INTEGRATION

BY TRIGONOMETRIC IDENTITIES

(WITHOUT ANSWERS)

Question 1

Carry out the following integrations:

1. $\int 3\sin^2 x \, dx$

2. $\int 4\cos^2 x \, dx$

3. $\int 3\sin x \cos x \, dx$

4. $\int (2-3\sin x)^2 \, dx$

5. $\int (1-\cos 2x)^2 \, dx$

6. $\int 2\tan^2 x \, dx$

7. $\int 5\cot^2 x \, dx$

8. $\int (2\tan x - \cot x)^2 \, dx$

9. $\int \frac{4\sin x}{\cos^2 x} \, dx$

10. $\int \frac{\cos x}{3\sin^2 x} \, dx$

Question 2

Integrate:

1. $\int (2+\sin x)^2 \, dx$
2. $\int \sin x\left(1+\sec^2 x\right) \, dx$
3. $\int (1-2\cos x)^2 \, dx$
4. $\int \frac{1}{\cos^2 x \tan^2 x} \, dx$
5. $\int 2+2\tan^2 x \, dx$
6. $\int \frac{1+\cos x}{\sin^2 x} \, dx$
7. $\int \frac{(1+\cos x)^2}{\sin^2 x} \, dx$
8. $\int 4\cos^2 x \, dx$
9. $\int 3\cot^2 x \, dx$
10. $\int (2\cos x-3\sin x)^2 \, dx$

Question 3

Integrate:

1. $\int \sin 2x \operatorname{cosec} x \, dx$

2. $\int \frac{1+\sin x}{\cos^2 x} \, dx$

3. $\int \tan^2 x \, dx$

4. $\int \frac{(1+\sin x)^2}{\cos^2 x} \, dx$

5. $\int \frac{\cos^2 x}{1+\sin x} \, dx$

6. $\int \frac{1}{1+\cos x} \, dx$

7. $\int \frac{(1+2\cos x)^2}{3\sin^2 x} \, dx$

8. $\int \sin x \sin 3x \, dx$

9. $\int \sin^2 2x \, dx$

10. $\int 2\cos 3x \sin x \, dx$

Question 4

Integrate:

1. $\displaystyle\int \frac{\cos 2x}{1-\cos^2 2x}\,dx$

2. $\displaystyle\int \cot^2 3x\,dx$

3. $\displaystyle\int \sin 2x \sec x\,dx$

4. $\displaystyle\int \frac{1}{\sin x \cos^2 x}\,dx$

5. $\displaystyle\int \frac{1}{\sec x - 1}\,dx$

6. $\displaystyle\int 1-\cot^2 x\,dx$

7. $\displaystyle\int \left(2\cos x - 3\right)^2\,dx$

8. $\displaystyle\int \left(3\sin x - \cos x\right)^2\,dx$

9. $\displaystyle\int \frac{1}{\cos x \sin^2 x}\,dx$

10. $\displaystyle\int \sin^2 x \sec^2 x\,dx$

Question 5

Integrate:

1. $\int \sin 3x \cos 2x \, dx$

2. $\int \frac{1}{\sin x \cos x} \, dx$

3. $\int \frac{1}{1-\sin x} \, dx$

4. $\int \sin^2 2x \, dx$

5. $\int \frac{\cos 2x}{\cos^2 x} \, dx$

6. $\int \cos^2 x \sin^2 x \, dx$

7. $\int \left(\sin x + 2\cos x\right)^2 \, dx$

8. $\int \frac{1}{\sin^2 x \cos^2 x} \, dx$

9. $\int \sqrt{\sin^2 x + \left(\cos x - 1\right)^2} \, dx$

10. $\int \frac{1-\cos x}{1+\cos x} \, dx$

Question 6

Integrate:

1. $\displaystyle\int \frac{1+\sin x}{1-\sin x}\,dx$

Question 7

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{2}} 4\sin^2 x \ dx = \pi$

2. $\displaystyle\int_0^{\frac{\pi}{6}} 24\cos^2 x \ dx = \pi + 3$

3. $\displaystyle\int_0^{\frac{\pi}{6}} 8\sin x\cos x \ dx = 1$

4. $\displaystyle\int_0^{\frac{\pi}{2}} \left(1-\sin x\right)^2 \ dx = \frac{3\pi}{2} - 4$

5. $\displaystyle\int_0^{\frac{\pi}{6}} \left(1-\cos 3x\right)^2 \ dx = \frac{\pi}{4} - \frac{2}{3}$

6. $\displaystyle\int_0^{\frac{\pi}{4}} 4\tan^2 x \ dx = 4 - \pi$

7. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(3\cot x + \tan x\right)^2 \ dx = \frac{2}{3}\left(10\sqrt{3} - \pi\right)$

8. $\displaystyle\int_0^{\frac{\pi}{4}} \left(\sec x + 4\cos x\right)^2 \ dx = 4\pi + 5$

9. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x} \ dx = 1$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \ dx = \sqrt{2} - 1$

Question 8

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 x \ dx = \frac{1}{8}(\pi + 2)$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \ dx = \frac{\pi}{4}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} (2\sin x - 3\cos x)^2 \ dx = \frac{1}{4}(13\pi - 24)$

4. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2 \ dx = 4\pi + 3\sqrt{3}$

5. $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 x \ dx = \frac{1}{4}(4 - \pi)$

6. $\displaystyle\int_0^{\frac{\pi}{6}} \sin x \sin 3x \ dx = \frac{\sqrt{3}}{16}$

7. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} \ dx = 1 + \sqrt{3}$

8. $\displaystyle\int_0^{\frac{\pi}{2}} \left(1 + \tan \tfrac{x}{2}\right)^2 \ dx = 2 + \ln 4$

9. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x \ dx = \frac{2}{3}$

10. $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cot^2 2x \ dx = \frac{1}{2} - \frac{\sqrt{3}}{6} - \frac{\pi}{24}$

Question 9

Carry out the following integrations:

1. $\displaystyle\int_0^{\frac{\pi}{12}} 6\sin^2\theta \ d\theta = \frac{1}{4}(\pi - 3)$

2. $\displaystyle\int_0^{\frac{\pi}{6}} \sin^3\theta \ d\theta = \frac{5}{24}$

3. $\displaystyle\int_0^{\frac{\pi}{12}} 10\sin 8\theta\cos 2\theta \, d\theta = \frac{1}{12}\left(16 + 3\sqrt{3}\right)$

4. $\displaystyle\int_0^{\frac{\pi}{4}} \left(\cos x + \sec x\right)^2 \, dx = \frac{5}{8}(\pi + 2)$

5. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sin x + \cot x\right)^2 \, dx = \frac{1}{8}\left(26 - \pi - 4\sqrt{2}\right)$