INTEGRATION

BY PARTIAL FRACTIONS

Question 1

Carry out each of the following integrations.

1. $\displaystyle\int \frac{17-4x}{(x-2)(x+1)}\,dx = 3\ln|x-2|-7\ln|x+1|+C$

2. $\displaystyle\int \frac{2-x}{(x+1)(2x-1)}\,dx = \frac{1}{2}\ln|2x-1|-\ln|x+1|+C$

3. $\displaystyle\int \frac{4}{(x-2)(2-3x)}\,dx = \ln\left|\frac{3x-2}{x-2}\right|+C$

4. $\displaystyle\int \frac{5x-7}{(x-1)(5x-3)}\,dx = 2\ln|5x-3|-\ln|x-1|+C$

5. $\displaystyle\int \frac{18x-1}{(2x+1)(3x-1)}\,dx = 2\ln|2x+1|+\ln|3x-1|+C$

6. $\displaystyle\int \frac{3x-5}{x(1-x)}\,dx = 2\ln|x-1|-5\ln|x|+C$

7. $\displaystyle\int \frac{7x-19}{x^2-2x-15}\,dx = 5\ln|x+3|+2\ln|x-5|+C$

8. $\displaystyle\int \frac{x^2+14x+1}{(x+3)(x-5)(x+7)}\,dx = \ln\left|\frac{x^2-2x-15}{x+7}\right|+C$

9. $\displaystyle\int \frac{7x+4}{(x-2)(x+1)^2}\,dx = 2\ln\left|\frac{x-2}{x+1}\right|-\frac{1}{x+1}+C$

10. $\displaystyle\int \frac{2x^2+x+8}{(x-2)(x+1)^2}\,dx = 2\ln|x-2|+\frac{3}{x+1}+C$



Question 2

Carry out each of the following integrations.

1. $\displaystyle\int \frac{3x-1}{(2x+1)(x-2)}\,dx = \frac{1}{2}\ln|2x+1| + \ln|x-2| + C$

2. $\displaystyle\int \frac{2}{(x-2)(x-4)}\,dx = \ln\left|\frac{x-4}{x-2}\right| + C$

3. $\displaystyle\int \frac{3}{(2+x)(1-x)}\,dx = \ln\left|\frac{2+x}{1-x}\right| + C$

4. $\displaystyle\int \frac{1}{(x+1)(x+2)}\,dx = \ln\left|\frac{x+1}{x+2}\right| + C$

5. $\displaystyle\int \frac{x+1}{9x^2-1}\,dx = \frac{2}{9}\ln|3x-1| - \frac{1}{9}\ln|3x+1| + C$

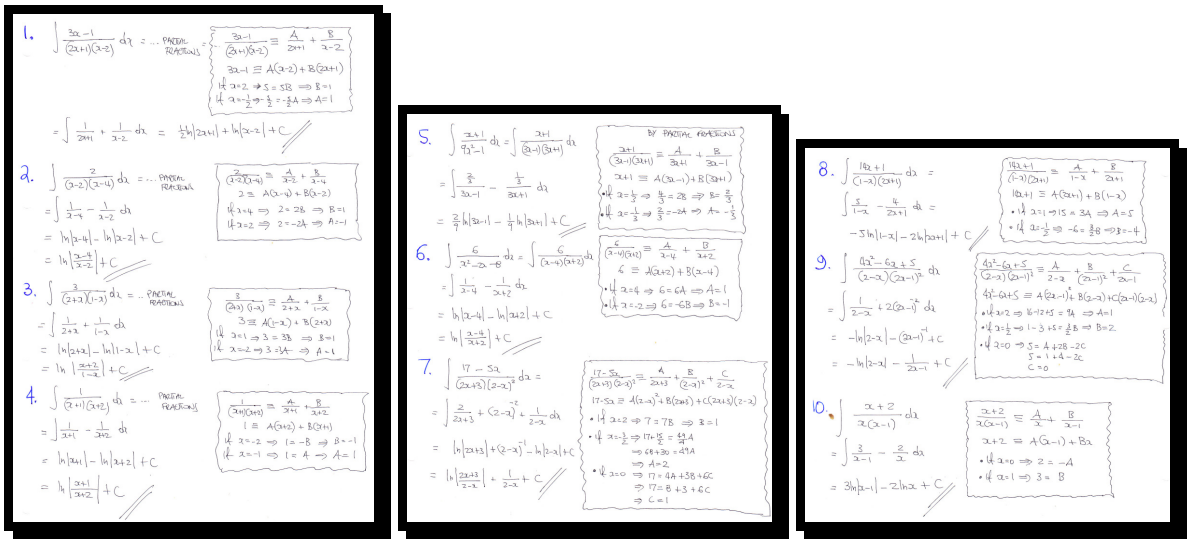
6. $\displaystyle\int \frac{6}{x^2-2x-8}\,dx = \ln\left|\frac{x-4}{x+2}\right| + C$

7. $\displaystyle\int \frac{17-5x}{(2x+3)(2-x)^2}\,dx = \ln\left|\frac{2x+3}{2-x}\right| + \frac{1}{2-x} + C$

8. $\displaystyle\int \frac{14x+1}{(1-x)(2x+1)}\,dx = -5\ln|1-x| - 2\ln|2x+1| + C$

9. $\displaystyle\int \frac{4x^2-6x+5}{(2-x)(2x-1)^2}\,dx = -\frac{1}{2x-1} - \ln|2-x| + C$

10. $\displaystyle\int \frac{x+2}{x(x-1)}\,dx = 3\ln|x-1| - 2\ln|x| + C$



Question 3

Carry out each of the following integrations.

1. $\displaystyle\int \frac{10x^2-23x+11}{(2-3x)(2x-1)^2}\,dx=-\frac{2}{2x-1}-\frac{1}{3}\ln|2-3x|-\frac{1}{2}\ln|2x-1|+C$

2. $\displaystyle\int \frac{1}{x^2(x-1)}\,dx=\frac{1}{x}+\ln\left|\frac{x-1}{x}\right|+C$

3. $\displaystyle\int \frac{8(x^2+1)}{(x-3)(x+1)^2}\,dx=5\ln|x-3|+3\ln|x+1|+\frac{4}{x+1}+C$

4. $\displaystyle\int \frac{1}{x(x-2)}\,dx=\frac{1}{2}\ln\left|\frac{x-2}{x}\right|+C$

5. $\displaystyle\int \frac{1}{x^2-4}\,dx=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

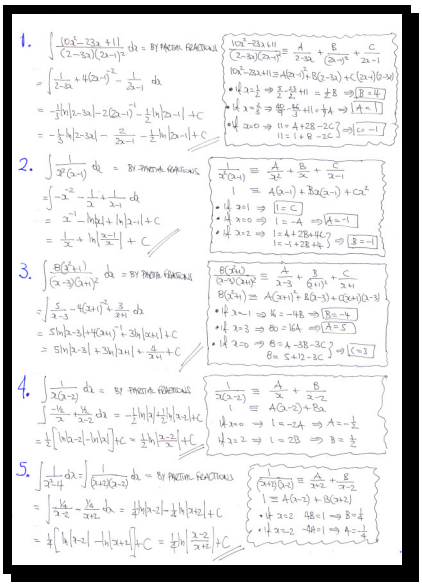
6. $\displaystyle\int \frac{4x^2-x+1}{(x-1)(2x-1)}\,dx=2x+4\ln|x-1|-\frac{3}{2}\ln|2x-1|+C$

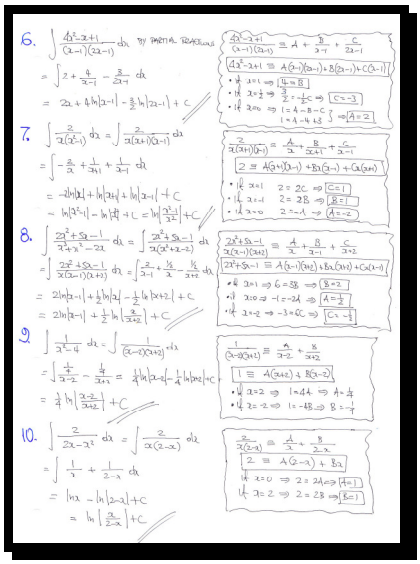
7. $\displaystyle\int \frac{2}{x(x^2-1)}\,dx=\ln\left|\frac{x^2-1}{x^2}\right|+C$

8. $\displaystyle\int \frac{2x^2+5x-1}{x^3+x^2-2x}\,dx=2\ln|x-1|+\frac{1}{2}\ln\left|\frac{x}{x+2}\right|+C$

9. $\displaystyle\int \frac{1}{x^2-4}\,dx=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

10. $\displaystyle\int \frac{2}{2x-x^2}\,dx=\ln\left|\frac{x}{2-x}\right|+C$





Question 4

Carry out each of the following integrations.

1. $\displaystyle\int_0^1 \frac{3x}{(x+1)(x-2)}\,dx = -\ln 2$

2. $\displaystyle\int_{\frac{1}{6}}^{\frac{1}{3}} \frac{14x+1}{(2x+1)(1-x)}\,dx = 3\ln\left(\frac{5}{4}\right)$

3. $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{(1-x)(1+x)^2}\,dx = \frac{1}{6}+\frac{1}{4}\ln 3$

4. $\displaystyle\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,dx = \ln 54$

5. $\displaystyle\int_0^{\frac{1}{2}} \frac{3-5x}{(1-x)(2-3x)}\,dx = \frac{4}{3}\ln 2$

6. $\displaystyle\int_{-1}^1 \frac{9+4x^2}{9-4x^2}\,dx = -2+3\ln 5$

7. $\displaystyle\int_0^1 \frac{18-4x-x^2}{(4-3x)(1+x)^2}\,dx = \frac{7}{3}\ln 2+\frac{3}{2}$

8. $\displaystyle\int_2^3 \frac{x^2+x+2}{x^2+2x-3}\,dx = 1+\ln\left(\frac{25}{18}\right)$

9. $\displaystyle\int_0^{\frac{1}{4}} \frac{4}{(2x+1)(1-2x)}\,dx = \ln 3$

10. $\displaystyle\int_0^1 \frac{17-5x}{(3+2x)(2-x)^2}\,dx = \frac{1}{2}+\ln\left(\frac{10}{3}\right)$



Question 5

Carry out each of the following integrations.

1. $\displaystyle\int_4^9 \frac{5x^2-8x+1}{2x(x-1)^2}\,dx = \ln\left(\frac{32}{3}\right)-\frac{5}{24}$

2. $\displaystyle\int_0^1 \frac{x^2}{x^2-4}\,dx = 1-\ln 3$

3. $\displaystyle\int_0^5 \frac{1}{(x+1)(x+2)(x+3)}\,dx = \ln\left(\frac{8}{7}\right)$

4. $\displaystyle\int_0^1 \frac{10}{(x+1)(x+3)(2x+1)}\,dx = 3\ln 3-3\ln 2$

5. $\displaystyle\int_0^4 \frac{13-2x}{(x+4)(2x+1)}\,dx = 4\ln 3-3\ln 2$

6. $\displaystyle\int_2^6 \frac{2x^2-x+11}{(x+2)(2x-3)}\,dx = 4+4\ln 3-3\ln 2$

7. $\displaystyle\int_0^2 \frac{25x+1}{(2x-1)(x+1)^2}\,dx = \frac{16}{3}$

8. $\displaystyle\int_5^8 \frac{2x^2}{x^2-16}\,dx = 6+4\ln 3$

9. $\displaystyle\int_2^3 \frac{x^2-3x+5}{(4-x)(1-x)^2}\,dx = \frac{1}{2}+\ln 2$

10. $\displaystyle\int_0^2 \frac{4x^3-12x^2-22x-3}{(4-x)(2x+1)}\,dx = \frac{1}{2}\ln\left(\frac{5}{64}\right)-6$

