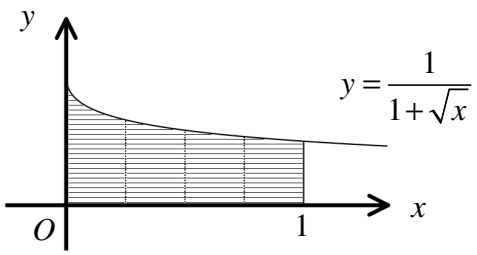
NUMERICAL INTEGRATION

THE TRAPEZIUM RULE

Question 1 (**)



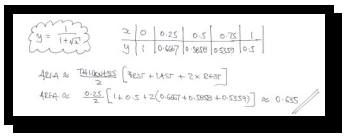
The figure above shows part of the curve C with equation

$$y=\frac{1}{1+\sqrt{x}},\ x\geq 0.$$

It is required to estimate the area of the shaded region which is bounded by C, the coordinate axes and the straight line with equation $x=1$.

Use the trapezium rule with 4 equally spaced strips to estimate the area of this region, giving the answer correct to 3 decimal places.

$\boxed{}$, $\boxed{\approx 0.635}$



Question 2 (**)

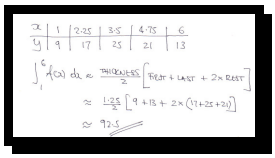
The values of y, for a curve with equation $y = f(x)$, have been tabulated below.

x	1	2.25	3.5	4.75	6
y	9	17	25	21	13

Use the trapezium rule with all the values from the above table to find an estimate for the integral

$$\int_1^6 f(x)\ dx .$$

$\square$, $\boxed{92.5}$



Question 3 (**)

The y values, for the curve with equation $y=\sqrt{x^3-x}$, have been tabulated below.

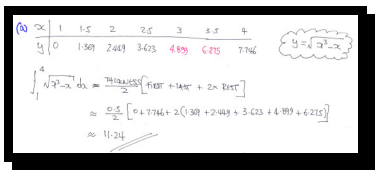
x	1	1.5	2	2.5	3	3.5	4
y	0	1.369	2.449	3.623			7.746

a) Complete the table.

b) Use the trapezium rule with all the values from the table above to find an estimate, correct to 2 decimal places, for the integral

$$\int_1^4 \sqrt{x^3-x}\ dx\,.$$

$\boxed{}$, $\boxed{4.899}$, $\boxed{6.275}$, $\boxed{\approx 11.24}$



Question 4 **(**+)**

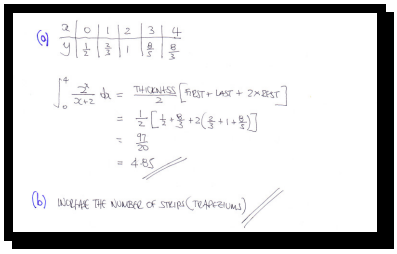
a) Use the trapezium rule with five equally spaced ordinates (four strips) to find the value of

$$\int_0^4 \frac{2^x}{x+2}\,dx\,,$$

giving the answer correct to three significant figures.

b) State how a better approximation to the value of the integral can be obtained using the trapezium rule.

[ ], [4.85]

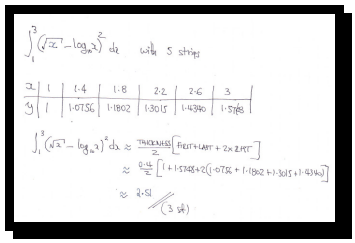


Question 5 **(**+)**

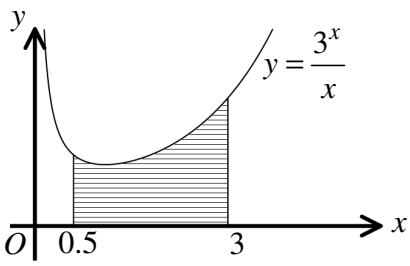
$$I=\int_1^3 \left(\sqrt{x}-\log_{10} x\right)^2\,dx\,.$$

Use the trapezium rule with 5 equally spaced strips to find an estimate for I.

[ ], [≈ 2.51]



Question 6 **(**+)**



The figure above shows part of the curve C with equation

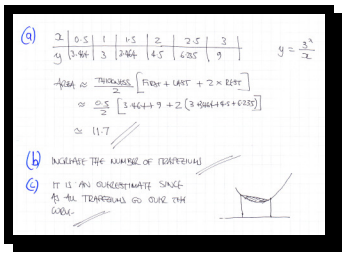
$$y = \frac{3^x}{x}, \ x \neq 0 .$$

a) Use the trapezium rule with 5 equally spaced strips to estimate, to three significant figures, the area bounded by C, the x axis and the straight lines with equations $x = 0.5$ and $x = 3$.

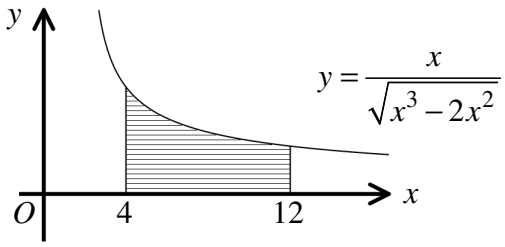
b) State how the accuracy of the estimate obtained in part (a) can be improved.

c) Explain with the aid of a diagram whether the estimate obtained in part (a) is an underestimate or an overestimate for the actual value for this area.

$\boxed{}$, $\boxed{\approx 11.7}$



Question 7 (+)**



The figure above shows part of the curve C with equation

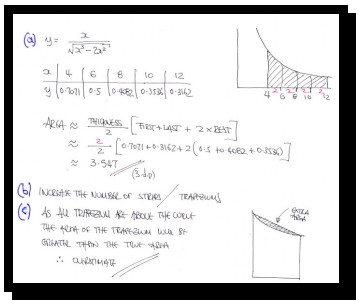
$$y = \frac{x}{\sqrt{x^3 - 2x^2}} .$$

a) Use the trapezium rule with 4 equally spaced strips to estimate, to three significant figures, the area bounded by C, the x axis and the vertical straight lines with equations $x = 4$ and $x = 12$.

b) State how the estimate obtained in part **(a)** can be improved.

c) Explain with the aid of a diagram whether the estimate obtained in part **(a)** is an underestimate or an overestimate for the actual value of this area.

$\boxed{}$, $\boxed{\approx 3.547}$

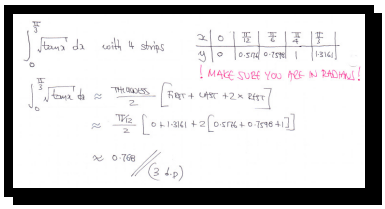


Question 8 **(**+)**

$$I=\int_0^{\frac{\pi}{3}}\sqrt{\tan x}\ \ dx$$

Use the trapezium rule with 4 equally spaced strips to find an estimate for I .

$$\boxed{\approx 0.768}$$

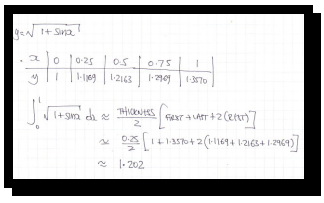


Question 9 **(**+)**

$$I=\int_0^{1}\sqrt{1+\sin x}\ \ dx$$

Use the trapezium rule with 4 equally spaced strips to estimate the approximate value of I , giving the answer correct to 3 decimal places

$$\boxed{\approx 1.202}$$

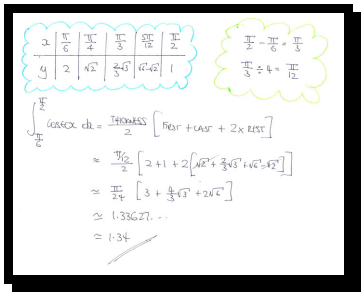


Question 10 (+)**

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \text{cosec}\, x \ \ dx$$

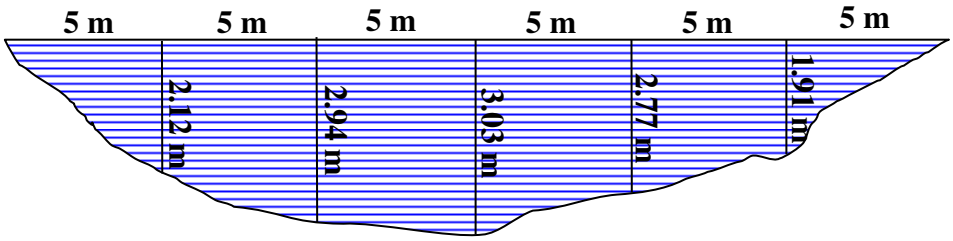
Use the trapezium rule with 4 equally spaced strips to find an estimate for I.

$\boxed{}$, $\boxed{\approx 1.34}$



Question 11 (***)

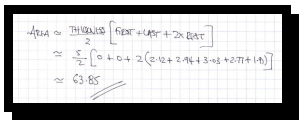
The figure below shows the cross section of a river.



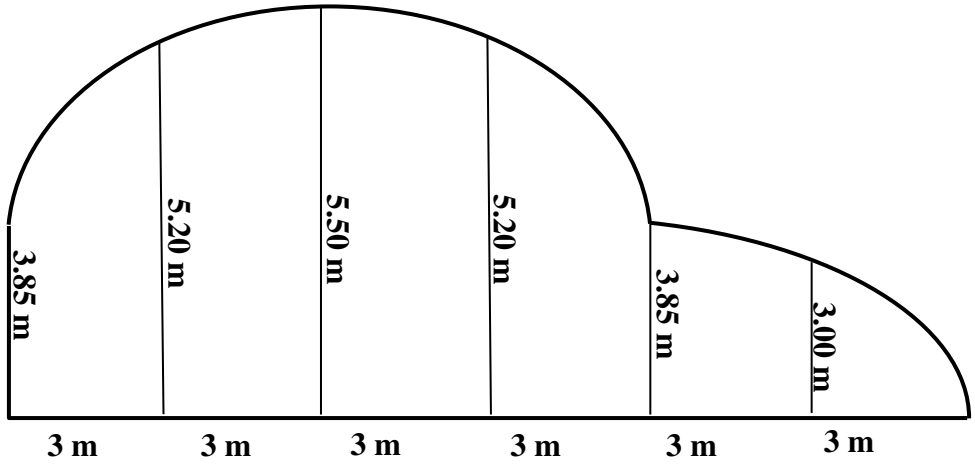
The depth of the river, in metres, from one river bank directly across to the other river bank, is recorded at 5 metre intervals.

Estimate the cross sectional area of the river, by using the trapezium rule with all the measurements provided in the above figure.

$\square$, $\approx 63.85 \text{ m}^2$



Question 12 (***)

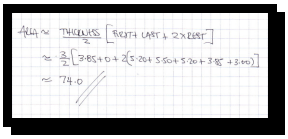


The figure above shows the cross section of a tunnel.

The height of the tunnel, in metres, from one end directly across to the other end, is recorded at 3 metre intervals.

Use the trapezium rule to estimate the cross sectional area of the tunnel.

$\square$, $\approx 74 \text{ m}^2$



Question 13 (*+)**

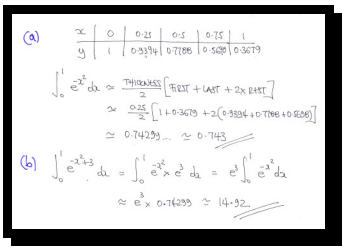
a) Use the trapezium rule with 4 equally spaced strips to find an estimate for

$$\int_0^1 \mathrm{e}^{-x^2}\ dx\,.$$

b) Use the answer of part **(a)** to find an estimate for

$$\int_0^1 \mathrm{e}^{-x^2+3}\ dx\,.$$

$\boxed{}$, $\boxed{\approx 0.743}$, $\boxed{\approx 14.92}$



Question 14 **(***+)**

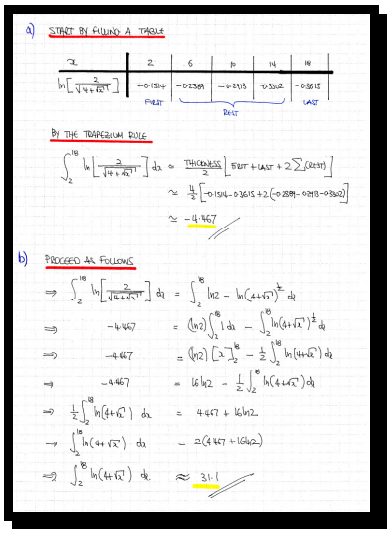
a) Use the trapezium rule with 5 equally spaced ordinates to estimate the value of the following integral.

$$\int_2^{18} \ln\left[\frac{2}{\sqrt{4+\sqrt{x}}}\right] dx .$$

b) Use the answer of part **(a)** to estimate the value of

$$\int_2^{18} \ln\left(4+\sqrt{x}\right) dx .$$

$\boxed{}$, $\boxed{\approx -4.467}$, $\boxed{\approx 31.1}$



Question 15 **(***+)**

a) Use the trapezium rule with 5 equally spaced ordinates to estimate the value of the following integral.

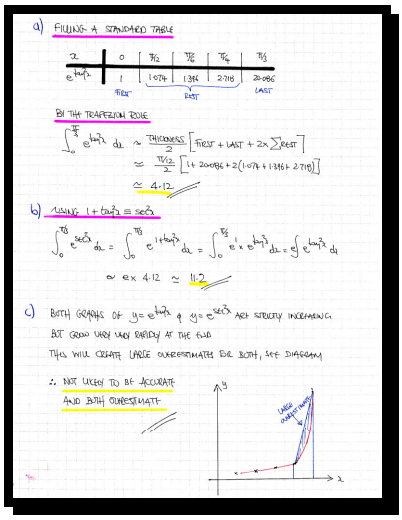
$$\int_0^{\frac{1}{3}\pi} \mathrm{e}^{\tan^2 x} \, dx \, .$$

b) Use the answer of part **(a)** to estimate the value of

$$\int_0^{\frac{1}{3}\pi} \mathrm{e}^{\sec^2 x} \, dx \, .$$

c) Discuss briefly whether the estimates of the previous parts of the question are likely to be accurate, stating further whether they are overestimates or underestimates to the true values of these integrals.

$\boxed{}$, $\boxed{\approx 4.12}$, $\boxed{\approx 11.2}$



Question 16 (****)

a) Use the trapezium rule with 4 equally spaced strips to find an estimate for

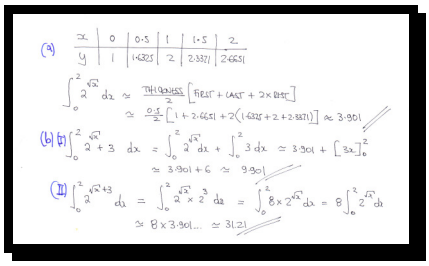
$$\int_0^2 2^{\sqrt{x}} \, dx .$$

b) Use the answer of part **(a)** to find estimates for …

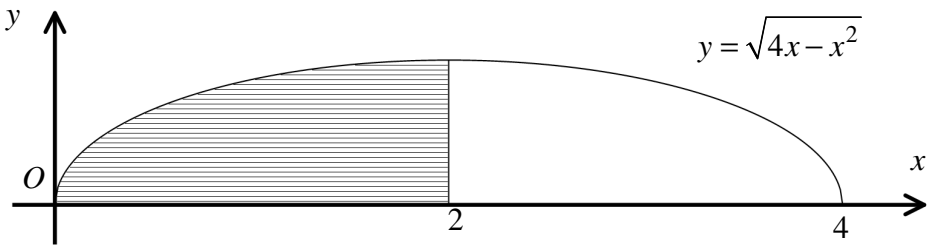
i. … $\displaystyle\int_0^2 2^{\sqrt{x}} + 3 \, dx .$

ii. … $\displaystyle\int_0^2 2^{\sqrt{x}+3} \, dx .$

$\boxed{}$, $\boxed{\approx 3.901}$, $\boxed{\approx 9.901}$, $\boxed{\approx 31.21}$



Question 17 **(****)**



The figure above shows part of the curve C with equation

$$y=\sqrt{4x-x^2}\ .$$

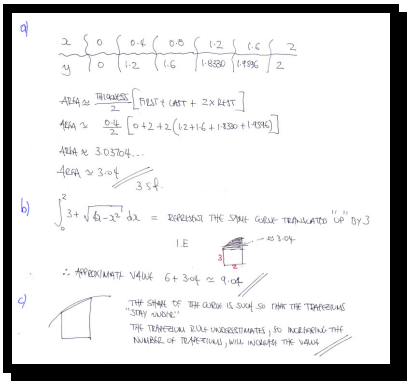
a) Use the trapezium rule with 5 equally spaced trapeziums to estimate, to three significant figures, the area bounded by C, the x axis and the vertical straight line with equation $x=2$.

b) Hence find an estimate for

$$\int_0^2 3+\sqrt{4x-x^2}\ \ dx\ .$$

a) State, with justification, whether the answer of part **(a)** will increase or decrease if more than 5 trapeziums are used.

$\boxed{}$, $\boxed{\approx 3.04}$, $\boxed{\approx 9.04}$



Question 18 (****)

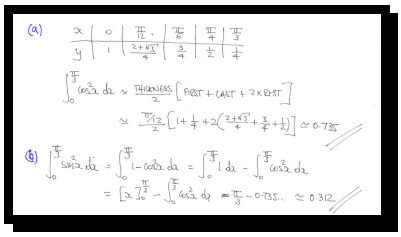
a) Use the trapezium rule with 4 equally spaced strips to find an estimate for

$$\int_0^{\frac{\pi}{3}} \cos^2 x \ \ dx \, .$$

b) Use the answer of part (a) to find an estimate for

$$\int_0^{\frac{\pi}{3}} \sin^2 x \ \ dx \, .$$

$\boxed{}$, $\boxed{\approx 0.735}$, $\boxed{\approx 0.312}$



Question 19 (****)

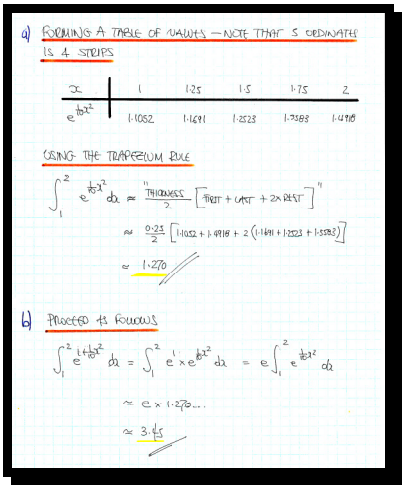
a) Find an estimate for the following integral, by using the trapezium rule with 5 equally spaced **ordinates.** to for

$$\int_1^2 \mathrm{e}^{\frac{1}{10}x^2}\,dx\,.$$

b) Use the answer of part **(a)** to find estimates for

$$\int_1^2 \mathrm{e}^{1+\frac{1}{10}x^2}\,dx\,.$$

$\boxed{}$, $\boxed{\approx 1.270}$, $\boxed{\approx 3.45}$



Question 20 **(****)**

a) **Use the trapezium rule** with 6 equally spaced strips to find an estimate, correct to 3 decimal places, for

$$\int_0^{1.2} \sin^2 x \ dx .$$

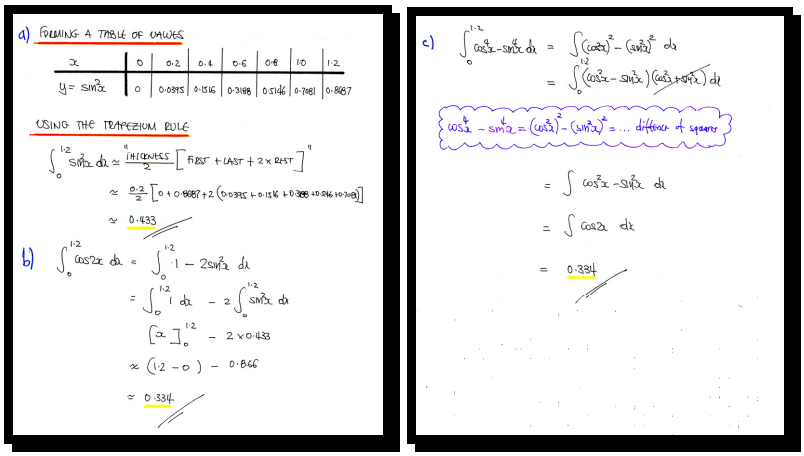
b) **Use the answer of part (a)** to find an estimate for

$$\int_0^{1.2} \cos 2x \ dx .$$

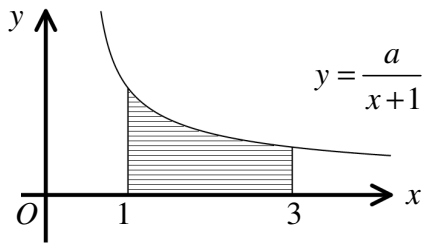
c) **Use the answer of part (b)** to find an estimate for

$$\int_0^{1.2} \left[\cos^4 x - \sin^4 x \right] dx .$$

$\boxed{}$, $\boxed{\approx 0.433}$, $\boxed{\approx 0.334}$, $\boxed{\approx 0.334}$



Question 21 (**+)**



The figure above shows part of the curve C with equation

$$y=\frac{a}{x+1},$$

where a is a positive integer.

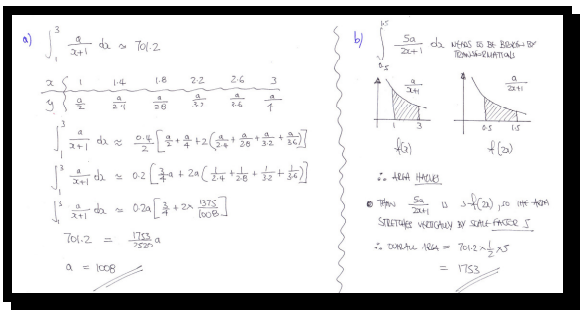
When the trapezium rule with 5 equally spaced strips is used, the area bounded by C, the x axis and the vertical straight lines with equations $x=1$ and $x=3$, is approximated to 701.2 square units.

a) Determine the value of a.

b) By considering suitable graph transformation, find an approximate value of

$$\int_{0.5}^{1.5}\frac{5a}{2x+1}\,dx.$$

$\boxed{}$, $\boxed{a=1008}$, $\boxed{\approx 1753}$



Question 22 (*****)

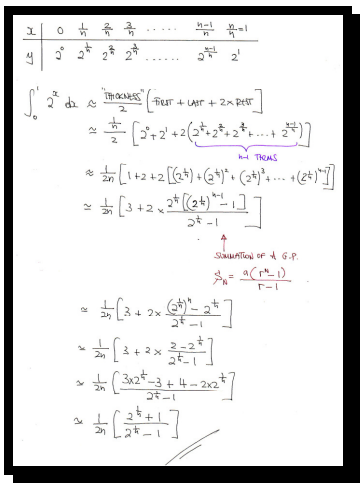
The trapezium rule with n equally spaced intervals is to be used to estimate the value of the following integral

$$\int_0^1 2^x \ dx \, .$$

Show that the value of this estimate is given by

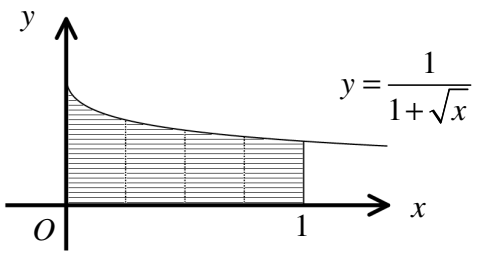
$$\frac{1}{2n}\left[\frac{2^{\frac{1}{n}}+1}{2^{\frac{1}{n}}-1}\right].$$

$\boxed{}$, $\boxed{\text{proof}}$



SIMPSON'S RULE

Question 1 (**)



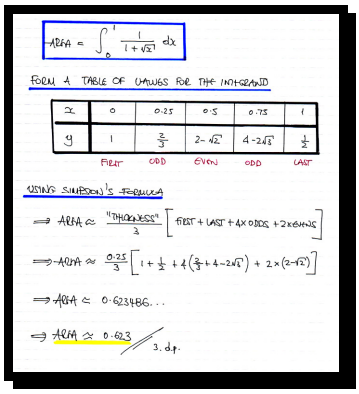
The figure above shows part of the curve C with equation

$$y=\frac{1}{1+\sqrt{x}}, \ x\geq 0 .$$

It is required to estimate the area of the shaded region bounded by C, the coordinate axes and the straight line with equation $x=1$.

Use Simpson's rule with 4 equally spaced strips to estimate the area of this region, giving the answer correct to 3 decimal places.

$\boxed{}$, $\boxed{0.623-0.624}$



Question 2 (**)

The values of y for the curve with equation $y=\dfrac{1}{\sqrt{x^3+1}}$ have been tabulated below.

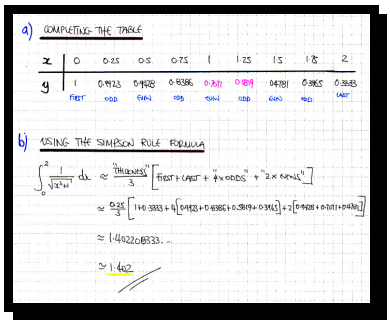
x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	1	0.9923	0.9428	0.8386			0.4781	0.3965	0.3333

a) Complete the table.

b) Use Simpson's rule with all the values from the table to find an estimate to 3 decimal places for the integral

$$\int_0^2 \frac{1}{\sqrt{x^3+1}}\,dx\,.$$

$\boxed{}$, $\boxed{0.7071}$, $\boxed{0.5819}$, $\boxed{1.402}$



Question 3 (**)

$$I=\int_0^2 \frac{1}{x^3+1}\,dx\,.$$

Use 3 equally spaced ordinates, to estimate the value of I …

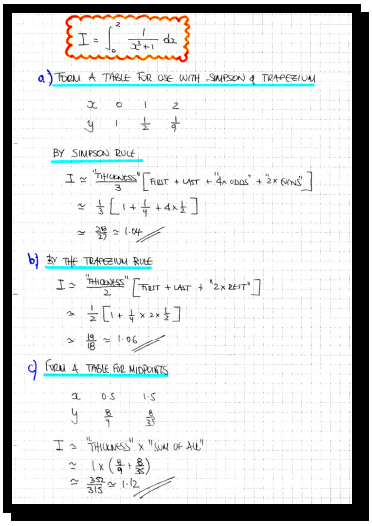
a) … by Simpson's rule.

b) … by the trapezium rule.

c) … by the mid-ordinate rule.

All steps in the calculations must be shown.

$\boxed{}$, $\boxed{1.04}$, $\boxed{1.06}$, $\boxed{1.12}$



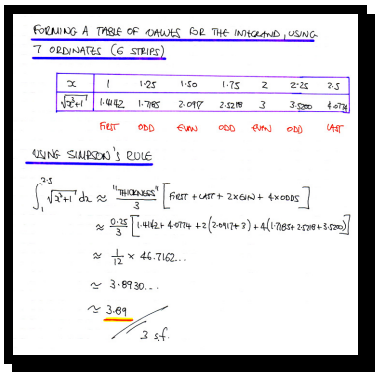
Question 4 (**)

$$I = \int_1^{2.5} \sqrt{x^3+1}\ dx\,.$$

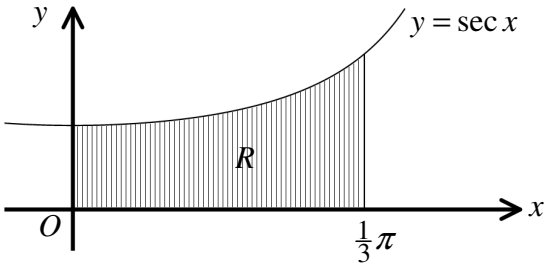
Use Simpson's rule with 6 equally spaced strips, to estimate the value of I.

All steps in the calculation must be shown and the final answer must be correct to 3 significant figures.

$\square$, $\boxed{3.89}$



Question 5 (**)



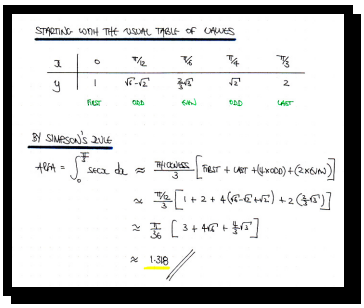
The figure above shows part of the curve with equation $y = \sec x$.

The region R, shown shaded in the figure, is bounded by the curve, the x axis and the straight line with equation $x = \frac{1}{3}\pi$.

Use Simpson's rule with 4 equally spaced intervals to estimate the area of R.

[*The answer must be supported with detailed calculations*.]

, 1.318

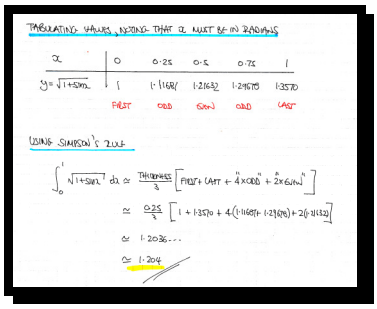


Question 6 (**)

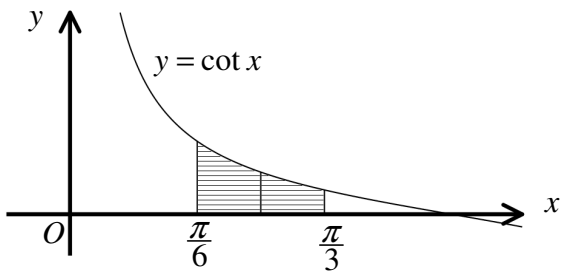
$$I=\int_0^1 \sqrt{1+\sin x}\ \ dx$$

Use Simpson's rule with 4 equally spaced strips to estimate the approximate value of I, giving the answer correct to 3 decimal places

$\boxed{}$, $\boxed{\approx 1.202}$



Question 7 (**)



The figure above shows part of the curve with equation $y = \cot x$.

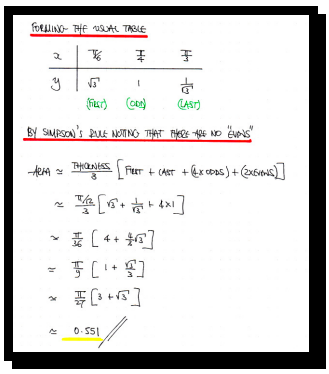
The region R, shown shaded in the figure, is bounded by the curve, the x axis and the straight lines with equations

$$x = \frac{\pi}{6} \qquad \text{and} \qquad x = \frac{\pi}{3}.$$

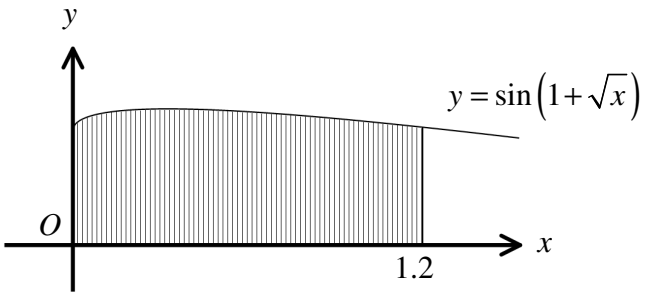
Use Simpson's rule with 3 equally spaced ordinates to estimate the area of R.

[*The answer must be supported with detailed calculations.*]

$\boxed{}$, $\boxed{0.551}$



Question 8 (+)**



The figure above shows part of the curve C with equation

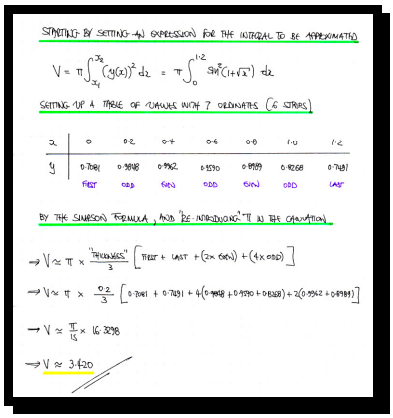
$$y=\sin\left(1+\sqrt{x}\right), \quad x\geq 0 .$$

It is required to estimate the volume of the solid of revolution, when the area of the shaded region bounded by C, the coordinate axes and the straight line with equation $x=1.2$ is fully revolved about the x axis.

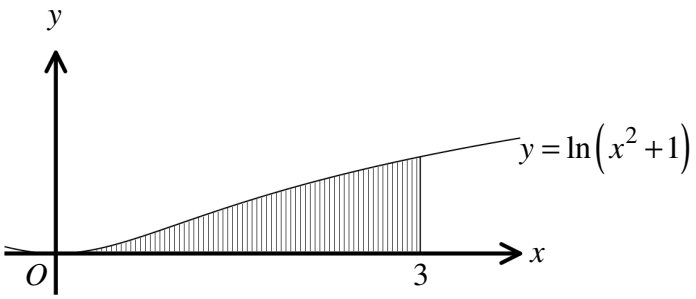
Use Simpson's rule with 7 equally spaced ordinates to find an approximation for the volume of this solid.

[*The answer must be supported with detailed calculations*.]

$\boxed{}$, $\boxed{3.42}$



Question 9 **(**+)**



The figure above shows part of the curve with equation

$$y = \ln\left(x^2 + 4\right).$$

The region R, shown shaded in the figure, is bounded by the curve, the x axis and the straight line with equation $x = 3$.

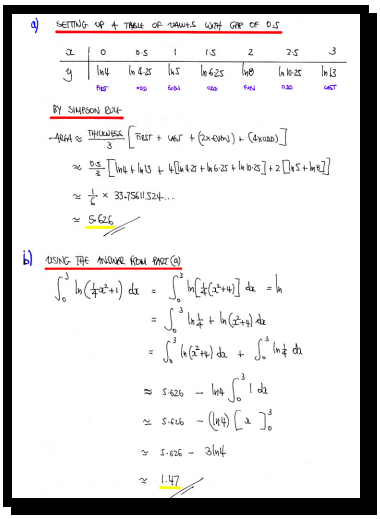
a) Use Simpson's rule with 7 equally spaced ordinates to estimate the area of R.

[*The answer must be supported with detailed calculations*.]

b) Deduce an estimate for the value of

$$\int_0^3 \ln\left(\frac{1}{4}x^2 + 1\right)\, dx.$$

$\boxed{}$, $\boxed{5.626}$, $\boxed{1.47}$


a) SETTING UP A TABLE OF VALUES WITH GAP OF 0.5

x	0	0.5	1	1.5	2	2.5	3
y	ln4	ln 4.25	ln5	ln 6.25	ln8	ln 10.25	ln13
	FIRST						LAST

BY SIMPSON RULE

$$\text{AREA} \approx \frac{\text{THICKNESS}}{3}\left[\text{FIRST} + \text{LAST} + (2\times\text{ODDS}) + (4\times\text{EVENS})\right]$$

$$\approx \frac{0.5}{3}\left[\ln 4 + \ln 13 + 4\left[\ln 4.25 + \ln 6.25 + \ln 10.25\right] + 2\left[\ln 5 + \ln 8\right]\right]$$

$$\approx \frac{1}{6}\times 33.75611524\ldots$$

$$\approx 5.626$$

b) USING THE ANSWER FROM PART (a)

$$\int_0^3 \ln\left(\tfrac{1}{4}x^2+1\right)dx = \int_0^3 \ln\left[\tfrac{1}{4}\left(x^2+4\right)\right]dx = \ln$$

$$= \int_0^3 \ln\tfrac{1}{4} + \ln\left(x^2+4\right)dx$$

$$= \int_0^3 \ln\left(x^2+4\right)dx + \int_0^3 \ln\tfrac{1}{4}\,dx$$

$$\approx 5.626 - \ln 4\int_0^3 1\,dx$$

$$\approx 5.626 - (\ln 4)\left[x\right]_0^3$$

$$\approx 5.626 - 3\ln 4$$

$$\approx 1.47$$

Question 10 (+)**

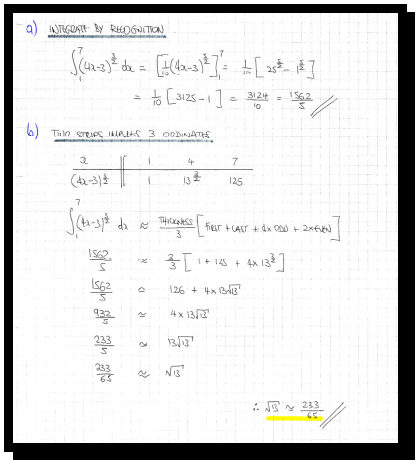
a) Find the exact value of the following integral

$$\int_1^7 (4x-3)^{\frac{3}{2}} \, dx \, , \quad x \geq 0 \, .$$

b) Use Simpson's rule with 2 strips and the answer of part **(a)** to show that

$$\sqrt{13} \approx \frac{233}{65} \, .$$

$\square$, $\boxed{\frac{1562}{5}}$



Question 11 (+)**

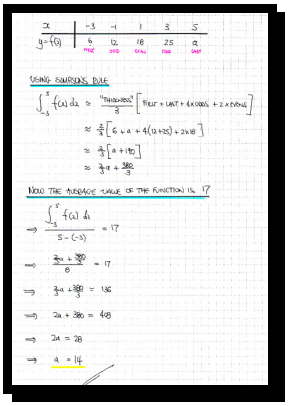
The values of y for the curve C with equation $y = f(x)$ have been tabulated below.

x	-3	-1	1	3	5
y	6	12	18	25	a

The average value of $f(x)$ in the interval $(-3,5)$ is 17.

Use Simpson's rule with all the values from the table to find an estimate for the value of the constant a.

$\boxed{}$, $\boxed{a = 14}$



Question 12 (+)**

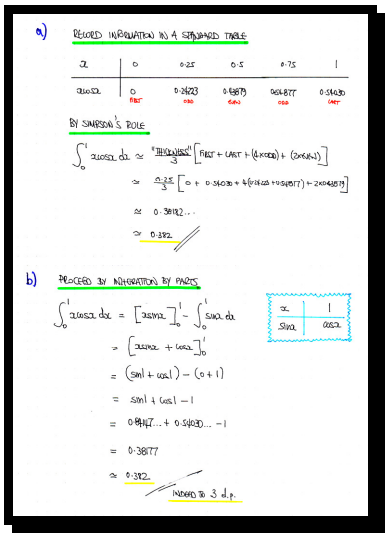
$$I = \int_0^1 x\cos x \ dx \, .$$

a) Use Simpson's rule with 4 equally spaced strips to estimate the value of I .

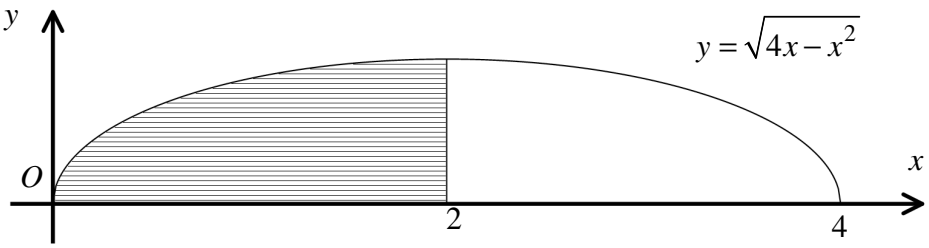
All steps in the calculation must be shown and the final answer must be correct to 3 decimal places.

b) Use integration by parts to show that the value of I found in part **(a)** is indeed correct to three decimal places.

$\boxed{}$, $\boxed{0.382}$



Question 13 (+)**



The figure above shows part of the curve C with equation

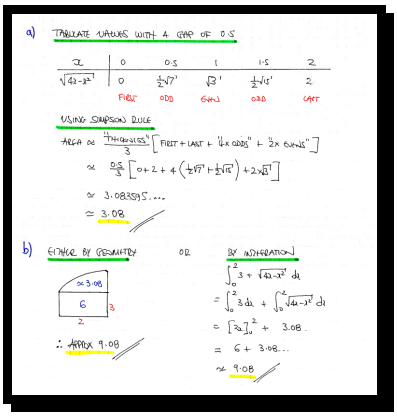
$$y=\sqrt{4x-x^2}\ .$$

a) Use Simpson's rule with 4 equally strips to estimate, to three significant figures, the area bounded by C, the x axis and the vertical straight line with equation $x=2$.

b) Hence find an estimate for

$$\int_0^2 3+\sqrt{4x-x^2}\ \ dx\ .$$

$\boxed{\ \ }$, $\boxed{\approx 3.08}$, $\boxed{\approx 9.08}$



Question 14 **(**+)**

a) Use Simpson's rule with 5 equally spaced ordinates to estimate the value of

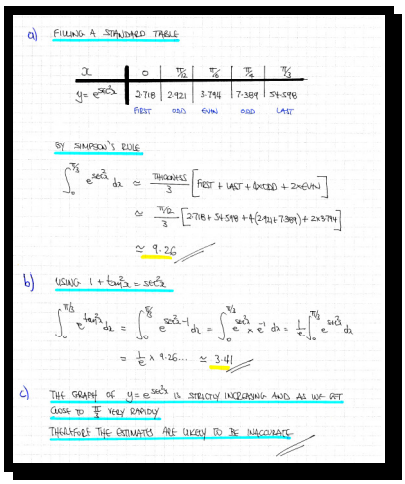
$$\int_0^{\frac{1}{3}\pi} \mathrm{e}^{\sec^2 x} \, dx \, .$$

b) Use the answer of part **(a)** to estimate the value of

$$\int_0^{\frac{1}{3}\pi} \mathrm{e}^{\tan^2 x} \, dx \, .$$

c) Explain whether the estimates of the previous parts of the question are likely to be accurate.

$\boxed{}$, $\boxed{\approx 9.26}$, $\boxed{\approx 3.41}$



Question 15 **(**+)**

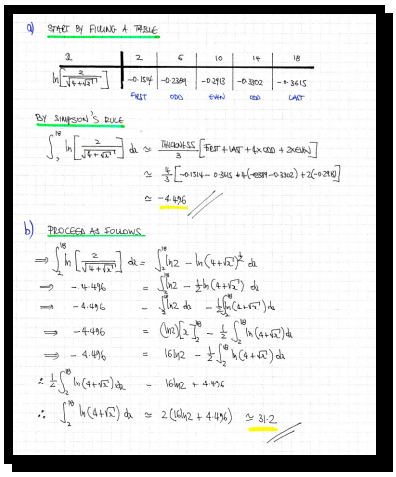
a) Use Simpson's rule with 5 equally spaced ordinates to estimate the value of

$$\int_2^{18} \ln\left[\frac{2}{\sqrt{4+\sqrt{x}}}\right] dx .$$

b) Use the answer of part **(a)** to estimate the value of

$$\int_2^{18} \ln\left(4+\sqrt{x}\right) dx .$$

$\boxed{}$, $\boxed{\approx -4.496}$, $\boxed{\approx 31.2}$



Question 16 **(**+)**

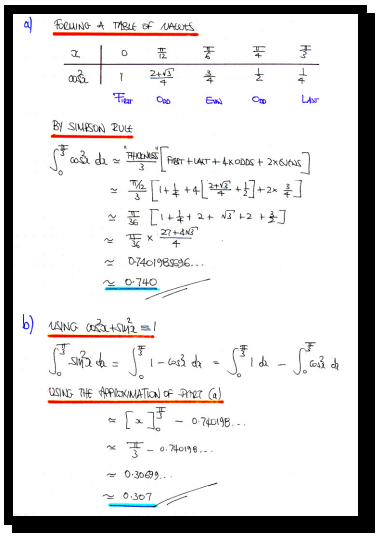
a) **Use Simpson's rule** with 4 equally spaced strips to find an estimate for

$$\int_0^{\frac{\pi}{3}} \cos^2 x \ dx .$$

b) **Use the answer of part (a)** to find an estimate for

$$\int_0^{\frac{\pi}{3}} \sin^2 x \ dx .$$

$\boxed{}$, $\boxed{\approx 0.740}$, $\boxed{\approx 0.307}$



Question 17 (+)**

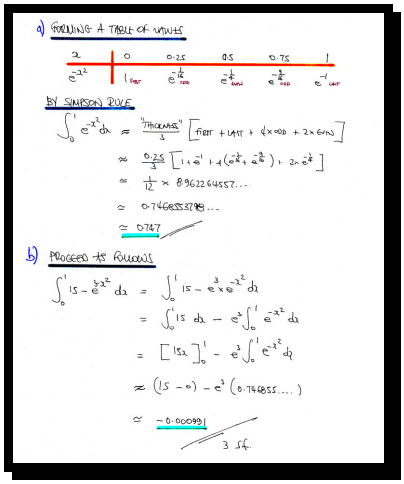
a) Use Simpson's rule with 4 equally spaced strips to find an estimate for

$$\int_0^1 \mathrm{e}^{-x^2}\ dx\,.$$

b) Use the answer of part **(a)** to find an estimate for

$$\int_0^1 15-\mathrm{e}^{3-x^2}\ dx\,.$$

$\boxed{}$, $\boxed{\approx 0.747}$, $\boxed{\approx 0.000991}$



Question 18 (*+)**

a) **Use the Simpson's rule** with 6 equally spaced strips to find an estimate, correct to 3 decimal places, for

$$\int_0^{1.2} \cos^2 x \ dx .$$

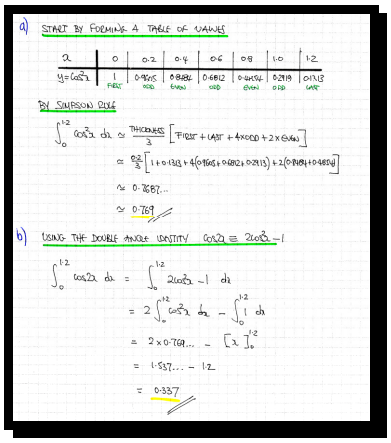
b) **Use the answer of part (a)** to find an estimate for

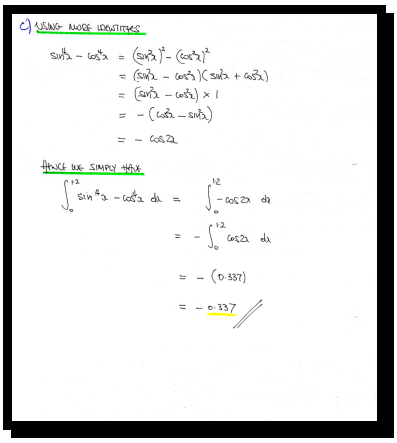
$$\int_0^{1.2} \cos 2x \ dx .$$

c) **Use the answer of part (b)** to find an estimate for

$$\int_0^{1.2} \left[\cos^4 x - \sin^4 x \right] dx .$$

$\boxed{}$, $\boxed{\approx 0.769}$, $\boxed{\approx 0.337}$, $\boxed{\approx -0.337}$





Question 19 (****)

$$I=\int_0^8 2^x \, dx$$

a) Use Simpson's rule with 8 equally spaced intervals to verify that

$$I \approx \frac{1105}{3}.$$

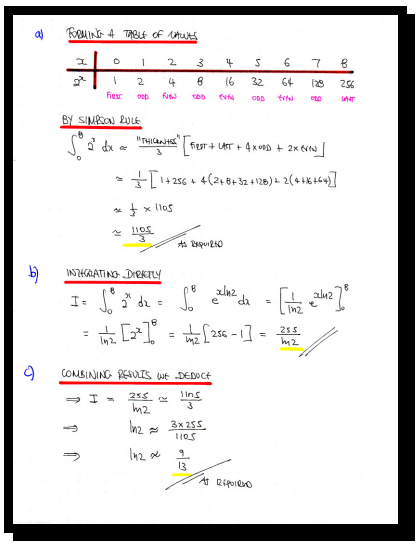
[The answer must be supported with detailed calculations.]

b) Find the exact value of I, by writing $2^x = \mathrm{e}^{x\ln 2}$.

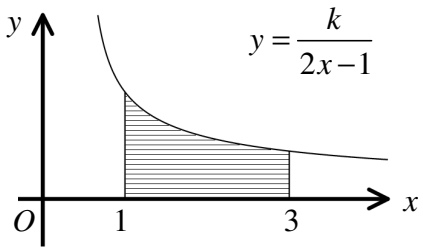
c) Hence show that

$$\ln 2 \approx \frac{9}{13}.$$

$\square$, $\boxed{I=\frac{255}{\ln 2}}$



Question 20 **(****+)**



The figure above shows part of the curve C with equation

$$y=\frac{k}{2x-1},$$

where k is a positive constant.

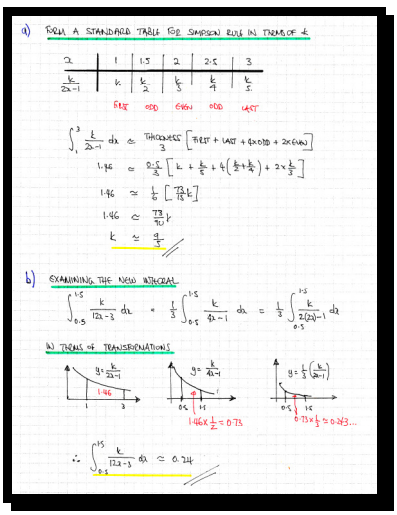
When Simpson's rule with 4 equally spaced strips is used, the area bounded by C, the x axis and the vertical straight lines with equations $x=1$ and $x=3$, is approximated to 30 square units.

a) Determine the value of k.

b) By considering suitable graph transformation, find an approximate value of

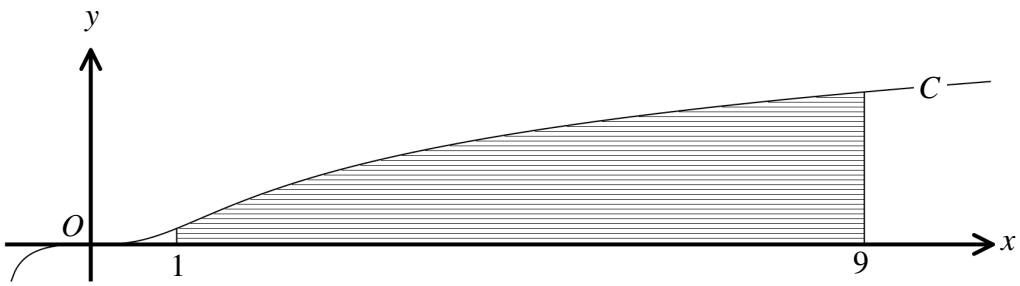
$$\int_{0.5}^{1.5}\frac{k}{12x-3}\,dx.$$

$\boxed{}$, $\boxed{k=\frac{9}{5}}$, $\boxed{\approx 0.24}$



MID-ORDINATE RULE

Question 1 (**)



The figure above shows part of the curve C with equation

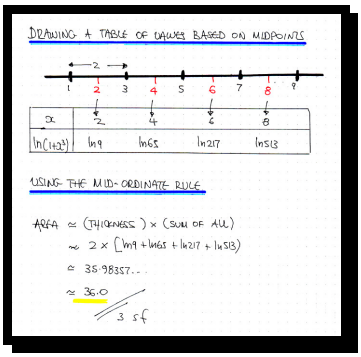
$$y=\ln\left(1+x^3\right),\ x>-1.$$

The area of the shaded region bounded by C, the x axis and the straight lines with equations $x=1$ and $x=9$ is to be estimated by the mid-ordinate rule using 4 equally spaced strips.

Find an estimate for the area of this region.

All steps in the calculation must be shown and the final answer must be correct to 3 significant figures.

$\boxed{}$, $\boxed{36.0}$



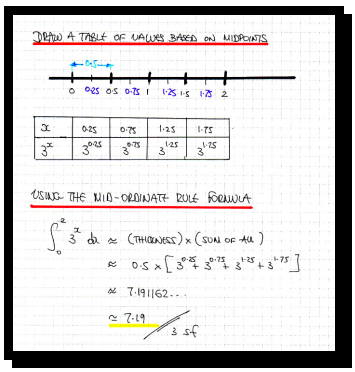
Question 2 (**)

$$I = \int_0^2 3^x \, dx$$

Use the mid-ordinate rule with 4 strips of equal width to obtain an estimate for I.

All steps in the calculation must be recorded and the final answer must be correct to three significant figures.

$\boxed{}$, $\boxed{7.19}$



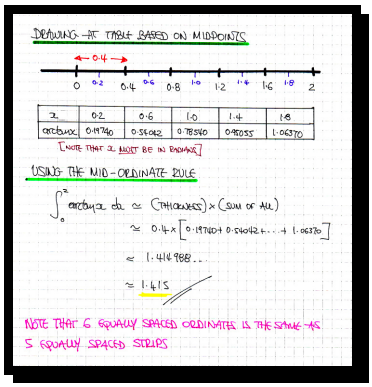
Question 3 (+)**

$$I = \int_0^2 \arctan x \ \ dx \ .$$

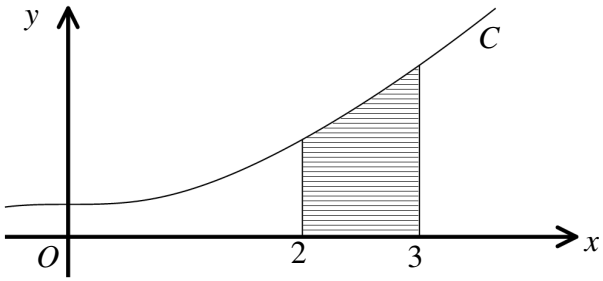
Use the mid-ordinate rule with 6 equally spaced ordinates to find an estimate for I .

All steps in the calculation must be shown and the final answer must be correct to 3 decimal places.

$\boxed{}$, $\boxed{1.415}$



Question 4 **(**+)**



The figure above shows part of the curve C with equation

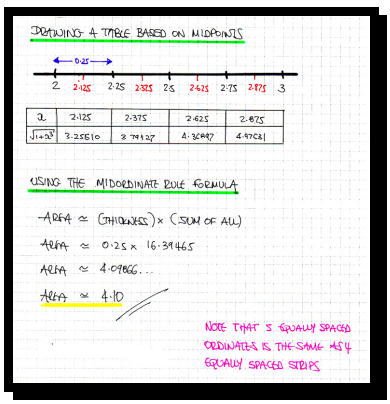
$$y=\sqrt{1+x^3}\ ,\ x\geq -1.$$

The shaded region is bounded by C, the x axis and the straight lines with equations $x=2$ and $x=3$ is to be estimated by the mid-ordinate rule using 5 equally spaced ordinates.

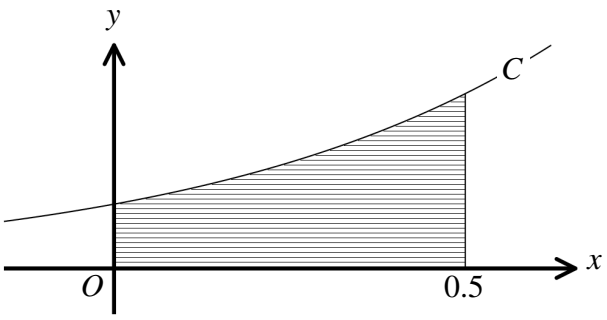
Calculate, correct to 2 decimal places the area of this region.

[The answer must be supported with a detailed method.]

$\boxed{}$, $\boxed{4.10}$



Question 5 (+)**



The figure above shows part of the curve C with equation

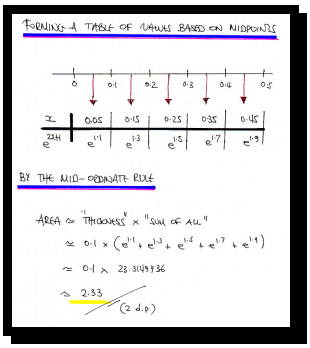
$$y = \mathrm{e}^{2x+1}, \ x \in \mathbb{R}.$$

Use the mid-ordinate rule with 6 equally spaced ordinates to estimate the area of the shaded region bounded by C, the x axis and the straight line with equation $x = 0.5$.

Give the answer correct to 2 decimal places.

[The answer must be supported with detailed calculations.]

$\boxed{}$, $\boxed{2.33}$



Question 6 (+)**

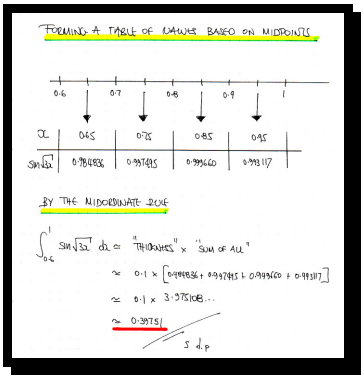
Use the mid-ordinate rule with 4 strips of equal width to find an estimate for

$$\int_{0.6}^{1} \sin\sqrt{3x}\ \ dx\,,$$

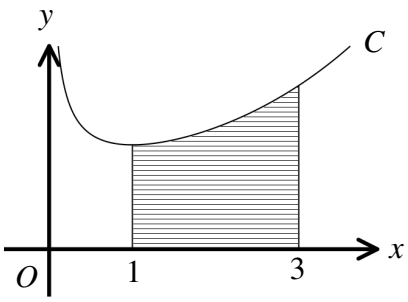
giving the final answer correct to five decimal places.

All steps in the calculations must be recorded.

$\boxed{}$, $\boxed{0.39751}$



Question 7 (***)



The figure above shows part of the curve C with equation

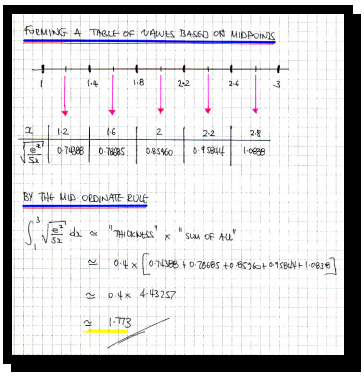
$$y=\sqrt{\frac{\mathrm{e}^x}{5x}}\,,\ x>0\,.$$

The area of the shaded region bounded by C, the x axis and the straight lines with equations $x=1$ and $x=3$ is to be estimated by the mid-ordinate rule using 5 equally spaced strips.

Find, correct to 3 decimal places, the area of this region.

[The answer must be supported with detailed calculations.]

$\boxed{}$, $\boxed{1.773}$



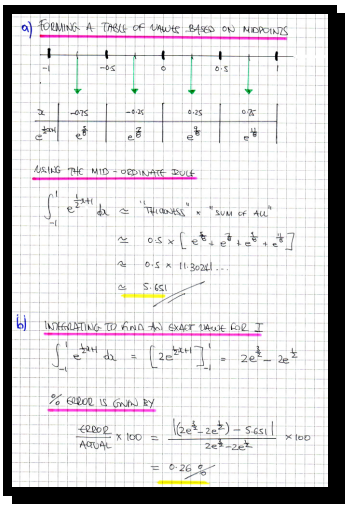
Question 8 **(***)**

$$I = \int_{-1}^{1} \mathrm{e}^{\frac{1}{2}x+1} \, dx$$

a) Use the mid-ordinate rule with 5 ordinates to find an estimate for I, giving the final answer correct to 3 decimal places.

b) Calculate the percentage error in the estimate of part **(a)**.

$\boxed{}$, $\boxed{5.651}$, $\boxed{0.26\%}$



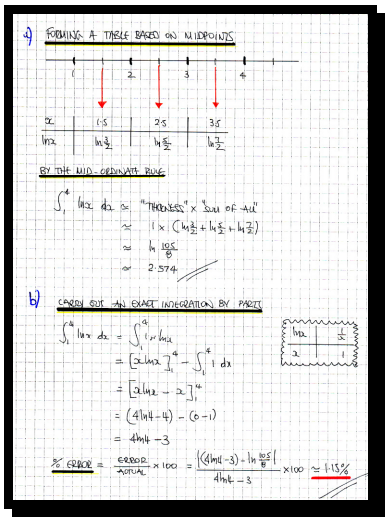
Question 9 **(***)**

$$I = \int_1^4 \ln x \ dx$$

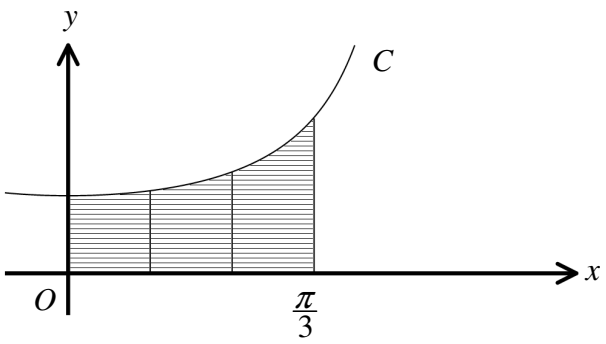
a) Use the mid-ordinate rule with 3 equally spaced strips to estimate the value of I, giving the final answer correct to 3 decimal places.

b) Calculate the percentage error in the estimate of part **(a)**.

$\boxed{}$, $\boxed{2.574}$, $\boxed{1.15\%}$



Question 10 (*+)**



The figure above shows part of the curve C with equation

$$y=\sec x,\quad 0\le x\le \frac{1}{3}\pi .$$

The shaded region bounded by C, the coordinate axes and the vertical straight line with equation $x=\frac{1}{3}\pi$ is to be estimated by the mid-ordinate rule using 3 equally spaced strips.

a) Find, correct to 3 decimal places, the area of this region.

The answer must be supported with detailed calculations.

b) Hence estimate the mean value of $y=\sec x$ in the interval $0\le x\le \frac{1}{3}\pi$.

$\boxed{}$, $\boxed{1.301}$, $\boxed{1.24}$

