A=1}$, $\boxed{B=\frac{1}{2}}$, $\boxed{C=-\frac{1}{2}}$



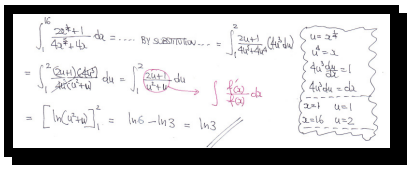
Question 118 (****)

Use the substitution $u=x^{\frac{1}{4}}$ to find

$$\int_1^{16}\frac{2x^{\frac{1}{4}}+1}{4x^{\frac{5}{4}}+4x}\,dx\,,$$

giving the final answer as an exact simplified natural logarithm.

$\boxed{\ln 3}$



Question 119 **(****)**

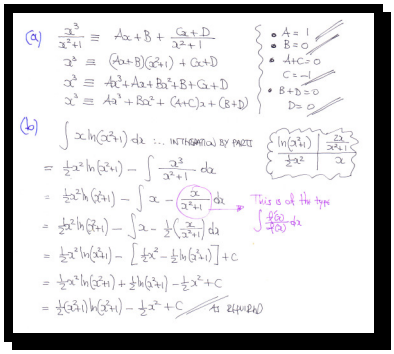
$$\frac{x^3}{x^2+1} \equiv Ax+B+\frac{Cx+D}{x^2+1}.$$

a) Determine the value of each of the constants A, B, C and D.

b) Use integration by parts to show that

$$\int x\ln\left(x^2+1\right)dx=\frac{1}{2}\left(x^2+1\right)\ln\left(x^2+1\right)-\frac{1}{2}x^2+C.$$

$\boxed{A=1}$, $\boxed{B=0}$, $\boxed{C=-1}$, $\boxed{D=0}$

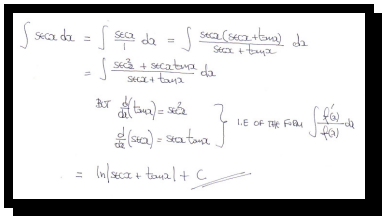


Question 120 **(****)**

By writing $\sec x$ as the fraction $\dfrac{\sec x}{1}$ and multiplying the numerator and the denominator by $\left(\sec x+\tan x\right)$, find

$$\int \sec x\,dx.$$

$\boxed{\ln\left|\sec x+\tan x\right|+C}$



Question 121 **(****)**

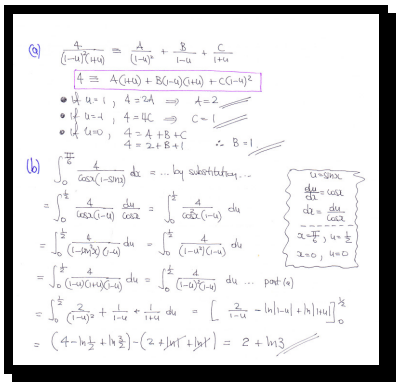
$$\frac{4}{(1-u)^2(1+u)} \equiv \frac{A}{(1-u)^2} + \frac{B}{1-u} + \frac{C}{1+u}.$$

a) Find the value of A, B and C in the above identity.

b) Hence by using a suitable substitution find the exact value of

$$\int_0^{\frac{\pi}{6}} \frac{4}{\cos x(1-\sin x)}\, dx.$$

$\boxed{\quad}$, $\boxed{A=2}$, $\boxed{B=1}$, $\boxed{C=1}$, $\boxed{2+\ln 3}$

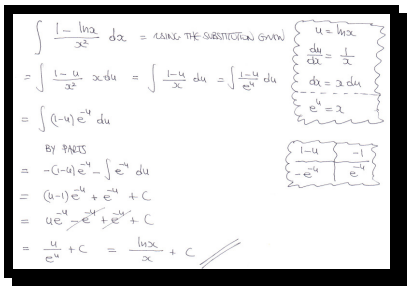


Question 122 (****)

Use the substitution $u = \ln x$, followed by integration by parts to find

$$\int \frac{1-\ln x}{x^2}\,dx .$$

$$\boxed{\frac{\ln x}{x}+C}$$



Question 123 (****)

$$y = \arctan x , \quad x \in \mathbb{R} .$$

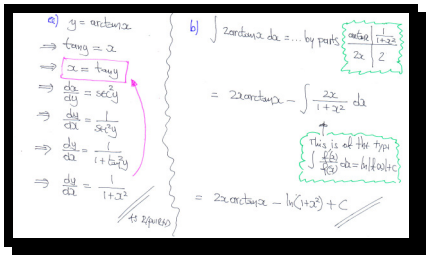
a) By writing the above equation as $x = \tan y$, show clearly that

$$\frac{dy}{dx} = \frac{1}{1+x^2} .$$

b) Use integration by parts and the result of part (a) to find

$$\int 2\arctan x \; dx .$$

$$\boxed{2x\arctan x - \ln\left(1+x^2\right)+C}$$



Question 124 **(****)**

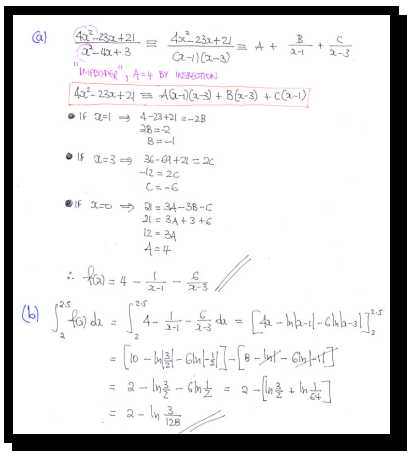
$$f(x)\equiv\frac{4x^2-23x+21}{x^2-4x+3},\ x\neq 1,\ x\neq 3.$$

a) Express $f(x)$ in partial fractions.

b) Hence find an exact value for

$$\int_2^{2.5} f(x)\ dx.$$

$$\boxed{f(x)\equiv 4-\frac{1}{x-1}-\frac{6}{x-3}},\quad \boxed{2-\ln\left(\frac{3}{128}\right)}$$

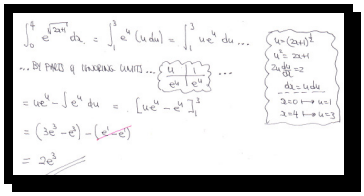


Question 125 (****)

By using the substitution $u=(2x+1)^{\frac{1}{2}}$, or otherwise, show that

$$\int_0^4 \mathrm{e}^{\sqrt{2x+1}}\ dx = 2\mathrm{e}^3 .$$

proof

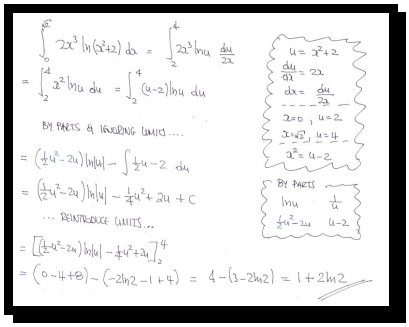


Question 126 (****)

Use the substitution $u=x^2+2$, followed by integration by parts to show that

$$\int 2x^3 \ln\left(x^2+2\right) dx = 1+2\ln 2 .$$

proof

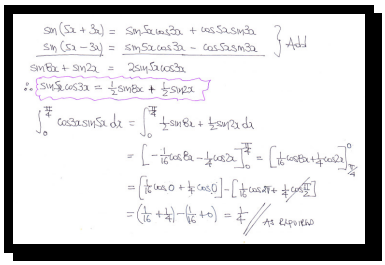


Question 127 **(****)**

By considering the trigonometric expansions of $\sin(5x+3x)$ and $\sin(5x-3x)$, show clearly that

$$\int_0^{\frac{\pi}{4}} \cos 3x \sin 5x \ dx = \frac{1}{4}.$$

proof

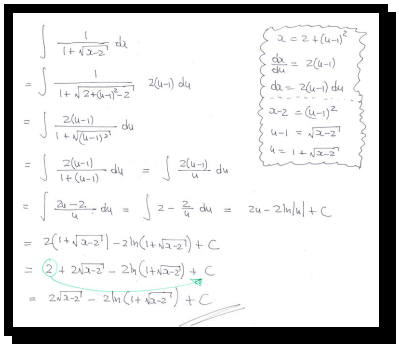


Question 128 **(****)**

By using the substitution $x = 2+(u-1)^2$, or otherwise, find

$$\int \frac{1}{1+\sqrt{x-2}} \, dx .$$

$$2\sqrt{x-2}+2\ln\left(1+\sqrt{x-2}\right)+C$$



Question 129 (****)

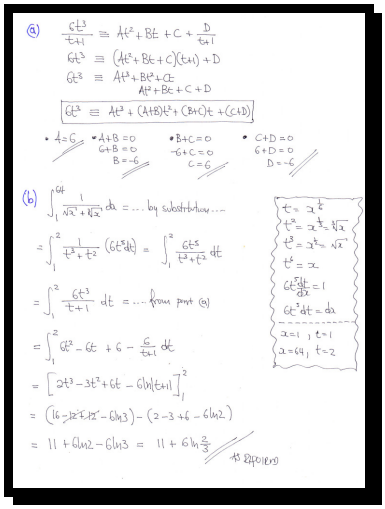
$$\frac{6t^3}{t+1} \equiv At^2 + Bt + C + \frac{D}{t+1}.$$

a) Determine the value of each of the constants A, B, C and D.

b) Use the substitution $t = x^{\frac{1}{6}}$ to show

$$\int_1^{64} \frac{1}{\sqrt{x}+\sqrt[3]{x}}\, dx = 11 + 6\ln\left(\frac{2}{3}\right).$$

$\boxed{A=6}$, $\boxed{B=-6}$, $\boxed{C=6}$, $\boxed{D=-6}$



Question 130 (****)

$$f(x)=8x\ln(2x+1),\ x>-\frac{1}{2}.$$

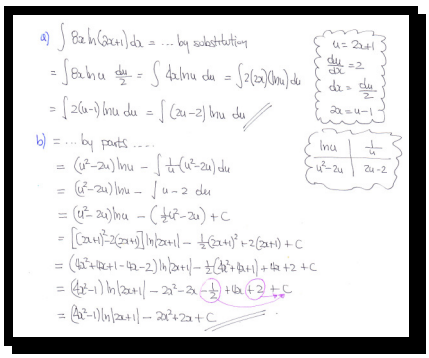
a) Use the substitution $u=2x+1$ to show that

$$\int f(x)\,dx=\int(2u-2)\ln u\ du.$$

b) Use integration by parts to show that

$$\int f(x)\,dx=\left(4x^2-1\right)\ln(2x+1)-2x^2+2x+C.$$

proof



Question 131 **(****)**

$$y=\frac{(1+\sin x)^2}{\cos^2 x}.$$

a) Calculate the two missing values of the following table.

x	$\frac{\pi}{6}$		$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{\pi}{3}$
y	3		5.8284	8.6784	13.9282

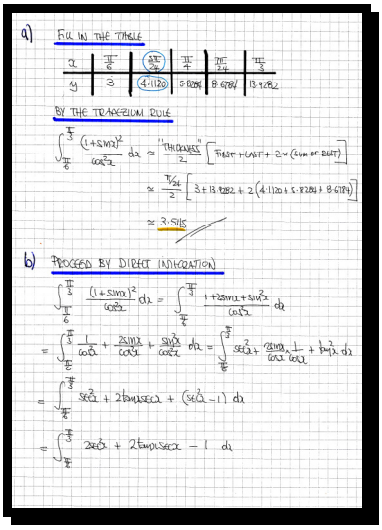
b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

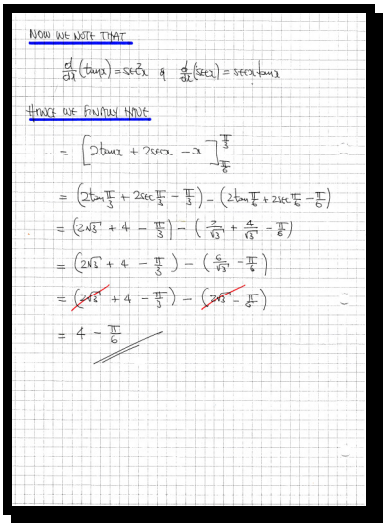
$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{(1+\sin x)^2}{\cos^2 x}\,dx.$$

c) Use trigonometric identities to find the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{(1+\sin x)^2}{\cos^2 x}\,dx.$$

$\boxed{}$, $\boxed{x=\frac{5\pi}{24}}$, $\boxed{4.112}$, $\boxed{4-\frac{1}{6}\pi}$





Question 132 (****)

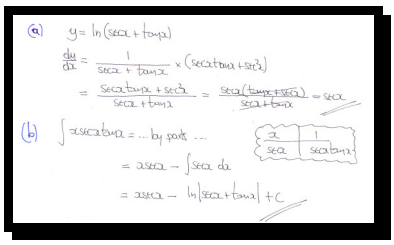
$$y = \ln\left(\sec x + \tan x\right).$$

a) Express $\dfrac{dy}{dx}$ as a single trigonometric function.

b) Hence find

$$\int x \sec x \tan x \, dx .$$

$$\boxed{x\sec x - \ln\left|\sec x + \tan x\right| + C}$$

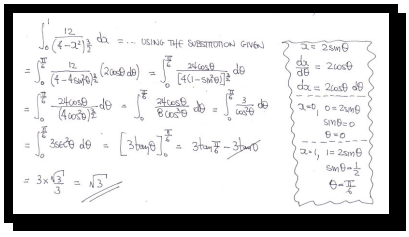


Question 133 (****)

Use the substitution $x = 2\sin\theta$ to find the exact value of

$$\int_0^1 \frac{12}{\left(4 - x^2\right)^{\frac{3}{2}}} \, dx .$$

$$\boxed{\sqrt{3}}$$



Question 134 (****)

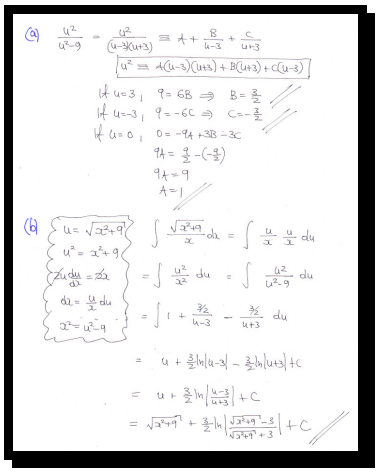
$$\frac{u^2}{u^2-9}\equiv A+\frac{B}{u-3}+\frac{C}{u+3}.$$

a) Find the value of A, B and C in the above identity.

b) By using the substitution $u=\sqrt{x^2+9}$, or otherwise, find

$$\int \frac{\sqrt{x^2+9}}{x}\,dx.$$

$\boxed{A=1}$, $\boxed{B=\frac{3}{2}}$, $\boxed{C=-\frac{3}{2}}$, $\boxed{\sqrt{x^2+9}+\frac{3}{2}\ln\left|\frac{\sqrt{x^2+9}-3}{\sqrt{x^2+9}+3}\right|+C}$

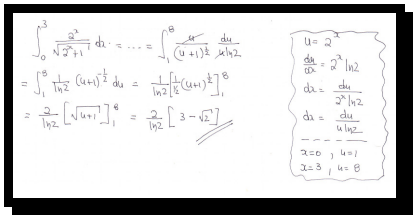


Question 135 (****)

By using the substitution $u = 2^x$, or otherwise, find an exact value for

$$\int_0^3 \frac{2^x}{\sqrt{2^x+1}}\, dx .$$

$$\boxed{\frac{2\left(3-\sqrt{2}\right)}{\ln 2}}$$



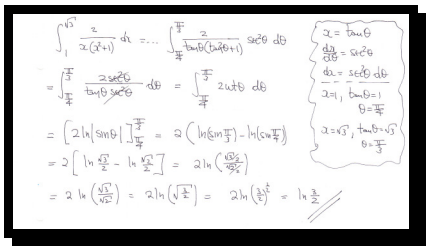
Question 136 (****)

Use the substitution $x = \tan\theta$ to show that

$$\int_1^{\sqrt{3}} \frac{2}{x\left(x^2+1\right)}\, dx = \ln\left(\frac{m}{n}\right),$$

where m and n are integers.

$$\boxed{\text{proof}}$$

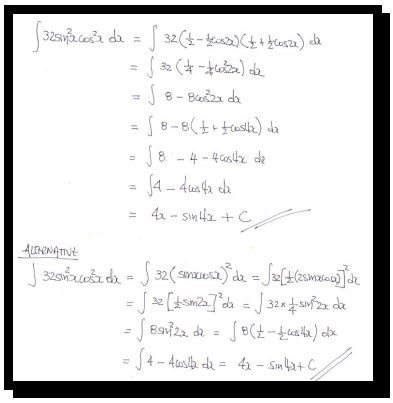


Question 137 (****)

Use trigonometric identities to find

$$\int 32\sin^2 x\cos^2 x\,dx\,.$$

$$\boxed{4x-\sin 4x+C}$$

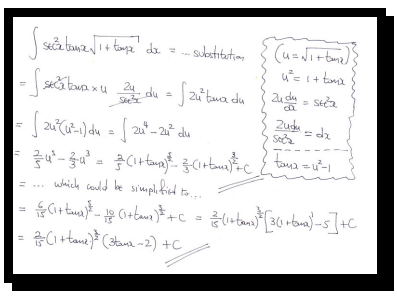


Question 138 (****)

By using the substitution $u^2=1+\tan x$, or otherwise, find

$$\int \sec^2 x\tan x\sqrt{1+\tan x}\,\,dx\,.$$

$$\boxed{\frac{2}{15}\left(3\tan x-2\right)\left(1+\tan x\right)^{\frac{3}{2}}}$$



Question 139 (****)

$$\frac{2u^2}{(u-1)(u+1)} \equiv A + \frac{B}{u+1} + \frac{C}{u-1}.$$

a) Find the value of A, B and C in the above identity.

b) By using the substitution $u^2 = x+1$, or otherwise, find an exact value for

$$\int_3^8 \frac{\sqrt{x+1}}{x}\, dx.$$

The table below shows some tabulated values for the equation $y = \dfrac{\sqrt{x+1}}{x}$, $3 \le x \le 8$.

x	3	4	5	6	7	8
y	0.6667	0.5590	0.4899		0.4041	0.3750

c) Complete the missing value in the table.

[continues overleaf]

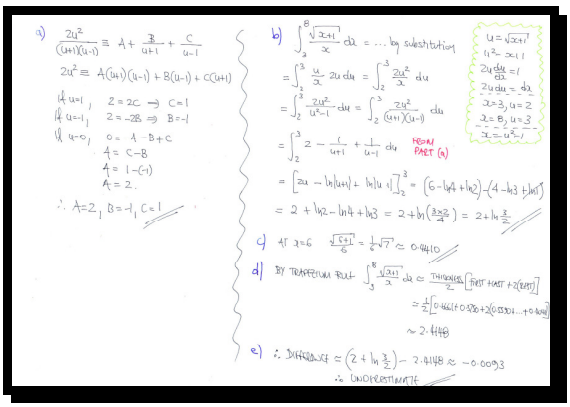
[continued from overleaf]

d) Use the trapezium rule with all the values from the table to find an approximate value for

$$\int_3^8 \frac{\sqrt{x+1}}{x}\, dx\,.$$

e) Calculate the difference between the exact value, found in part (b), and the trapezium rule estimate, found in part (d), and hence state whether the trapezium rule produces an overestimate or an underestimate.

$\boxed{}$, $\boxed{A=2}$, $\boxed{B=-1}$, $\boxed{C=-1}$, $\boxed{2+\ln\left(\frac{3}{2}\right)}$, $\boxed{0.4410}$, $\boxed{2.4148}$, $\boxed{0.0093}$



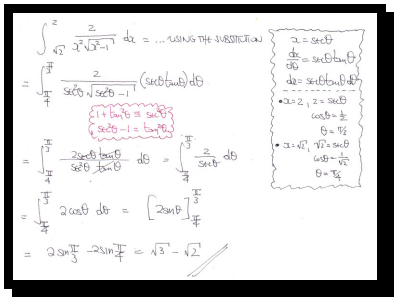
Question 140 (****)

Use the substitution $x = \sec\theta$ to show that

$$\int_{\sqrt{2}}^{2} \frac{2}{x^2\sqrt{x^2-1}}\,dx = \sqrt{m}-\sqrt{n}\,,$$

where m and n are integers.

$$\boxed{\sqrt{3}-\sqrt{2}}$$

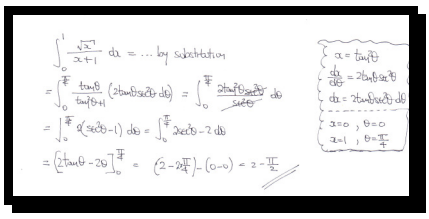


Question 141 (****)

Use the substitution $x = \tan^2\theta$ to find an exact value for

$$\int_{0}^{1} \frac{\sqrt{x}}{x+1}\,dx\,.$$

$$\boxed{2-\frac{\pi}{2}}$$



Question 142 **(****)**

$$y=\frac{\mathrm{e}^{2x}}{\mathrm{e}^{x}+1},\ x\in\mathbb{R}$$

a) Calculate the missing values of x and y in the following table.

x	$\ln 2$	x_2	x_3	x_4	$\ln 8$
y	1.333	y_2	y_3	y_4	7.111

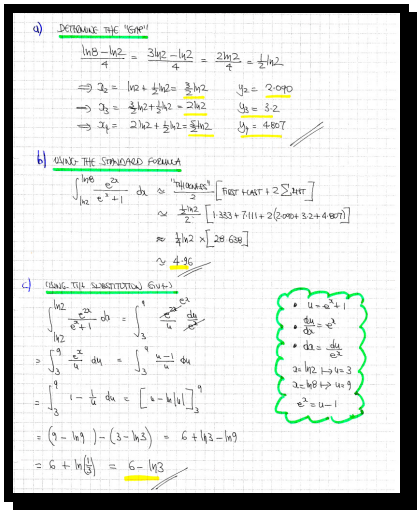
b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

$$\int_{\ln 2}^{\ln 8}\frac{\mathrm{e}^{2x}}{\mathrm{e}^{x}+1}\,dx.$$

c) Use the substitution $u=\mathrm{e}^{x}+1$ to find an exact simplified value for

$$\int_{\ln 2}^{\ln 8}\frac{\mathrm{e}^{2x}}{\mathrm{e}^{x}+1}\,dx.$$

$\boxed{}$, $\boxed{\frac{3}{2}\ln 2,\ 2\ln 2,\ \frac{5}{2}\ln 2,\ 2.090,\ 3.2,\ 4.807}$, $\boxed{\approx 4.96}$ $\boxed{6-\ln 3}$



Question 143 (****)

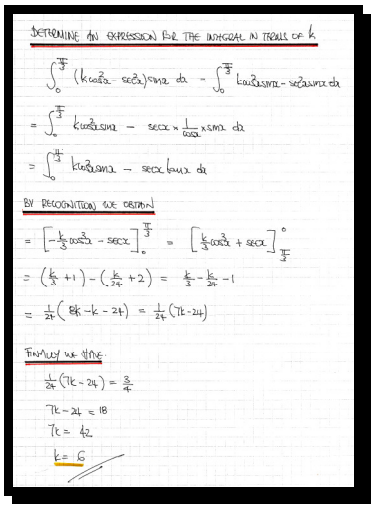
It is given that the value of

$$\int_0^{\frac{1}{3}\pi} \left(k\cos^2 x - \sec^2 x\right)\sin x \ dx,$$

is 2, where k is a non zero constant.

Determine the value of k.

$\boxed{}$, $\boxed{k=6}$



Question 144 (****)

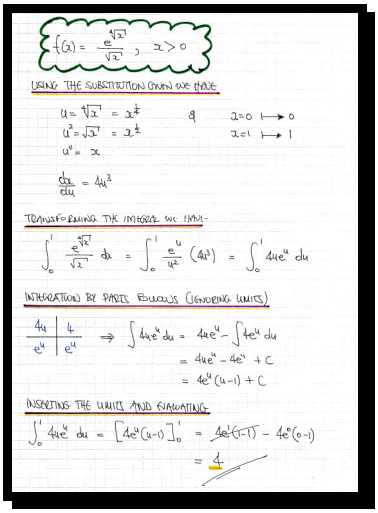
$$f(x)=\frac{e^{\sqrt[4]{x}}}{\sqrt{x}},\quad x\in\mathbb{R},\ x>0.$$

Find the value of

$$\int_0^1 f(x)\ dx,$$

given further that the integral exists.

$\square$, 4



Question 145 (****)

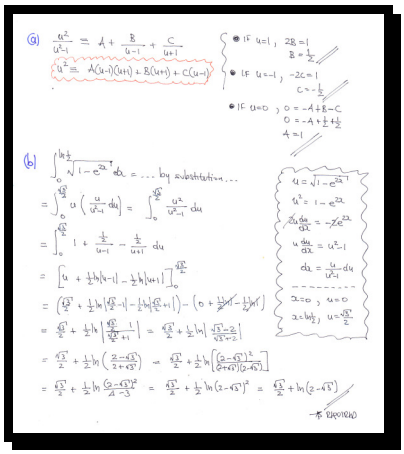
$$\frac{u^2}{u^2-1} \equiv A+\frac{B}{u-1}+\frac{C}{u+1}.$$

a) Find the value of A, B and C in the above identity.

b) Use the substitution $u=\sqrt{1-\mathrm{e}^{2x}}$ to show

$$\int_0^{\ln\frac{1}{2}} \sqrt{1-\mathrm{e}^{2x}}\ dx = \frac{\sqrt{3}}{2}+\ln\left(2-\sqrt{3}\right).$$

$\boxed{A=1}$, $\boxed{B=\frac{1}{2}}$, $\boxed{C=-\frac{1}{2}}$

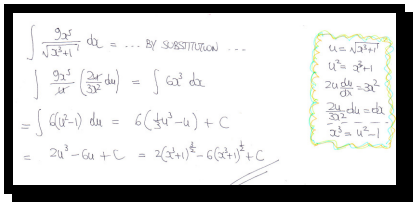


Question 146 (****)

By using the substitution $u=\sqrt{x^3+1}$, or otherwise, find an expression for

$$\int \frac{9x^5}{\sqrt{x^3+1}}\,dx\,.$$

$$\boxed{2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}+C}$$



Question 147 (****)

It is given that

$$\sin\left(A+B\right)\equiv\sin A\cos B+\cos A\sin B\,.$$

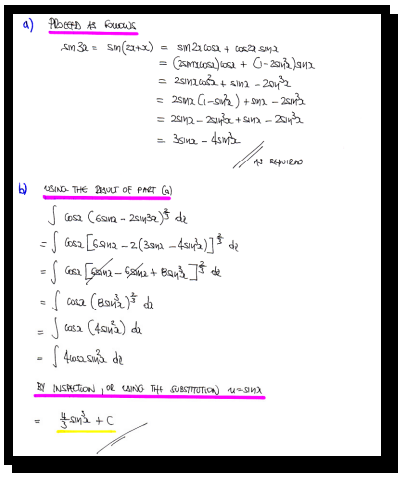
a) Use the above trigonometric identity to show that

$$\sin 3x\equiv 3\sin x-4\sin^3 x\,.$$

b) Hence find

$$\int\cos x\left(6\sin x-2\sin 3x\right)^{\frac{2}{3}}dx\,.$$

$\boxed{}$, $\boxed{\frac{4}{3}\sin^3 x+C}$

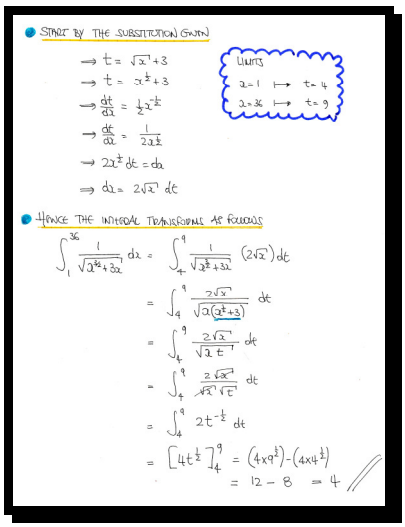


Question 148 (****)

Use the substitution $t=3+\sqrt{x}$ to find the value of the following integral

$$\int_1^{36} \frac{1}{\sqrt{x^{\frac{3}{2}}+3x}}\,dx\,.$$

$\boxed{}$, $\boxed{4}$

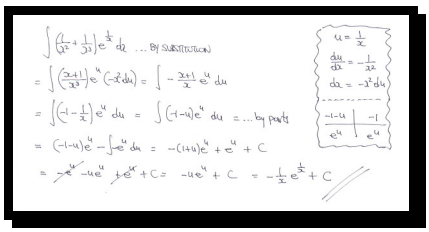


Question 149 (****)

By using the substitution $u=\frac{1}{x}$, or otherwise, show that

$$\int_0^4\left(\frac{1}{x^2}+\frac{1}{x^3}\right)e^{\frac{1}{x}}\,dx=-\frac{1}{x}e^{\frac{1}{x}}+C\,.$$

proof

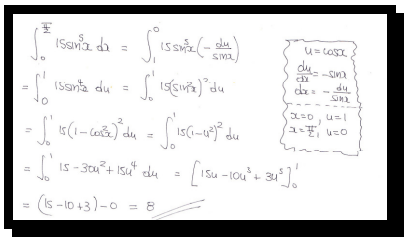


Question 150 (****)

By using the substitution $u=\cos x$, or otherwise, show clearly that

$$\int_0^{\frac{\pi}{2}} 15\cos^5 x \ dx = 8 .$$

proof



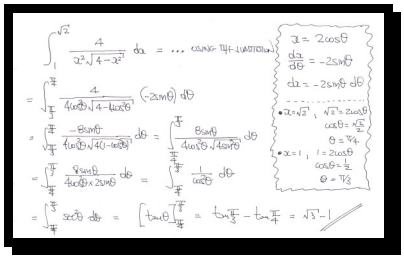
Question 151 (****)

Use the substitution $x=2\cos\theta$ to show that

$$\int_1^{\sqrt{2}} \frac{4}{x^2\sqrt{4-x^2}} \ dx = \sqrt{m}-n ,$$

where m and n are integers.

proof



Question 152 **(****)**

$$y=\frac{4x+3}{3x+4},\ x\neq -\frac{4}{3}.$$

a) Calculate the five missing values of x and y in the following table.

x	0				32
y	$\frac{3}{4}$	$\frac{35}{29}$	$\frac{67}{52}$		

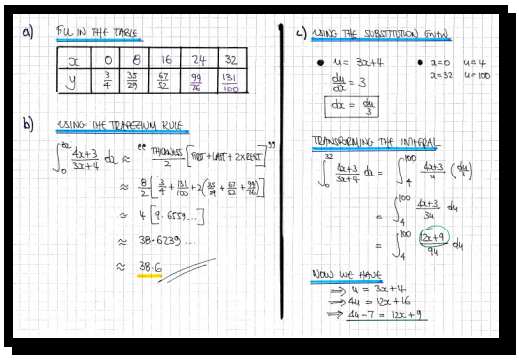
b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

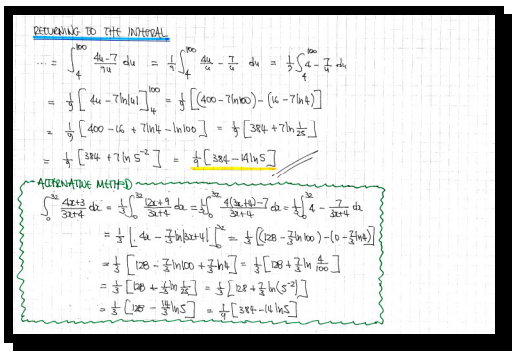
$$\int_0^{32}\frac{4x+3}{3x+4}\,dx.$$

c) Use the substitution $u=3x+4$ to find the exact value of

$$\int_0^{32}\frac{4x+3}{3x+4}\,dx.$$

$\boxed{}$, $\boxed{\begin{matrix}8,\ 16,\ 24\\ \frac{99}{76},\ \frac{131}{100}\end{matrix}}$, $\boxed{38.6}$, $\boxed{\frac{128}{3}-\frac{14}{9}\ln 5 = \frac{1}{9}\left[384-14\ln 5\right]}$





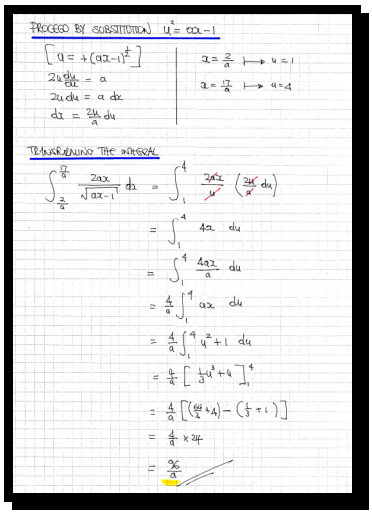
Question 153 (****)

Determine, in terms of a, the value of the following integral.

$$\int_{\frac{2}{a}}^{\frac{17}{a}} \frac{2ax}{\sqrt{ax-1}}\,dx\,,\ a\neq 0\,.$$

You may find the substitution $u^2=ax-1$ useful in this question.

$\square$, $\boxed{\dfrac{96}{a}}$

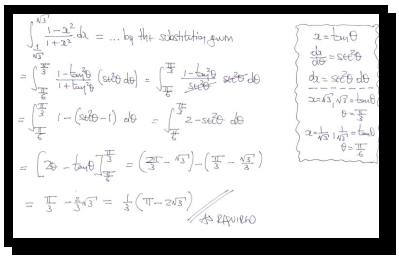


Question 154 (****)

Use the substitution $x=\tan\theta$ to show that

$$\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1-x^2}{1+x^2}\,dx=\frac{1}{3}\left(\pi-2\sqrt{3}\right).$$

proof

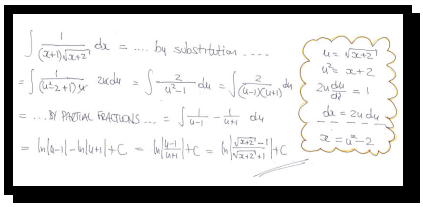


Question 155 (****)

By using the substitution $u=\sqrt{x+2}$, or otherwise, find an expression for

$$\int \frac{1}{(x+1)\sqrt{x+2}}\,dx.$$

$$\ln\left|\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right|+C$$



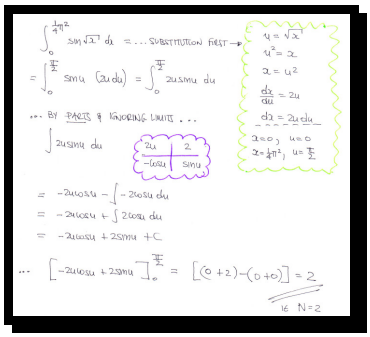
Question 156 (****)

Use appropriate integration techniques to show that

$$\int_0^{\frac{1}{4}\pi^2} \sin\sqrt{x}\ \ dx = N\ ,$$

where N is a positive integer.

$\boxed{\ \ }$, $\boxed{N=2}$

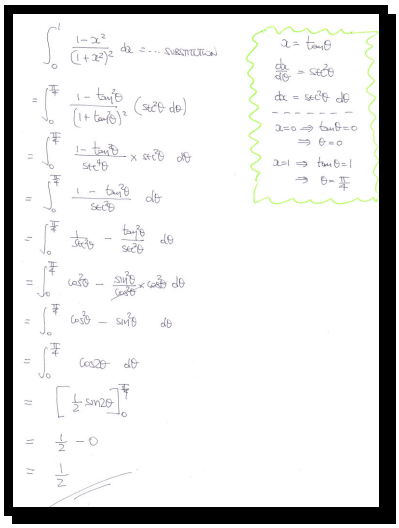


Question 157 (****)

By using the substitution $x = \tan\theta$, or otherwise, find the value of

$$\int_0^1 \frac{1-x^2}{\left(1-x^2\right)^2}\, dx\,.$$

$$\boxed{\frac{1}{2}}$$



Question 158 **(****)**

$$y=\left(1+\cot^2 x\right)\sec^2 x\,,\quad 0<x<\tfrac{1}{2}\pi\,.$$

a) Calculate the three missing values of x in the following table.

x	$\frac{1}{6}\pi$				$\frac{1}{3}\pi$
y	$\frac{16}{3}$	$32-16\sqrt{3}$	4	$32-16\sqrt{3}$	$\frac{16}{3}$

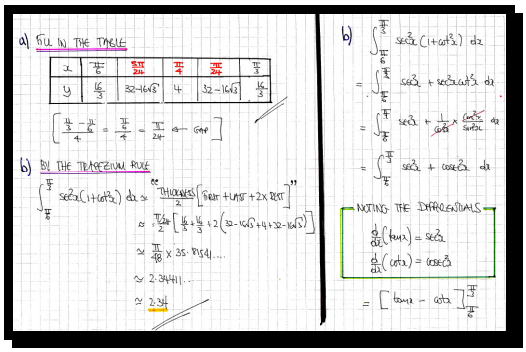
b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

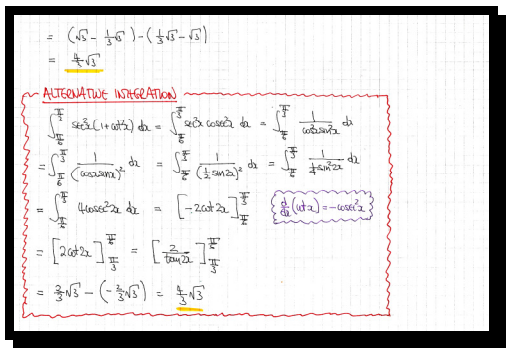
$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\left(1+\cot^2 x\right)\sec^2 x\ dx\,.$$

c) Use an appropriate integration method to find an exact simplified value for

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\left(1+\cot^2 x\right)\sec^2 x\ dx\,.$$

$\square$, $\boxed{\frac{5\pi}{24},\ \frac{\pi}{4},\ \frac{7\pi}{24}}$, $\boxed{2.34}$, $\boxed{\frac{4}{3}\sqrt{3}}$





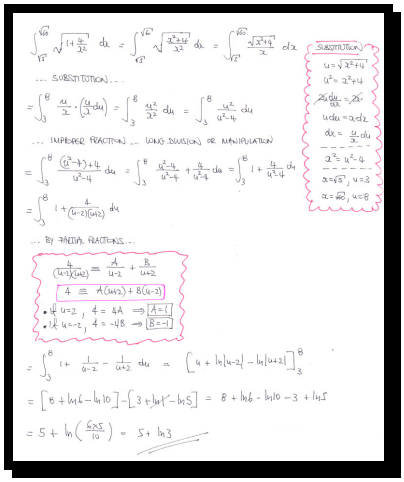
Question 159 **(****+)**

Use appropriate integration techniques to evaluate

$$\int_{\sqrt{5}}^{\sqrt{60}} \sqrt{1+\frac{4}{x^2}}\ dx\,.$$

Give the answer in the form $a+b\ln 3$, where a and b are positive integers.

$5+\ln 3$

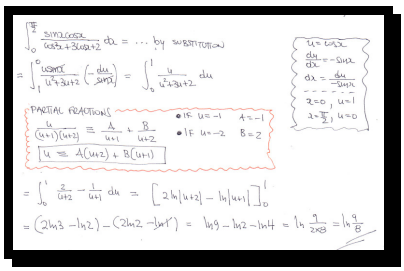


Question 160 **(****+)**

Use a suitable substitution to show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\cos^2 x+3\cos x+2}\ dx=\ln\left(\frac{9}{8}\right).$$

proof

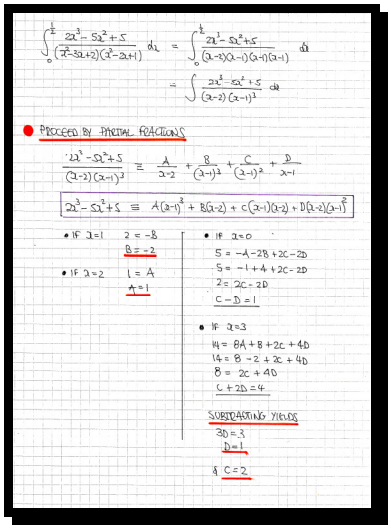


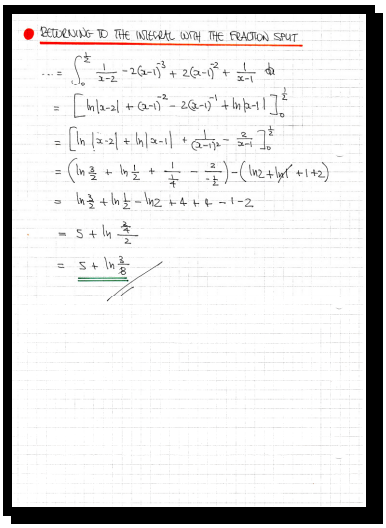
Question 161 (****)

Use partial fractions to determine, in exact simplified form, the value of the following integral.

$$\int_0^{\frac{1}{2}} \frac{2x^3-5x^2+5}{\left(x^2-3x+2\right)\left(x^2-2x+1\right)}\,dx\,.$$

$\square$, $\boxed{5+\ln\left(\frac{3}{8}\right)}$



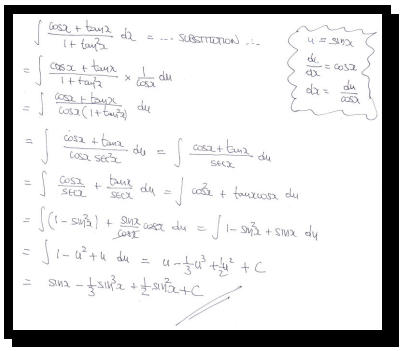


Question 162 (****)

Use the substitution $u = \sin x$ to find an expression for

$$\int \frac{\cos x + \tan x}{1 + \tan^2 x}\, dx .$$

$$\boxed{\sin x - \frac{1}{3}\sin^3 x + \frac{1}{2}\sin^2 x + C}$$

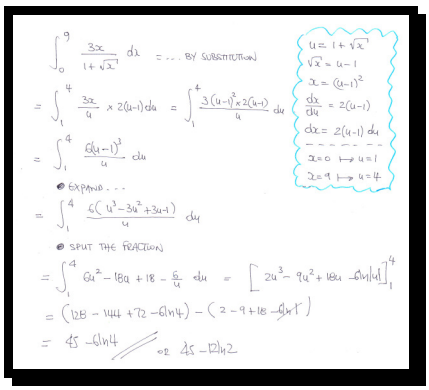


Question 163 (**+)**

Use the substitution $u = 1 + \sqrt{x}$ to evaluate

$$\int_0^9 \frac{3x}{1 + \sqrt{x}}\, dx .$$

$$\boxed{45 - 12\ln 2}$$



Question 164 **(****+)**

$$y=\frac{x^2}{2x+1},\ x\neq -\frac{1}{2}$$

a) Calculate the two missing values of y in the following table.

x	0	0.1	0.2	0.3	0.4	0.5
y	0	$\frac{1}{120}$	$\frac{1}{35}$			$\frac{1}{8}$

b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate, correct to 4 significant figures, for the following integral.

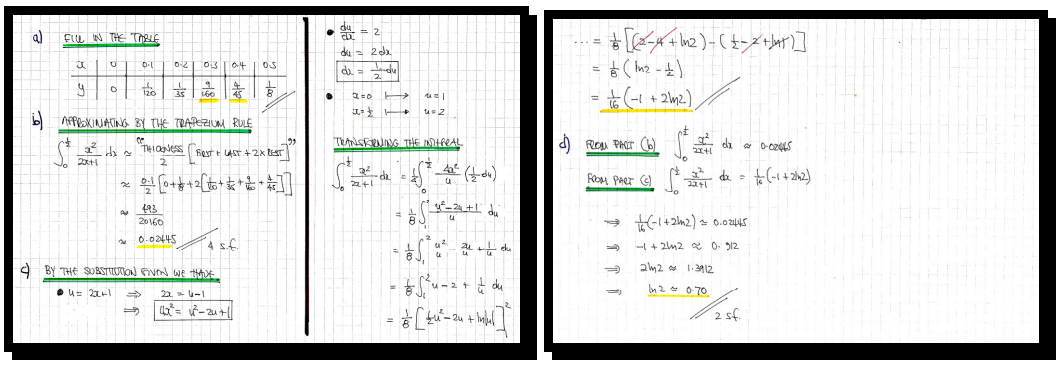
$$\int_0^{\frac{1}{2}} \frac{x^2}{2x+1}\,dx\,.$$

d) Use the substitution $u=2x+1$ to find an exact simplified value for

$$\int_0^{\frac{1}{2}} \frac{x^2}{2x+1}\,dx\,.$$

e) Hence deduce, by referring to parts **(b) and (c)**, the approximate value of $\ln 2$ correct to 2 significant figures.

$\square$, $\boxed{\frac{9}{160},\frac{4}{45}}$, $\boxed{0.02445}$, $\boxed{\frac{1}{16}\left[-1+2\ln 2\right]}$, $\boxed{\ln 2\approx 0.70}$



Question 165 (**+)**

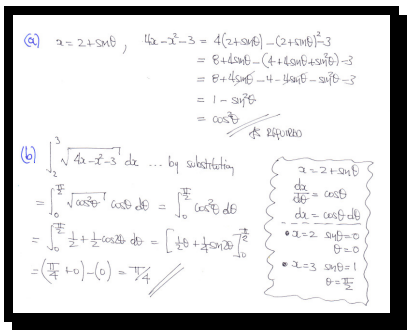
$$f(x)=-x^2+4x-3\,,\ 1\le x\le 3\,.$$

a) Show clearly that $f(2+\sin\theta)=\cos^2\theta$.

b) Hence find the exact value of

$$\int_2^3 \sqrt{f(x)}\ dx\,.$$

$$\frac{\pi}{4}$$



Question 166 (**+)**

$$f(x)=\ln\left(\frac{1+x}{1-x}\right),\ |x|<1.$$

a) Show that $f(x)$ is an odd function.

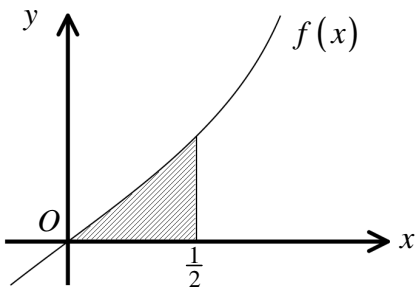
b) Find an expression for $f'(x)$ as a single simplified fraction, showing further that $f'(x)$ is an even function.

c) Determine an expression for $f^{-1}(x)$.

[continues overleaf]

[continued from overleaf]

The figure below shows part of the graph of $f(x)$.

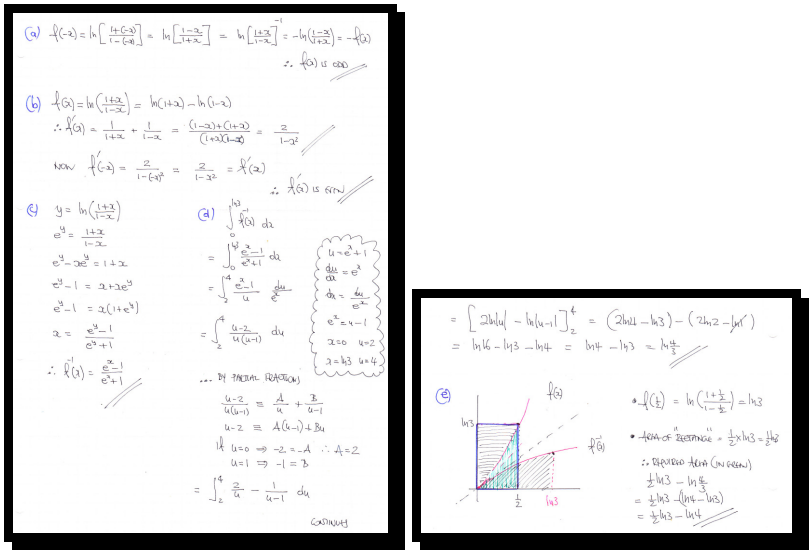


d) Use the substitution $u = \mathrm{e}^x + 1$ to find the exact value of

$$\int_0^{\ln 3} f^{-1}(x)\ dx .$$

e) Hence find an exact value for the area of the shaded region, bounded by $f(x)$, the coordinate axes and the line $x = \frac{1}{2}$.

$$\boxed{f'(x) = \frac{2}{1-x^2}}, \quad \boxed{f'(x) = \frac{\mathrm{e}^x - 1}{\mathrm{e}^x + 1}}, \quad \boxed{\ln\left(\frac{4}{3}\right)}, \quad \boxed{\text{area} = \frac{1}{2}\ln 3 - 2\ln 2 \approx 0.262}$$



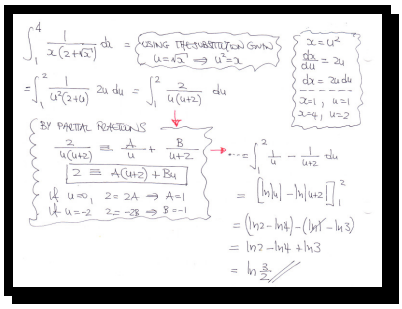
Question 167 (**+)**

By using the substitution $u=\sqrt{x}$ find

$$\int_1^4 \frac{1}{x\left(2+\sqrt{x}\right)}\,dx,$$

giving the answer as an exact single natural logarithm.

$\ln\left(\frac{3}{2}\right)$

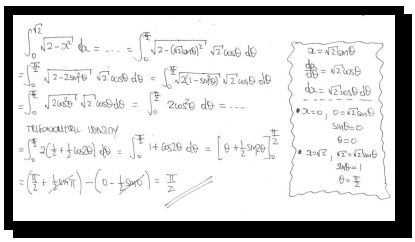


Question 168 (**+)**

Use the substitution $x=\sqrt{2}\sin\theta$ to show that

$$\int_0^{\sqrt{2}} \sqrt{2-x^2}\,dx=\frac{\pi}{2}.$$

proof



Question 169 (**+)**

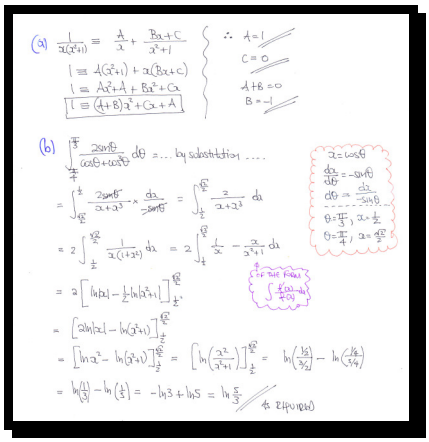
$$\frac{1}{x\left(x^2+1\right)}\equiv\frac{A}{x}+\frac{Bx+C}{x^2+1}.$$

a) Find the value of each of the constants A, B and C.

b) Use the substitution $x=\cos\theta$ to show

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2\sin\theta}{\cos\theta+\cos^3\theta}\,d\theta=\ln\left(\frac{5}{3}\right).$$

$\boxed{A=1}$, $\boxed{B=-1}$, $\boxed{C=0}$

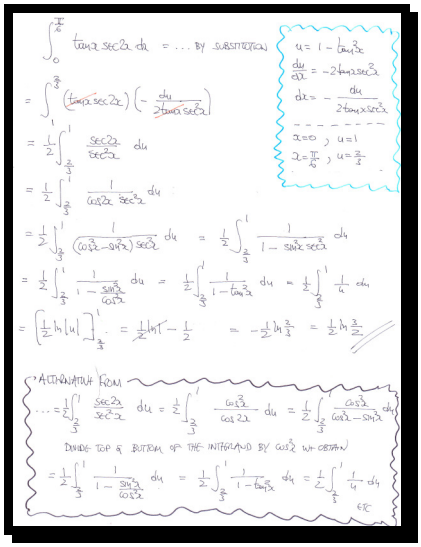


Question 170 **(****+)**

By using the substitution $u=1-\tan^2 x$, or otherwise, find the exact value of

$$\int_0^{\frac{\pi}{6}} \tan x \sec 2x \ \ dx .$$

$$\boxed{\frac{1}{2}\ln\left(\frac{3}{2}\right)}$$



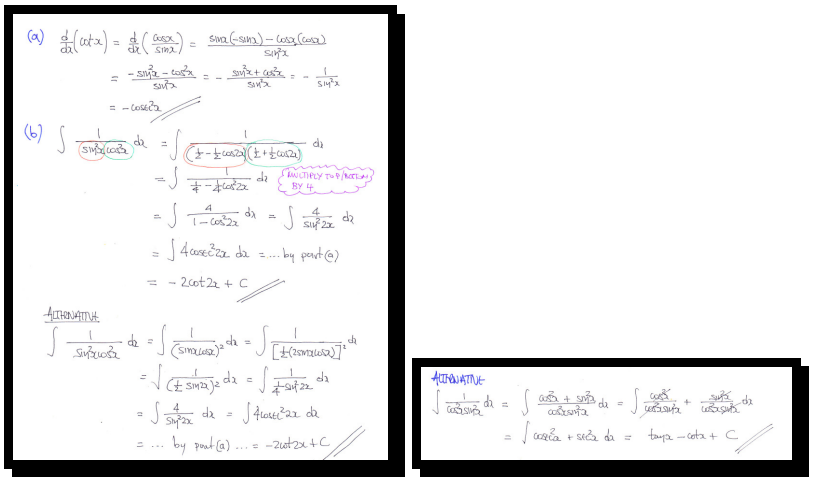
Question 171 **(****+)**

a) Show clearly that $\dfrac{d}{dx}(\cot x)=-\text{cosec}^2 x$.

b) Use trigonometric identities to find

$$\int \frac{1}{\sin^2 x\cos^2 x}\,dx\,.$$

$$\boxed{-2\cot 2x+C}$$

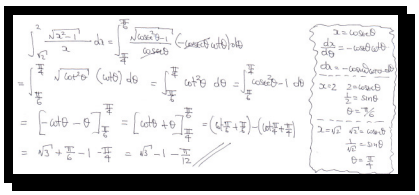


Question 172 **(****+)**

Use the substitution $x=\text{cosec}\,\theta$ to find the exact value of

$$\int_{\sqrt{2}}^{2} \frac{\sqrt{x^2-1}}{x}\,dx\,.$$

$$\boxed{\sqrt{3}-1-\frac{\pi}{12}}$$



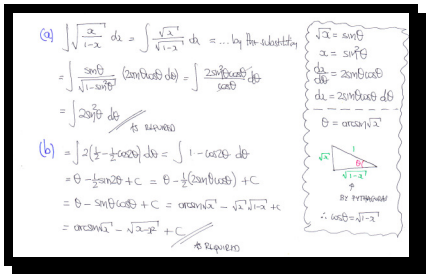
Question 173 (**+)**

a) Write down an expression for $\dfrac{d}{dx}\left(\mathrm{e}^{\cos x}\right)$.

b) By using integration by parts, or otherwise, show that

$$\int \mathrm{e}^{\cos x}\cos x\sin x\,dx=\mathrm{e}^{x}\left(1-\cos x\right)+\text{constant}\,.$$

$$\boxed{\frac{d}{dx}\left(\mathrm{e}^{\cos x}\right)=-\mathrm{e}^{\cos x}\sin x}$$

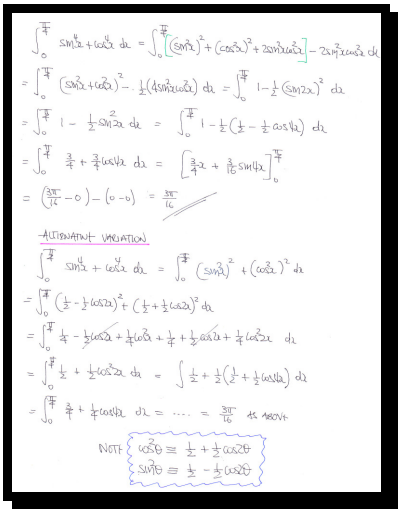


Question 174 (**+)**

By using trigonometric identities, show that

$$\int_0^{\frac{\pi}{4}} \sin^4 x + \cos^4 x \ dx = \frac{3\pi}{16}.$$

proof

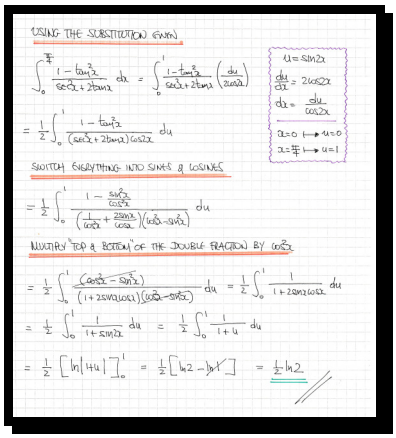


Question 175 (**+)**

By using the substitution $u=\sin 2x$, or otherwise, find an exact simplified value for the following trigonometric integral.

$$\int_0^{\frac{1}{4}\pi} \frac{1-\tan^2 x}{\sec^2 x+2\tan x}\,dx\,.$$

$\boxed{}$, $\boxed{\frac{1}{2}\ln 2}$

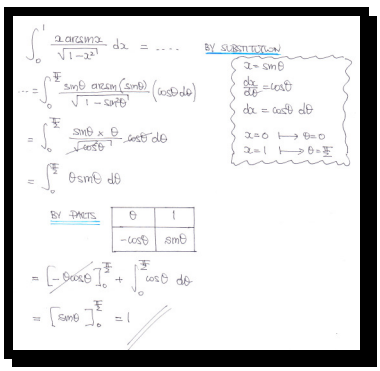


Question 176 (**+)**

By using a suitable substitution, or otherwise, find the value of

$$\int_0^1 \frac{x\arcsin x}{\sqrt{1-x^2}}\,dx\,.$$

$\boxed{1}$



Question 177 **(****+)**

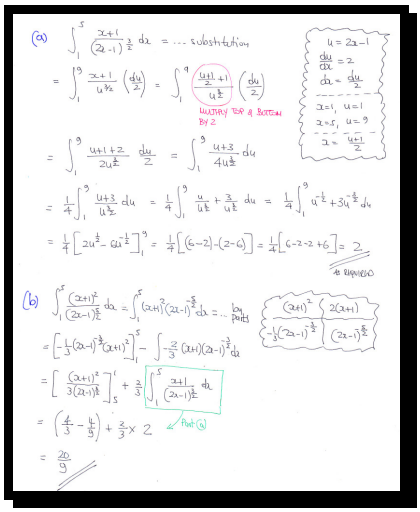
a) Use the substitution $u=2x-1$ to show that

$$\int_1^5 \frac{x+1}{(2x-1)^{\frac{3}{2}}}\,dx=2\,.$$

b) By using integration by parts and the result of part **(a)**, find the value of

$$\int_1^5 \frac{(x+1)^2}{(2x-1)^{\frac{5}{2}}}\,dx\,.$$

$$\frac{20}{9}$$

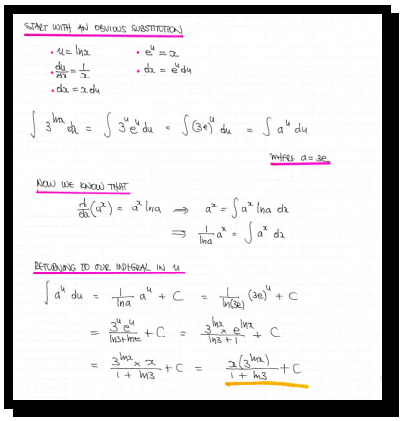


Question 178 (**+)**

Use the substitution $u = \ln x$ to show that

$$\int 3^{\ln x}\, dx = \frac{x\left(3^{\ln x}\right)}{1+\ln 3} + \text{constant}.$$

V, [ ], proof



Question 179 **(****+)**

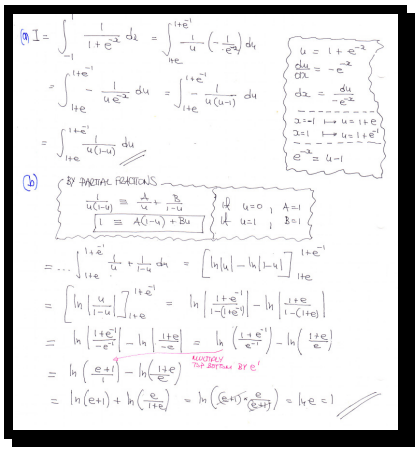
$$J=\int_{-1}^{1}\frac{1}{1+\mathrm{e}^{-x}}\,dx\,.$$

a) Show that the substitution $u=1+\mathrm{e}^{-x}$ transforms J into

$$\int_{1+\mathrm{e}}^{1+\mathrm{e}^{-1}}\frac{1}{u(1-u)}\,du\,.$$

b) By expressing $\dfrac{1}{u(1-u)}$ into partial fractions show clearly that $J=1$.

proof

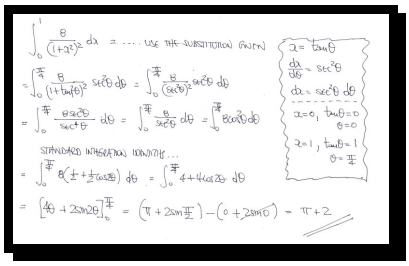


Question 180 (**+)**

Use the substitution $x = \tan\theta$ to find the exact value of

$$\int_0^1 \frac{8}{\left(1+x^2\right)^2}\, dx .$$

$\boxed{}$, $\boxed{\pi+2}$

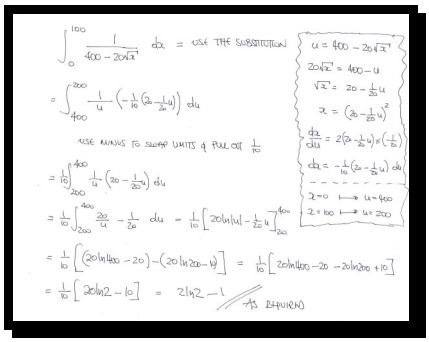


Question 181 (**+)**

Use the substitution $u = 400 - 20\sqrt{x}$ to show that

$$\int_0^{100} \frac{1}{400-20\sqrt{x}}\, dx = -1+2\ln 2 .$$

$\boxed{\text{proof}}$



Question 182 **(****+)**

It is given that

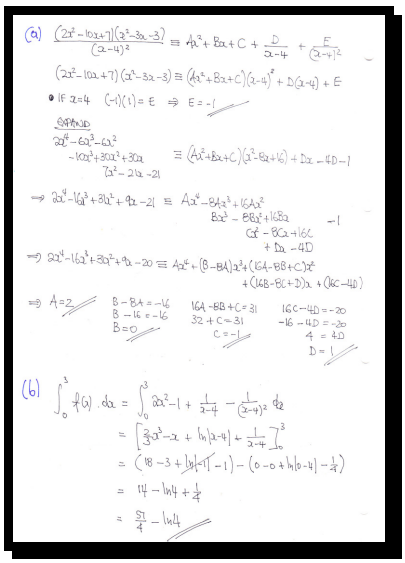
$$\frac{\left(2x^2-10x+7\right)\left(x^2-3x-3\right)}{\left(x-4\right)^2}\equiv Ax^2+Bx+C+\frac{D}{x-4}+\frac{E}{\left(x-4\right)^2}.$$

a) Find the value of A, B, C, D and E in the above identity.

b) Hence find the exact value of

$$\int_0^3 f\left(x\right)\,dx.$$

$\boxed{A=2}$, $\boxed{B=0}$, $\boxed{C=-1}$, $\boxed{D=1}$, $\boxed{E=-1}$, $\boxed{\frac{57}{4}-\ln 4}$



Question 183 **(****+)**

$$x=\frac{1}{2}\left(-1+4\tan\theta\right),\ -\frac{1}{2}\pi<\theta<\frac{1}{2}\pi\ .$$

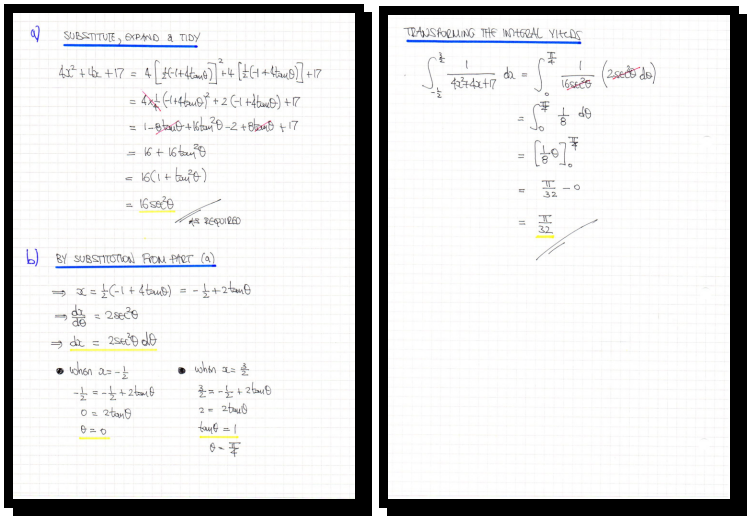
a) Use trigonometric identities to show that

$$4x^2+4x+17=16\sec^2\theta\,.$$

b) Hence find the exact value of

$$\int_{-\frac{1}{2}}^{\frac{3}{2}}\frac{1}{4x^2+4x+17}\,dx\,.$$

$\square$, $\boxed{\dfrac{\pi}{32}}$



Question 184 **(****+)**

It is given that

$$\sin\left(A+B\right)\equiv\sin A\cos B+\cos A\sin B\,.$$

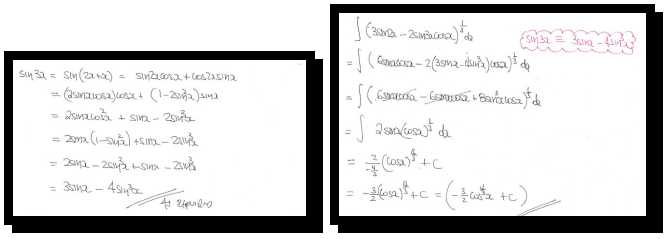
Use the above trigonometric identity to show that

$$\sin 3x\equiv 3\sin x-4\sin^3 x\,,$$

and hence find

$$\int\sqrt[3]{3\sin 2x-2\sin 3x\cos x}\,dx\,.$$

$$\boxed{-\frac{3}{2}\sin^{\frac{4}{3}}x+C}$$

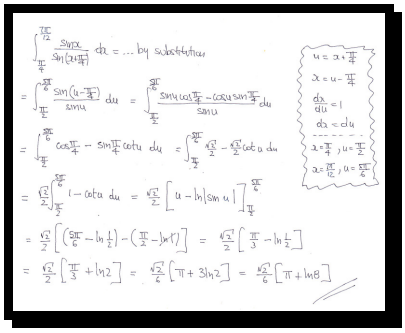


Question 185 (**+)**

Use the substitution $u = x + \dfrac{\pi}{4}$ to show that

$$\int_{\frac{\pi}{4}}^{\frac{7\pi}{12}} \frac{\sin x}{\sin\left(x+\dfrac{\pi}{4}\right)}\, dx = \frac{\sqrt{2}}{6}\left(\pi + \ln 8\right).$$

proof

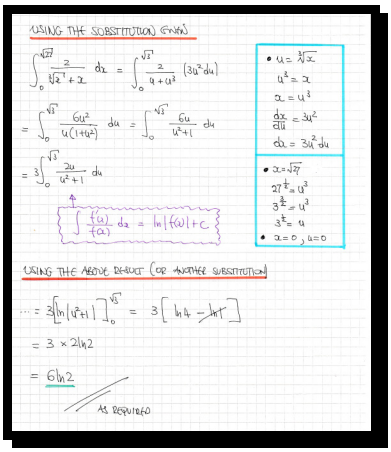


Question 186 (**+)**

By using the substitution $u=\sqrt[3]{x}$, or otherwise, show that

$$\int_0^{\sqrt{27}} \frac{2}{x+\sqrt[3]{x}}\,dx = 6\ln 2 .$$

[ ], [proof]



Question 187 **(****+)**

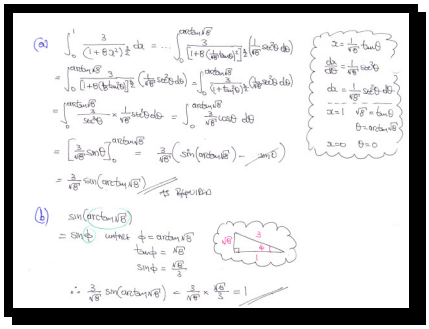
$$I=\int_0^1 \frac{3}{\left(1+8x^2\right)^{\frac{3}{2}}}\,dx$$

a) Use the substitution $x=\frac{1}{\sqrt{8}}\tan\theta$ to show that

$$I=\frac{3}{\sqrt{8}}\sin\left(\arctan\sqrt{8}\right).$$

b) Show, presenting detailed calculations, that $I=1$.

proof



Question 188 **(****+)**

$$I=\int_1^2 \frac{1}{x^2-x\sqrt{x^2-1}}\,dx\,.$$

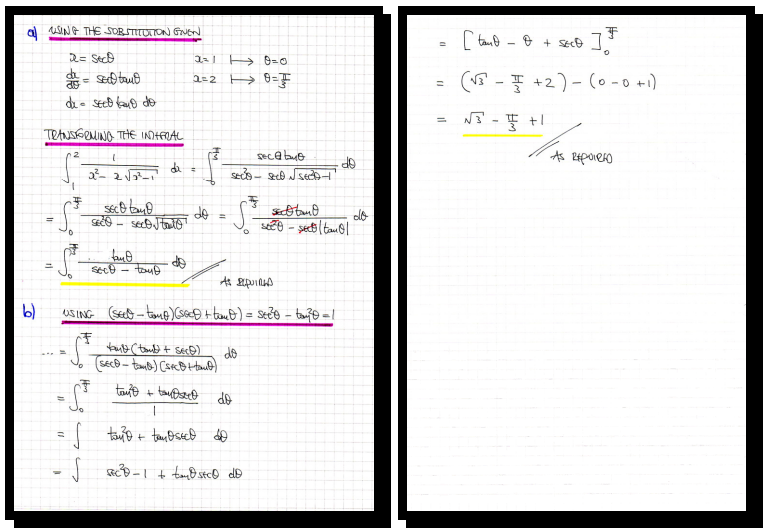
a) Show that the substitution $x=\sec\theta$ transforms I to

$$I=\int_0^{\frac{1}{3}\pi} \frac{\tan\theta}{\sec\theta-\tan\theta}\,d\theta\,.$$

b) Hence use trigonometric identities to show that

$$I=1+\sqrt{3}-\frac{1}{3}\pi\,.$$

[ ], [proof]

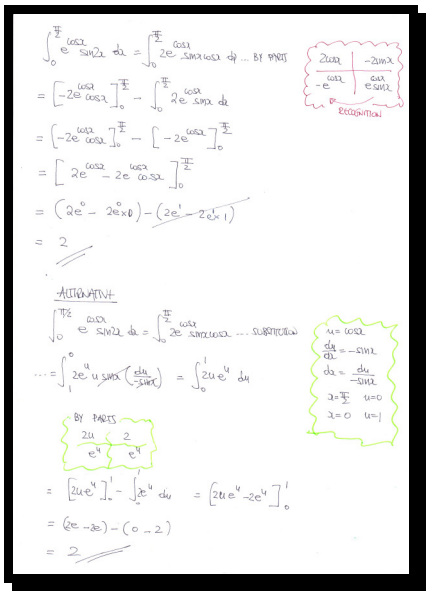


Question 189 (**+)**

Use integration by parts to find the value of

$$\int_0^{\frac{\pi}{2}} \mathrm{e}^{\cos x} \sin 2x \ \ dx \, .$$

2



Question 190 **(****+)**

$$\sin 2x \equiv \frac{2\tan x}{1+\tan^2 x}.$$

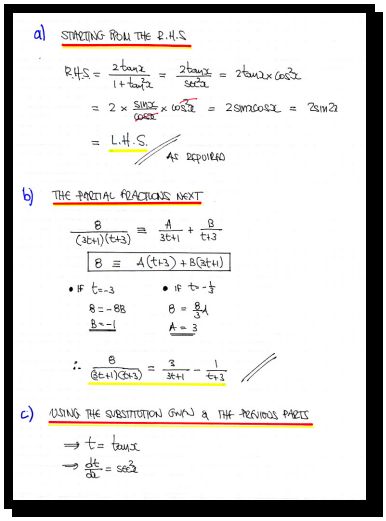
a) Prove the validity of the above trigonometric identity.

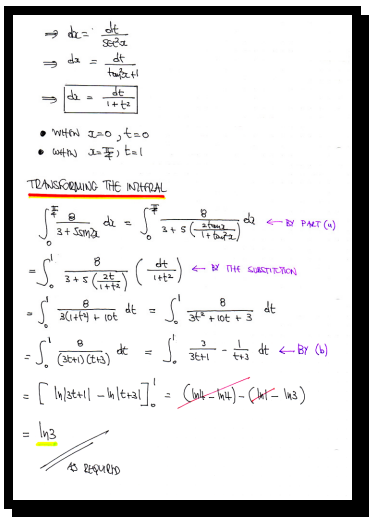
b) Express $\dfrac{8}{(3t+1)(t+3)}$ into partial fractions.

c) Hence use the substitution $t=\tan x$ to show that

$$\int_0^{\frac{\pi}{4}} \frac{8}{3+5\sin 2x}\,dx = \ln 3.$$

$\square$, $\boxed{\dfrac{8}{(3t+1)(t+3)} = \dfrac{3}{3t+1} - \dfrac{1}{t+3}}$





Question 191 **(****+)**

It is given that for some constants A and B

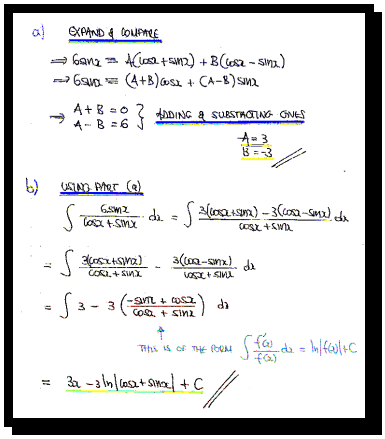
$$6\sin x \equiv A(\cos x+\sin x)+B(\cos x-\sin x).$$

a) Find the value of A and the value of B.

b) Hence find

$$\int \frac{6\sin x}{\cos x+\sin x}\,dx.$$

$\boxed{}$, $\boxed{A=3}$, $\boxed{B=-3}$, $\boxed{3x-3\ln|\cos x+\sin x|+C}$

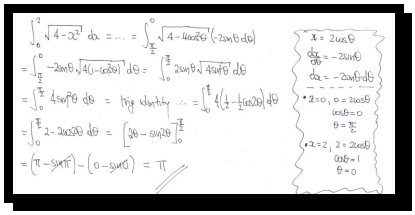


Question 192 **(****+)**

Use the substitution $x=2\sin\theta$ to show that

$$\int_0^2 \sqrt{4-x^2}\,dx=\pi.$$

proof

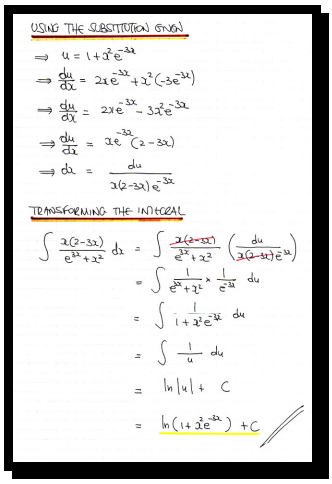


Question 193 (**+)**

Use the substitution $u = 1 + x^2 \mathrm{e}^{-3x}$ to find an expression for

$$\int \frac{x(2-3x)}{\mathrm{e}^{3x} + x^2} \, dx \, .$$

$\square$, $\boxed{\ln\left(1 + x^2 e^{-3x}\right) + C}$


USING THE SUBSTITUTION GIVEN

$\Rightarrow u = 1 + x^2 e^{-3x}$

$\Rightarrow \frac{du}{dx} = 2x e^{-3x} + x^2(-3e^{-3x})$

$\Rightarrow \frac{du}{dx} = 2x e^{-3x} - 3x^2 e^{-3x}$

$\Rightarrow \frac{du}{dx} = x e^{-3x}(2-3x)$

$\Rightarrow dx = \frac{du}{x(2-3x)e^{-3x}}$

TRANSFORMING THE INTEGRAL

$$\int \frac{x(2-3x)}{e^{3x}+x^2} dx = \int \frac{x(2-3x)}{e^{3x}+x^2} \left(\frac{du}{x(2-3x)e^{-3x}}\right)$$

$$= \int \frac{1}{e^{3x}+x^2} \times \frac{1}{e^{-3x}} du$$

$$= \int \frac{1}{1+x^2 e^{-3x}} du$$

$$= \int \frac{1}{u} du$$

$$= \ln|u| + C$$

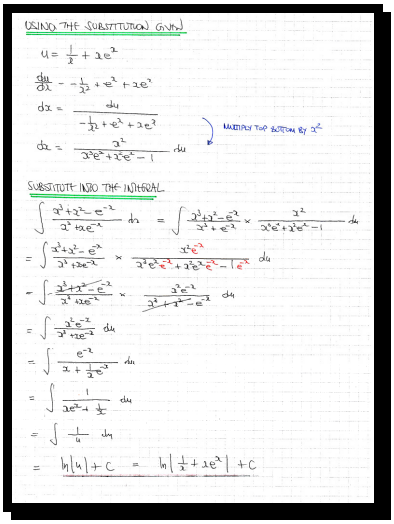
$$= \ln(1+x^2 e^{-3x}) + C$$

Question 194 (**+)**

Use the substitution $u=\dfrac{1}{x}+x\mathrm{e}^{x}$ to find an expression for

$$\int \frac{x^3+x^2-\mathrm{e}^{-x}}{x^3+x\mathrm{e}^{-x}}\,dx\,.$$

$\boxed{}$, $\boxed{\ln\left|\dfrac{1}{x}+x\mathrm{e}^{x}\right|+C}$



Question 195 **(****+)**

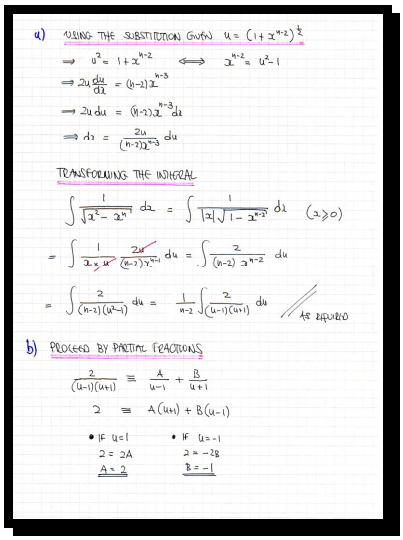
$$\int \frac{1}{\sqrt{x^2+x^n}}\,dx\,,\ n\neq 2\,,\ x\geq 0\,.$$

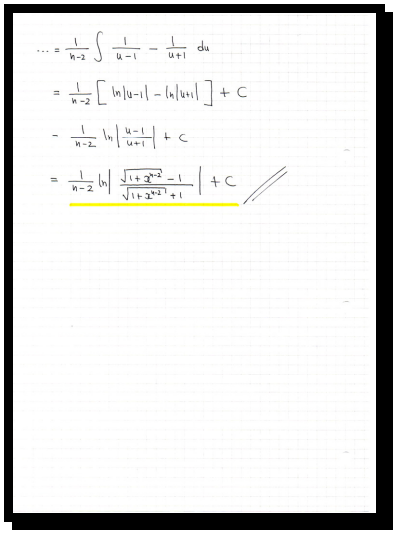
a) Show that the substitution $u^2=1+x^{n-2}$ transforms above integral into

$$\frac{1}{n-2}\int \frac{2}{(u-1)(u+1)}\,du\,.$$

b) Use partial fractions to find, in terms of x and n, an integrated expression for the original integral.

$\square$, $\dfrac{1}{n-2}\ln\left|\dfrac{\sqrt{1+x^{n-2}}-1}{\sqrt{1+x^{n-2}}+1}\right|+C$





Question 196 **(****+)**

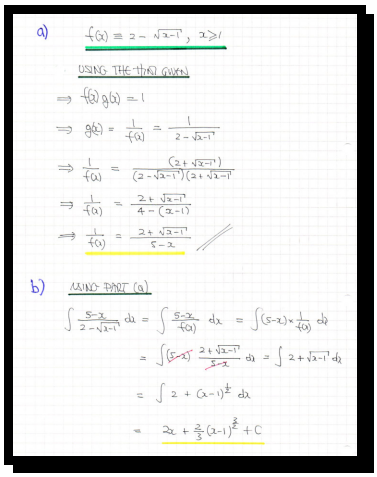
$$f(x)\equiv 2-\sqrt{x-1}\,,\ x\geq 1.$$

a) Find a simplified expression for $g(x)$ so that $f(x)g(x)=1$.

b) Hence, or otherwise, find

$$\int \frac{5-x}{2-\sqrt{x-1}}\,dx\,.$$

$\square$, $\boxed{g(x)\equiv\dfrac{2+\sqrt{x-1}}{5-x}}$, $\boxed{2x+\frac{1}{2}(x-1)^{\frac{3}{2}}+C}$

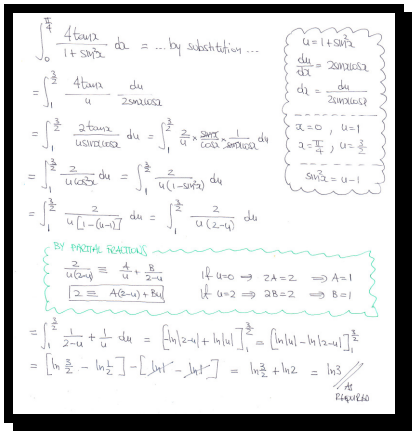


Question 197 (**+)**

By using the substitution $u=1+\sin^2 x$, or otherwise, show clearly that

$$\int_0^{\frac{\pi}{4}} \frac{4\tan x}{1+\sin^2 x}\,dx=\ln 3\,.$$

proof



Question 198 **(****+)**

$$\sec x \equiv \frac{\cos x}{1-\sin^2 x}.$$

a) Prove the validity of the above trigonometric identity.

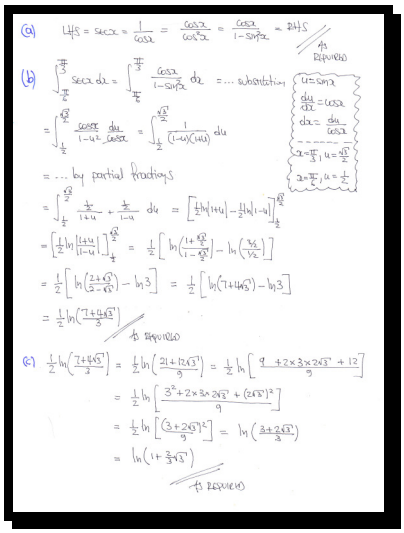
b) Use the substitution $u = \sin x$ to show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec x \ dx = \frac{1}{2}\ln\left(\frac{7+4\sqrt{3}}{3}\right).$$

c) Show clearly that

$$\frac{1}{2}\ln\left(\frac{7+4\sqrt{3}}{3}\right) = \ln\left(1+\frac{2}{3}\sqrt{3}\right)$$

proof

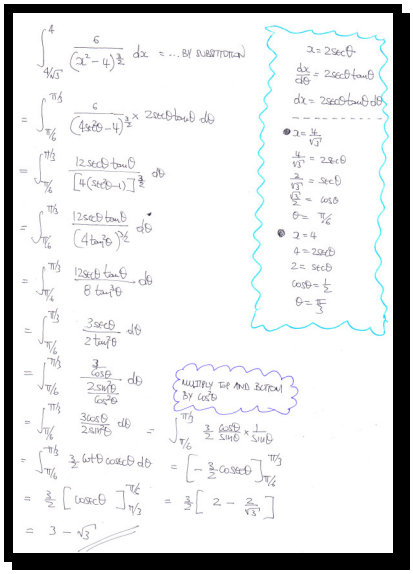


Question 199 (**+)**

Use the substitution $x = 2\sec\theta$, to find the exact value of

$$\int_{\frac{4}{\sqrt{3}}}^{4} \frac{6}{\left(x^2-4\right)^{\frac{3}{2}}}\,dx\,.$$

$\boxed{3-\sqrt{3}}$

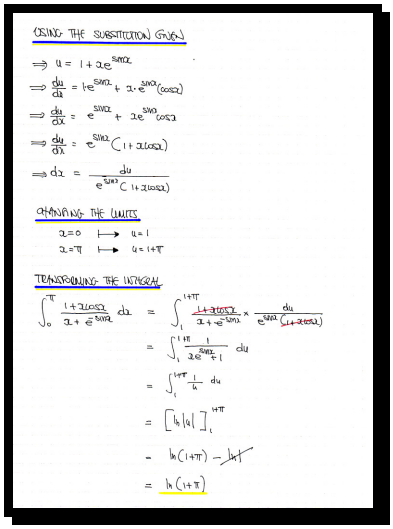


Question 200 (**+)**

Use the substitution $u = 1 + x\mathrm{e}^{\sin x}$ to find an exact simplified value for the following definite integral.

$$\int_0^{\pi} \frac{1 + x\cos x}{x + \mathrm{e}^{-\sin x}}\, dx .$$

$\square$, $\boxed{\ln(1+\pi)}$


USING THE SUBSTITUTION GIVEN

$\Rightarrow u = 1 + x\mathrm{e}^{\sin x}$

$\Rightarrow \frac{du}{dx} = 1\mathrm{e}^{\sin x} + x\cdot\mathrm{e}^{\sin x}(\cos x)$

$\Rightarrow \frac{du}{dx} = \mathrm{e}^{\sin x} + x\mathrm{e}^{\sin x}\cos x$

$\Rightarrow \frac{du}{dx} = \mathrm{e}^{\sin x}(1 + x\cos x)$

$\Rightarrow dx = \frac{du}{\mathrm{e}^{\sin x}(1 + x\cos x)}$

CHANGING THE LIMITS

$x = 0 \longmapsto u = 1$

$x = \pi \longmapsto u = 1 + \pi$

TRANSFORMING THE INTEGRAL

$$\int_0^{\pi} \frac{1 + x\cos x}{x + \mathrm{e}^{\sin x}}\, dx = \int_1^{1+\pi} \frac{1 + x\cos x}{x + \mathrm{e}^{\sin x}} \times \frac{du}{\mathrm{e}^{\sin x}(1 + x\cos x)}$$

$$= \int_1^{1+\pi} \frac{1}{x\mathrm{e}^{\sin x} + 1}\, du$$

$$= \int_1^{1+\pi} \frac{1}{u}\, du$$

$$= \Big[\ln|u|\Big]_1^{1+\pi}$$

$$= \ln(1+\pi) - \ln 1$$

$$= \ln(1+\pi)$$

Question 201 **(****+)**

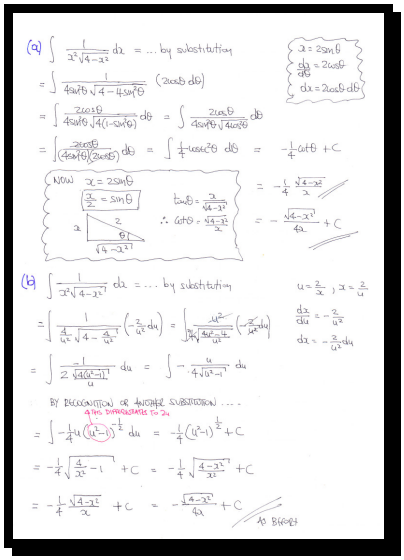
$$I=\int \frac{1}{x^2\sqrt{4-x^2}}\,dx\,.$$

a) Use the substitution $x=2\sin\theta$ to show clearly that

$$I=-\frac{\sqrt{4-x^2}}{4x}+C\,.$$

b) Verify the answer to part **(a)** by using the substitution $u=\dfrac{2}{x}$.

proof

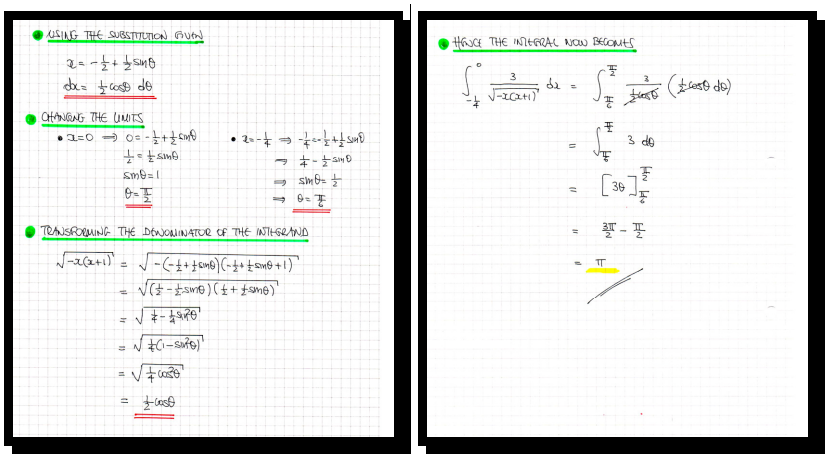


Question 202 (**+)**

By using the substitution $x=-\frac{1}{2}+\frac{1}{2}\sin\theta$, or otherwise, find the exact value of

$$\int_{-\frac{1}{4}}^{0}\frac{3}{\sqrt{-x(x+1)}}\,dx\,.$$

$\boxed{}$, $\boxed{\pi}$

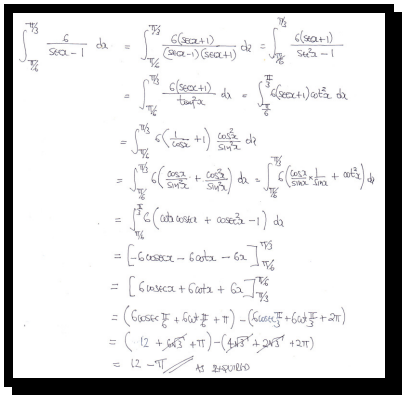


Question 203 (**+)**

By using multiplying the numerator and denominator of the integrand by $(\sec x+1)$, and manipulating it further by various trigonometric identities, show clearly that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{6}{\sec x-1}\, dx = 12-\pi .$$

proof

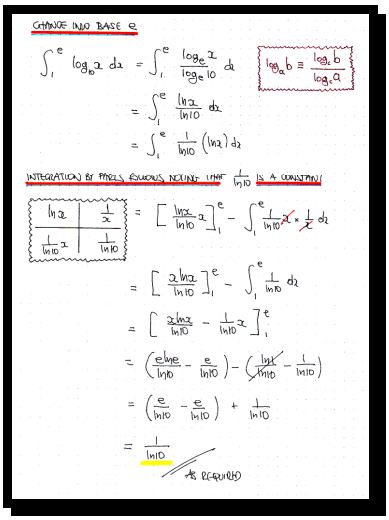


Question 204 **(****+)**

By changing the base of the logarithmic integrand into base e and further using integration by parts, show that

$$\int_1^{\mathrm{e}} \log_{10} x \ dx = \frac{1}{\ln 10}.$$

$\boxed{}$, $\boxed{\text{proof}}$



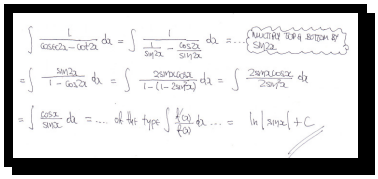
Question 205 **(****+)**

Use trigonometric identities to find

$$\int \frac{1}{\operatorname{cosec} 2x - \cot 2x} \, dx,$$

giving the answer in the form $\ln\left|f(x)\right|$

$\boxed{\ln\left|\sin x\right| + C}$

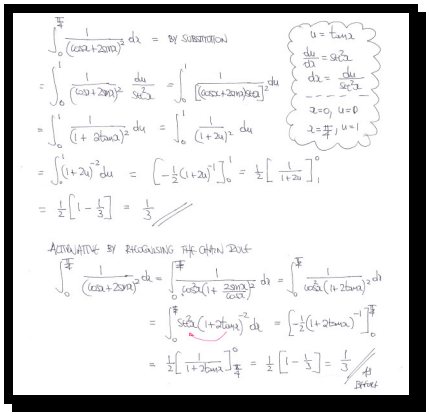


Question 206 (**+)**

By using the substitution $u = \tan x$, or otherwise, find the exact value of

$$\int_0^{\frac{\pi}{4}} \frac{1}{(\cos x + 2\sin x)^2}\, dx .$$

$\boxed{\frac{1}{3}}$

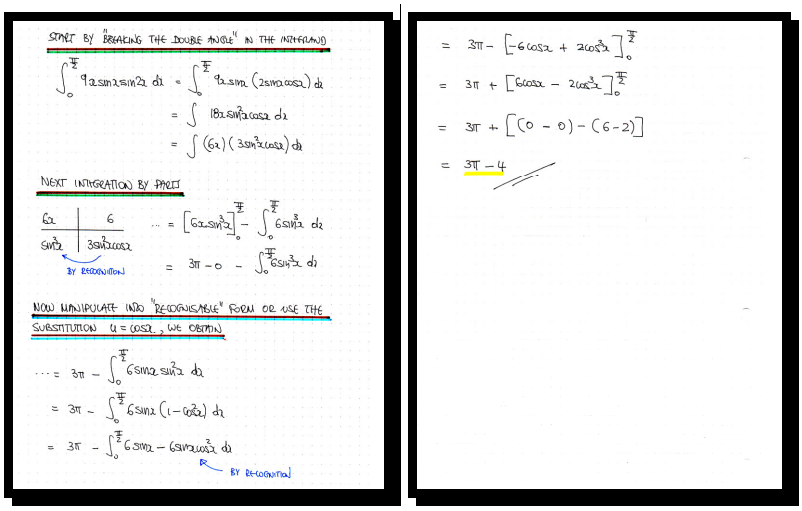


Question 207 (**+)**

Use trigonometric identities and integration by parts to find an exact value for

$$\int_0^{\frac{\pi}{2}} 9x\sin x\sin 2x \ \ dx\,.$$

$\boxed{}$, $\boxed{3\pi-4}$



Question 208 **(****+)**

$$I \equiv \int \frac{1}{1+\sin 2x}\, dx\,.$$

a) Integrate I by multiplying the numerator and denominator of the integrand by $(1-\sin 2x)$.

b) Hence evaluate

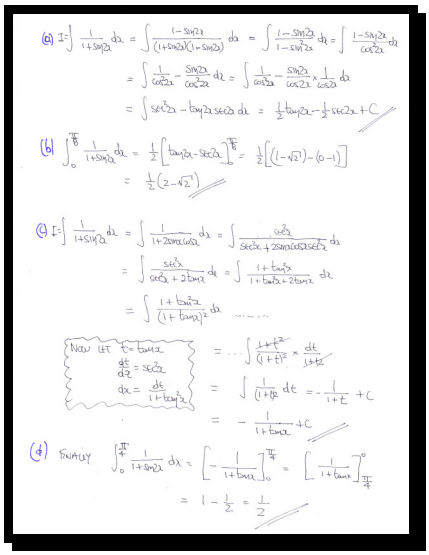
$$\int_0^{\frac{\pi}{8}} \frac{1}{1+\sin 2x}\, dx\,.$$

c) Use the substitution $t = \tan x$ to integrate I .

d) Hence evaluate

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin 2x}\, dx\,.$$

$$\boxed{\frac{1}{2}\tan 2x - \frac{1}{2}\sec 2x + C}\,,\quad \boxed{\frac{1}{2}\left(2-\sqrt{2}\right)}\,,\quad \boxed{-\frac{1}{1+\tan x}+C}\,,\quad \boxed{\frac{1}{2}}$$

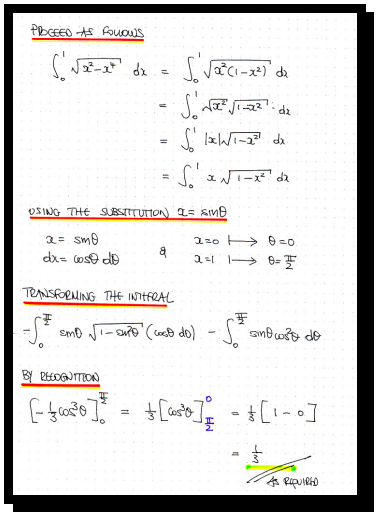


Question 209 (**+)**

Show clearly that

$$\int_0^1 \sqrt{x^2-x^4}\ dx \quad = \quad \frac{1}{3}.$$

[ ], [proof]

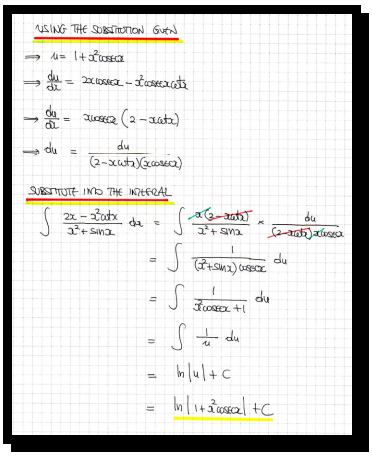


Question 210 (**+)**

Use the substitution $u = 1 + x^2 \operatorname{cosec} x$ to find an expression for

$$\int \frac{2x - x^2 \cot x}{x^2 + \sin x} \, dx .$$

$\boxed{}$, $\boxed{\ln\left|1 + x^2 \operatorname{cosec} x\right| + C}$



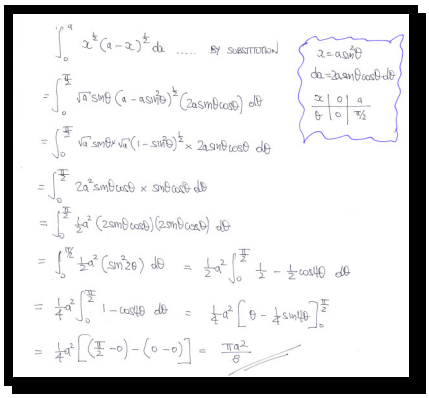
Question 211 (**+)**

Use a suitable trigonometric substitution to find an exact simplified value for

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ dx,$$

where a is a positive constant.

$$\boxed{\frac{1}{8}\pi a^2}$$



Question 212 **(****+)**

$$f(u)\equiv\frac{1}{u^2+5u+6}.$$

a) Express $f(u)$ into partial fractions.

$$I=\int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}}\frac{1}{(\sin x+2\cos x)(\sin x+3\cos x)}\,dx.$$

b) Express I in the form

$$I=\int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}}\frac{\sec^2 x}{g(\tan x)}\,dx,$$

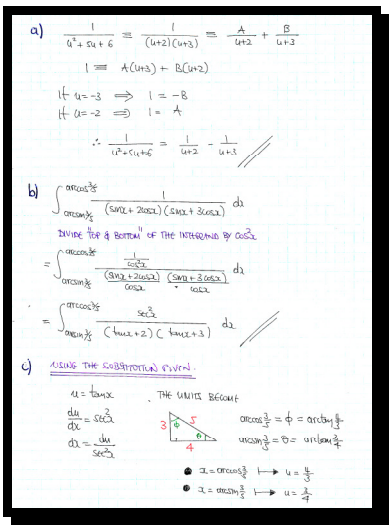
where g is a function to be found.

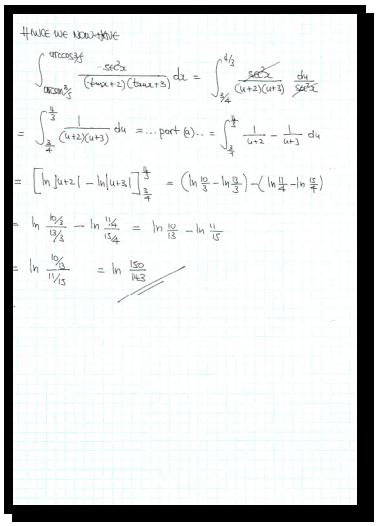
c) Hence show that

$$I=\ln\left(\frac{a}{b}\right),$$

where a and b are positive integers to be found.

$\square$, $f(u)\equiv\dfrac{1}{u+2}-\dfrac{1}{u+3}$, $g(\tan x)\equiv(2+\tan x)(3+\tan x)$, $I=\ln\left(\dfrac{150}{143}\right)$



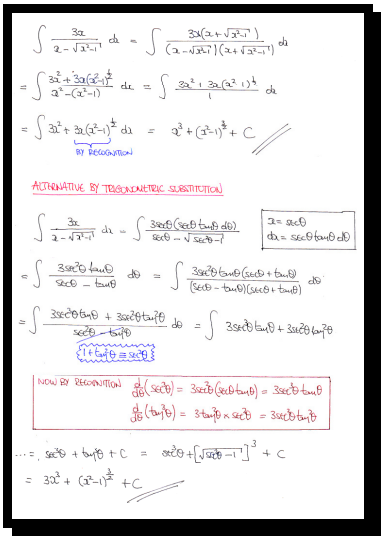


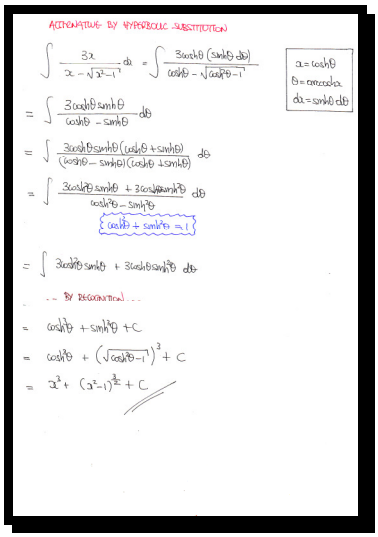
Question 213 (**+)**

Find in exact simplified form an expression for

$$\int \frac{3x}{x-\sqrt{x^2-1}}\,dx\,.$$

$\square$, $\boxed{x^3+\left(x^2-1\right)^{\frac{3}{2}}+C}$



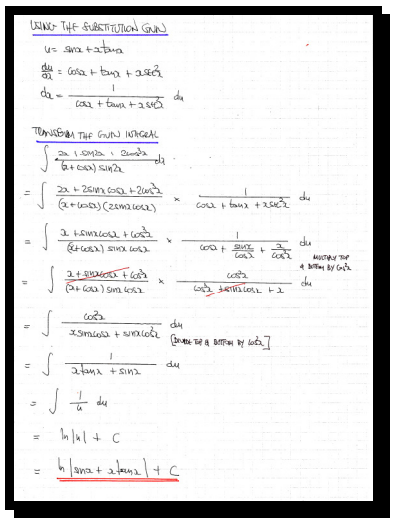


Question 214 (**+)**

Use the substitution $u = \sin x + x\tan x$ to find an expression for

$$\int \frac{2x+\sin 2x+2\cos^3 x}{(x+\cos x)\sin 2x}\,dx\,.$$

$\boxed{}$, $\boxed{\ln\left|\sin x + x\tan x\right| + C}$

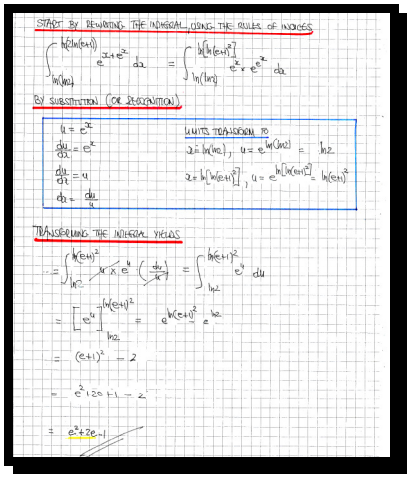


Question 215 **(****+)**

Find, in exact simplified form, the value of

$$\int_{\ln(\ln 2)}^{\ln\left[2\ln(e+1)\right]} e^{x+e^x}\,dx\,.$$

$\square$, $\boxed{e^2+2e-1}$

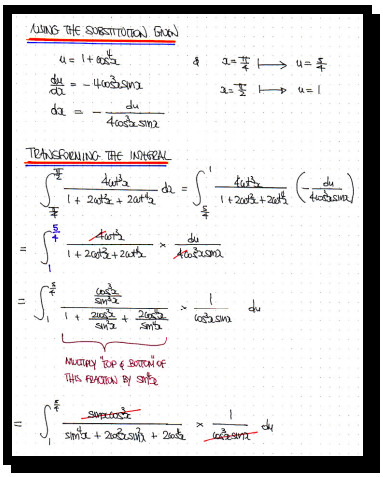


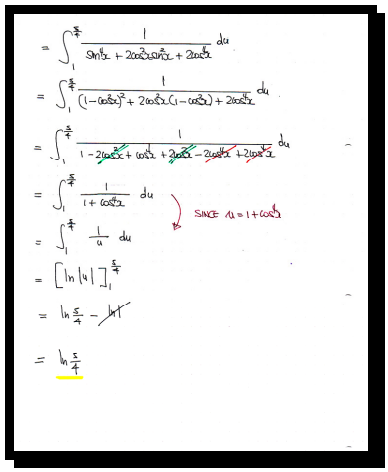
Question 216 **(*****)**

By using the substitution $u=1+\cos^4 x$, or otherwise, find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{4\cot^3 x}{1+2\cot^2 x+2\cot^4 x}\,dx\,.$$

$\square$, $\boxed{\ln\left(\frac{5}{4}\right)}$





Question 217 **(*****)**

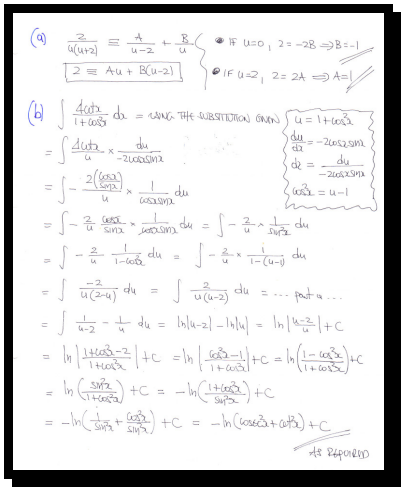
$$\frac{2}{u(u-2)} \equiv \frac{A}{u-2} + \frac{B}{u}.$$

a) Find the value of each of the constants A and B.

b) By using the substitution $u = 1+\cos^2 x$, or otherwise, show clearly that

$$\int \frac{4\cot x}{1+\cos^2 x}\, dx = -\ln\left(\operatorname{cosec}^2 x + \cot^2 x\right) + C.$$

$\boxed{A=1}$, $\boxed{B=1}$

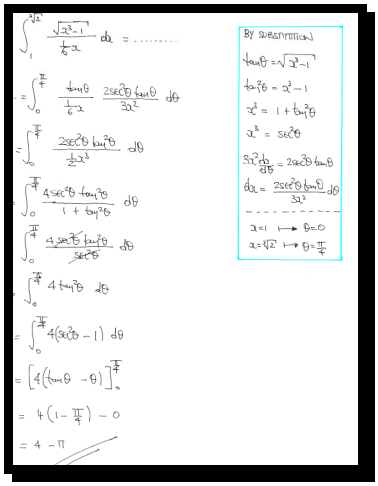


Question 218 (*****)

By using the substitution $\tan\theta = \sqrt{x^3-1}$, or otherwise, find an exact value for the following integral.

$$\int_1^{\sqrt[3]{2}} \frac{\sqrt{x^3-1}}{\frac{1}{6}x}\,dx .$$

$\boxed{}$, $\boxed{4-\pi}$

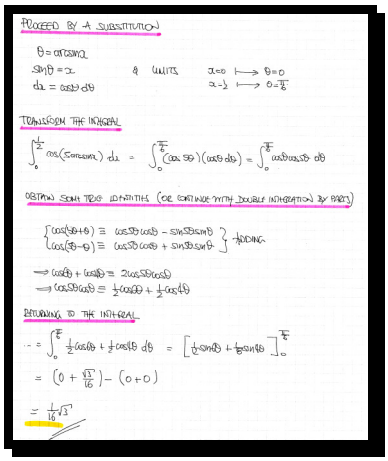


Question 219 (*****)

Use appropriate integration methods to find an exact simplified value for

$$\int_0^{\frac{1}{2}} \cos\left(5\arcsin x\right)\ dx\ .$$

V, □, $\boxed{\dfrac{\sqrt{3}}{16}}$

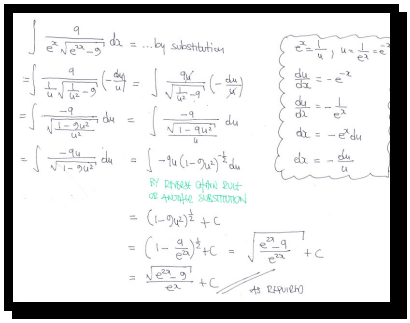


Question 220 (*****)

By using the substitution $\mathrm{e}^x = \dfrac{1}{u}$, or otherwise, show clearly that

$$\int \frac{9}{\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-9}}\,dx = \frac{\sqrt{\mathrm{e}^{2x}-9}}{\mathrm{e}^x} + C\,.$$

proof

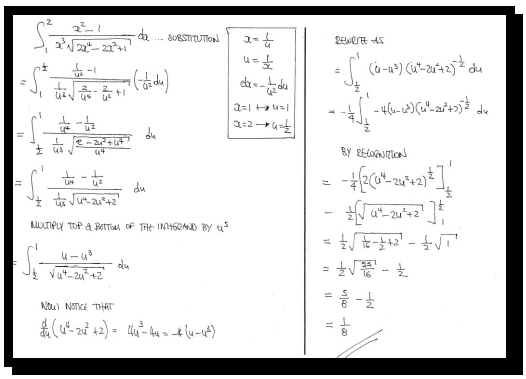


Question 221 (*****)

By using a reciprocal substitution, or otherwise, find the value of the following integral.

$$\int_1^2 \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}\,dx\,.$$

$\square$, $\boxed{\dfrac{1}{8}}$

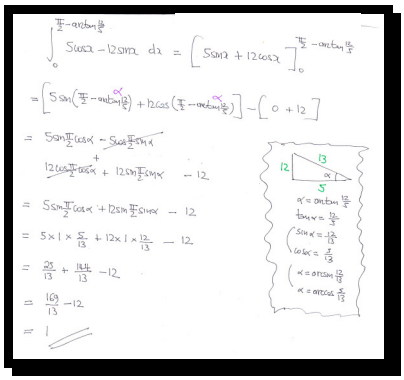


Question 222 (*****) non calculator

Show clearly that

$$\int_0^{\frac{\pi}{2}-\arctan\frac{12}{5}} 5\cos x-12\sin x \ dx \ = \ 1 .$$

proof

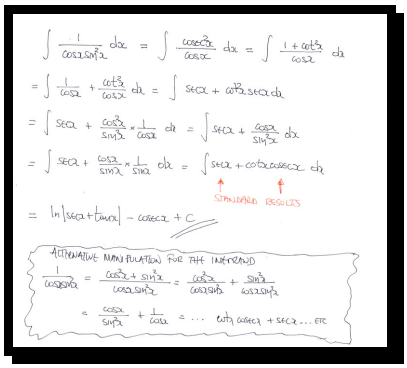


Question 223 (*****)

Use trigonometric identities to find

$$\int \frac{1}{\cos x \sin^2 x} \, dx .$$

$$\ln\left|\sec x+\tan x\right|-\operatorname{cosec} x+C$$



Question 224 **(*****)**

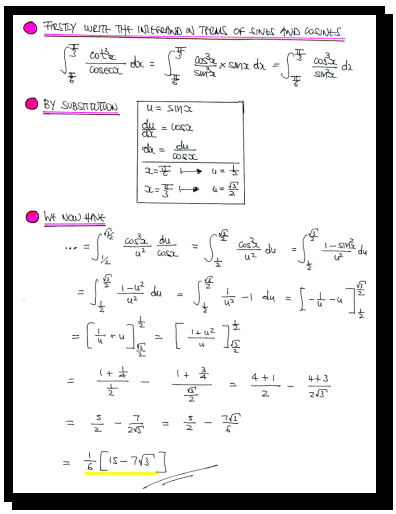
$$I=\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\frac{\cot^3 x}{\operatorname{cosec} x}\,dx\,.$$

Use appropriate integration techniques to show that

$$I=\frac{1}{6}\left[a+b\sqrt{3}\right],$$

where a and b are integers to be found.

$\square$, $\boxed{I=\frac{1}{6}\left[15-7\sqrt{3}\right]}$

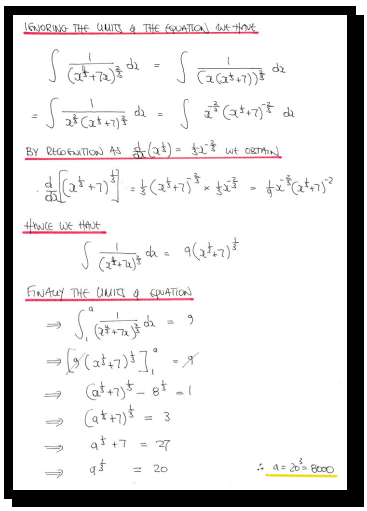


Question 225 **(*****)**

$$I=\int_1^a \frac{1}{\left(x^{\frac{4}{3}}+7x\right)^{\frac{2}{3}}}\,dx\,.$$

Given that $I=9$, determine the value a.

$\boxed{}$, $\boxed{a=8000}$

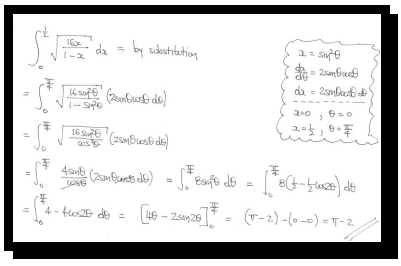


Question 226 **(*****)**

By using a suitable trigonometric substitution, show clearly that

$$\int_0^{\frac{1}{2}} \sqrt{\frac{16x}{1-x}}\,dx=\pi-2\,.$$

$\boxed{\text{proof}}$

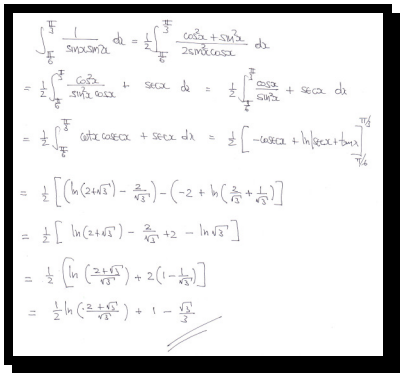


Question 227 (*****)

Find an exact simplified value for

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin x \sin 2x}\, dx .$$

$$\boxed{\frac{1}{2}\ln\left[\frac{2+\sqrt{3}}{\sqrt{3}}\right]+1-\frac{\sqrt{3}}{3}}$$



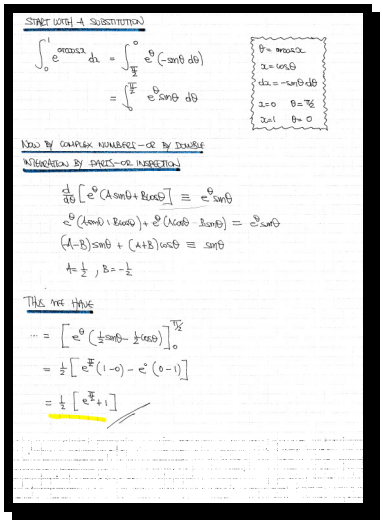
Question 228 (*****)

Evaluate the following definite integral.

$$\int_0^1 \mathrm{e}^{\arccos x}\ dx\,.$$

Give the answer in exact simplified form.

$\square$, proof

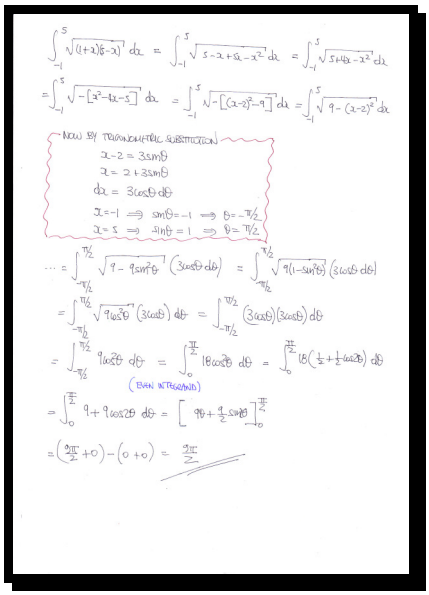


Question 229 (*****)

Use a suitable trigonometric substitution to find the exact value of

$$\int_{-1}^{5} \sqrt{(1+x)(5-x)}\ dx\,.$$

$$\boxed{\frac{9\pi}{2}}$$

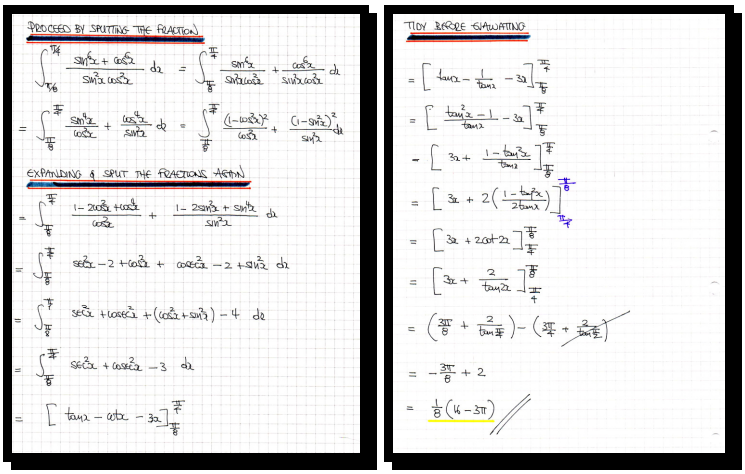


Question 230 (*****)

By using trigonometric identities, show that

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}\,dx = \frac{1}{8}(16-3\pi).$$

, proof

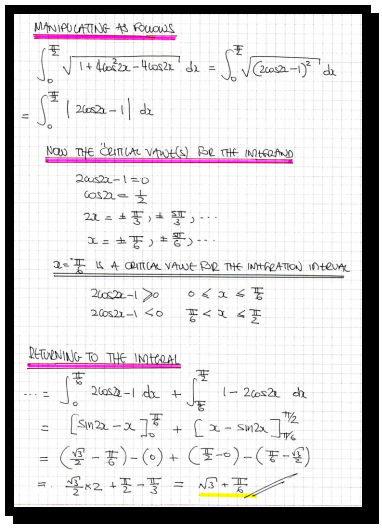


Question 231 **(*****)**

Find, in exact simplified form, the value of the following integral.

$$\int_0^{\frac{1}{2}\pi} \sqrt{1+4\cos^2 2x-4\cos 2x}\ \ dx\,.$$

$\boxed{}$, $\boxed{\frac{1}{6}\pi+\sqrt{3}}$

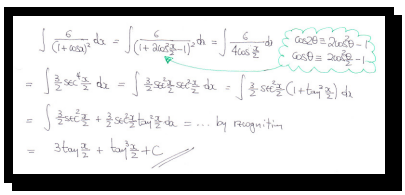


Question 232 **(*****)**

By using a suitable cosine double angle trigonometric identity find

$$\int \frac{6}{(1+\cos x)^2}\ dx\,.$$

$\boxed{3\tan\frac{x}{2}+\tan^3\frac{x}{2}+C}$

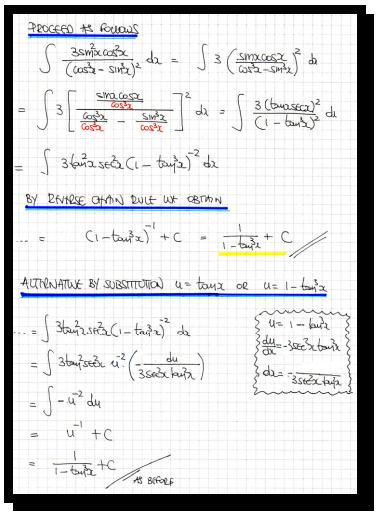


Question 233 **(*****)**

By expressing the integrand in the form $\sec^2 x\, f(\tan x)$, or otherwise, find a simplified expression for the following integral.

$$\int \frac{3\sin^2 x\cos^2 x}{\left(\cos^3 x-\sin^3 x\right)^2}\,dx\,.$$

$\square$, $\boxed{\dfrac{1}{1-\tan^3 x}+C}$



Question 234 **(*****)**

$$\sec x \equiv \frac{1+\tan^2\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}.$$

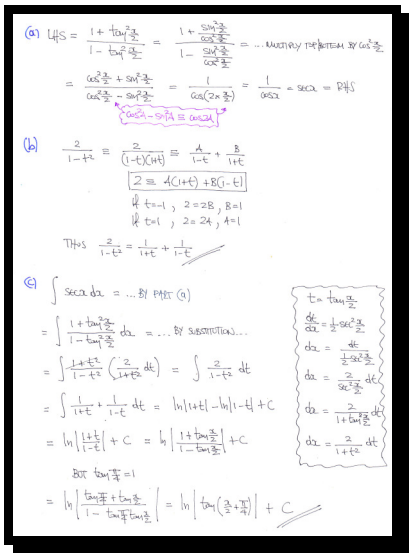
a) Prove the validity of the above trigonometric identity.

b) Express $\dfrac{2}{1-t^2}$ into partial fractions.

c) Hence use the substitution $t=\tan\left(\frac{x}{2}\right)$ to show that

$$\int \sec x \, dx = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C .$$

$$\boxed{\frac{2}{1-t^2}=\frac{1}{1+t}+\frac{1}{1-t}}$$

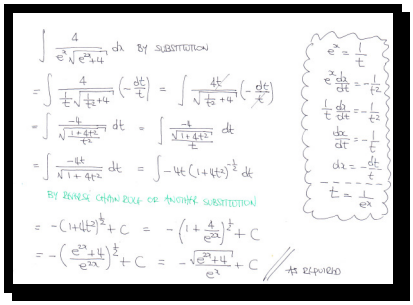


Question 235 (*****)

By using the substitution $\mathrm{e}^x = \dfrac{1}{t}$, or otherwise, show clearly that

$$\int \frac{4}{\mathrm{e}^x\sqrt{\mathrm{e}^{2x}+4}}\,dx = -\frac{\sqrt{\mathrm{e}^{2x}+4}}{\mathrm{e}^x} + C\,.$$

proof

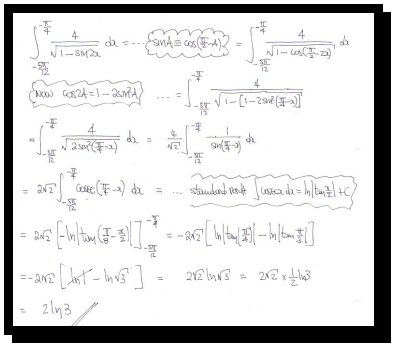


Question 236 (*****)

Use the fact that $\sin A \equiv \cos\left(\dfrac{\pi}{2} - A\right)$ and other trigonometric identities to show that

$$\int_{-\frac{5\pi}{12}}^{-\frac{\pi}{4}} \frac{4}{\sqrt{1-\sin 2x}}\,dx = 2\ln 3\,.$$

proof

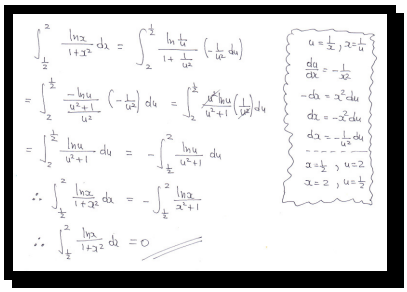


Question 237 (*****)

Use the substitution $x=\dfrac{1}{u}$ to find the value of

$$\int_{\frac{1}{2}}^{2} \frac{\ln x}{1+x^2}\,dx\,.$$

$\boxed{0}$

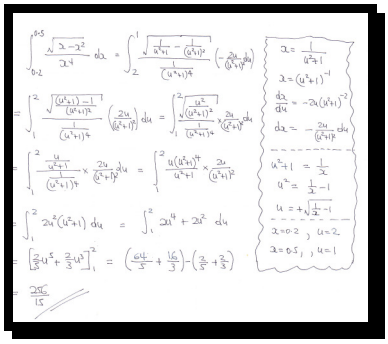


Question 238 (*****)

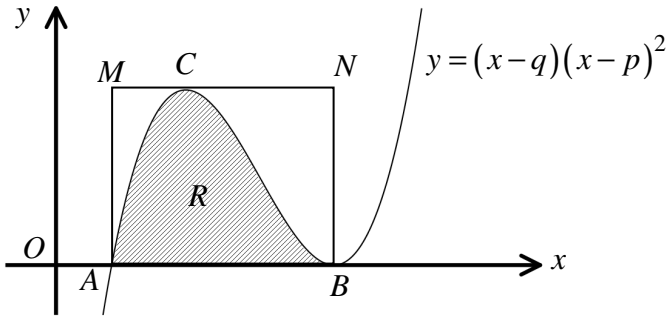
Use the substitution $x=\dfrac{1}{u^2+1}$ to show that

$$\int_{0.2}^{0.5} \frac{\sqrt{x-x^2}}{x^4}\,dx=\frac{256}{15}\,.$$

$\boxed{\text{proof}}$



Question 239 **(*****)**



The figure above shows the graph of the curve with equation

$$y=(x-q)(x-p)^2,$$

where p and q are positive constants.

The curve meets the x axis at the points A and B. The region R, shown shaded in the figure, is bounded by the curve and the x axis.

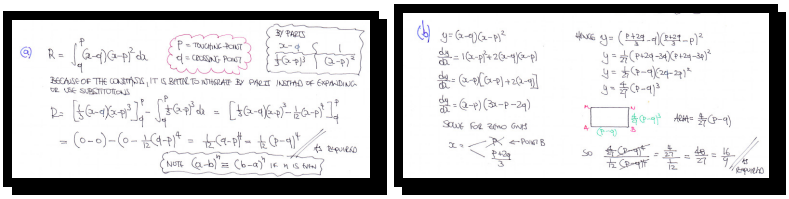
a) Show that the area of the shaded region is

$$\frac{1}{12}(p-q)^4.$$

The point C is the local maximum of the curve. The rectangle $AMCNB$ is such so that MCN is parallel to the x axis and both AM and BN are parallel to the y axis.

b) Show that the area of the rectangle $AMCNB$ is $\frac{16}{9}$ times as large as the area of R, regardless of the values of p and q.

proof

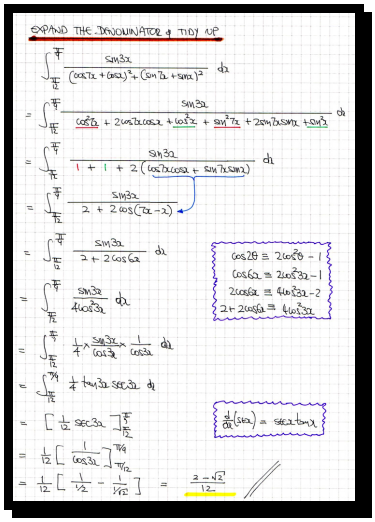


Question 240 (*****)

$$f(x)=\frac{\sin 3x}{(\cos 7x+\cos x)^2+(\sin 7x+\sin x)^2},\ x\in\mathbb{R}.$$

Use trigonometric identities to find the exact value of $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{9}\pi} f(x)\ dx$.

$\square$, $\boxed{\dfrac{2-\sqrt{2}}{12}}$

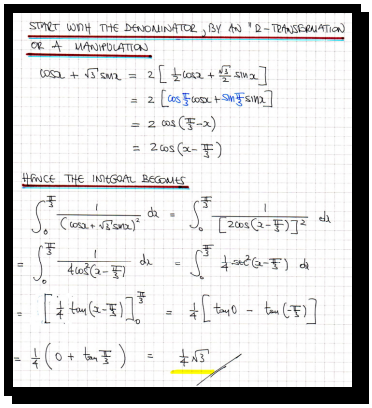


Question 241 (*****)

Use a suitable trigonometric manipulation to find an exact simplified answer for the following integral.

$$\int_0^{\frac{\pi}{3}} \frac{1}{\left(\cos x+\sqrt{3}\sin x\right)^2}\,dx\,.$$

$\boxed{}$, $\boxed{\frac{1}{4}\sqrt{3}}$



Question 242 **(*****)**

$$f(x)=3\sin x-\cos x+3,\ x\in\mathbb{R}.$$

$$g(x)=\sin x+\cos x,\ x\in\mathbb{R}.$$

a) Express $f(x)$ in the form

$$A\times g(x)+B\times g'(x)+3,$$

where A and B are constants.

b) Express $g(x)$ in the form

$$R\cos(x-\varphi),$$

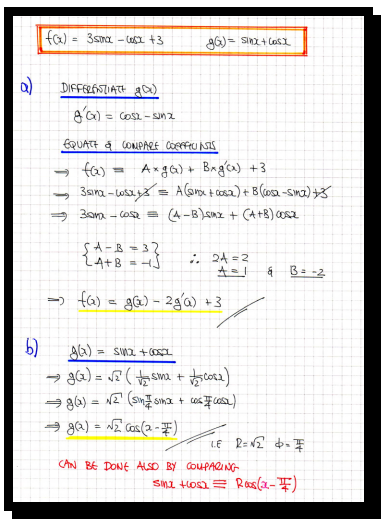
where R and φ are positive constants.

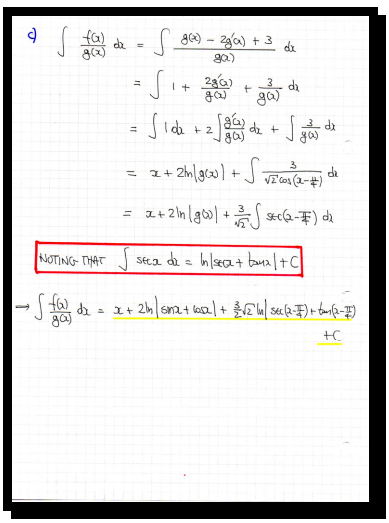
c) Hence find a simplified expression for

$$\int \frac{f(x)}{g(x)}\,dx.$$

$\boxed{}$, $\boxed{A=1}$, $\boxed{B=-2}$, $\boxed{R=\sqrt{2}}$, $\boxed{\varphi=\frac{1}{4}\pi}$,

$$\boxed{x-2\ln\left|\sin x+\cos x\right|+\frac{3}{2}\sqrt{2}\ln\left|\sec\left(x-\frac{1}{4}\pi\right)+\tan\left(x-\frac{1}{4}\pi\right)\right|+C}$$



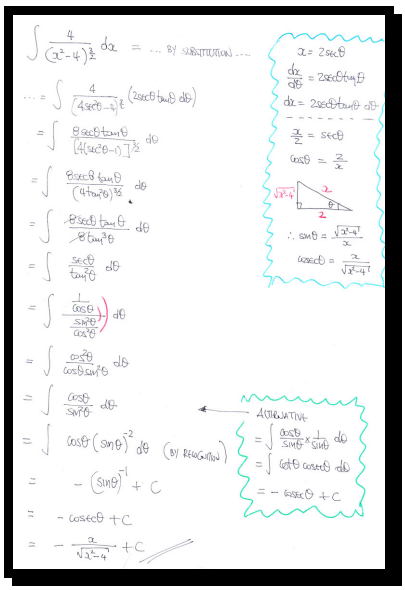


Question 243 **(*****)**

Use the substitution $x = 2\sec\theta$, to find a simplified expression for

$$\int \frac{6}{\left(x^2-4\right)^{\frac{3}{2}}}\,dx\,.$$

$$\boxed{-\frac{x}{\sqrt{x^2-4}}+C}$$

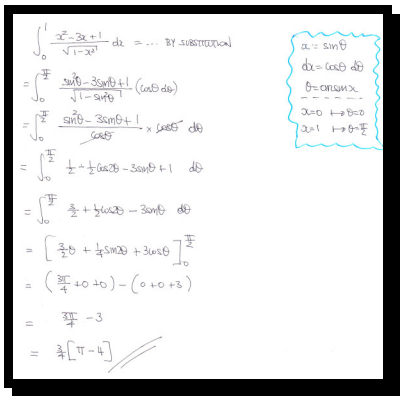


Question 244 **(*****)**

Find an exact simplified value for

$$\int_0^1 \frac{x^2-3x+1}{\sqrt{1-x^2}}\,dx\,.$$

$$\boxed{\frac{3}{4}[\pi-4]}$$

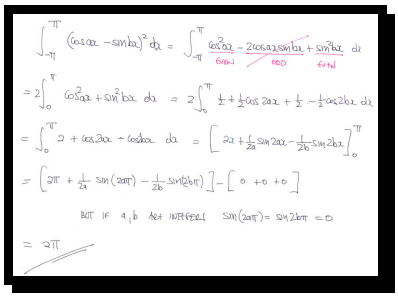


Question 245 **(*****)**

Given that a and b are integers, evaluate

$$\int_{-\pi}^{\pi} (\cos ax-\sin bx)^2\,dx\,.$$

$$\boxed{2\pi}$$



Question 246 **(*****)**

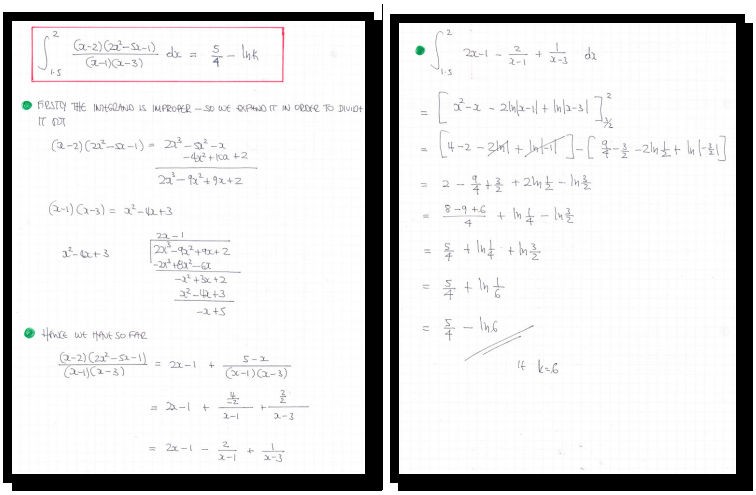
$$I=\int_{1.5}^{2}\frac{(x-2)\left(2x^2-5x-1\right)}{(x-1)(x-3)}\,dx\,.$$

Use appropriate integrations techniques to show that

$$I=\frac{5}{4}-\ln k\,,$$

where k is a positive integer.

$\boxed{}$, $\boxed{k=6}$



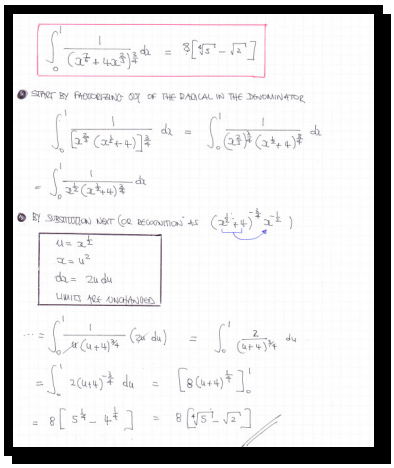
Question 247 (*****)

$$I=\int_0^1 \left(x^{\frac{7}{6}}+4x^{\frac{2}{3}}\right)^{-\frac{3}{4}}\,dx\,.$$

Use appropriate integration techniques to show that

$$I=8\left[\sqrt[4]{5}\,-\sqrt{2}\right].$$

[ ], [proof]



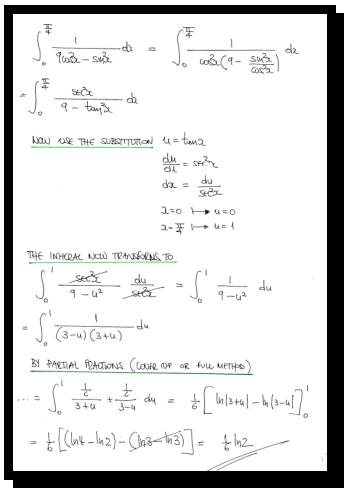
Question 248 (*****)

$$I=\int_0^{\frac{1}{4}\pi}\frac{1}{9\cos^2 x-\sin^2 x}\,dx\,.$$

By using a tangent substitution, or otherwise, show that

$$I=\frac{1}{6}\ln 2\,.$$

, proof



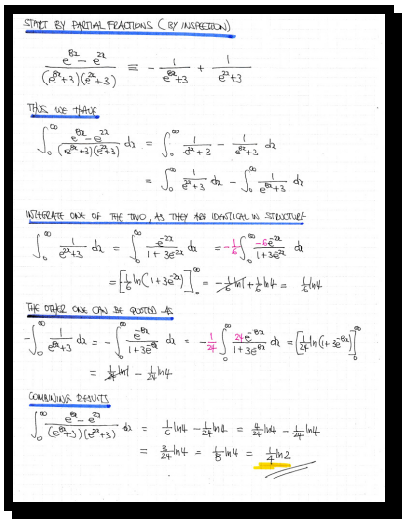
Question 249 (*****)

Find an exact simplified value for the following definite integral.

$$\int_0^{\infty} \frac{e^{8x}-e^{2x}}{\left(e^{8x}+3\right)\left(e^{2x}+3\right)}\,dx\,.$$

You may assume without proof that the integral converges.

V, □, $\boxed{\frac{1}{4}\ln 2}$



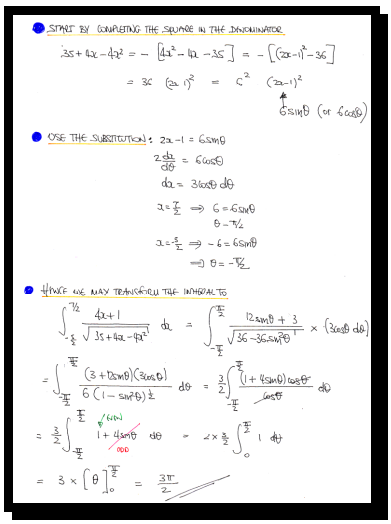
Question 250 (*****)

$$I=\int_{-\frac{5}{2}}^{\frac{7}{2}}\frac{4x+1}{\sqrt{35+4x-4x^2}}\,dx\,.$$

By writing $35+4x-4x^2$ in completed the square form, followed by a suitable trigonometric substitution, show that

$$I=\frac{3}{2}\pi\,.$$

, proof



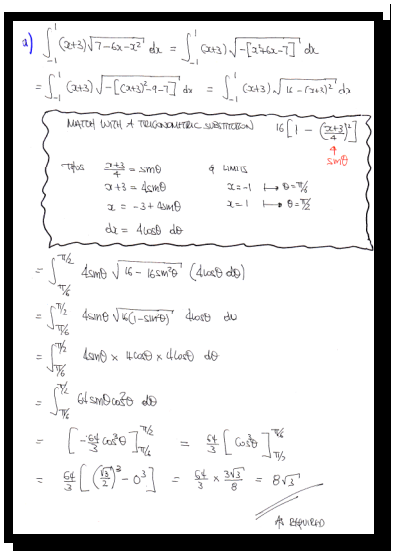
Question 251 **(*****)**

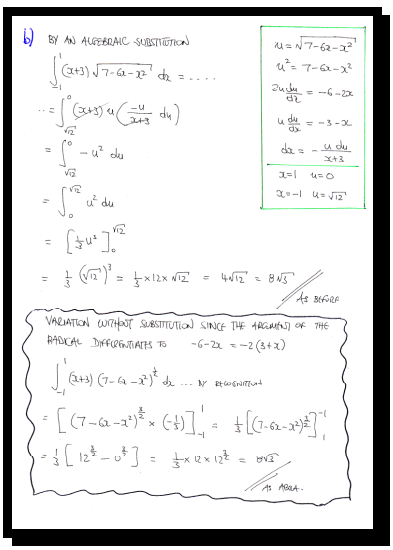
$$I=\int_{-1}^{1}(x+3)\sqrt{7-6x-x^2}\ \ dx$$

a) Use a suitable trigonometric substitution to show that $I=8\sqrt{3}$.

b) Verify the answer of part **(a)** by an alternative method.

[ ], [proof]



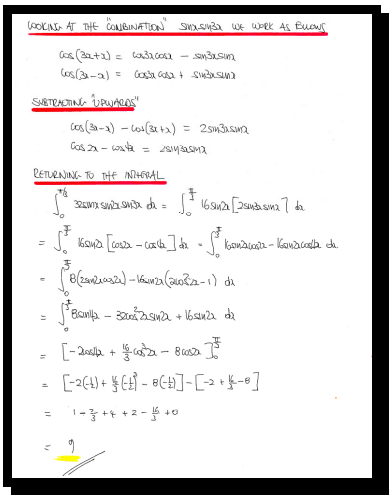


Question 252 **(*****)**

Use trigonometric identities to find the value of

$$\int_0^{\frac{\pi}{3}} 32\sin x\sin 2x\sin 3x \ \ dx\,.$$

$\boxed{}$, $\boxed{9}$

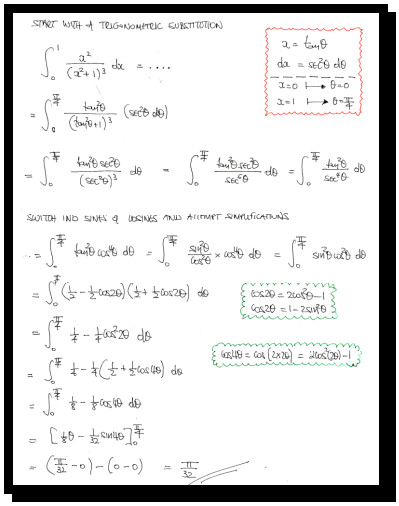


Question 253 (*****)

Use suitable integration techniques to show that

$$\int_0^1 \frac{x^2}{\left(x^2+1\right)^3}\,dx = \frac{\pi}{32}.$$

[ ], proof

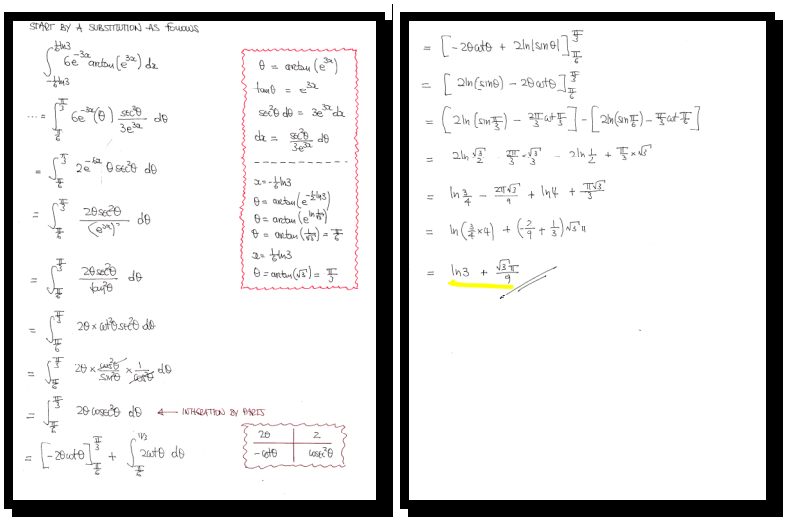


Question 254 **(*****)**

Use suitable integration techniques to show that

$$\int_{-\frac{1}{6}\ln 3}^{\frac{1}{6}\ln 3} 6\mathrm{e}^{-3x}\arctan\left(\mathrm{e}^{3x}\right)\,dx \;=\; \ln 3+\frac{\pi\sqrt{3}}{9}.$$

[ ], [proof]



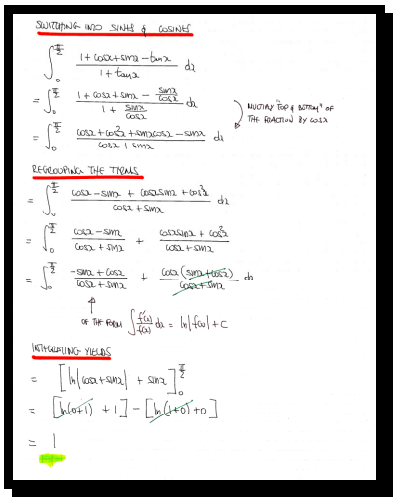
Question 255 **(*****)**

Use suitable integration techniques to show that

$$\int_0^{\frac{1}{2}\pi} \frac{1+\cos x+\sin x-\tan x}{1+\tan x}\,dx \quad = \quad 1\,.$$

You may assume that the above integral converges.

[ ], proof

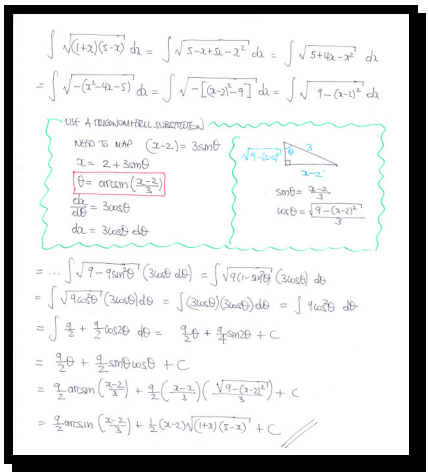


Question 256 (*****)

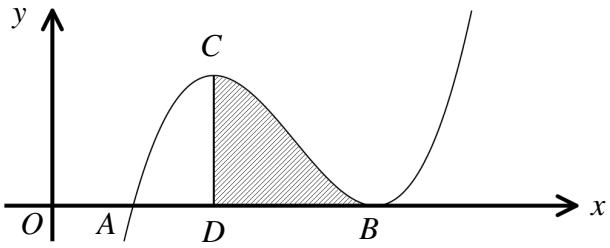
Use a suitable trigonometric substitution to find a simplified expression for

$$\int \sqrt{(1+x)(5-x)}\ dx\,.$$

$$\boxed{\frac{9}{2}\arcsin\left(\frac{x-2}{3}\right)+\frac{1}{2}(x-2)\sqrt{(1+x)(5-x)}+C}$$



Question 257 (*****)



The figure above shows a cubic curve that crosses the x axis at $A(a,0)$ and touches the x axis at $B(b,0)$, where a and b are positive constants. The point C is a local maximum of the curve.

a) Find the x coordinate of C, in terms of a and b.

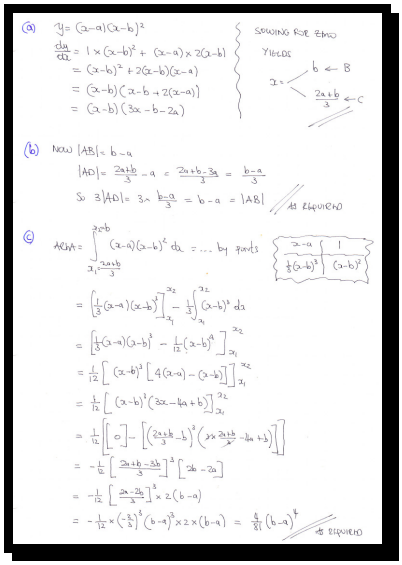
The point D lies on the x axis so that CD is parallel to the y axis.

b) Show that $|AB|=3|AD|$.

The region R is bounded by the curve, the line segment CD and the x axis.

c) Use integration by parts to show that the area of R is $\frac{4}{81}(b-a)^4$.

$$\boxed{x=\tfrac{1}{3}(2a+b)}$$

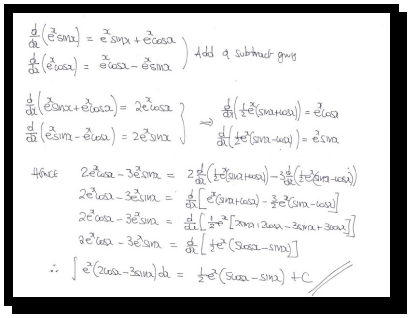


Question 258 (*****)

By considering the derivatives of $e^x \sin x$ and $e^x \cos x$, find

$$\int e^x (2\cos x - 3\sin x)\, dx .$$

$$\boxed{\frac{1}{2}e^x (5\cos x - \sin x) + C}$$

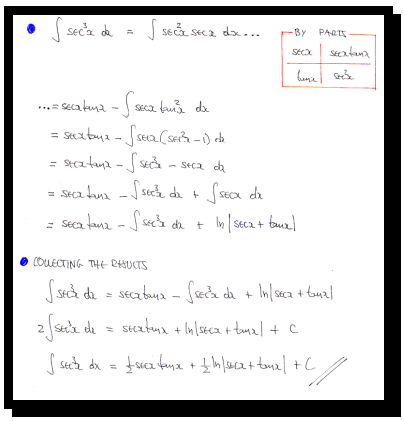


Question 259 (*****)

Use integration by parts and suitable trigonometric identities to find

$$\int \sec^3 x\, dx .$$

$$\boxed{\quad}, \quad \boxed{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln\left|\sec x + \tan x\right| + C}$$

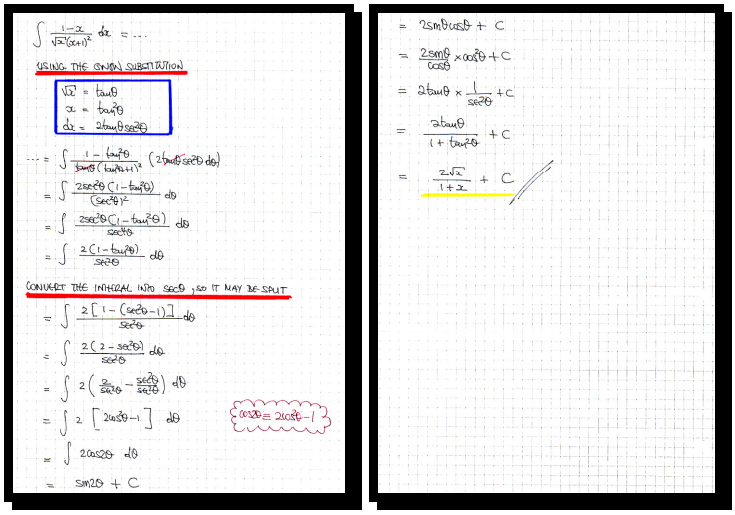


Question 260 **(*****)**

By using the substitution $\sqrt{x}=\tan\theta$, or otherwise, find a simplified expression for the following integral.

$$\int \frac{1-x}{\sqrt{x}\left(x+1\right)^2}\,dx\,.$$

$\square$, $\boxed{\dfrac{2\sqrt{x}}{x+1}+C}$



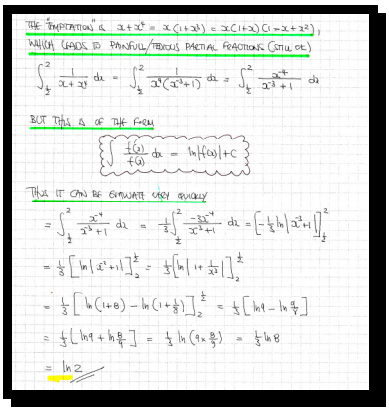
Question 261 (*****)

Find the value of the following definite integral.

$$\int_{\frac{1}{2}}^{2} \frac{1}{x+x^4}\,dx\ .$$

Give the answer in the form $\ln k$, where k is a positive integer.

$\boxed{}$, $\boxed{k=2}$

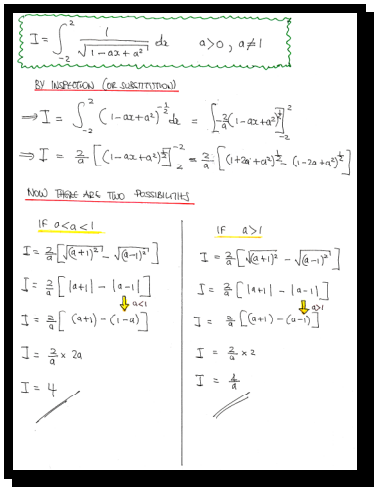


Question 262 (*****)

$$I=\int_{-2}^{2}\frac{1}{\sqrt{1-ax+a^2}}\,dx\,,\ a>0\,,\ a\neq 0\,.$$

Find the two possible values of I, giving the answer in terms of a where appropriate.

$$\boxed{}\,,\qquad \begin{array}{|ll|}\hline I=4 & 0<a<1 \\ I=\dfrac{4}{a} & a>1 \\ \hline \end{array}$$



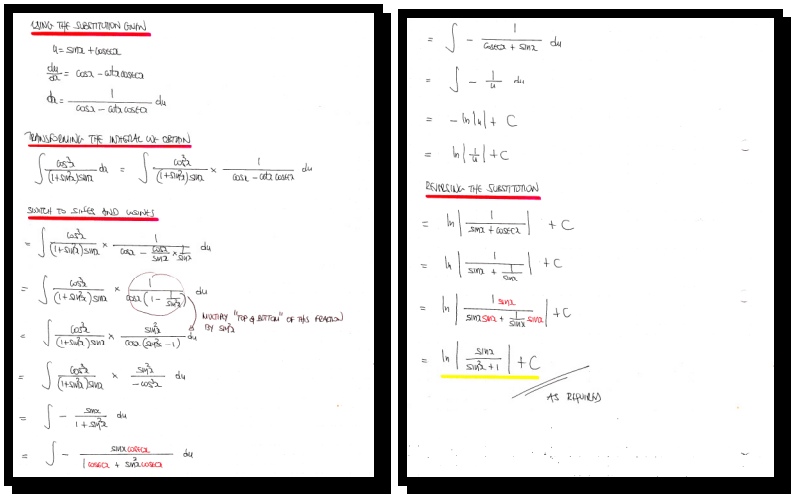
Question 263 **(*****)**

$$I = \int \frac{\cos^3 x}{\left(1+\sin^2 x\right)\sin x}\, dx\,.$$

By using the substitution $u = \sin x + \operatorname{cosec} x$, or otherwise, show that

$$I = \ln\left|\frac{\sin x}{1+\sin^2 x}\right| + \text{constant}$$

[ ], [proof]



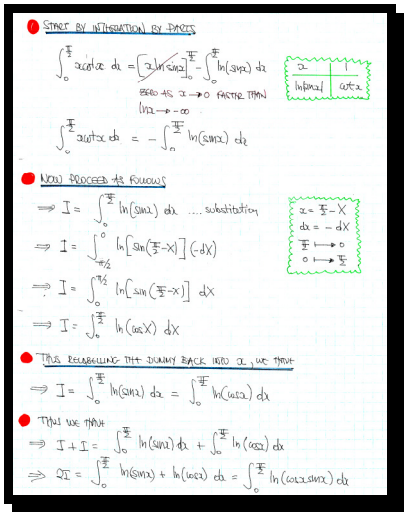
Question 264 **(*****)**

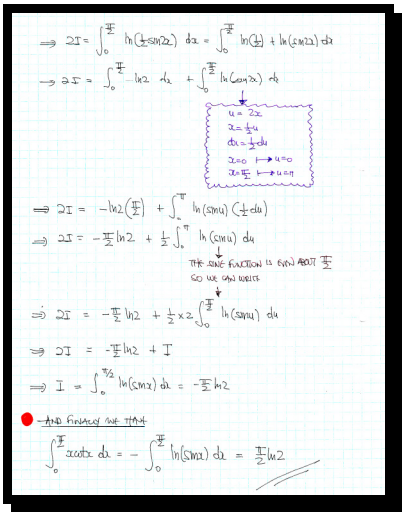
$$I = \int_0^{\frac{1}{2}\pi} x\cot x \ \ dx \, .$$

Use appropriate integration techniques to show that

$$I = \frac{1}{2}\pi \ln 2 \, .$$

[ ], proof



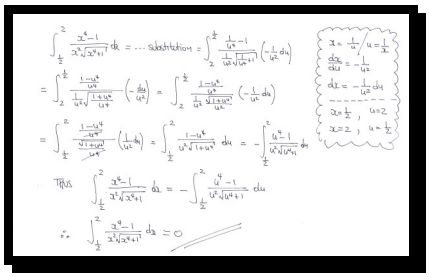


Question 265 (*****)

Use the substitution $x=\dfrac{1}{u}$ to find the value of

$$\int_{\frac{1}{2}}^{2} \frac{x^4-1}{x^2\sqrt{x^4+1}}\,dx\,.$$

$\boxed{0}$

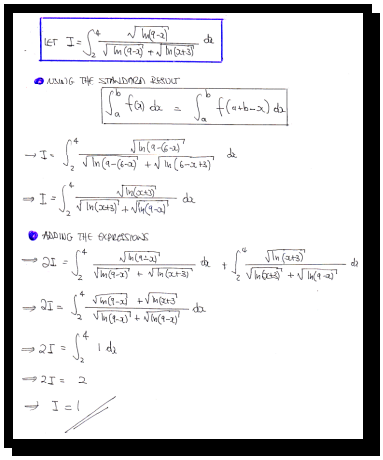


Question 266 (*****)

Use a suitable substitution to find the value of

$$\int_{2}^{4} \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x)}+\sqrt{\ln(3+x)}}\,dx\,.$$

$\boxed{}$, $\boxed{1}$

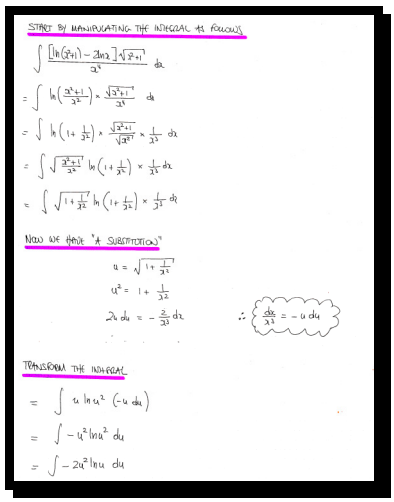


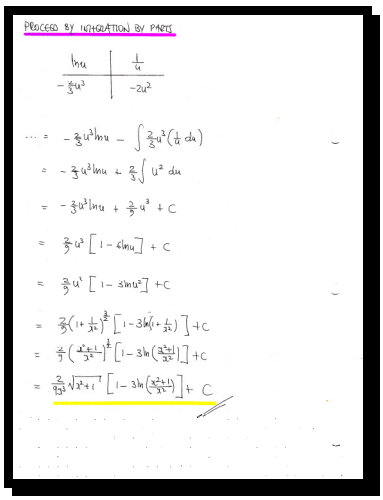
Question 267 **(*****)**

Use an appropriate substitution followed by integration by parts to find a simplified expression for

$$\int \frac{\left[\ln\left(x^2+1\right)-2\ln x\right]\sqrt{x^2+1}}{x^4}\,dx\,.$$

$$\square\,,\quad \boxed{\frac{2}{9x^3}\left(x^2+1\right)^{\frac{3}{2}}\left[1-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+C}$$



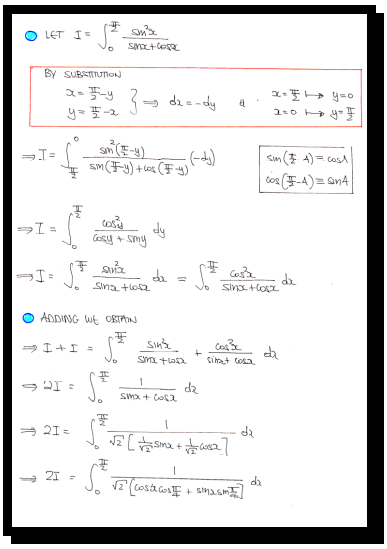


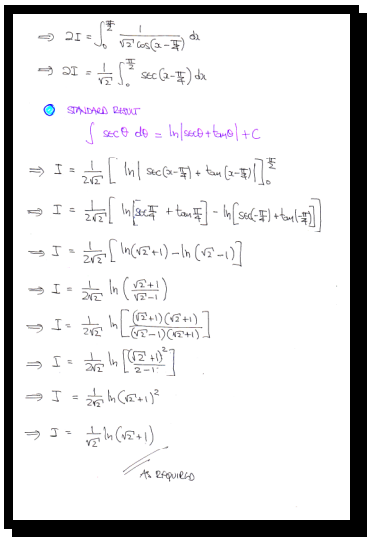
Question 268 (*****)

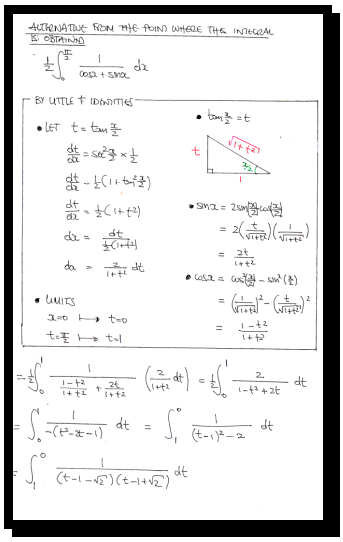
Use appropriate integration techniques to show that

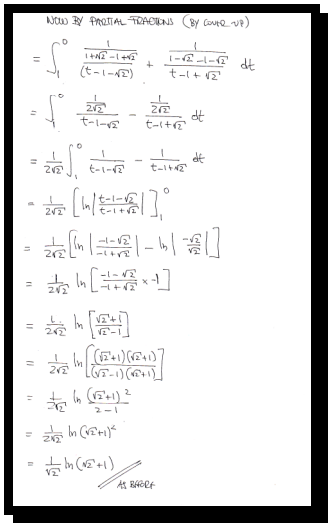
$$\int_0^{\frac{1}{2}\pi} \frac{\sin^2 x}{\sin x+\cos x}\,dx = \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right).$$

, proof







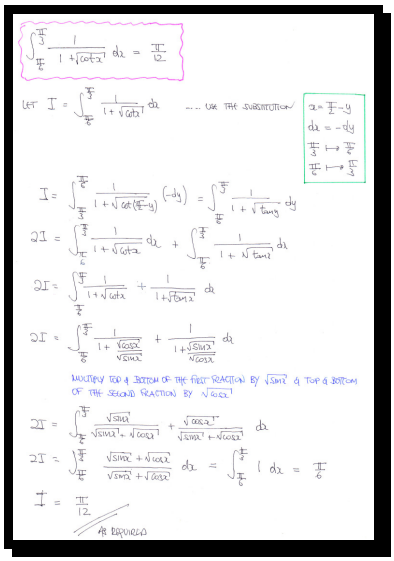


Question 269 **(*****)**

Use appropriate integration techniques to show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1}{1+\sqrt{\cot x}}\,dx = \frac{\pi}{12}.$$

proof

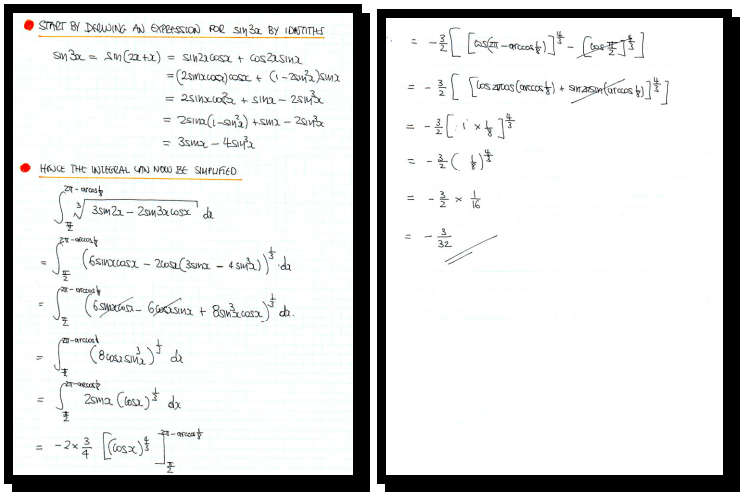


Question 270 **(*****)**

Use appropriate integration techniques to find an exact answer for the following definite integral.

$$\int_{\frac{1}{2}\pi}^{2\pi-\arccos\frac{1}{8}} \sqrt[3]{3\sin 2x-2\sin 3x\cos x}\ \ dx$$

$\square$, $\boxed{-\frac{3}{32}}$



Question 271 **(*****)**

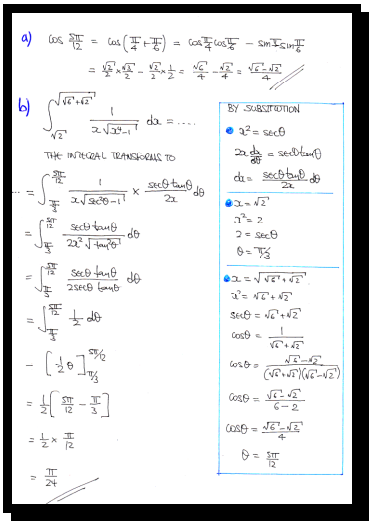
a) Use the compound angle identity $\cos(A+B)$ to show that

$$\cos\left(\frac{5\pi}{12}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$$

b) Use a suitable trigonometric substitution to find the exact value of

$$\int_{\sqrt{2}}^{\sqrt{\sqrt{6}+\sqrt{2}}} \frac{2}{x\sqrt{x^4-1}}\,dx\,.$$

$\square$, $\boxed{\dfrac{\pi}{24}}$



Question 272 (*****)

It is given that the functions of x, $u(x)$ and $v(x)$ satisfy

$$\int u(x)v(x)\,dx=\left[\int u(x)dx\right]\times\left[\int v(x)\,dx\right],\text{ for } x\in\mathbb{R},\ x\neq 0,\ x\neq 1.$$

a) Show clearly that

$$\frac{\int u(x)\,dx}{u(x)}+\frac{\int v(x)\,dx}{v(x)}=1.$$

b) Given further that

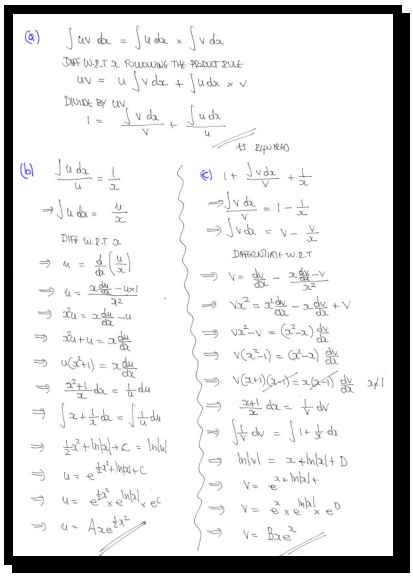
$$\frac{\int u(x)\,dx}{u(x)}=\frac{1}{x},$$

show that

$$u(x)=Ax\mathrm{e}^{\frac{1}{2}x^2},\text{ where } A \text{ is an arbitrary constant.}$$

c) Determine a similar expression for $v(x)$.

$$\boxed{v(x)=Bx\mathrm{e}^{x}}$$

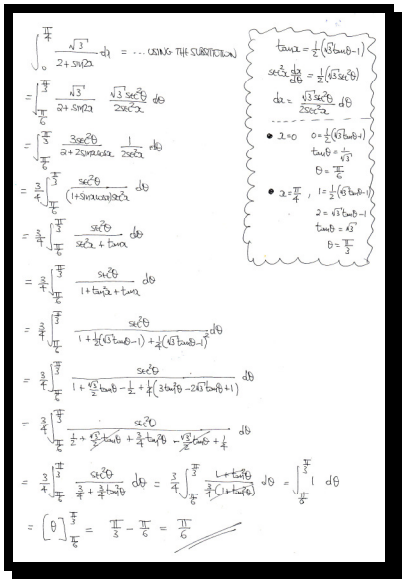


Question 273 (*****)

Use the substitution $\tan x = \frac{1}{2}\left(-1+\sqrt{3}\tan\theta\right)$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{3}}{2+\sin 2x}\, dx .$$

$\square$, $\boxed{\frac{\pi}{6}}$



Question 274 **(*****)**

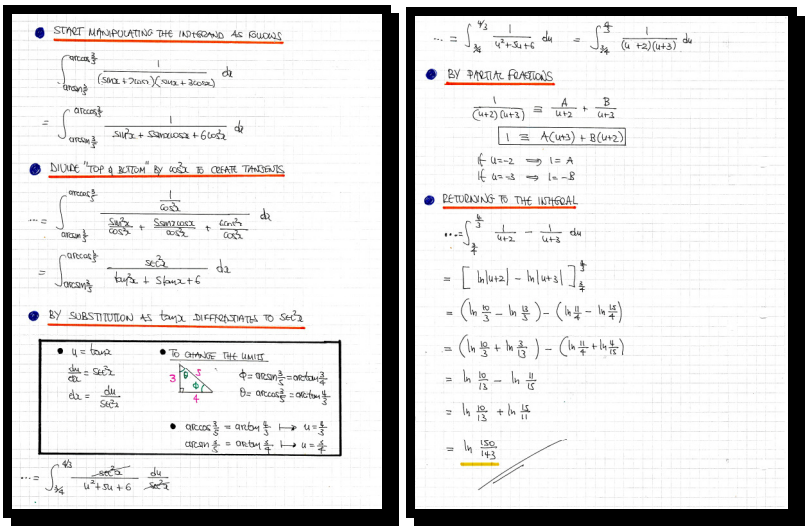
$$I=\int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}}\frac{1}{(\sin x+2\cos x)(\sin x+3\cos x)}\,dx\,.$$

Use appropriate integration techniques to show that

$$I=\ln\left(\frac{a}{b}\right),$$

where a and b are positive integers to be found.

$\square$, $\boxed{I=\ln\left(\frac{150}{143}\right)}$

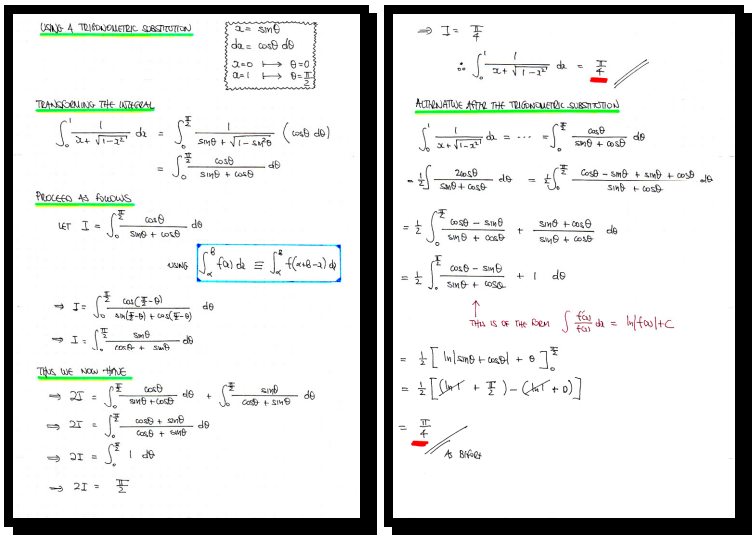


Question 275 (*****)

Use appropriate integration techniques to show that

$$\int_0^1 \frac{1}{x+\sqrt{1-x^2}}\,dx=\frac{\pi}{4}.$$

[ ], proof



Question 276 (*****)

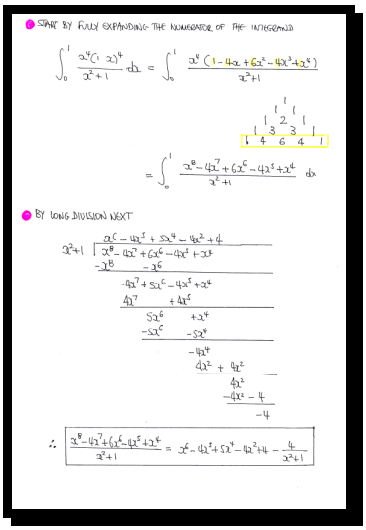
Use polynomial division to find the exact value of

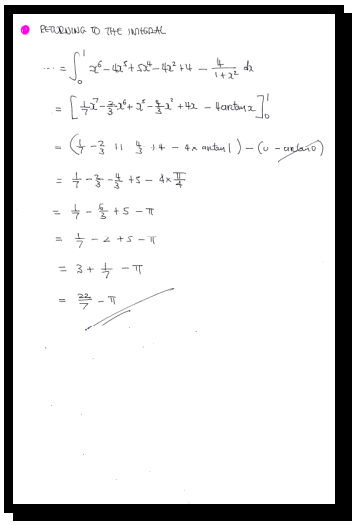
$$\int_0^1 \frac{x^4(1-x)^4}{x^2+1}\,dx\,.$$

You may assume that

$$\int \frac{1}{1+x^2}\,dx = \arctan x + \text{constant}\,.$$

$\square$, $\boxed{\dfrac{22}{7}-\pi}$





Question 277 **(*****)**

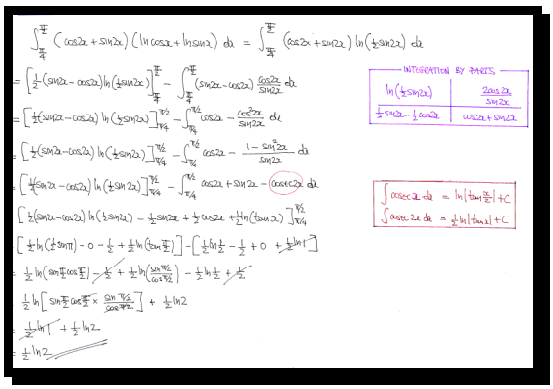
Use integration by parts to find a simplified exact value for

$$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \left(\cos 2x + \sin 2x\right)\left(\ln\cos x + \ln\sin x\right)\ dx .$$

You may assume that

$$\int \operatorname{cosec} x\ dx = \ln\left|\tan\left(\frac{1}{2}x\right)\right| + \text{constant} .$$

$\square$, $\boxed{\frac{1}{2}\ln 2}$



Question 278 **(*****)**

It is given that

$$x^2+x+2=(u-x)^2 .$$

a) Show clearly that …

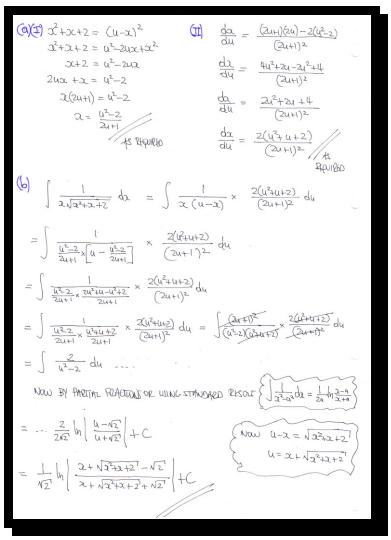
i. … $x=\dfrac{u^2-2}{2u+1}$.

ii. … $\dfrac{dx}{du}=\dfrac{2\left(u^2+u+2\right)}{(2u+1)^2}$.

b) Find a simplified expression for

$$\int \frac{1}{x\sqrt{x^2+x+2}}\,dx \ .$$

$$\boxed{}, \ \boxed{\frac{1}{\sqrt{2}}\ln\left|\frac{x+\sqrt{x^2+x+2}-\sqrt{2}}{x+\sqrt{x^2+x+2}+\sqrt{2}}\right|+C}$$



Question 279 (*****)

It is given that a and b are distinct real constants and λ is a real parameter.

a) Starting by the relationship between two functions of x, $f(x)$ and $g(x)$

$$\left[\lambda f(x)+g(x)\right]^2 \geq 0,$$

show clearly that

$$\lambda^2 \int_a^b \left[f(x)\right]^2 \, dx + 2\lambda \int_a^b f(x)\,g(x) \, dx + \int_a^b \left[g(x)\right]^2 \, dx \geq 0.$$

b) Deduce the Cauchy Schwarz inequality for integrals

$$\left[\int_a^b f(x)\,g(x) \, dx\right]^2 \leq \left[\int_a^b \left[f(x)\right]^2 \, dx\right]\left[\int_a^b \left[g(x)\right]^2 \, dx\right].$$

[continues overleaf]

[continued from overleaf]

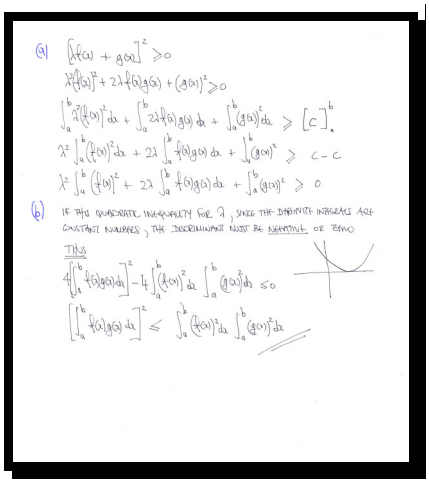
c) By letting $f(x)=\sqrt{\sin x}$ and $g(x)=1$, show that

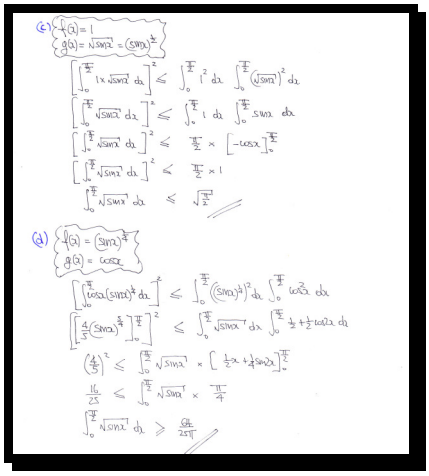
$$\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\ dx \leq \sqrt{\frac{\pi}{2}}\,.$$

d) By letting $f(x)=\sqrt{\sqrt{\sin x}}$ and $g(x)=\cos x$, show that

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\ dx \geq \frac{64}{25\pi}\,.$$

proof



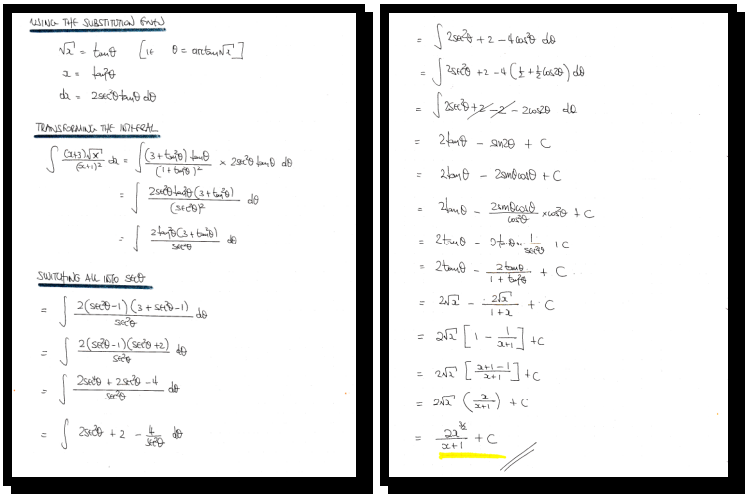


Question 280 (*****)

By using the substitution $\sqrt{x}=\tan\theta$, or otherwise, find

$$\int \frac{(x+3)\sqrt{x}}{(x+1)^2}\,dx.$$

$$\boxed{}\,,\quad \boxed{\frac{2x^{\frac{3}{2}}}{x+1}+C}$$

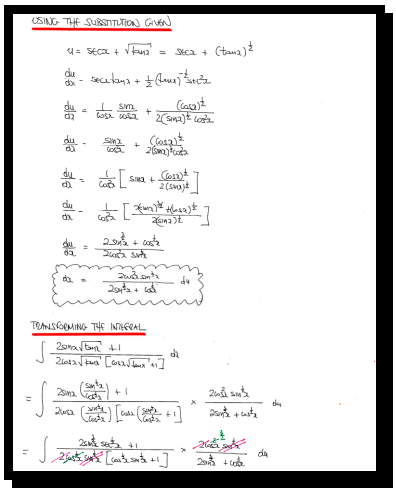


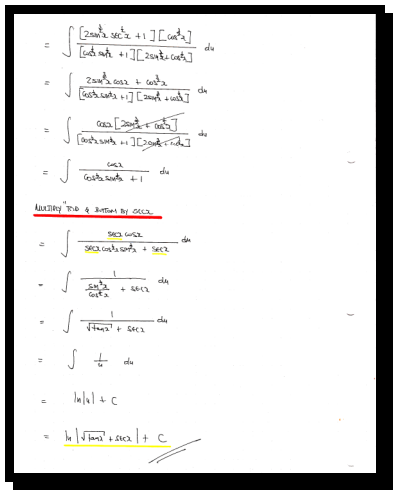
Question 281 **(*****)**

By using the substitution $u=\sec x+\sqrt{\tan x}$, or otherwise, find

$$\int \frac{1+2\sin x\sqrt{\tan x}}{2\left[1+\cos x\sqrt{\tan x}\right]\cos x\sqrt{\tan x}}\,dx\,.$$

$\boxed{}$, $\boxed{\ln\left|\sec x+\sqrt{\tan x}\right|+C}$





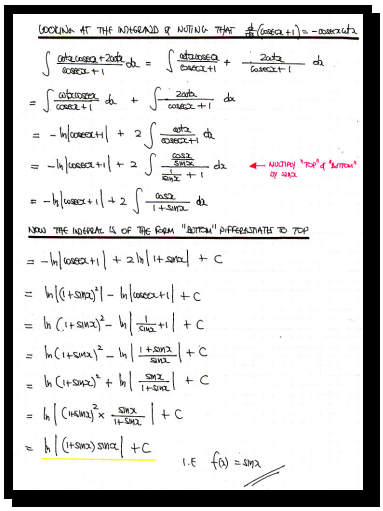
Question 282 **(*****)**

It is given that

$$\int \frac{\cot x \operatorname{cosec} x + 2\cot x}{1+\operatorname{cosec} x}\,dx \equiv \ln\left|\left[1+f(x)\right]f(x)\right| + \text{constant}.$$

Using integration techniques, determine an expression for $f(x)$.

$\square$, $\boxed{f(x)=\sin x}$



Question 283 **(*****)**

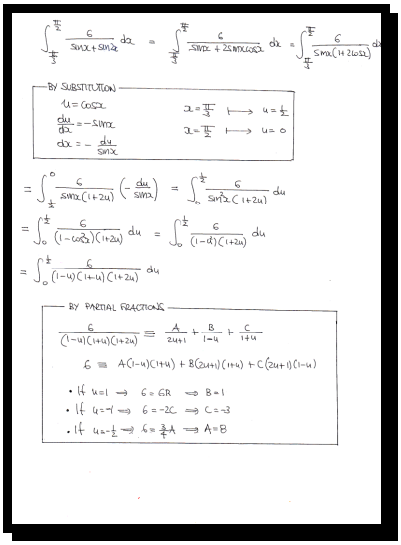
$$I=\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi}\frac{6}{\sin x+\sin 2x}\,dx$$

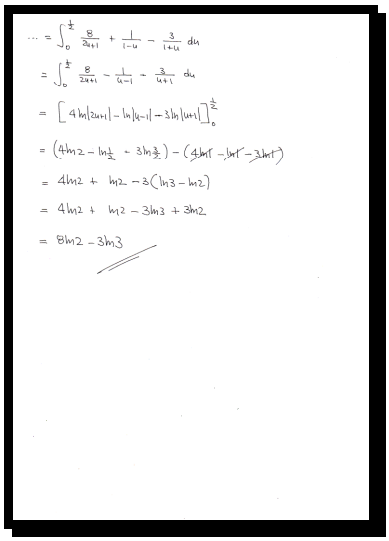
Use appropriate integration techniques to show that

$$I=A\ln N+B\ln M\ ,$$

where A, B, N and M are integers to be found.

$\boxed{}$, $\boxed{I=8\ln 2-3\ln 3}$



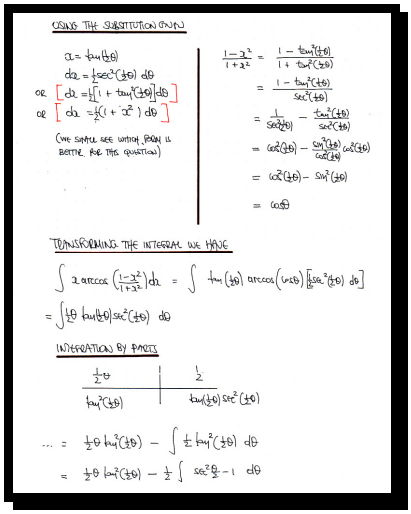


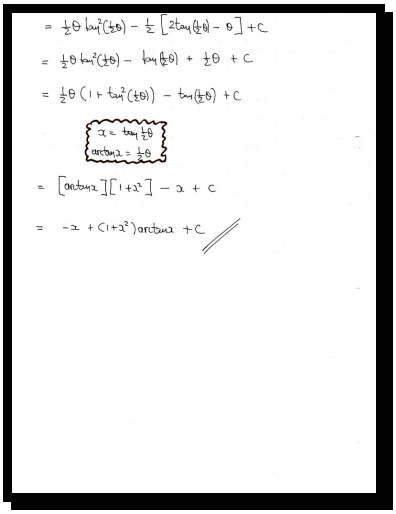
Question 284 **(*****)**

Use the substitution $x=\tan\left(\frac{1}{2}\theta\right)$, to find a simplified expression for

$$\int x\arccos\left[\frac{1-x^2}{1+x^2}\right]dx.$$

$\square$, $\boxed{-x+\left(1+x^2\right)\arctan x+\text{constant}}$



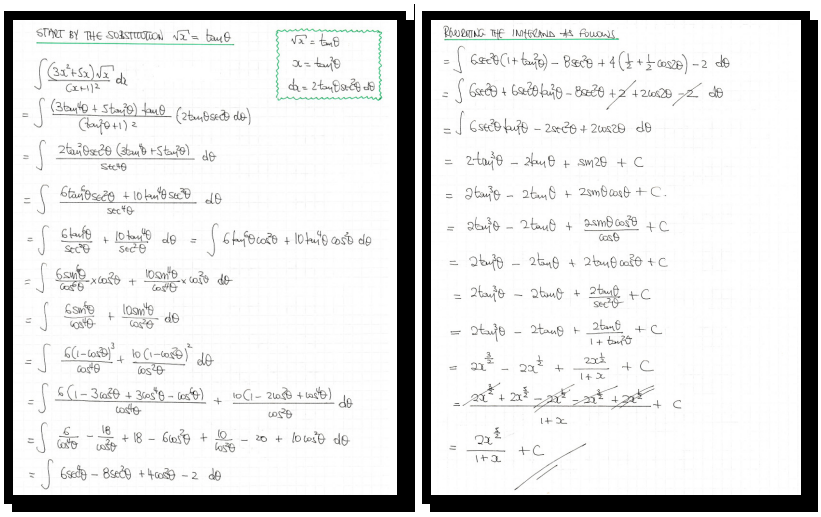


Question 285 **(*****)**

By using a suitable trigonometric substitution, or otherwise, find

$$\int \frac{\left(3x^2+5x\right)\sqrt{x}}{\left(x+1\right)^2}\,dx\,.$$

$\square$, $\boxed{\dfrac{2x^{\frac{5}{2}}}{x+1}+C}$



Question 286 (*****)

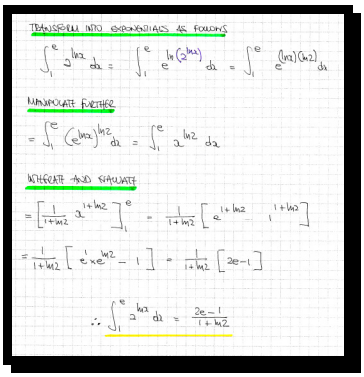
The function f is defined as

$$f(x) \equiv 2^{\ln x}, \quad x \in [1, \infty).$$

Show, with details workings, that

$$\int_1^{\mathrm{e}} f(x)\, dx = \frac{2\mathrm{e}-1}{1+\ln 2}.$$

[ ], [proof]



Question 287 (*****)

A function F is defined by the integral

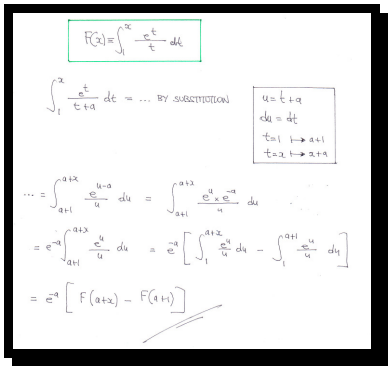
$$F(x) \equiv \int_1^x \frac{\mathrm{e}^t}{t}\, dt\,,\ x \geq 1.$$

Find a simplified expression, in terms of F, for

$$\int_1^x \frac{\mathrm{e}^t}{t+a}\, dt\,,$$

where a is a positive constant.

$\square$, $\displaystyle\int_1^x \frac{\mathrm{e}^t}{t+a}\, dt = \mathrm{e}^{-a}\left[F(x+a)-F(a+1)\right]$



Question 288 **(*****)**

It is given that

$$u^2=\frac{1-x^2}{(1-x)^2},\ x\neq\pm1.$$

a) Show clearly that …

i. … $x=\dfrac{u^2-1}{u^2+1}$.

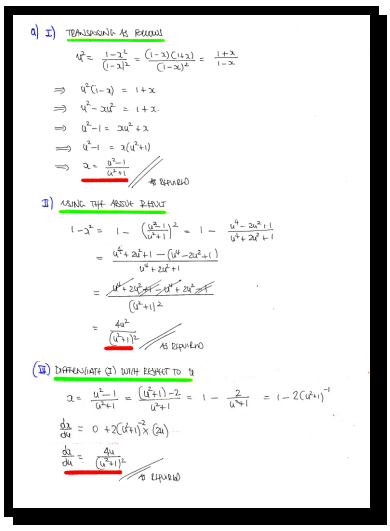
ii. … $1-x^2=\dfrac{4u^2}{\left(u^2+1\right)^2}$

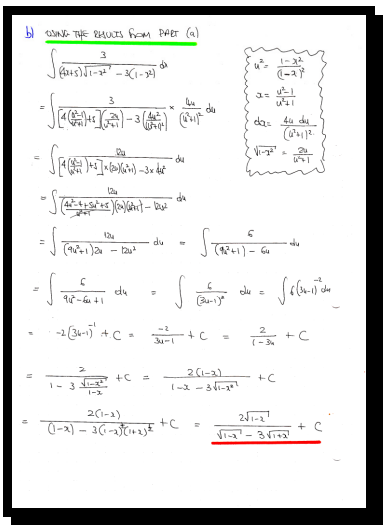
iii. … $\dfrac{dx}{du}=\dfrac{4u}{\left(u^2+1\right)^2}$.

b) Hence show further that

$$\int\frac{3}{(4x+5)\sqrt{1-x^2}-3\left(1-x^2\right)}\,dx \quad = \quad \frac{2\sqrt{1-x}}{\sqrt{1-x}-3\sqrt{1+x}}+\text{constant}.$$

, proof



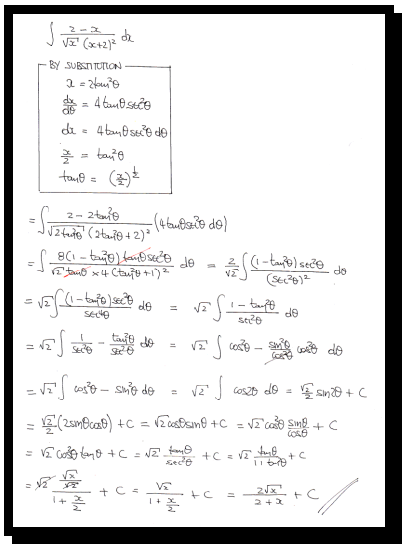


Question 289 (*****)

By using the substitution $x = 2\tan^2\theta$, or otherwise, find

$$\int \frac{2-x}{\sqrt{x}(x+2)^2}\,dx\,.$$

$\square$, $\boxed{\dfrac{2\sqrt{x}}{x+2}+C}$



Question 290 **(*****)**

It is given that

$$\sqrt{5-4x-x^2}=(1-x)u\,,\ x\neq 1,\ x\neq -5\,.$$

a) Show clearly that …

i. … $x=\dfrac{u^2-5}{u^2+1}$.

ii. … $dx=\dfrac{12u}{\left(u^2+1\right)^2}du$.

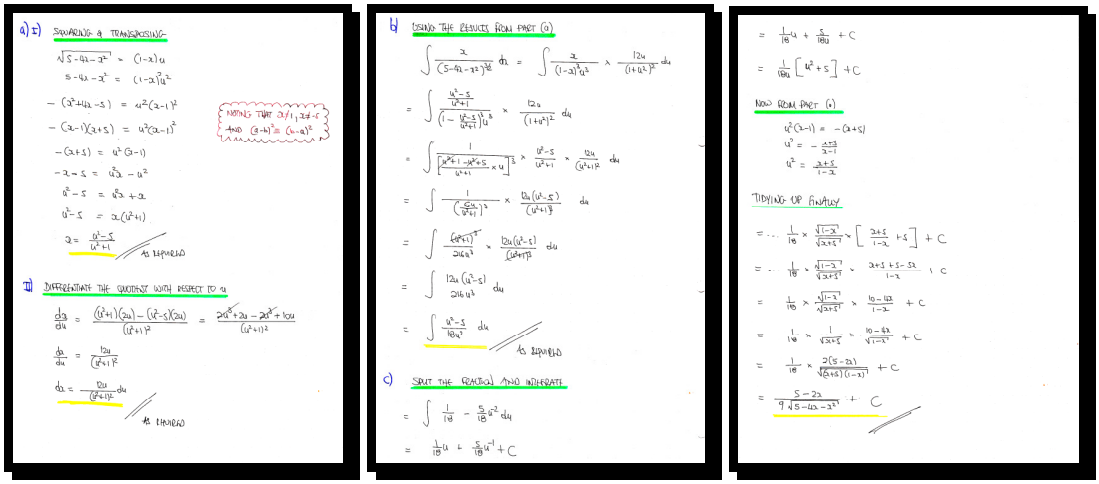
b) Hence show further that

$$\int\frac{x}{\left(5-4x-x^2\right)^{\frac{3}{2}}}dx=\int\frac{u^2-5}{18u^2}du\,.$$

c) Find a simplified expression for

$$\int\frac{x}{\left(5-4x-x^2\right)^{\frac{3}{2}}}dx\ .$$

$\boxed{}$, $\boxed{\dfrac{5-2x}{9\sqrt{5-4x-x^2}}+C}$

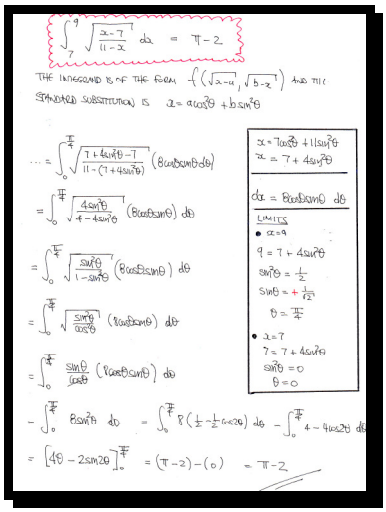


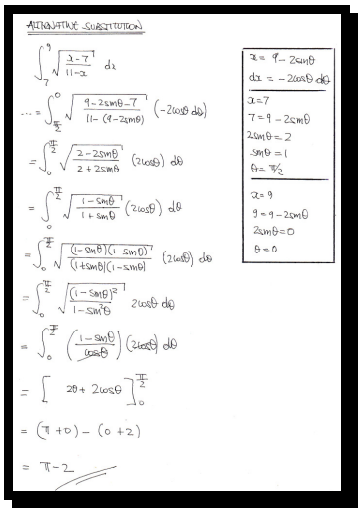
Question 291 **(*****)**

By using an appropriate trigonometric substitution, or otherwise, find an exact value for the following integral.

$$\int_7^9 \sqrt{\frac{x-7}{11-x}}\, dx\,.$$

$\boxed{}$, $\boxed{\pi-2}$





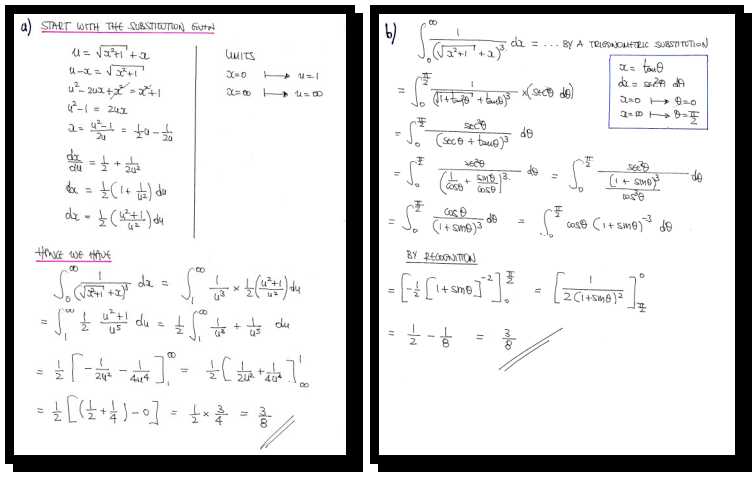
Question 292 **(*****)**

$$I=\int_0^{\infty}\frac{1}{\left(x+\sqrt{x^2+1}\right)^2}\,dx\,.$$

a) Use the substitution $u=x+\sqrt{x^2+1}$ to find the value of I.

b) Verify the answer to part **(a)** by a trigonometric substitution.

$\boxed{}$, $\boxed{\frac{3}{8}}$

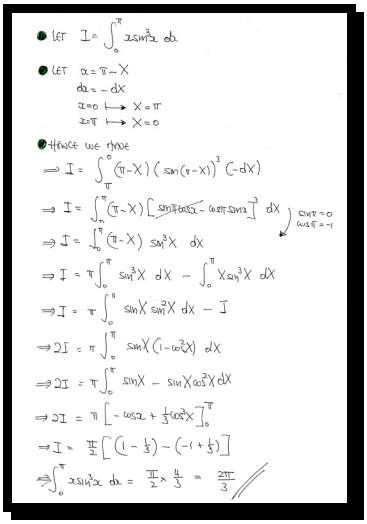


Question 293 (*****)

Find an exact value for the following integral.

$$\int_0^{\pi} x\sin^3 x \ dx .$$

$\square$, $\boxed{\dfrac{2\pi}{3}}$

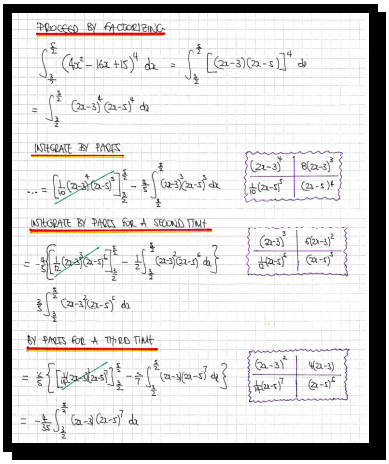


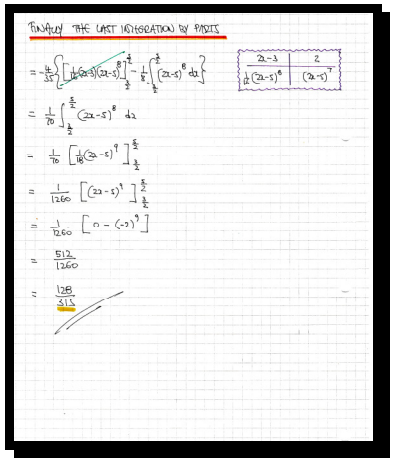
Question 294 (*****)

Determine, as an exact simplified fraction, the value of the following integral.

$$\int_{\frac{3}{2}}^{\frac{5}{2}} \left(4x^2-16x+15\right)^4 \, dx \, .$$

$\square$, $\dfrac{128}{315}$





Question 295 (*****)

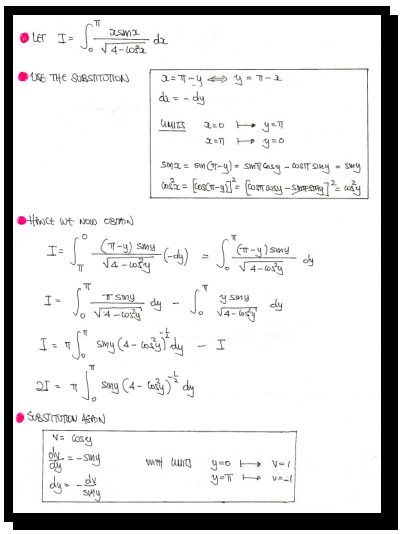
Find an exact value for

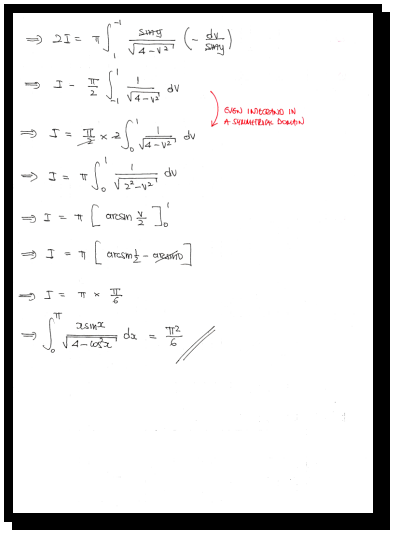
$$\int_0^{\pi} \frac{x\sin x}{\sqrt{4-\cos^2 x}}\,dx\,.$$

You my assume without proof that

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx \;=\; \arcsin\left(\frac{x}{a}\right)+\text{constant}\,.$$

$\square$, $\boxed{\dfrac{\pi^2}{6}}$





Question 296 (*****)

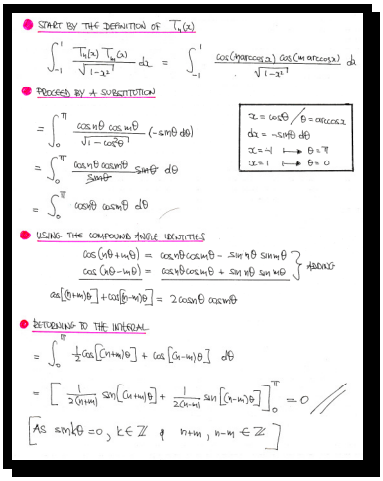
A family of functions, known as the Chebyshev polynomials of the first kind $T_n(x)$, is defined as

$$T_n(x)=\cos\left(n\arccos x\right),\ -1\le x\le 1,\ n\in\mathbb{N}.$$

Evaluate the following integral

$$\int_{-1}^{1}\frac{T_n(x)\,T_m(x)}{\sqrt{1-x^2}}\,dx.$$

$\square$, $\boxed{0}$



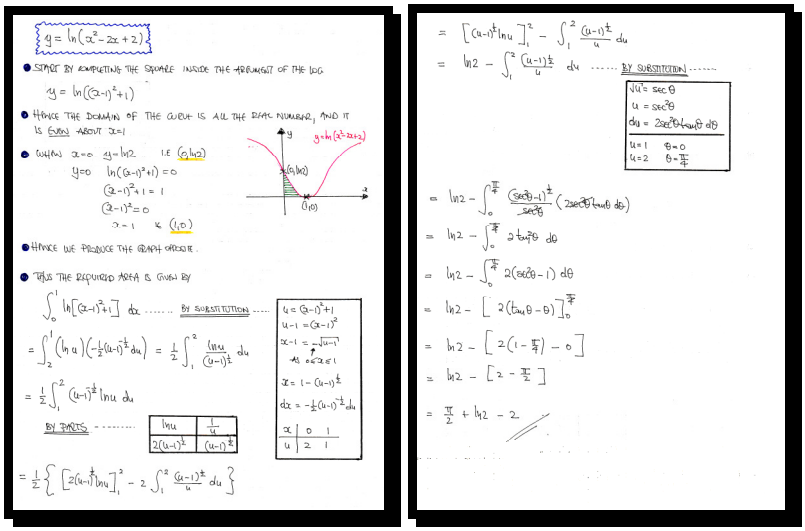
Question 297 (*****)

The function $y=f\left(x\right)$ is defined in the largest possible real domain by

$$f\left(x\right)\equiv\ln\left[x^2-2x+2\right].$$

Sketch the graph of $f\left(x\right)$ and determine an exact simplified value for the area of the finite region bounded by the graph of $f\left(x\right)$ and the coordinate axes.

$\square$, $\boxed{\frac{1}{2}\pi-2+\ln 2}$



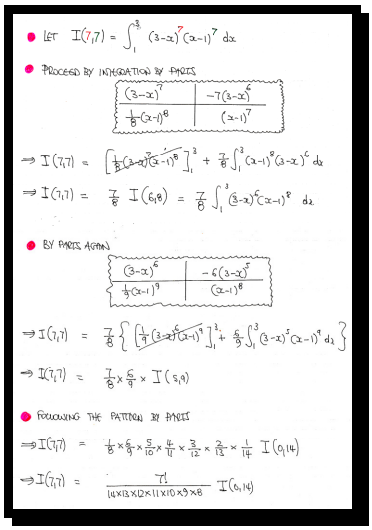
Question 298 (*****)

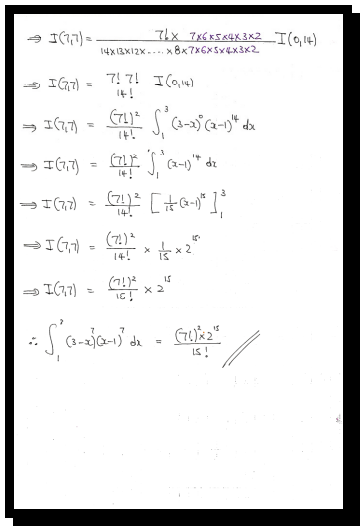
$$I=\int_1^3 (3-x)^7(x-1)^7\ dx\,.$$

Show that

$$I=\frac{(7!)^2\times 2^{15}}{15!}\,.$$

[ ], proof



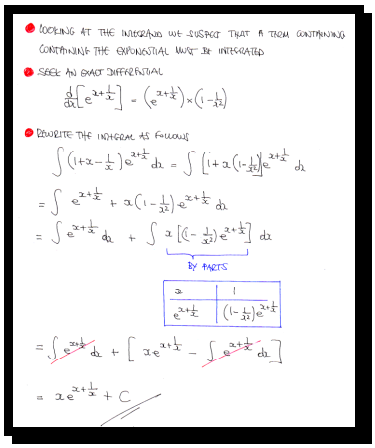


Question 299 **(*****)**

Use integration by parts to find a simplified expression for

$$\int \left(1+x-\frac{1}{x}\right)\mathrm{e}^{x+\frac{1}{x}}\,dx\,.$$

$\boxed{}$, $\boxed{x\mathrm{e}^{x+\frac{1}{x}}+C}$

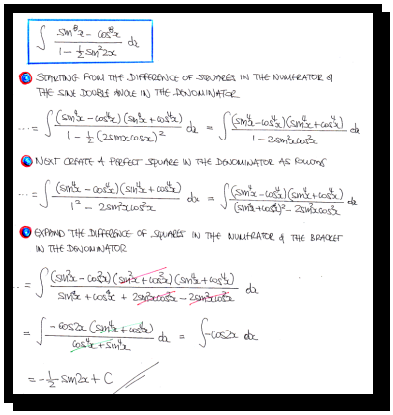


Question 300 **(*****)**

Use trigonometric identities to find a simplified expression for

$$\int \frac{\sin^8 x-\cos^8 x}{1-\frac{1}{2}\sin^2 2x}\,dx\,.$$

$\boxed{}$, $\boxed{-\frac{1}{2}\sin 2x+C}$

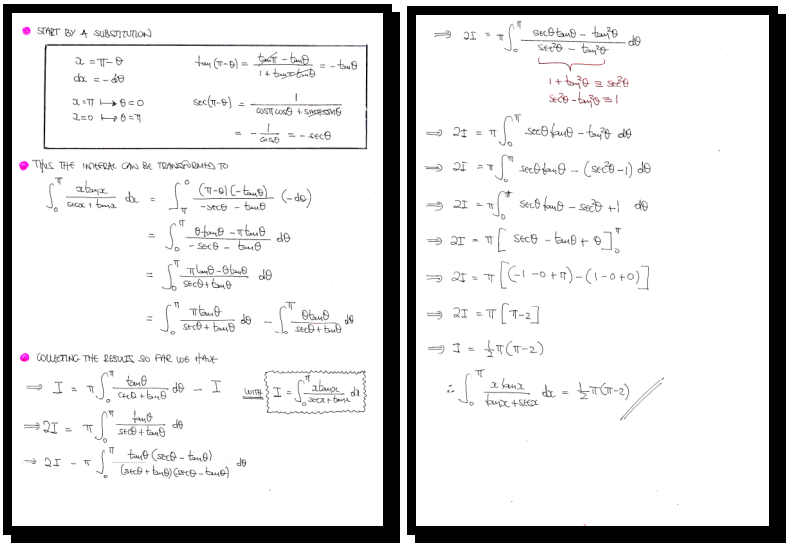


Question 301 **(*****)**

By using an appropriate substitution followed by trigonometric identities, show that

$$\int_0^{\pi} \frac{x\tan x}{\tan x+\sec x}\,dx = \frac{1}{2}\pi(\pi-2).$$

, proof

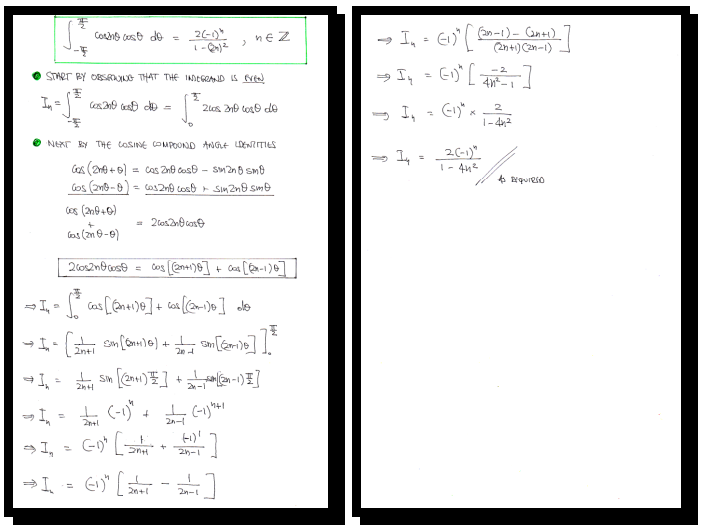


Question 302 (*****)

Show that if n is an integer, then

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos(2n\theta)\cos\theta \ d\theta = \frac{2(-1)^n}{1-(2n)^2}.$$

, proof

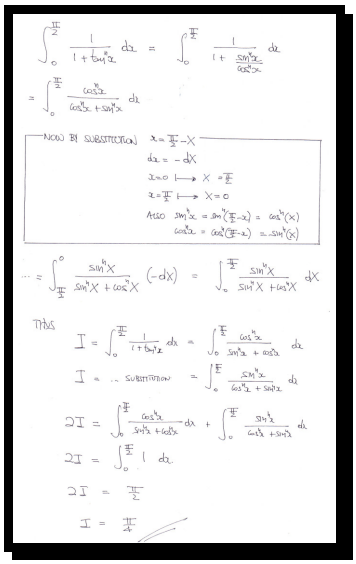


Question 303 (*****)

$$\int_0^{\frac{1}{2}\pi} \frac{1}{1+\tan^n x}\, dx \quad , \ n \in \mathbb{Q} .$$

Find the value of the above integral, for all values of n

$$\boxed{\frac{\pi}{4}}$$

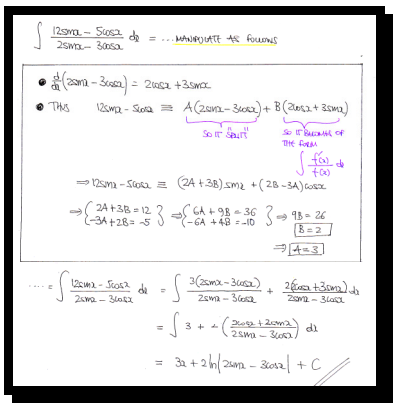


Question 304 (*****)

By suitably rewriting the numerator of the integrand, find a simplified expression for the following integral.

$$\int \frac{12\sin x-5\cos x}{2\sin x-3\cos x}\,dx\,.$$

$\boxed{}$, $\boxed{3x+2\ln\left|2\sin x-3\cos x\right|+C}$

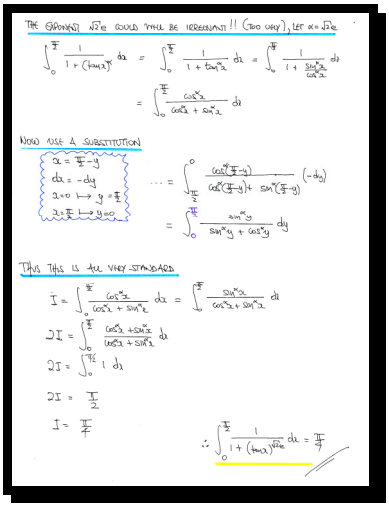


Question 305 (*****)

Find an exact value for the following integral

$$\int_0^{\frac{1}{2}\pi} \frac{1}{1+(\tan x)^{\sqrt{2}\,\mathrm{e}}}\,dx\,.$$

$\square$, $\boxed{\frac{1}{4}\pi}$



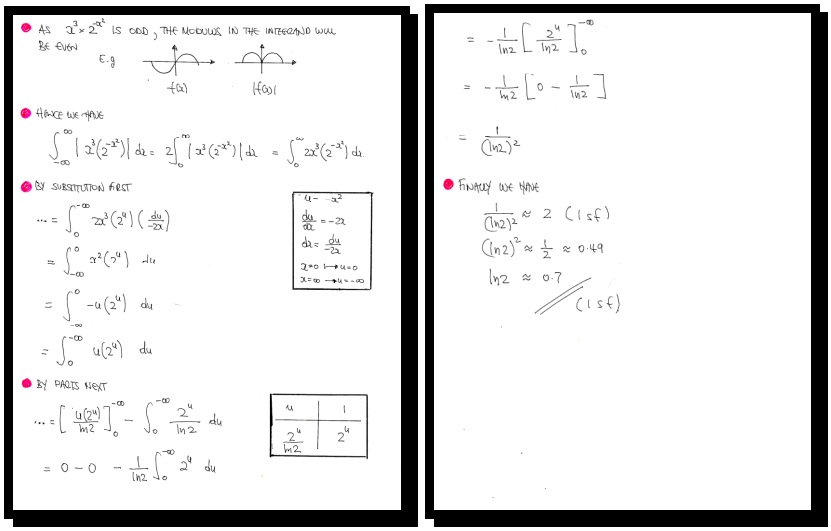
Question 306 (*****)

$$I=\int_{-\infty}^{\infty}\left|x^3\left(2^{-x^2}\right)\right|\,dx$$

It is given that $I\approx 2$.

Use this fact to estimate the value of $\ln 2$ correct to 1 significant figure.

$\boxed{}$, $\boxed{\ln 2\approx 0.7}$



Question 307 **(*****)**

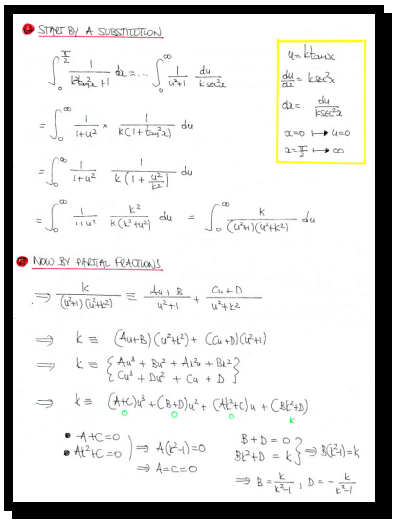
The definite integral I is defined in terms of the constant k, where $k \neq 0$, $k \neq \pm 1$.

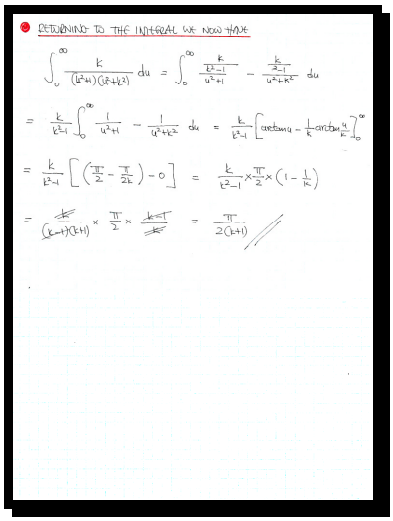
$$I = \int_0^{\frac{1}{2}\pi} \frac{1}{1+k^2 \tan^2 x} \, dx .$$

Use appropriate integration techniques to show that

$$I = \frac{\pi}{2(k+1)} .$$

[ ], [proof]



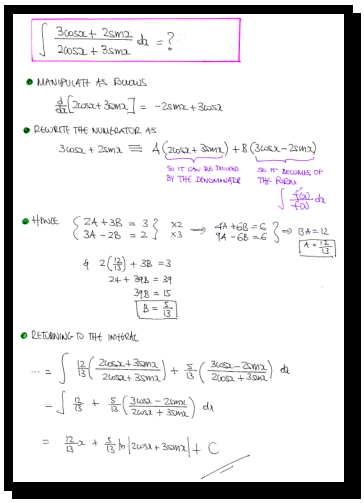


Question 308 (*****)

By suitably rewriting the numerator of the integrand, find a simplified expression for the following integral.

$$\int \frac{3\cos x+2\sin x}{2\cos x+3\sin x}\,dx\,.$$

$\square$, $\boxed{\frac{12}{13}x+\frac{5}{13}\ln\left|2\cos x+3\sin x\right|+C}$

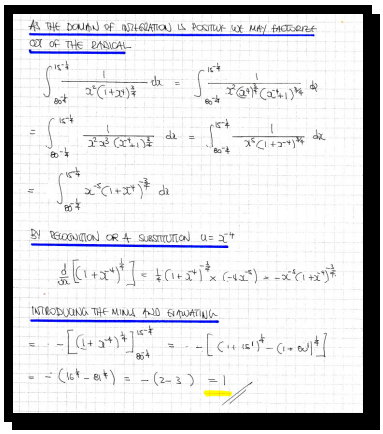


Question 309 **(*****)**

Find the value of the following definite integral.

$$\int_{80^{-\frac{1}{4}}}^{15^{-\frac{1}{4}}} \frac{1}{x^2\left(1+x^4\right)^{\frac{3}{4}}}\,dx\ .$$

$\square$, $\boxed{1}$



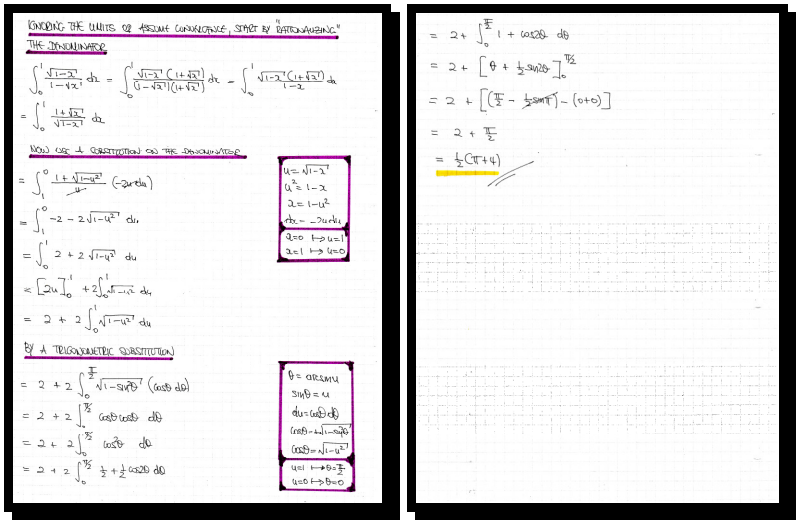
Question 310 (*****)

Find in exact simplified form the value of

$$\int_0^1 \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx\,.$$

You may assume that the integral converges.

V, □, $\boxed{\frac{1}{2}(\pi+4)}$

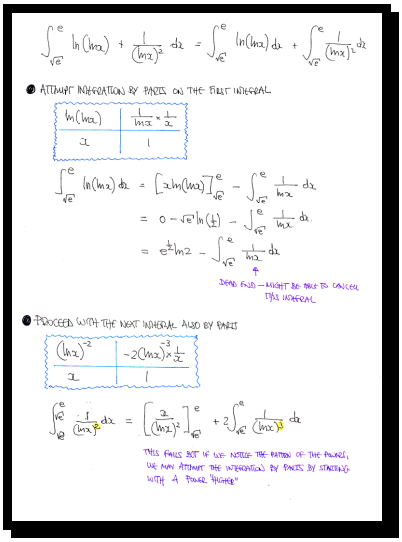


Question 311 (*****)

Find an exact simplified value for

$$\int_{\sqrt{e}}^{e} \ln\left(\ln x\right)+\frac{1}{\left(\ln x\right)^2}\ dx\,.$$

$\square$, $\boxed{e^{\frac{1}{2}}\left(2+\ln 2\right)-e}$



$$\int_{\sqrt{e}}^{e} \ln(\ln x)+\frac{1}{(\ln x)^2}\,dx = \int_{\sqrt{e}}^{e} \ln(\ln x)\,dx + \int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx$$

ATTEMPT INTEGRATION BY PARTS ON THE FIRST INTEGRAL

$\ln(\ln x)$	$\frac{1}{\ln x}\times\frac{1}{x}$
x	1

$$\int_{\sqrt{e}}^{e} \ln(\ln x)\,dx = \left[x\ln(\ln x)\right]_{\sqrt{e}}^{e} - \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx$$
$$= 0 - \sqrt{e}\ln\left(\tfrac{1}{2}\right) - \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx$$
$$= e^{\frac{1}{2}}\ln 2 - \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx$$

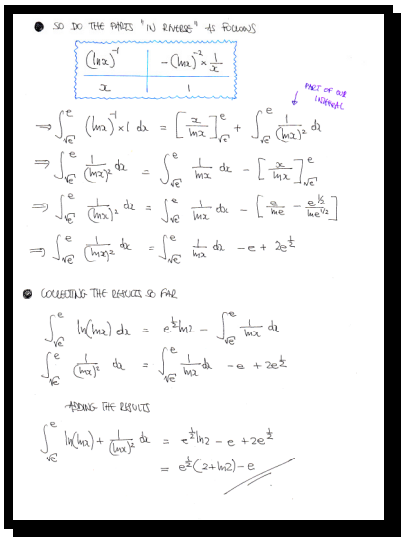
DOING EVEN — MIGHT BE ABLE TO CANCEL THIS INTEGRAL

PROCEED WITH THE NEXT INTEGRAL ALSO BY PARTS

$(\ln x)^{-2}$	$-2(\ln x)^{-3}\times\frac{1}{x}$
x	1

$$\int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx = \left[\frac{x}{(\ln x)^2}\right]_{\sqrt{e}}^{e} + 2\int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^3}\,dx$$

THIS FAILS BUT IF WE NOTICE THE PATTERN OF THE POWERS, WE NOW ATTEMPT THE INTEGRATION BY PARTS BY STARTING WITH A POWER "HIGHER"



SO DO THE PARTS "IN REVERSE" AS FOLLOWS

$(\ln x)^{-1}$	$-(\ln x)^{-2}\times\frac{1}{x}$
x	1

PART OF OUR INTEGRAL

$$\Rightarrow \int_{\sqrt{e}}^{e} (\ln x)^{-1}\times 1\,dx = \left[\frac{x}{\ln x}\right]_{\sqrt{e}}^{e} + \int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx$$
$$\Rightarrow \int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx = \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx - \left[\frac{x}{\ln x}\right]_{\sqrt{e}}^{e}$$
$$\Rightarrow \int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx = \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx - \left[\frac{e}{\ln e} - \frac{e^{\frac{1}{2}}}{\ln e^{\frac{1}{2}}}\right]$$
$$\Rightarrow \int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx = \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx - e + 2e^{\frac{1}{2}}$$

COLLECTING THE RESULTS SO FAR

$$\int_{\sqrt{e}}^{e} \ln(\ln x)\,dx = e^{\frac{1}{2}}\ln 2 - \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx$$
$$\int_{\sqrt{e}}^{e} \frac{1}{(\ln x)^2}\,dx = \int_{\sqrt{e}}^{e} \frac{1}{\ln x}\,dx - e + 2e^{\frac{1}{2}}$$

ADDING THE RESULTS

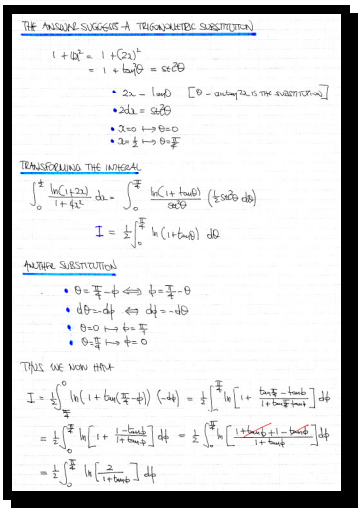
$$\int_{\sqrt{e}}^{e} \ln(\ln x)+\frac{1}{(\ln x)^2}\,dx = e^{\frac{1}{2}}\ln 2 - e + 2e^{\frac{1}{2}}$$
$$= e^{\frac{1}{2}}(2+\ln 2) - e$$

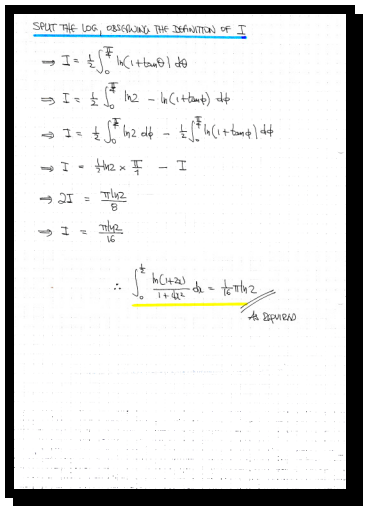
Question 312 (*****)

By using appropriate substitutions, or otherwise, show that

$$\int_0^{\frac{1}{2}} \frac{\ln\left(1+2x\right)}{1+4x^2}\,dx = \frac{\pi \ln 2}{16}.$$

[ ], [proof]



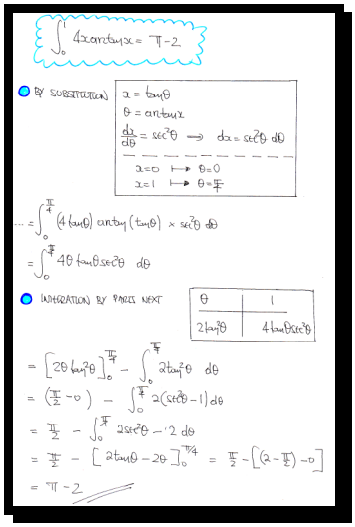


Question 313 (*****)

Use appropriate integration techniques to show that

$$\int_0^1 4x\arctan x \ dx = \pi - 2 .$$

[ ], [proof]



Question 314 **(*****)**

$$I = \int_0^{\frac{1}{2}} \frac{\ln(1+2x)}{1+4x^2}\, dx\,.$$

a) Use an appropriate trigonometric substitution to show that

$$I = \int_0^{\frac{\pi}{4}} \frac{1}{2}\ln\sqrt{2} + \frac{1}{2}\ln\left[\frac{\cos\left(\theta-\frac{1}{4}\pi\right)}{\cos\theta}\right] d\theta\,.$$

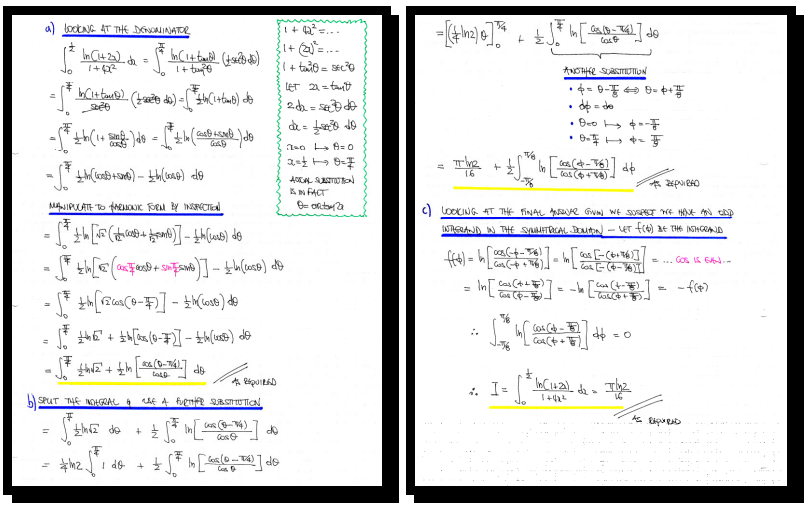
b) Show further that

$$I = \frac{\pi\ln 2}{16} + \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{1}{2}\ln\left[\frac{\cos\left(\varphi-\frac{1}{8}\pi\right)}{\cos\left(\varphi+\frac{1}{8}\pi\right)}\right] d\varphi\,.$$

c) Deduce that

$$I = \frac{\pi\ln 2}{16}\,.$$

V, [ ], proof



Question 315 (*****)

$$I = \int \frac{2x+1}{\sqrt{x-1}}\, dx\,.$$

a) Show that

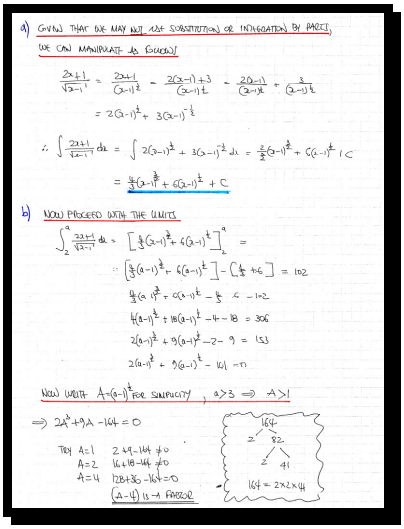
$$I = \frac{4}{3}(x-1)^{\frac{3}{2}} + 6(x-1)^{\frac{1}{2}} + \text{constant}\,.$$

You may not use any substitution or integration by parts.

b) Determine the value of a, given that

$$\int_2^a \frac{2x+1}{\sqrt{x-1}}\, dx = 102\,.$$

$\boxed{}$, $\boxed{a=17}$


a) GIVEN THAT WE MAY NOT USE SUBSTITUTION OR INTEGRATION BY PARTS, WE CAN MANIPULATE AS FOLLOWS:

$\frac{2x+1}{\sqrt{x-1}} = \frac{2x+1}{(x-1)^{\frac{1}{2}}} = \frac{2(x-1)+3}{(x-1)^{\frac{1}{2}}} = \frac{2(x-1)}{(x-1)^{\frac{1}{2}}} + \frac{3}{(x-1)^{\frac{1}{2}}}$

$= 2(x-1)^{\frac{1}{2}} + 3(x-1)^{-\frac{1}{2}}$

$\therefore \int \frac{2x+1}{\sqrt{x-1}}\, dx = \int 2(x-1)^{\frac{1}{2}} + 3(x-1)^{-\frac{1}{2}}\, dx = \frac{2}{3}(x-1)^{\frac{3}{2}} + 6(x-1)^{\frac{1}{2}} + C$

$= \frac{4}{3}(x-1)^{\frac{3}{2}} + 6(x-1)^{\frac{1}{2}} + C$

b) NOW PROCEED WITH THE LIMITS

$\int_2^a \frac{2x+1}{\sqrt{x-1}}\, dx = \left[\frac{4}{3}(x-1)^{\frac{3}{2}} + 6(x-1)^{\frac{1}{2}}\right]_2^a =$

$\therefore \left[\frac{4}{3}(a-1)^{\frac{3}{2}} + 6(a-1)^{\frac{1}{2}}\right] - \left[\frac{4}{3}+6\right] = 102$

$\frac{4}{3}(a-1)^{\frac{3}{2}} + 6(a-1)^{\frac{1}{2}} - \frac{4}{3} - 6 = 102$

$4(a-1)^{\frac{3}{2}} + 18(a-1)^{\frac{1}{2}} - 4 - 18 = 306$

$2(a-1)^{\frac{3}{2}} + 9(a-1)^{\frac{1}{2}} - 2 - 9 = 153$

$2(a-1)^{\frac{3}{2}} + 9(a-1)^{\frac{1}{2}} - 164 = 0$

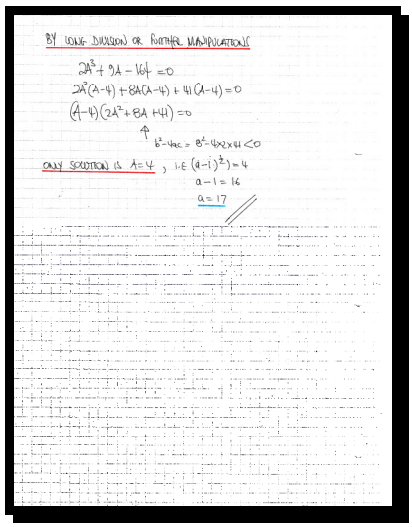
NOW WRITE $A = (a-1)^{\frac{1}{2}}$ FOR SIMPLICITY, $a > 3 \Rightarrow A > 1$

$\Rightarrow 2A^3 + 9A - 164 = 0$

TRY $A=1$ $2+9-164 \neq 0$
$A=2$ $16+18-164 \neq 0$
$A=4$ $128+36-164=0$

$(A-4)$ IS A FACTOR

164 = 2 × 82 = 2 × 2 × 41 = 2 × 2 × 41


BY LONG DIVISION OR FURTHER MANIPULATION:

$2A^3 + 9A - 164 = 0$
$2A^2(A-4) + 8A(A-4) + 41(A-4) = 0$
$(A-4)(2A^2 + 8A + 41) = 0$

$b^2 - 4ac = 8^2 - 4 \times 2 \times 41 < 0$

ONLY SOLUTION IS $A=4$, i.e. $(a-1)^{\frac{1}{2}} = 4$

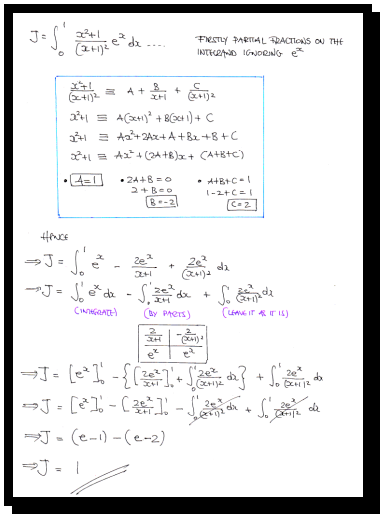
$a - 1 = 16$
$a = 17$

Question 316 (*****)

$$J=\int_0^1 \frac{\left(x^2+1\right)\mathrm{e}^x}{\left(x+1\right)^2}\,dx\,.$$

Show that $J=1$

[ ] , [proof]

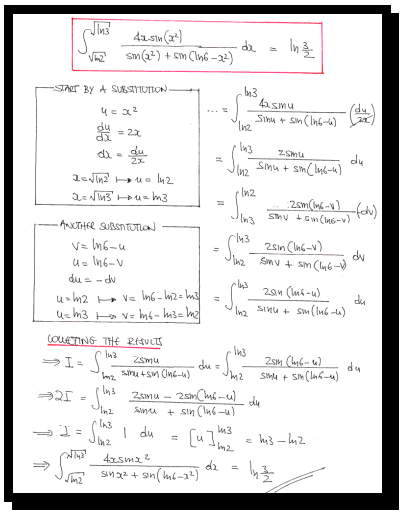


Question 317 (*****)

By using an appropriate substitution or substitutions, show that

$$\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{4x\sin\left(x^2\right)}{\sin\left(x^2\right)+\sin\left(\ln 6-x^2\right)}\,dx = \ln\left(\frac{3}{2}\right).$$

[ ] , [proof]



Question 318 (*****)

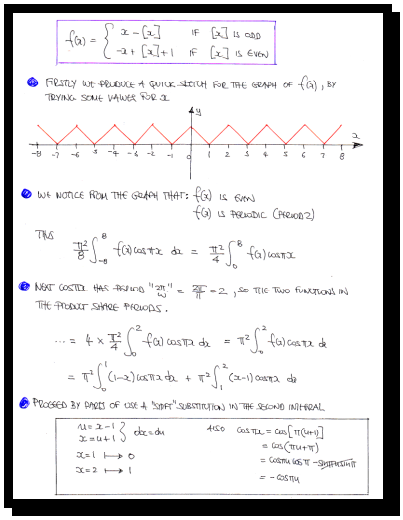
$$f(x)=\begin{cases} x-[x] & x\in\mathbb{R},\ [x]=2k+1,\ k\in\mathbb{Z} \\ -x+[x]+1 & x\in\mathbb{R}\ \ [x]=2k,\ \ k\in\mathbb{Z} \end{cases},$$

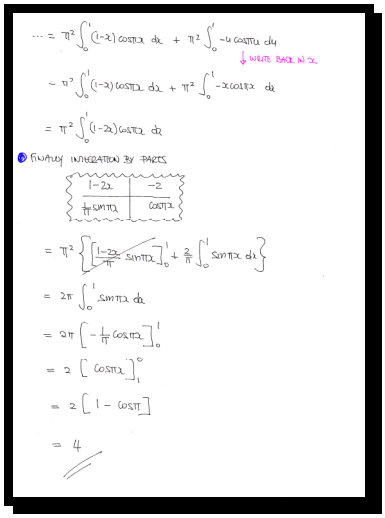
where $[x]$ is defined as the greatest integer less or equal to x.

Find the value of

$$\frac{\pi^2}{8}\int_{-8}^{8} f(x)\cos(\pi x)\,dx.$$

$\square$, 4



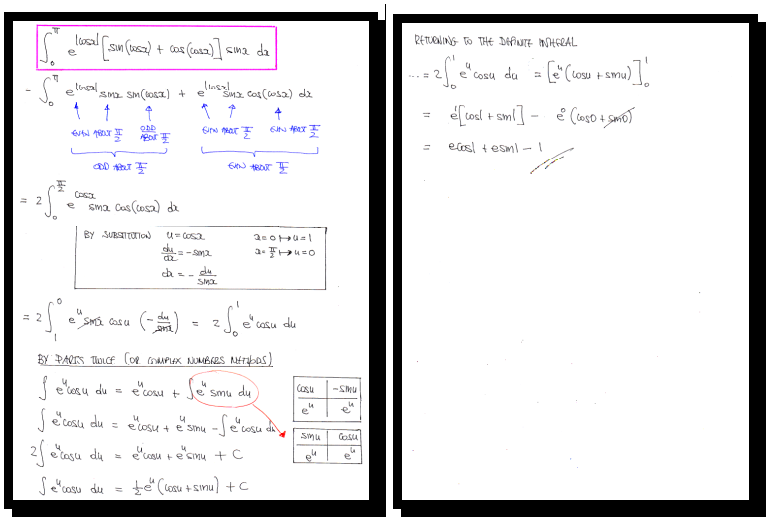


Question 319 (*****)

By using symmetry arguments, find the exact value of the following integral

$$\int_0^{\pi} e^{|\cos x|}\left[\sin\left(\cos x\right)+\cos\left(\cos x\right)\right]\sin x \ dx \ .$$

$\boxed{}$, $\boxed{e\left(\cos 1+\sin 1\right)-1}$



$$\int_0^{\pi} e^{|\cos x|}\left[\sin(\cos x)+\cos(\cos x)\right]\sin x\ dx$$

$$= \int_0^{\pi} e^{|\cos x|}\sin x\sin(\cos x) + e^{|\cos x|}\sin x\cos(\cos x)\ dx$$

ODD ABOUT $\frac{\pi}{2}$ (SIN ABOUT $\frac{\pi}{2}$ × ODD ABOUT $\frac{\pi}{2}$); EVEN ABOUT $\frac{\pi}{2}$ (SIN ABOUT $\frac{\pi}{2}$ × SIN ABOUT $\frac{\pi}{2}$)

$$= 2\int_0^{\frac{\pi}{2}} e^{\cos x}\sin x\cos(\cos x)\ dx$$

BY SUBSTITUTION: $u=\cos x$, $\frac{du}{dx}=-\sin x$, $dx=-\frac{du}{\sin x}$; $x=0\mapsto u=1$, $x=\frac{\pi}{2}\mapsto u=0$

$$= 2\int_1^0 e^{u}\sin x\cos u\left(-\frac{du}{\sin x}\right) = 2\int_0^1 e^{u}\cos u\ du$$

BY PARTS TWICE (OR COMPLEX NUMBERS METHODS)

$$\int e^{u}\cos u\ du = e^{u}\cos u + \int e^{u}\sin u\ du$$

$\cos u$	$-\sin u$
e^{u}	e^{u}

$$\int e^{u}\cos u\ du = e^{u}\cos u + e^{u}\sin u - \int e^{u}\cos u\ du$$

$\sin u$	$\cos u$
e^{u}	e^{u}

$$2\int e^{u}\cos u\ du = e^{u}\cos u + e^{u}\sin u + C$$

$$\int e^{u}\cos u\ du = \frac{1}{2}e^{u}\left(\cos u+\sin u\right)+C$$

RETURNING TO THE DEFINITE INTEGRAL

$$\ldots = 2\int_0^1 e^{u}\cos u\ du = \left[e^{u}\left(\cos u+\sin u\right)\right]_0^1$$

$$= e\left[\cos 1+\sin 1\right] - e^{0}\left(\cos 0+\sin 0\right)$$

$$= e\cos 1 + e\sin 1 - 1$$

Question 320 (*****)

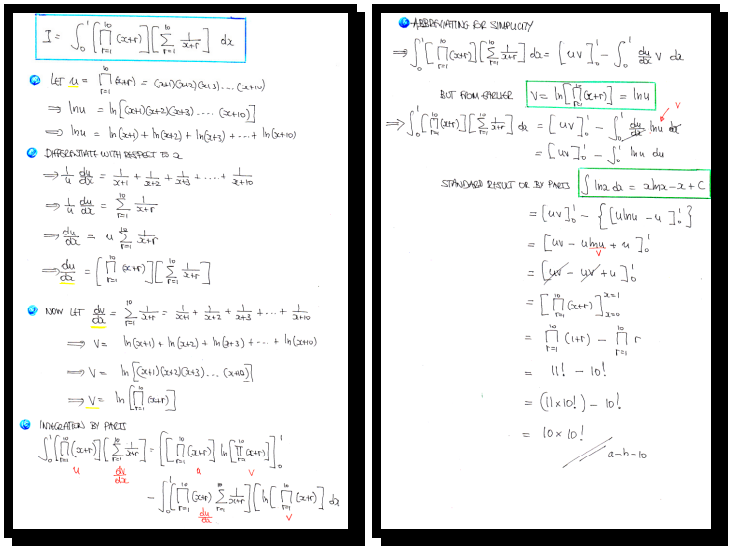
$$I=\int_0^1 \left[\prod_{r=1}^{10}(x+r)\right]\left[\sum_{r=1}^{10}\left(\frac{1}{x+r}\right)\right]dx\,.$$

Show by a detailed method that

$$I=a\times b!,$$

where a and b are positive integers to be found.

$\boxed{}$, $\boxed{a=b=10}$



Question 321 **(*****)**

$$I=\int \sqrt{\tan x}\ dx\,.$$

a) Use a suitable substitution to show that

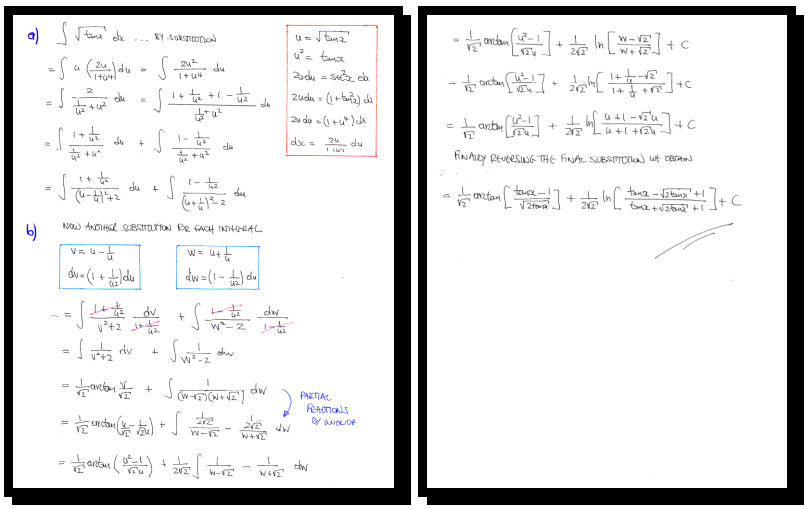
$$I=\int \frac{1+\dfrac{1}{u^2}}{\left(u-\dfrac{1}{u}\right)^2+2}\,dx+I=\int \frac{1-\dfrac{1}{u^2}}{\left(u+\dfrac{1}{u}\right)^2-2}\,dx\,.$$

b) By using a further substitution in each of the integrals of part **(a)** find a simplified expression for I, in terms of x.

You may assume without proof that

$$\int \frac{1}{x^2+a^2}\,dx=\frac{1}{a}\arctan\left[\frac{x}{a}\right]+\text{constant}\,.$$

$\square$, $\boxed{\dfrac{1}{\sqrt{2}}\arctan\left[\dfrac{\tan x-1}{\sqrt{2\tan x}}\right]+\dfrac{1}{2\sqrt{2}}\ln\left[\dfrac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right]+C}$

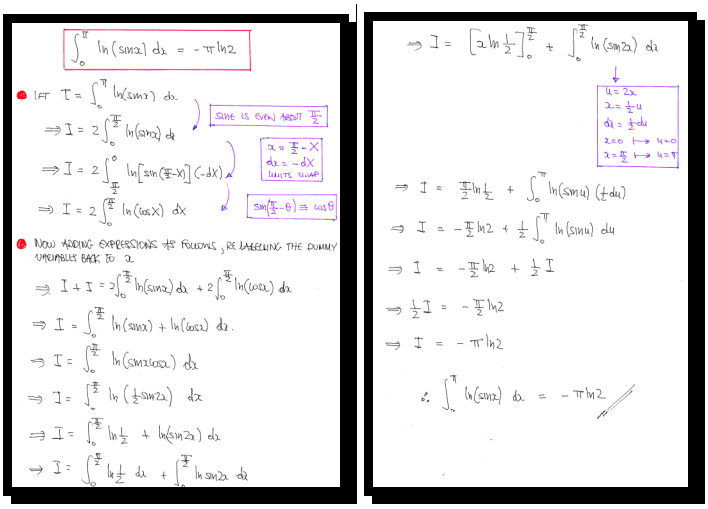


Question 322 (*****)

By using an appropriate substitution or substitutions, show that

$$\int_0^{\pi} \ln\left(\sin x\right)\, dx = -\pi \ln 2 .$$

[ ], proof



Question 323 **(*****)**

It is given that

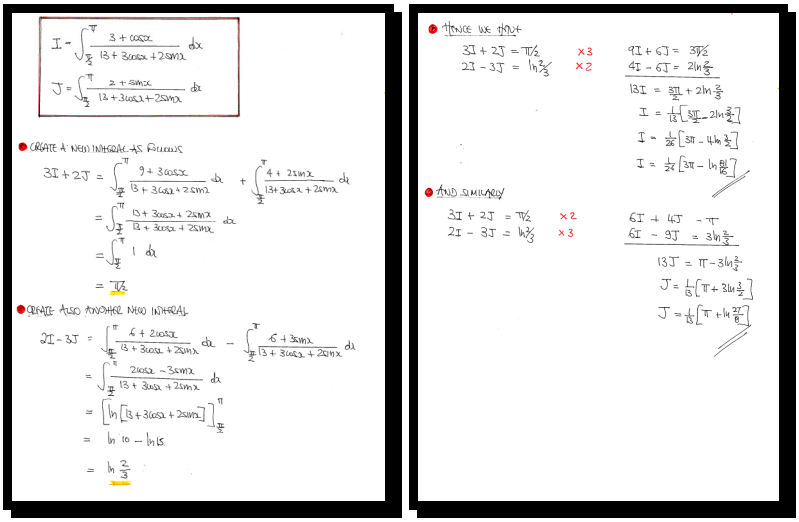
$$I=\int_{\frac{1}{2}\pi}^{\pi}\frac{3+\cos x}{13+3\cos x+2\sin x}\,dx \quad \text{and} \quad J=\int_{\frac{1}{2}\pi}^{\pi}\frac{2+\sin x}{13+3\cos x+2\sin x}\,dx\,.$$

By considering two linear combinations in I and J, show that

$$I=\frac{1}{26}\left[3\pi-\ln\left(\frac{81}{16}\right)\right],$$

and find a similar expression for J.

$\square$, $\boxed{I=\frac{1}{13}\left[\pi+\ln\left(\frac{27}{8}\right)\right]}$

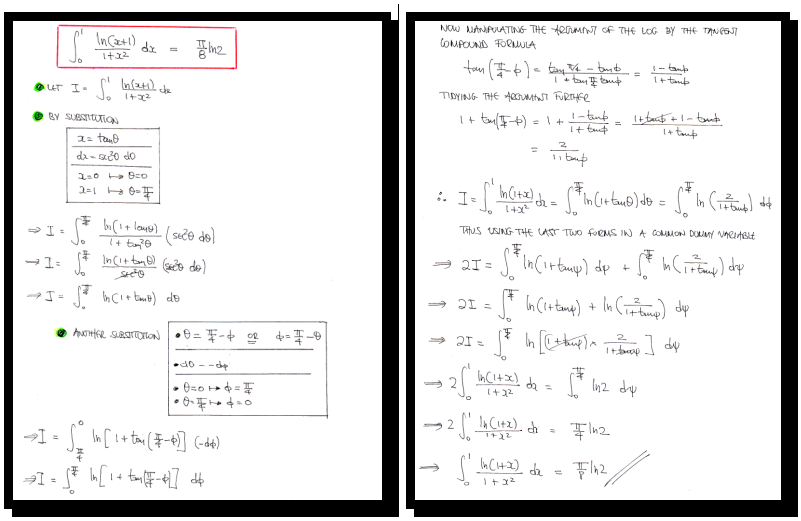


Question 324 (*****)

By using an appropriate substitution or substitutions, show that

$$\int_0^1 \frac{\ln(x+1)}{1+x^2}\,dx = \frac{\pi \ln 2}{8}.$$

[ ], [proof]



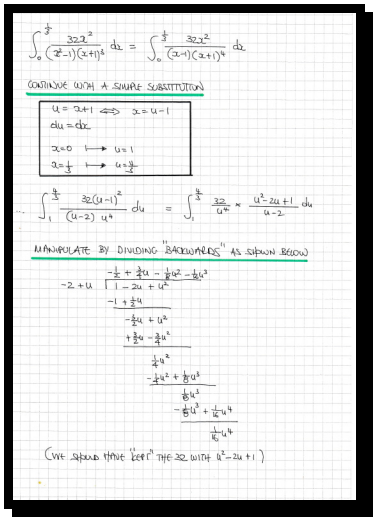
Question 325 **(*****)**

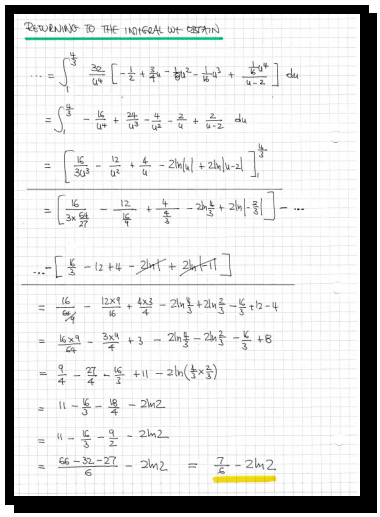
It is given that

$$I=\int_0^{\frac{1}{3}}\frac{32x^2}{\left(x^2-1\right)\left(x+1\right)^3}\,dx\,.$$

Show that $I=\dfrac{7}{6}-2\ln 2$.

[ ], proof





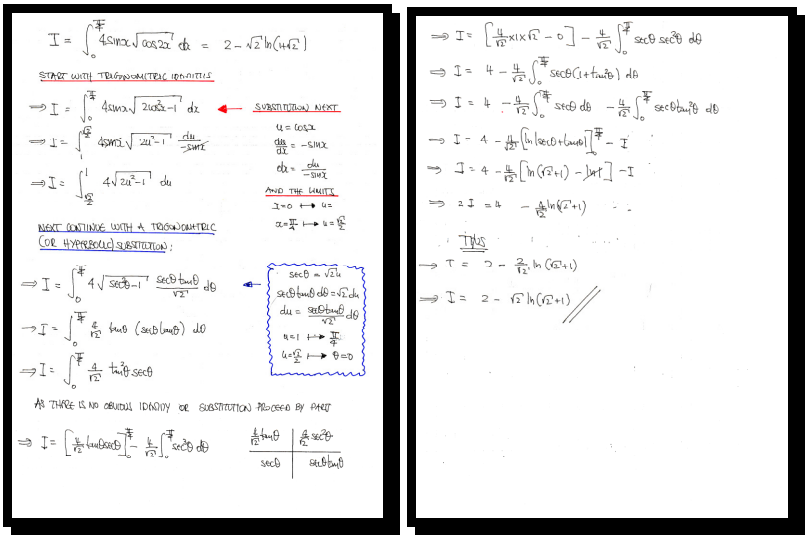
Question 326 (*****)

$$I = \int_0^{\frac{1}{2}\pi} 4\sin x\sqrt{\cos 2x}\ dx\,.$$

By using an appropriate substitution or substitutions, show that

$$I = 2-\sqrt{2}\ln\left(1+\sqrt{2}\right).$$

, proof

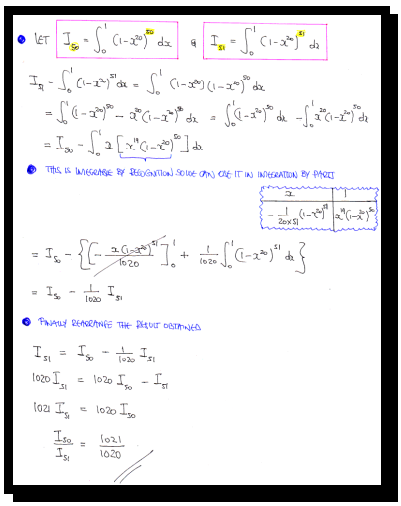


Question 327 **(*****)**

Find as an exact fraction the value of I,

$$I=\frac{\displaystyle\int_0^1\left(1-x^{20}\right)^{50}\,dx}{\displaystyle\int_0^1\left(1-x^{20}\right)^{51}\,dx}.$$

$\square$, $\boxed{\dfrac{1021}{1020}}$



Question 328 **(*****)**

The integral I is defined as

$$I=\int_0^{\pi}\frac{\sin^2 x}{1+\cos^2 x}\,dx\,.$$

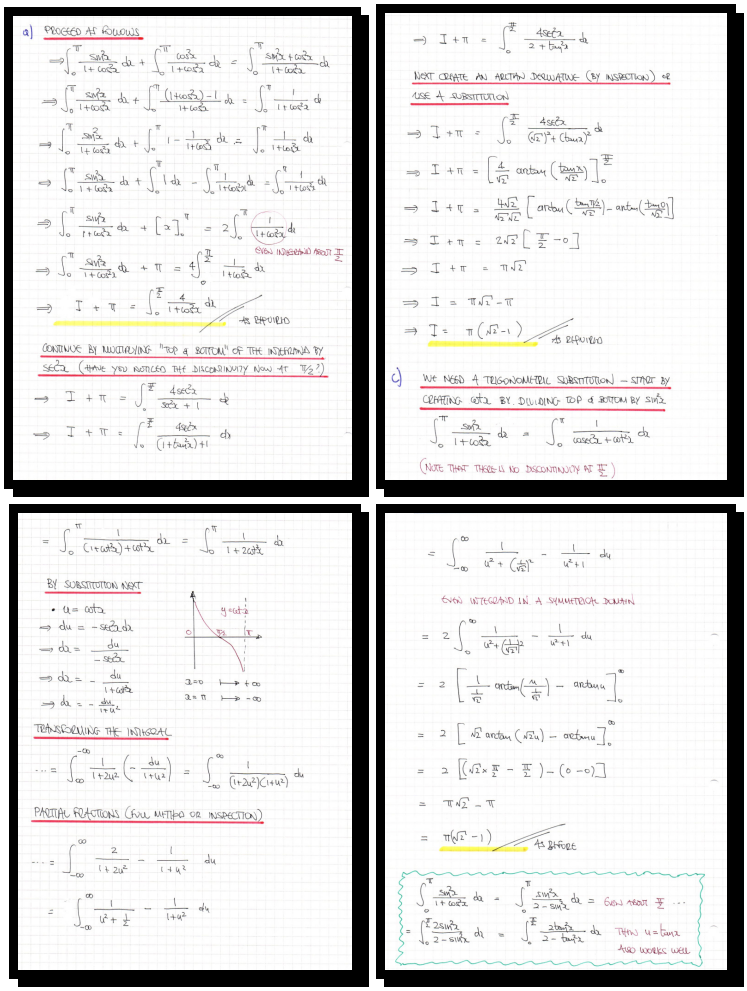
a) Show by a detailed method that

$$I+\pi=\int_o^{\frac{1}{2}\pi}\frac{4}{1+\cos^2 x}\,dx\,.$$

b) Hence, find the value of I in exact simplified form.

c) Verify the answer obtained in part **(b)** by an alternative method by first writing the integrand of I as a function of $\cot^2 x$.

$\square$, $\boxed{I=\pi\left(\sqrt{2}-1\right)}$

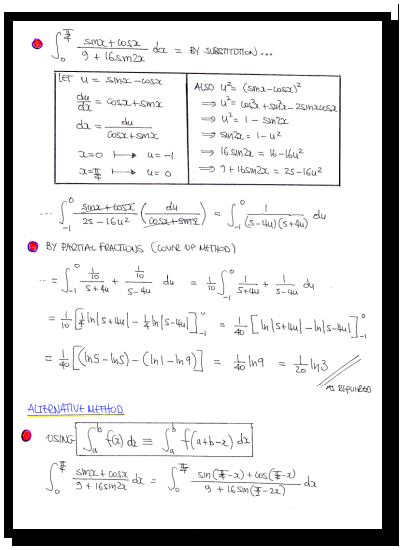


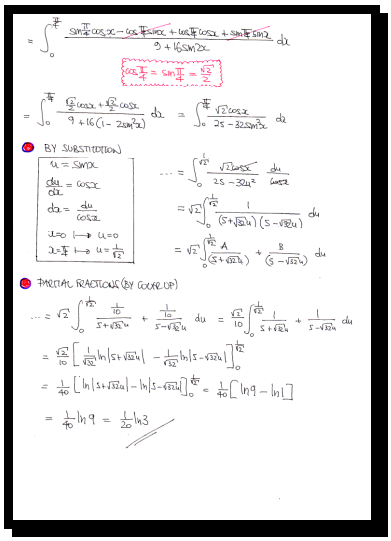
Question 329 **(*****)**

By using an appropriate substitution or substitutions, followed by partial fractions show that

$$\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx = \frac{\ln 3}{20}.$$

[ ], proof





Question 330 (*****)

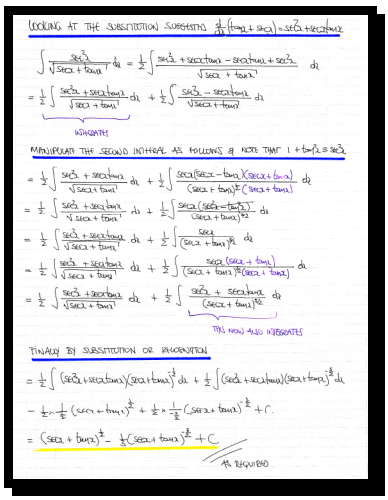
$$I = \int \frac{\sec^2 x}{\sqrt{\sec x + \tan x}}\, dx\,.$$

Without using a verification approach, show that

$$I = (\sec x + \tan x)^{\frac{1}{2}} - \frac{1}{3}(\sec x + \tan x)^{-\frac{3}{2}} \ + \ \text{constant}\,.$$

You may consider the substitution $u = \sec x + \tan x$ useful at some stages in the manipulation of the integrand.

□, proof

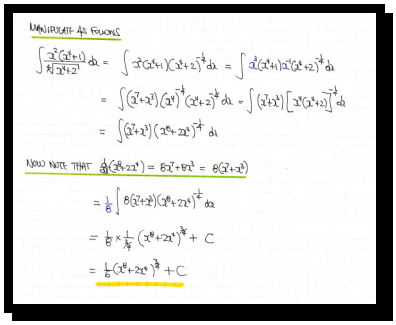


Question 331 (*****)

Use an appropriate integration method to determine an antiderivative for the following indefinite integral.

$$\int \frac{x^2\left(x^4+1\right)}{\sqrt[4]{x^4+1}}\,dx \ .$$

V, ☐, $\boxed{\frac{1}{6}\left(x^8+2x^4\right)^{\frac{1}{2}}+C}$



Question 332 **(*****)**

Use partial fractions followed by integration by parts to show that

$$\int_0^{\infty} \left[\frac{x^2+3x+3}{(x+1)^3} \right] \mathrm{e}^{-x} \sin x \ dx = \frac{1}{2}.$$

, proof

