INTEGRATION

BY TRIGONOMETRIC IDENTITIES

Question 1

Carry out the following integrations:

1. $\int 3\sin^2 x \, dx = \frac{3}{2}x - \frac{3}{4}\sin 2x + C$

2. $\int 4\cos^2 x \, dx = 2x + \sin 2x + C$

3. $\int 3\sin x \cos x \, dx = -\frac{3}{4}\cos 2x + C$

4. $\int (2 - 3\sin x)^2 \, dx = \frac{17}{2}x + 12\cos x - \frac{9}{4}\sin 2x + C$

5. $\int (1 - \cos 2x)^2 \, dx = \frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x + C$

6. $\int 2\tan^2 x \, dx = 2\tan x - 2x + C$

7. $\int 5\cot^2 x \, dx = -5\cot x - 5x + C$

8. $\int (2\tan x - \cot x)^2 \, dx = 4\tan x - \cot x - 9x + C$

9. $\int \frac{4\sin x}{\cos^2 x} \, dx = 4\sec x + C$

10. $\int \frac{\cos x}{3\sin^2 x} \, dx = -\frac{1}{3}\operatorname{cosec} x + C$

1. $\int 3\sin^2 x\,dx = \int 3\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx = \int \frac{3}{2}-\frac{3}{2}\cos 2x\,dx$
$= \frac{3}{2}x-\frac{3}{4}\sin 2x + C$

2. $\int 4\cos^2 x\,dx = \int 4\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx = \int 2+2\cos 2x\,dx$
$= 2x+\sin 2x + C$

3. $\int 3\sin x\cos x\,dx = \int \frac{3}{2}(2\sin x\cos x)\,dx = \int \frac{3}{2}\sin 2x\,dx = -\frac{3}{4}\cos 2x + C$

4. $\int (2-3\sin x)^2\,dx = \int 4-12\sin x+9\sin^2 x\,dx = \int 4-12\sin x+9\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx$
$= \int 4-12\sin x+\frac{9}{2}-\frac{9}{2}\cos 2x\,dx = \int \frac{17}{2}-12\sin x-\frac{9}{2}\cos 2x\,dx$
$= \frac{17}{2}x+12\cos x-\frac{9}{4}\sin 2x + C$

5. $\int (1-\cos 2x)^2\,dx = \int 1-2\cos 2x+\cos^2 2x\,dx$
$= \int 1-2\cos 2x+\left(\frac{1}{2}+\frac{1}{2}\cos 4x\right)dx$
$= \int 1-2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos 4x\,dx$
$= \int \frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\,dx$
$= \frac{3}{2}x-\sin 2x+\frac{1}{8}\sin 4x + C$

6. $\int 2\tan^2 x\,dx = \int 2\left(\sec^2 x-1\right)dx = \int 2\sec^2 x-2\,dx = 2\tan x-2x+C$

7. $\int 5\sec^2 x\,dx = \int 5\left(\sec^2 x-1\right)dx = \int 5\sec^2 x-5\,dx = 5\tan x-5x+C$

8. $\int (2\tan x-\cot x)^2\,dx = \int 4\tan^2 x-4\tan x\cot x+\cot^2 x\,dx$
$= \int 4\left(\sec^2 x-1\right)-4\tan x\left(\frac{1}{\tan x}\right)+\left(\csc^2 x-1\right)dx$
$= \int 4\sec^2 x-4-4+\csc^2 x-1\,dx$
$= \int 4\sec^2 x+\csc^2 x-9\,dx$
$= 4\tan x-\cot x-9x+C$

9. $\int \frac{4\sin x}{\cos^2 x}\,dx = \int \frac{4\sin x}{\cos x}\times\frac{1}{\cos x}\,dx = \int 4\tan x\sec x\,dx = 4\sec x+C$

10. $\int \frac{\cos x}{3\sin^2 x}\,dx = \int \frac{1}{3}\times\frac{\cos x}{\sin x}\times\frac{1}{\sin x}\,dx = \int \frac{1}{3}\cot x\csc x\,dx = -\frac{1}{3}\csc x+C$

Question 2

Carry out the following integrations:

1. $\displaystyle\int \left(2+\sin x\right)^2 \, dx = \frac{9}{2}x - 4\cos x - \frac{1}{4}\sin 2x + C$

2. $\displaystyle\int \sin x\left(1+\sec^2 x\right) \, dx = \sec x - \cos x + C$

3. $\displaystyle\int \left(1-2\cos x\right)^2 \, dx = 3x - 4\sin x + \sin 2x + C$

4. $\displaystyle\int \frac{1}{\cos^2 x \tan^2 x} \, dx = -\cot x + C$

5. $\displaystyle\int 2+2\tan^2 x \, dx = 2\tan x + C$

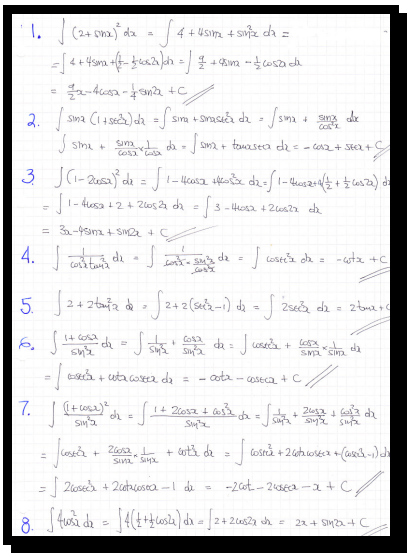
6. $\displaystyle\int \frac{1+\cos x}{\sin^2 x} \, dx = -\cot x - \operatorname{cosec} x + C$

7. $\displaystyle\int \frac{\left(1+\cos x\right)^2}{\sin^2 x} \, dx = -2\cot x - x - 2\operatorname{cosec} x + C$

8. $\displaystyle\int 4\cos^2 x \, dx = 2x + \sin 2x + C$

9. $\displaystyle\int 3\cot^2 x \, dx = -3\cot x - 3x + C$

10. $\displaystyle\int \left(2\cos x - 3\sin x\right)^2 \, dx = \frac{13}{2}x - \frac{5}{4}\sin 2x + 3\cos 2x + C$



1. $\int (2+\sin x)^2\,dx = \int 4+4\sin x+\sin^2 x\,dx =$
$= \int 4+4\sin x+\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx = \int \frac{9}{2}+4\sin x-\frac{1}{2}\cos 2x\,dx$
$= \frac{9}{2}x-4\cos x-\frac{1}{4}\sin 2x+C$

2. $\int \sin x\left(1+\sec^2 x\right)dx = \int \sin x+\sin x\sec^2 x\,dx = \int \sin x+\frac{\sin x}{\cos^2 x}\,dx$
$\int \sin x+\frac{\sin x}{\cos x}\times\frac{1}{\cos x}\,dx = \int \sin x+\tan x\sec x\,dx = -\cos x+\sec x+C$

3. $\int (1-2\cos x)^2\,dx = \int 1-4\cos x+4\cos^2 x\,dx = \int 1-4\cos x+4\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx$
$= \int 1-4\cos x+2+2\cos 2x\,dx = \int 3-4\cos x+2\cos 2x\,dx$
$= 3x-4\sin x+\sin 2x+C$

4. $\int \frac{1}{\cos^2 x\tan^2 x}\,dx = \int \frac{1}{\cos^2 x\times\frac{\sin^2 x}{\cos^2 x}}\,dx = \int \csc^2 x\,dx = -\cot x+C$

5. $\int 2+2\tan^2 x\,dx = \int 2+2\left(\sec^2 x-1\right)dx = \int 2\sec^2 x\,dx = 2\tan x+C$

6. $\int \frac{1+\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}+\frac{\cos x}{\sin^2 x}\,dx = \int \csc^2 x+\frac{\cos x}{\sin x}\times\frac{1}{\sin x}\,dx$
$= \int \csc^2 x+\cot x\csc x\,dx = -\cot x-\csc x+C$

7. $\int \frac{(1+\cos x)^2}{\sin^2 x}\,dx = \int \frac{1+2\cos x+\cos^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}+\frac{2\cos x}{\sin^2 x}+\frac{\cos^2 x}{\sin^2 x}\,dx$
$= \int \csc^2 x+\frac{2\cos x}{\sin x}\times\frac{1}{\sin x}+\cot^2 x\,dx = \int \csc^2 x+2\cot x\csc x+\left(\csc^2 x-1\right)dx$
$= \int 2\csc^2 x+2\cot x\csc x-1\,dx = -2\cot x-2\csc x-x+C$

8. $\int 4\cos^2 x\,dx = \int 4\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx = \int 2+2\cos 2x\,dx = 2x+\sin 2x+C$

9. $\int 3\cot^2 x\,dx = \int 3\left(\csc^2 x-1\right)dx = \int 3\csc^2 x-3\,dx$
$= -3\cot x-3x+C$

10. $\int (2\cos x-3\sin x)^2\,dx = \int 4\cos^2 x-12\cos x\sin x+9\sin^2 x\,dx$
$= \int 4\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)-6\sin 2x+9\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx$
$= \int 2+2\cos 2x-6\sin 2x+\frac{9}{2}-\frac{9}{2}\cos 2x\,dx = \int \frac{13}{2}-\frac{5}{2}\cos 2x-6\sin 2x\,dx$
$= \frac{13}{2}x-\frac{5}{4}\sin 2x+3\cos 2x+C$

Question 3

Carry out the following integrations:

1. $\displaystyle\int \sin 2x \operatorname{cosec} x \, dx = 2\sin x + C$

2. $\displaystyle\int \frac{1+\sin x}{\cos^2 x} \, dx = \sec x + \tan x + C$

3. $\displaystyle\int \tan^2 x \, dx = \tan x - x + C$

4. $\displaystyle\int \frac{(1+\sin x)^2}{\cos^2 x} \, dx = 2\tan x + 2\sec x - x + C$

5. $\displaystyle\int \frac{\cos^2 x}{1+\sin x} \, dx = x + \cos x + C$

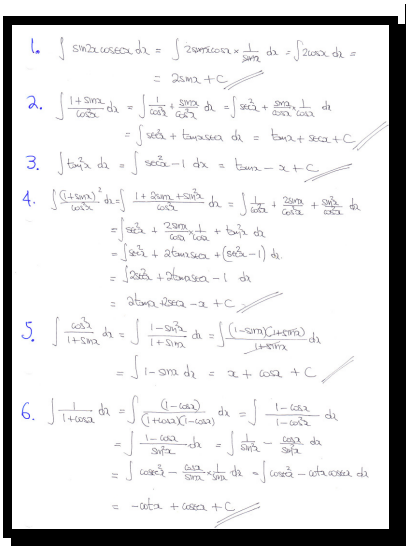
6. $\displaystyle\int \frac{1}{1+\cos x} \, dx = \operatorname{cosec} x - \cot x + C$

7. $\displaystyle\int \frac{(1+2\cos x)^2}{3\sin^2 x} \, dx = -\frac{5}{3}\cot x - \frac{4}{3}\operatorname{cosec} x - \frac{4}{3}x + C$

8. $\displaystyle\int \sin x \sin 3x \, dx = \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$

9. $\displaystyle\int \sin^2 2x \, dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

10. $\displaystyle\int 2\cos 3x \sin x \, dx = \frac{1}{2}\cos 2x - \frac{1}{4}\cos 4x + C$



1. $\int \sin 2x \operatorname{cosec} x \, dx = \int 2\sin x \cos x \times \frac{1}{\sin x} \, dx = \int 2\cos x \, dx =$
$= 2\sin x + C$

2. $\int \frac{1+\sin x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \, dx = \int \sec^2 x + \frac{\sin x}{\cos x} \frac{1}{\cos x} \, dx$
$= \int \sec^2 x + \tan x \sec x \, dx = \tan x + \sec x + C$

3. $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx = \tan x - x + C$

4. $\int \frac{(1+\tan x)^2}{\cos^2 x} \, dx = \int \frac{1 + 2\sin x + \sin^2 x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \, dx$
$= \int \sec^2 x + \frac{2\sin x}{\cos x} \frac{1}{\cos x} + \tan^2 x \, dx$
$= \int \sec^2 x + 2\tan x \sec x + \left(\sec^2 x - 1\right) dx$
$= \int 2\sec^2 x + 2\tan x \sec x - 1 \, dx$
$= 2\tan x + 2\sec x - x + C$

5. $\int \frac{\cos^2 x}{1+\sin x} \, dx = \int \frac{1-\sin^2 x}{1+\sin x} \, dx = \int \frac{(1-\sin x)(1+\sin x)}{1+\sin x} \, dx$
$= \int 1 - \sin x \, dx = x + \cos x + C$

6. $\int \frac{1}{1+\cos x} \, dx = \int \frac{(1-\cos x)}{(1+\cos x)(1-\cos x)} \, dx = \int \frac{1-\cos x}{1-\cos^2 x} \, dx$
$= \int \frac{1-\cos x}{\sin^2 x} \, dx = \int \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} \, dx$
$= \int \operatorname{cosec}^2 x - \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \, dx = \int \operatorname{cosec}^2 x - \cot x \operatorname{cosec} x \, dx$
$= -\cot x + \operatorname{cosec} x + C$

7. $\int \frac{(1+2\cos x)^2}{3\sin^2 x} \, dx = \int \frac{1 + 4\cos x + 4\cos^2 x}{3\sin^2 x} \, dx = \int \frac{1}{3\sin^2 x} + \frac{4\cos x}{3\sin^2 x} + \frac{4\cos^2 x}{3\sin^2 x} \, dx$
$= \int \frac{1}{3}\operatorname{cosec}^2 x + \frac{4}{3}\frac{\cos x}{\sin^2 x}\frac{1}{\sin x} + \frac{4}{3}\cot^2 x \, dx$
$= \int \frac{1}{3}\operatorname{cosec}^2 x + \frac{4}{3}\cot x \operatorname{cosec} x + \frac{4}{3}\left(\operatorname{cosec}^2 x - 1\right) dx$
$= \int \frac{5}{3}\operatorname{cosec}^2 x + \frac{4}{3}\cot x \operatorname{cosec} x - \frac{4}{3} \, dx$
$= -\frac{5}{3}\cot x - \frac{4}{3}\operatorname{cosec} x - \frac{4}{3}x + C$

8. $\cos(3x+x) = \cos 3x \cos x - \sin 3x \sin x$
$\cos(3x-x) = \cos 3x \cos x + \sin 3x \sin x$
$\cos 2x - \cos 4x = 2\sin 3x \sin x$ ← subtract "rows"
$\sin 3x \sin x = \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x$
$\therefore \int \sin 3x \sin x \, dx = \int \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x \, dx = \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$

9. $\int \sin^2 2x \, dx = \int \frac{1}{2} - \frac{1}{2}\cos 4x \, dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

10. $\sin(3x+x) = \sin 3x \cos x + \cos 3x \sin x$
$\sin(3x-x) = \sin 3x \cos x - \cos 3x \sin x$
$\sin 4x - \sin 2x = 2\cos 3x \sin x$
$\therefore 2\cos 3x \sin x = \sin 4x - \sin 2x$
$\therefore \int 2\cos 3x \sin x \, dx = \int \sin 4x - \sin 2x \, dx$
$= -\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x + C$

Question 4

Carry out the following integrations:

1. $\displaystyle\int \frac{\cos 2x}{1-\cos^2 2x}\,dx = -\frac{1}{2}\operatorname{cosec} 2x + C$

2. $\displaystyle\int \cot^2 3x\,dx = -x - \frac{1}{3}\cot 3x + C$

3. $\displaystyle\int \sin 2x \sec x\,dx = -2\cos x + C$

4. $\displaystyle\int \frac{1}{\sin x \cos^2 x}\,dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + \sec x + C$

5. $\displaystyle\int \frac{1}{\sec x - 1}\,dx = -x - \cot x - \operatorname{cosec} x + C$

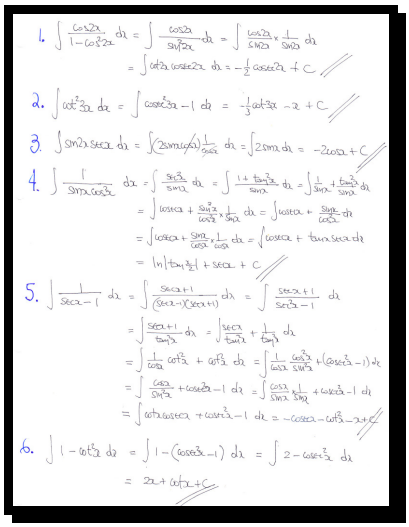
6. $\displaystyle\int 1-\cot^2 x\,dx = 2x + \cot x + C$

7. $\displaystyle\int (2\cos x - 3)^2\,dx = 11x + \sin 2x - 12\sin x + C$

8. $\displaystyle\int (3\sin x - \cos x)^2\,dx = 5x - 2\sin 2x + \frac{3}{2}\cos 2x + C = 5x - 2\sin 2x - 3\sin^2 x + C$

9. $\displaystyle\int \frac{1}{\cos x \sin^2 x}\,dx = \ln|\sec x + \tan x| - \operatorname{cosec} x + C$

10. $\displaystyle\int \sin^2 x \sec^2 x\,dx = \tan x - x + C$



1. $\int \frac{\cos 2x}{1-\cos^2 2x}\,dx = \int \frac{\cos 2x}{\sin^2 2x}\,dx = \int \frac{\cos 2x}{\sin 2x}\times\frac{1}{\sin 2x}\,dx$
$= \int \cot 2x \operatorname{cosec} 2x\,dx = -\frac{1}{2}\operatorname{cosec} 2x + C$

2. $\int \cot^2 3x\,dx = \int \operatorname{cosec}^2 3x - 1\,dx = -\frac{1}{3}\cot 3x - x + C$

3. $\int \sin 2x \sec x\,dx = \int (2\sin x \cos x)\frac{1}{\cos x}\,dx = \int 2\sin x\,dx = -2\cos x + C$

4. $\int \frac{1}{\sin x \cos^2 x}\,dx = \int \frac{\sec^2 x}{\sin x}\,dx = \int \frac{1+\tan^2 x}{\sin x}\,dx = \int \frac{1}{\sin x} + \frac{\tan^2 x}{\sin x}\,dx$
$= \int \operatorname{cosec} x + \frac{\sin^2 x}{\cos^2 x}\times\frac{1}{\sin x}\,dx = \int \operatorname{cosec} x + \frac{\sin x}{\cos^2 x}\,dx$
$= \int \operatorname{cosec} x + \frac{\sin x}{\cos x}\times\frac{1}{\cos x}\,dx = \int \operatorname{cosec} x + \tan x \sec x\,dx$
$= \ln\left|\tan\frac{x}{2}\right| + \sec x + C$

5. $\int \frac{1}{\sec x - 1}\,dx = \int \frac{\sec x + 1}{(\sec x - 1)(\sec x + 1)}\,dx = \int \frac{\sec x + 1}{\sec^2 x - 1}\,dx$
$= \int \frac{\sec x + 1}{\tan^2 x}\,dx = \int \frac{\sec x}{\tan^2 x} + \frac{1}{\tan^2 x}\,dx$
$= \int \frac{1}{\cos x}\cot^2 x + \cot^2 x\,dx = \int \frac{1}{\cos x}\frac{\cos^2 x}{\sin^2 x} + (\operatorname{cosec}^2 x - 1)\,dx$
$= \int \frac{\cos x}{\sin^2 x} + \operatorname{cosec}^2 x - 1\,dx = \int \frac{\cos x}{\sin x}\frac{1}{\sin x} + \operatorname{cosec}^2 x - 1\,dx$
$= \int \cot x \operatorname{cosec} x + \operatorname{cosec}^2 x - 1\,dx = -\operatorname{cosec} x - \cot x - x + C$

6. $\int 1 - \cot^2 x\,dx = \int 1 - (\operatorname{cosec}^2 x - 1)\,dx = \int 2 - \operatorname{cosec}^2 x\,dx$
$= 2x + \cot x + C$

7. $\int (2\cos x - 3)^2\,dx = \int 4\cos^2 x - 12\cos x + 9\,dx$
$= \int 4\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) - 12\cos x + 9\,dx = \int 11 + 2\cos 2x - 12\cos x\,dx$
$= 11x + \sin 2x - 12\sin x + C$

8. $\int (3\sin x - \cos x)^2\,dx = \int 9\sin^2 x - 6\sin x\cos x + \cos^2 x\,dx$
$= \int 9\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) - 3(2\sin x\cos x) + \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$
$= \int 5 - 4\cos 2x - 3\sin 2x\,dx$
$= 5x - 2\sin 2x + \frac{3}{2}\cos 2x + C$

9. $\int \frac{1}{\cos x \sin^2 x}\,dx = \int \frac{\operatorname{cosec}^2 x}{\cos x}\,dx = \int \frac{1+\cot^2 x}{\cos x}\,dx = \int \frac{1}{\cos x} + \frac{\cot^2 x}{\cos x}\,dx$
$= \int \sec x + \frac{\cot^2 x}{\sin^2 x}\times\frac{1}{\cos x}\,dx = \int \sec x + \frac{\cos x}{\sin^2 x}\,dx$
$= \int \sec x + \frac{\cos x}{\sin x}\times\frac{1}{\sin x}\,dx = \int \sec x + \cot x \operatorname{cosec} x\,dx$
$= \ln\left|\sec x + \tan x\right| - \operatorname{cosec} x + C$

10. $\int \sin^2 x \sec^3 x\,dx = \int \sin^2 x \times \frac{1}{\cos^2 x}\,dx = \int \tan^2 x\,dx$
$= \int \sec^2 x - 1\,dx = \tan x - x + C$

Question 5

Carry out the following integrations:

1. $\displaystyle\int \sin 3x\cos 2x\ dx = -\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$

2. $\displaystyle\int \frac{1}{\sin x\cos x}\ dx = -\frac{1}{2}\ln\left|\operatorname{cosec} 2x + \cot 2x\right| + C = \ln\left|\tan x\right| + C$

3. $\displaystyle\int \frac{1}{1-\sin x}\ dx = \sec x + \tan x + C$

4. $\displaystyle\int \sin^2 2x\ dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

5. $\displaystyle\int \frac{\cos 2x}{\cos^2 x}\ dx = 2x - \tan x + C$

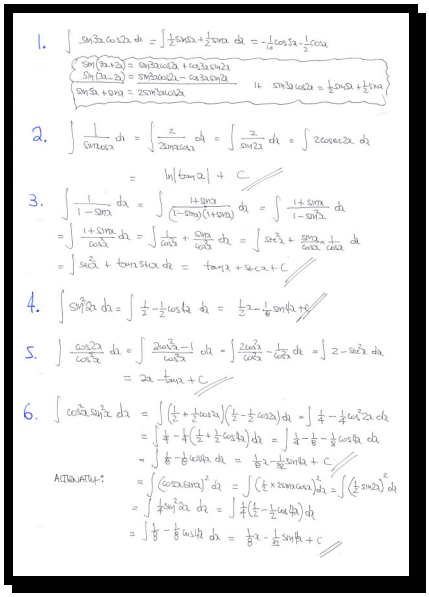
6. $\displaystyle\int \cos^2 x\sin^2 x\ dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

7. $\displaystyle\int \left(\sin x + 2\cos x\right)^2\ dx = \frac{5}{2}x + 2\sin^2 x + \frac{3}{4}\sin 2x + C$

8. $\displaystyle\int \frac{1}{\sin^2 x\cos^2 x}\ dx = -2\cot 2x + C$

9. $\displaystyle\int \sqrt{\sin^2 x + \left(\cos x - 1\right)^2}\ dx = -4\cos\left(\frac{x}{2}\right) + C$

10. $\displaystyle\int \frac{1-\cos x}{1+\cos x}\ dx = 2\tan\left(\frac{x}{2}\right) - x + C = -2\cot x - x + 2\operatorname{cosec} x + C$



1. $\int \sin 3x \cos 2x \, dx = \int \frac{1}{2}\sin 5x + \frac{1}{2}\sin x \, dx = -\frac{1}{10}\cos 5x - \frac{1}{2}\cos x$

$\sin(3x+2x) = \sin 3x\cos 2x + \cos 3x \sin 2x$
$\sin(3x-2x) = \sin 3x\cos 2x - \cos 3x\sin 2x$
$\sin 5x + \sin x = 2\sin 3x\cos 2x$ $\quad \therefore \sin 3x\cos 2x = \frac{1}{2}\sin 5x + \frac{1}{2}\sin x$

2. $\int \frac{1}{\sin x \cos x} \, dx = \int \frac{2}{2\sin x\cos x} \, dx = \int \frac{2}{\sin 2x} \, dx = \int 2\operatorname{cosec} 2x \, dx$
$= \ln|\tan x| + C$

3. $\int \frac{1}{1-\sin x} \, dx = \int \frac{1+\sin x}{(1-\sin x)(1+\sin x)} \, dx = \int \frac{1+\sin x}{1-\sin^2 x} \, dx$
$= \int \frac{1+\sin x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \, dx = \int \sec^2 x + \frac{\sin x}{\cos x}\frac{1}{\cos x} \, dx$
$= \int \sec^2 x + \tan x \sec x \, dx = \tan x + \sec x + C$

4. $\int \sin^2 2x \, dx = \int \frac{1}{2} - \frac{1}{2}\cos 4x \, dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

5. $\int \frac{\cos 2x}{\cos^2 x} \, dx = \int \frac{2\cos^2 x - 1}{\cos^2 x} \, dx = \int \frac{2\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} \, dx = \int 2 - \sec^2 x \, dx$
$= 2x - \tan x + C$

6. $\int \cos^2 3x \sin^2 x \, dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 6x\right)\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx = \int \frac{1}{4} - \frac{1}{4}\cos^2 2x \, dx$
$= \int \frac{1}{4} - \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx = \int \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4x \, dx$
$= \int \frac{1}{8} - \frac{1}{8}\cos 4x \, dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

ALTERNATIVE:
$= \int (\cos 3x \sin x)^2 \, dx = \int \left(\frac{1}{2} \times 2\sin x\cos 3x\right)^2 dx = \int \left(\frac{1}{2}\sin 2x\right)^2 dx$
$= \int \frac{1}{4}\sin^2 2x \, dx = \int \frac{1}{4}\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx$
$= \int \frac{1}{8} - \frac{1}{8}\cos 4x \, dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

7. $\int (\sin x + 2\cos x)^2 \, dx = \int \sin^2 x + 4\sin x\cos x + 4\cos^2 x \, dx$
$= \int \frac{1}{2} - \frac{1}{2}\cos 2x + 2\sin 2x + 4\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$
$= \int \frac{5}{2} + \frac{3}{2}\cos 2x + 2\sin 2x \, dx = \frac{5}{2}x + \frac{3}{4}\sin 2x - \cos 2x + C$

8. $\int \frac{1}{\sin^2 x \cos^2 x} \, dx = \int \frac{1}{(\sin x\cos x)^2} \, dx = \int \frac{1}{\left(\frac{1}{2}\times 2\sin x\cos x\right)^2} \, dx = \int \frac{1}{\left(\frac{1}{2}\sin 2x\right)^2} \, dx$
$= \int \frac{1}{\frac{1}{4}\sin^2 2x} \, dx = \int 4\operatorname{cosec}^2 2x \, dx = -2\cot 2x + C$

ALTERNATIVE $= \int \frac{1}{\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)} \, dx = \int \frac{1}{\frac{1}{4} - \frac{1}{4}\cos^2 2x} \, dx$
$= \int \frac{4}{1-\cos^2 2x} \, dx = \int \frac{4}{\sin^2 2x} \, dx = \int 4\operatorname{cosec}^2 2x$
$= -2\cot 2x + C$

9. $\int \sqrt{\sin^2 x + (\cos x - 1)^2} \, dx = \int \sqrt{\sin^2 x + \cos^2 x - 2\cos x + 1} \, dx$
$= \int \sqrt{1 + 1 - 2\cos x} \, dx = \int \sqrt{2 - 2\cos x} \, dx$
$\left(\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow \cos\left(2\times\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)\right)$
$= \int \sqrt{2 - 2\left(1 - 2\sin^2\frac{x}{2}\right)} \, dx = \int \sqrt{4\sin^2\frac{x}{2}} \, dx$
$= \int 2\sin\frac{x}{2} \, dx = -4\cos\frac{x}{2} + C$

10. $\int \frac{1-\cos x}{1+\cos x} \, dx = \int \frac{(1-\cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)} \, dx = \int \frac{1 - 2\cos x + \cos^2 x}{1-\cos^2 x} \, dx$
$= \int \frac{1}{\sin^2 x} - \frac{2\cos x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \, dx = \int \operatorname{cosec}^2 x - 2\frac{\cos x}{\sin x}\frac{1}{\sin x} + \cot^2 x \, dx$
$= \int \operatorname{cosec}^2 x - 2\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x - 1 \, dx$
$= \int 2\operatorname{cosec}^2 x - 2\cot x \operatorname{cosec} x - 1 \, dx$
$= -2\cot x + 2\operatorname{cosec} x - x + C$

Can also be done by "little t" substitution

Question 6

Carry out the following integrations:

1. $\displaystyle\int \frac{1+\sin x}{1-\sin x}\,dx = 2\tan x - x + 2\sec x + C$

2.

Question 7

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{2}} 4\sin^2 x \ dx = \pi$

2. $\displaystyle\int_0^{\frac{\pi}{6}} 24\cos^2 x \ dx = \pi + 3$

3. $\displaystyle\int_0^{\frac{\pi}{6}} 8\sin x \cos x \ dx = 1$

4. $\displaystyle\int_0^{\frac{\pi}{2}} \left(1-\sin x\right)^2 \ dx = \frac{3\pi}{2} - 4$

5. $\displaystyle\int_0^{\frac{\pi}{6}} \left(1-\cos 3x\right)^2 \ dx = \frac{\pi}{4} - \frac{2}{3}$

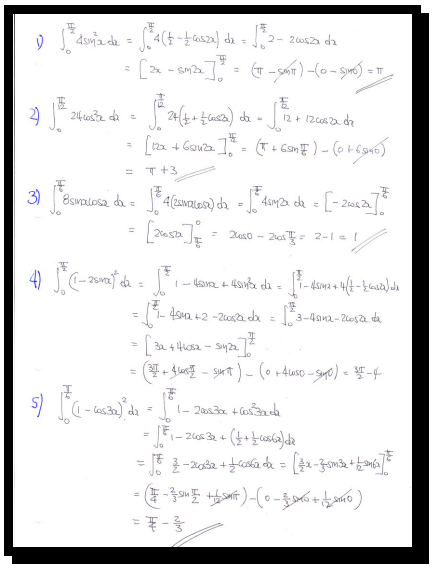
6. $\displaystyle\int_0^{\frac{\pi}{4}} 4\tan^2 x \ dx = 4 - \pi$

7. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(3\cot x + \tan x\right)^2 \ dx = \frac{2}{3}\left(10\sqrt{3} - \pi\right)$

8. $\displaystyle\int_0^{\frac{\pi}{4}} \left(\sec x + 4\cos x\right)^2 \ dx = 4\pi + 5$

9. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x} \ dx = 1$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \ dx = \sqrt{2} - 1$



1) $\int_0^{\frac{\pi}{2}} 4\sin^2 2x \, dx = \int_0^{\frac{\pi}{2}} 4\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx = \int_0^{\frac{\pi}{2}} 2 - 2\cos 4x \, dx$

$= \left[2x - \sin 4x\right]_0^{\frac{\pi}{2}} = (\pi - \sin 2\pi) - (0 - \sin 0) = \pi$

2) $\int_0^{\frac{\pi}{12}} 24\cos^2 3x \, dx = \int_0^{\frac{\pi}{12}} 24\left(\frac{1}{2} + \frac{1}{2}\cos 6x\right) dx = \int_0^{\frac{\pi}{12}} 12 + 12\cos 6x \, dx$

$= \left[12x + 6\sin 6x\right]_0^{\frac{\pi}{12}} = \left(\pi + 6\sin\frac{\pi}{2}\right) - (0 + 6\sin 0)$

$= \pi + 3$

3) $\int_0^{\frac{\pi}{6}} 8\sin x\cos x \, dx = \int_0^{\frac{\pi}{6}} 4(2\sin x\cos x) \, dx = \int_0^{\frac{\pi}{6}} 4\sin 2x \, dx = \left[-2\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left[2\cos 2x\right]_{\frac{\pi}{6}}^{0} = 2\cos 0 - 2\cos\frac{\pi}{3} = 2 - 1 = 1$

4) $\int_0^{\frac{\pi}{6}} (1 - 2\sin x)^2 \, dx = \int_0^{\frac{\pi}{6}} 1 - 4\sin x + 4\sin^2 x \, dx = \int_0^{\frac{\pi}{6}} 1 - 4\sin x + 4\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$= \int_0^{\frac{\pi}{6}} 1 - 4\sin x + 2 - 2\cos 2x \, dx = \int_0^{\frac{\pi}{6}} 3 - 4\sin x - 2\cos 2x \, dx$

$= \left[3x + 4\cos x - \sin 2x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{3\pi}{2} + 4\cos\frac{\pi}{2} - \sin\pi\right) - (0 + 4\cos 0 - \sin 0) = \frac{3\pi}{2} - 4$

5) $\int_0^{\frac{\pi}{6}} (1 - \cos 3x)^2 \, dx = \int_0^{\frac{\pi}{6}} 1 - 2\cos 3x + \cos^2 3x \, dx$

$= \int_0^{\frac{\pi}{6}} 1 - 2\cos 3x + \left(\frac{1}{2} + \frac{1}{2}\cos 6x\right) dx$

$= \int_0^{\frac{\pi}{6}} \frac{3}{2} - 2\cos 3x + \frac{1}{2}\cos 6x \, dx = \left[\frac{3}{2}x - \frac{2}{3}\sin 3x + \frac{1}{12}\sin 6x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{\pi}{4} - \frac{2}{3}\sin\frac{\pi}{2} + \frac{1}{12}\sin\pi\right) - \left(0 - \frac{2}{3}\sin 0 + \frac{1}{12}\sin 0\right)$

$= \frac{\pi}{4} - \frac{2}{3}$

6) $\int_0^{\frac{\pi}{4}} 4\tan^2 x \, dx = \int_0^{\frac{\pi}{4}} 4(\sec^2 x - 1) \, dx = \int_0^{\frac{\pi}{4}} 4\sec^2 x - 4 \, dx$

$= \left[4\tan x - 4x\right]_0^{\frac{\pi}{4}} = \left(4\tan\frac{\pi}{4} - \pi\right) - \left(4\tan 0 - 0\right)$

$= 4 - \pi$

7) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (3\cot x + \tan x)^2 \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 9\cot^2 x + 6\cot x\tan x + \tan^2 x \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 9(\operatorname{cosec}^2 x - 1) + 6 + (\sec^2 x - 1) \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 9\operatorname{cosec}^2 x + \sec^2 x - 4 \, dx$

$= \left[-9\cot x + \tan x - 4x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \left[9\cot x + 4x - \tan x\right]_{\frac{\pi}{3}}^{\frac{\pi}{6}}$

$= \left(9\cot\frac{\pi}{6} + \frac{4\pi}{3} - \tan\frac{\pi}{6}\right) - \left(9\cot\frac{\pi}{3} + \frac{4\pi}{3} - \tan\frac{\pi}{3}\right)$

$= \left(9\sqrt{3} + \frac{2\pi}{3} - \frac{\sqrt{3}}{3}\right) - \left(3\sqrt{3} + \frac{4\pi}{3} - \sqrt{3}\right)$

$= 7\sqrt{3} - \frac{1}{3}\sqrt{3} - \frac{2\pi}{3} = \frac{20}{3}\sqrt{3} - \frac{2\pi}{3} = \frac{2}{3}(10\sqrt{3} - \pi)$

8) $\int_0^{\frac{\pi}{4}} (\sec x + 4\cos x)^2 \, dx = \int_0^{\frac{\pi}{4}} \sec^2 x + 8\sec x\cos x + 16\cos^2 x \, dx$

$= \int_0^{\frac{\pi}{4}} \sec^2 x + 8 + 16\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$

$= \int_0^{\frac{\pi}{4}} \sec^2 x + 16 + 8\cos 2x \, dx$

$= \left[\tan x + 16x + 4\sin 2x\right]_0^{\frac{\pi}{4}}$

$= \left(\tan\frac{\pi}{4} + 4\pi + 4\sin\frac{\pi}{2}\right) - \left(\tan 0 + 4\sin 0\right)$

$= 1 + 4\pi + 4$

$= 4\pi + 5$

9. $\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x} \, dx = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \, dx = \int_0^{\frac{\pi}{3}} \tan x\sec x \, dx$

$= \left[\sec x\right]_0^{\frac{\pi}{3}} = \sec\frac{\pi}{3} - \sec 0$

$= 2 - 1 = 1$

10. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\operatorname{cosec} x \, dx$

$= \left[-\operatorname{cosec} x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left[\operatorname{cosec} x\right]_{\frac{\pi}{2}}^{\frac{\pi}{4}}$

$= \operatorname{cosec}\frac{\pi}{4} - \operatorname{cosec}\frac{\pi}{2} = \sqrt{2} - 1$

Question 8

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 x \ dx = \frac{1}{8}(\pi + 2)$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \ dx = \frac{\pi}{4}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} (2\sin x - 3\cos x)^2 \ dx = \frac{1}{4}(13\pi - 24)$

4. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 \ dx = 4\pi + 3\sqrt{3}$

5. $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 x \ dx = \frac{1}{4}(4 - \pi)$

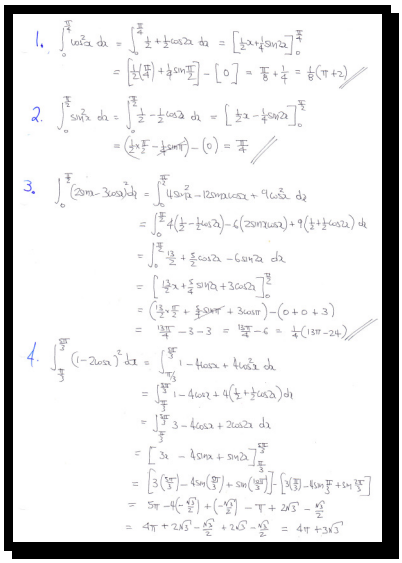
6. $\displaystyle\int_0^{\frac{\pi}{6}} \sin x \sin 3x \ dx = \frac{\sqrt{3}}{16}$

7. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} \ dx = 1 + \sqrt{3}$

8. $\displaystyle\int_0^{\frac{\pi}{2}} \left(1 + \tan \tfrac{x}{2}\right)^2 \ dx = 2 + \ln 4$

9. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x \ dx = \frac{2}{3}$

10. $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cot^2 2x \ dx = \frac{1}{2} - \frac{\sqrt{3}}{6} - \frac{\pi}{24}$



1. $\int_0^{\frac{\pi}{4}} \cos^2 2x \, dx = \int_0^{\frac{\pi}{4}} \frac{1}{2} + \frac{1}{2}\cos 4x \, dx = \left[\frac{1}{2}x + \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{4}}$
$= \left(\frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{8}\sin\pi\right) - \left[0\right] = \frac{\pi}{8} + \frac{1}{4} = \frac{1}{8}(\pi + 2)$

2. $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$
$= \left(\frac{1}{2}\times\frac{\pi}{2} - \frac{1}{4}\sin\pi\right) - \left(0\right) = \frac{\pi}{4}$

3. $\int_0^{\frac{\pi}{2}} (2\sin x - 3\cos x)^2 dx = \int_0^{\frac{\pi}{2}} 4\sin^2 x - 12\sin x\cos x + 9\cos^2 x \, dx$
$= \int_0^{\frac{\pi}{2}} 4\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) - 6(2\sin x\cos x) + 9\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$
$= \int_0^{\frac{\pi}{2}} \frac{13}{2} + \frac{5}{2}\cos 2x - 6\sin 2x \, dx$
$= \left[\frac{13}{2}x + \frac{5}{4}\sin 2x + 3\cos 2x\right]_0^{\frac{\pi}{2}}$
$= \left(\frac{13}{2}\times\frac{\pi}{2} + \frac{5}{4}\sin\pi + 3\cos\pi\right) - (0 + 0 + 3)$
$= \frac{13\pi}{4} - 3 - 3 = \frac{13\pi}{4} - 6 = \frac{1}{4}(13\pi - 24)$

4. $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 dx = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 1 - 4\cos x + 4\cos^2 x \, dx$
$= \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 1 - 4\cos x + 4\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$
$= \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 3 - 4\cos x + 2\cos 2x \, dx$
$= \left[3x - 4\sin x + \sin 2x\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$
$= \left[3\left(\frac{5\pi}{3}\right) - 4\sin\left(\frac{5\pi}{3}\right) + \sin\left(\frac{10\pi}{3}\right)\right] - \left[3\left(\frac{\pi}{3}\right) - 4\sin\frac{\pi}{3} + \sin\frac{2\pi}{3}\right]$
$= 5\pi - 4\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) - \pi + 2\sqrt{3} - \frac{\sqrt{3}}{2}$
$= 4\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2} + 2\sqrt{3} - \frac{\sqrt{3}}{2} = 4\pi + 3\sqrt{3}$

5. $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \int_0^{\frac{\pi}{4}} \sec^2 x - 1 \, dx = \left[\tan x - x\right]_0^{\frac{\pi}{4}}$
$= \left[\tan\frac{\pi}{4} - \frac{\pi}{4}\right] - \left[0\right] = 1 - \frac{\pi}{4} = \frac{1}{4}(4 - \pi)$

6. $\cos(3x + x) = \cos 3x\cos x - \sin 3x\sin x$
$\cos(3x - x) = \cos 3x\cos x + \sin 3x\sin x$
SUBTRACT EQUATIONS "UPWARDS"
$\cos 2x - \cos 4x = 2\sin 3x\sin x$
$\int_0^{\frac{\pi}{6}} \sin x\sin 3x \, dx = \int_0^{\frac{\pi}{6}} \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x \, dx$
$= \left[\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{6}}$
$= \left(\frac{1}{4}\sin\frac{\pi}{3} - \frac{1}{8}\sin\frac{2\pi}{3}\right) - \left(0\right)$
$= \frac{1}{4}\times\frac{\sqrt{3}}{2} - \frac{1}{8}\times\frac{\sqrt{3}}{2} = \frac{1}{16}\sqrt{3}$

7. $\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} dx = \int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)} dx = \int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{1 - \sin^2 x} dx$
$= \int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{\cos^2 x} dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} dx$
$= \int_0^{\frac{\pi}{3}} \sec^2 x + \frac{\sin x}{\cos^2 x}\times\frac{1}{\cos x} dx = \int_0^{\frac{\pi}{3}} \sec^2 x + \tan x\sec x \, dx$
$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{3}} = \left(\tan\frac{\pi}{3} + \sec\frac{\pi}{3}\right) - (0 + 1)$
$= \sqrt{3} + 2 - 1 = \sqrt{3} + 1$

8. $\int_0^{\frac{\pi}{2}} \left(1 + \tan\frac{x}{2}\right)^2 dx = \int_0^{\frac{\pi}{2}} 1 + 2\tan\frac{x}{2} + \tan^2\frac{x}{2} \, dx$
$= \int_0^{\frac{\pi}{2}} 2 + 2\tan\frac{x}{2} + \sec^2\frac{x}{2} - 1 \, dx$
$= \left[4\ln\left|\sec\frac{x}{2}\right| + 2\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$
$\left[4\ln|\sec\frac{\pi}{4}| + 2\tan\left(\frac{\pi}{4}\right)\right] - \left[4\ln|\sec 0| + 0\right]$
$= 4\ln\sqrt{2} + 2 = 4\ln 2^{\frac{1}{2}} + 2$
$= \ln\left(2^1\right)^2 + 2 = \ln 4 + 2$

9. $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx = \int_0^{\frac{\pi}{2}} \cos^2 x\cos x \, dx = \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)\cos x \, dx$
$= \int_0^{\frac{\pi}{2}} \cos x - \sin^2 x\cos x \, dx = \left[\sin x - \frac{1}{3}\sin^3 x\right]_0^{\frac{\pi}{2}}$
$= \left[\sin\frac{\pi}{2} - \frac{1}{3}\sin^3\frac{\pi}{2}\right] - \left[0\right] = 1 - \frac{1}{3} = \frac{2}{3}$

10. $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cot^2 2x \, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \operatorname{cosec}^2 2x - 1 \, dx = \left[-\frac{1}{2}\cot 2x - x\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$
$= \left[\frac{1}{2}\cot 2x + x\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$
$= \left(\frac{1}{2}\cot\frac{\pi}{3} + \frac{\pi}{6}\right) - \left(\frac{1}{2}\cot\frac{\pi}{4} + \frac{\pi}{8}\right)$
$= \frac{1}{2}\times\frac{1}{\sqrt{3}} + \frac{\pi}{6} - \frac{1}{2} - \frac{\pi}{8}$
$= \frac{1}{2\sqrt{3}} - \frac{1}{2} + \frac{\pi}{24}$

Question 9

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{12}} 6\sin^2\theta \ d\theta = \frac{1}{4}(\pi - 3)$

2. $\displaystyle\int_0^{\frac{\pi}{6}} \sin^3\theta \ d\theta = \frac{5}{24}$

3. $\displaystyle\int_0^{\frac{\pi}{12}} 10\sin 8\theta \cos 2\theta \, d\theta = \frac{1}{12}\left(16 + 3\sqrt{3}\right)$

4. $\displaystyle\int_0^{\frac{\pi}{4}} \left(\cos x + \sec x\right)^2 \, dx = \frac{5}{8}(\pi + 2)$

5. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sin x + \cot x\right)^2 \, dx = \frac{1}{8}\left(26 - \pi - 4\sqrt{2}\right)$