INTEGRATION

by substitution

Question 1

Carry out the following integrations **by substitution** only.

1. $\displaystyle\int 4x(2x-1)^4\,dx=\frac{1}{6}(2x-1)^6+\frac{1}{5}(2x-1)^5+C$

2. $\displaystyle\int \frac{2x}{2x+1}\,dx=\frac{1}{2}(2x+1)-\frac{1}{2}\ln|2x+1|+C$

3. $\displaystyle\int x\left(4-x^2\right)^{-\frac{1}{2}}\,dx=-\left(4-x^2\right)^{\frac{1}{2}}+C$

4. $\displaystyle\int \frac{4x}{6x^2-1}\,dx=\frac{1}{3}\ln|6x^2-1|+C$

5. $\displaystyle\int x(3x-1)^4\,dx=\frac{1}{54}(3x-1)^6+\frac{1}{45}(3x-1)^5+C$

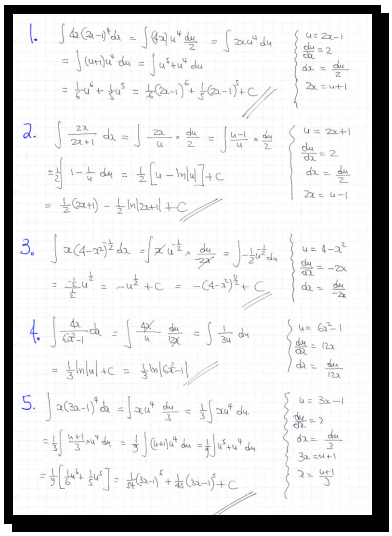
6. $\displaystyle\int \frac{8x}{\sqrt{4x-1}}\,dx=\frac{1}{3}(4x-1)^{\frac{3}{2}}+(4x-1)^{\frac{1}{2}}+C$

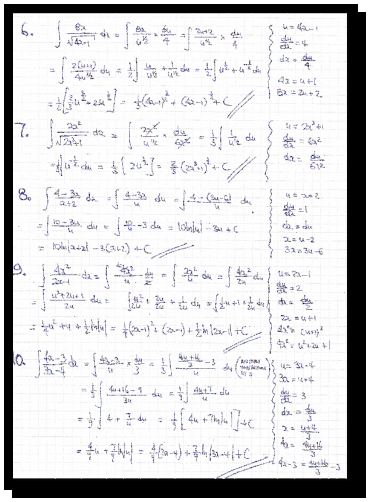
7. $\displaystyle\int \frac{2x^2}{\sqrt{2x^3+1}}\,dx=\frac{2}{3}\left(2x^3+1\right)^{\frac{1}{2}}+C$

8. $\displaystyle\int \frac{4-3x}{x+2}\,dx=10\ln|x+2|-3(x+2)+C$

9. $\displaystyle\int \frac{4x^2}{2x-1}\,dx=\frac{1}{4}(2x-1)^2+(2x-1)+\frac{1}{2}\ln|2x-1|+C$

10. $\displaystyle\int \frac{4x-3}{3x-4}\,dx=\frac{4}{9}(3x-4)+\frac{7}{9}\ln|3x-4|+C$





Question 2

Carry out the following integrations **by substitution** only.

1. $\displaystyle\int 6x(3x-1)^3\,dx=\frac{2}{15}(3x-1)^5+\frac{1}{6}(3x-1)^4+C$

2. $\displaystyle\int \frac{5x}{5x-1}\,dx=\frac{1}{5}(5x-1)+\frac{1}{5}\ln|5x-1|+C$

3. $\displaystyle\int 3x\left(x^2+1\right)^{\frac{1}{2}}\,dx=\left(x^2+1\right)^{\frac{3}{2}}+C$

4. $\displaystyle\int \frac{3x^2}{2x^3+1}\,dx=\frac{1}{2}\ln|2x^3+1|+C$

5. $\displaystyle\int x(2x-1)^5\,dx=\frac{1}{24}(2x-1)^6+\frac{1}{28}(2x-1)^7+C$

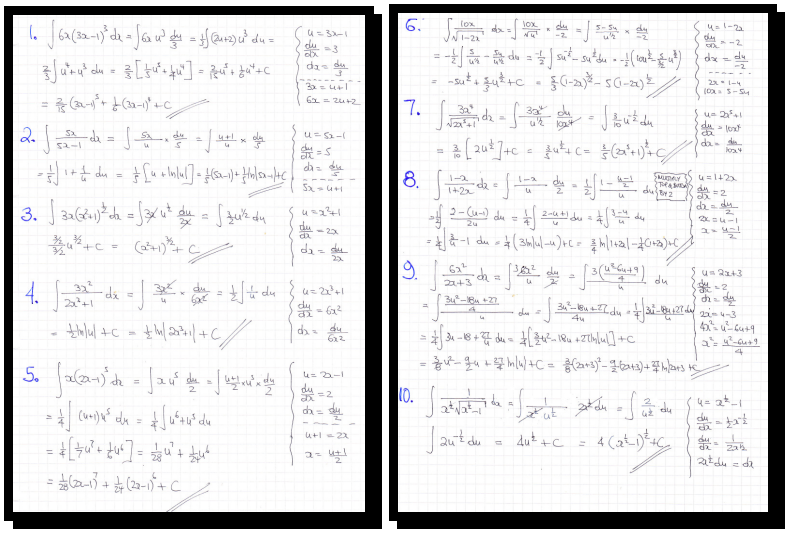
6. $\displaystyle\int \frac{10x}{\sqrt{1-2x}}\,dx=\frac{5}{3}(1-2x)^{\frac{3}{2}}-5(1-2x)^{\frac{1}{2}}+C$

7. $\displaystyle\int \frac{3x^4}{\sqrt{2x^5+1}}\,dx=\frac{3}{5}\left(2x^5+1\right)^{\frac{1}{2}}+C$

8. $\displaystyle\int \frac{1-x}{1+2x}\,dx=\frac{3}{4}\ln|1+2x|-\frac{1}{4}(1+2x)+C$

9. $\displaystyle\int \frac{6x^2}{2x+3}\,dx=\frac{3}{8}(2x+3)^2-\frac{9}{2}(2x+3)+\frac{27}{4}\ln|2x+3|+C$

10. $\displaystyle\int \frac{1}{x^{\frac{1}{2}}\sqrt{x^{\frac{1}{2}}-1}}\,dx=4\sqrt{x^{\frac{1}{2}}-1}+C$



Question 3

Carry out the following integrations **by substitution** only.

1. $\displaystyle\int 10x(5x-3)^3\,dx=\frac{2}{25}(5x-3)^5+\frac{3}{10}(5x-3)^4+C$

2. $\displaystyle\int \frac{12x}{2x-1}\,dx=3(2x-1)+3\ln|2x-1|+C$

3. $\displaystyle\int x\left(x^2-1\right)^{\frac{5}{2}}\,dx=\frac{1}{7}\left(x^2-1\right)^{\frac{7}{2}}+C$

4. $\displaystyle\int \frac{5x^5}{2x^6+7}\,dx=\frac{5}{12}\ln\left|2x^6+7\right|+C$

5. $\displaystyle\int 2x(1-5x)^4\,dx=-\frac{2}{125}(1-5x)^5+\frac{1}{75}(1-5x)^6+C$

6. $\displaystyle\int \frac{9x}{\sqrt{4x^2+1}}\,dx=\frac{9}{4}\sqrt{4x^2+1}+C$

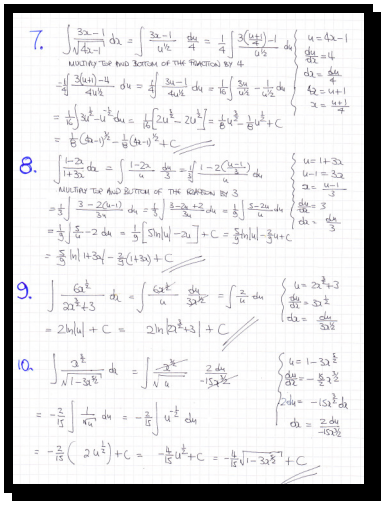
7. $\displaystyle\int \frac{3x-1}{\sqrt{4x-1}}\,dx=\frac{1}{8}(4x-1)^{\frac{3}{2}}-\frac{1}{8}(4x-1)^{\frac{1}{2}}+C$

8. $\displaystyle\int \frac{1-2x}{1+3x}\,dx=\frac{5}{9}\ln|1+3x|-\frac{2}{9}(1+3x)+C$

9. $\displaystyle\int \frac{6x^{\frac{1}{2}}}{2x^{\frac{3}{2}}+3}\,dx=2\ln\left|2x^{\frac{3}{2}}+3\right|+C$

10. $\displaystyle\int \frac{x^{\frac{3}{2}}}{\sqrt{1-3x^{\frac{5}{2}}}}\,dx=-\frac{4}{15}\sqrt{1-3x^{\frac{5}{2}}}+C$





Question 4

Carry out the following integrations to the answers given, **by using substitution** only.

1. $\displaystyle\int_0^{\frac{1}{2}} 8x(2x-1)^4 \, dx = \frac{1}{15}$

2. $\displaystyle\int_2^3 \frac{3x}{3x-5} \, dx = 1+\frac{10}{3}\ln 2$

3. $\displaystyle\int_0^1 x\left(1-x^2\right)^{\frac{3}{2}} \, dx = \frac{1}{5}$

4. $\displaystyle\int_0^1 \frac{4x}{x^2+1} \, dx = 2\ln 2$

5. $\displaystyle\int_1^3 2x(3x-1)^4 \, dx = \frac{55808}{5}$

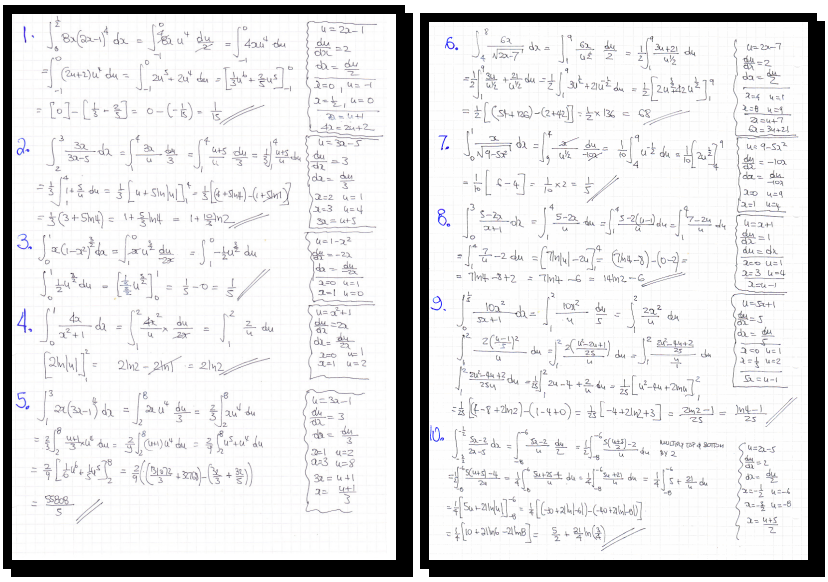
6. $\displaystyle\int_4^8 \frac{6x}{\sqrt{2x-7}} \, dx = 68$

7. $\displaystyle\int_0^1 \frac{x}{\sqrt{9-5x^2}} \, dx = \frac{1}{5}$

8. $\displaystyle\int_0^3 \frac{5-2x}{x+1} \, dx = 14\ln 2-6$

9. $\displaystyle\int_0^{\frac{1}{5}} \frac{10x^2}{5x+1} \, dx = \frac{\ln 4-1}{25}$

10. $\displaystyle\int_{-\frac{3}{2}}^{-\frac{1}{2}} \frac{5x-2}{2x-5} \, dx = \frac{5}{2}+\frac{21}{4}\ln\left(\frac{3}{4}\right)$



Question 5

Carry out the following integrations to the answers given, **by using substitution** only.

1. $\displaystyle\int_0^{\frac{3}{2}} 2x\left(2x-3\right)^4 \, dx = \frac{243}{20}$

2. $\displaystyle\int_0^{2} \frac{4x}{4x+1} \, dx = 2-\frac{1}{2}\ln 3$

3. $\displaystyle\int_0^{1} x^2\left(1-x^3\right)^{\frac{9}{2}} \, dx = \frac{2}{33}$

4. $\displaystyle\int_0^{4} \frac{12x}{x^2+9} \, dx = 12\ln\left(\tfrac{5}{3}\right)$

5. $\displaystyle\int_1^{2} 2x\left(3x-1\right)^4 \, dx = \frac{3569}{5}$

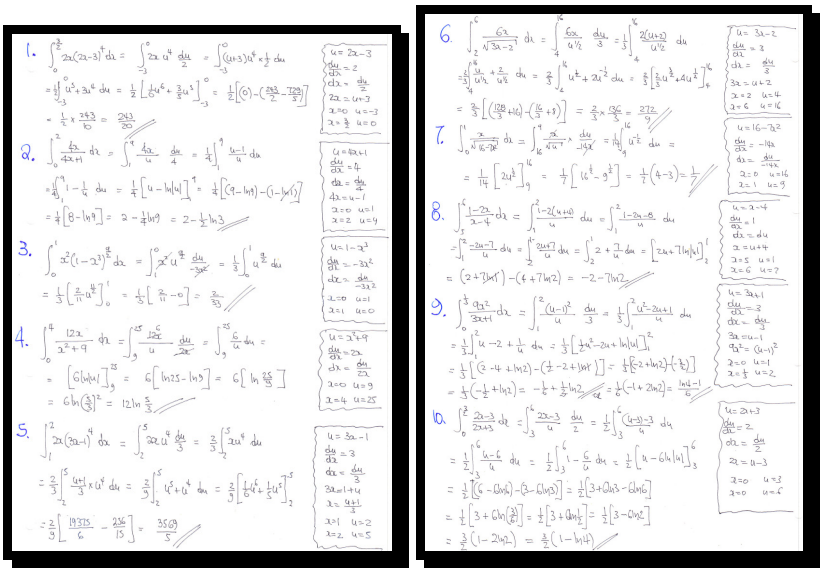
6. $\displaystyle\int_2^{6} \frac{6x}{\sqrt{3x-2}} \, dx = \frac{272}{9}$

7. $\displaystyle\int_0^{1} \frac{x}{\sqrt{16-7x^2}} \, dx = \frac{1}{7}$

8. $\displaystyle\int_5^{6} \frac{1-2x}{x-4} \, dx = -2-7\ln 2$

9. $\displaystyle\int_0^{\frac{1}{3}} \frac{9x^2}{3x+1} \, dx = \frac{\ln 4-1}{6}$

10. $\displaystyle\int_0^{\frac{3}{2}} \frac{2x-3}{2x+3} \, dx = \frac{3}{2}\left(1-\ln 4\right)$



Question 6

Carry out the following integrations.

1. $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx=\frac{2}{3}(x+1)^{\frac{3}{2}}-2(x+1)^{\frac{1}{2}}+C=\frac{2}{3}(x-2)\sqrt{x+1}+C$

2. $\displaystyle\int \frac{2x}{(2x+1)^3}\,dx=-\frac{1}{2}(2x+1)^{-1}+\frac{1}{4}(2x+1)^{-2}+C=-\frac{4x+1}{4(2x+1)^2}+C$

3. $\displaystyle\int \frac{x}{x+1}\,dx=x-\ln|x+1|+C=x+1-\ln|x+1|+C$

4. $\displaystyle\int \frac{x}{\sqrt{x-1}}\,dx=\frac{2}{3}(x-1)^{\frac{3}{2}}+2(x-1)^{\frac{1}{2}}+C=\frac{2}{3}(x+2)\sqrt{x-1}+C$

5. $\displaystyle\int \frac{4x+1}{2x-5}\,dx=2x+\frac{11}{2}\ln|2x-5|+C$

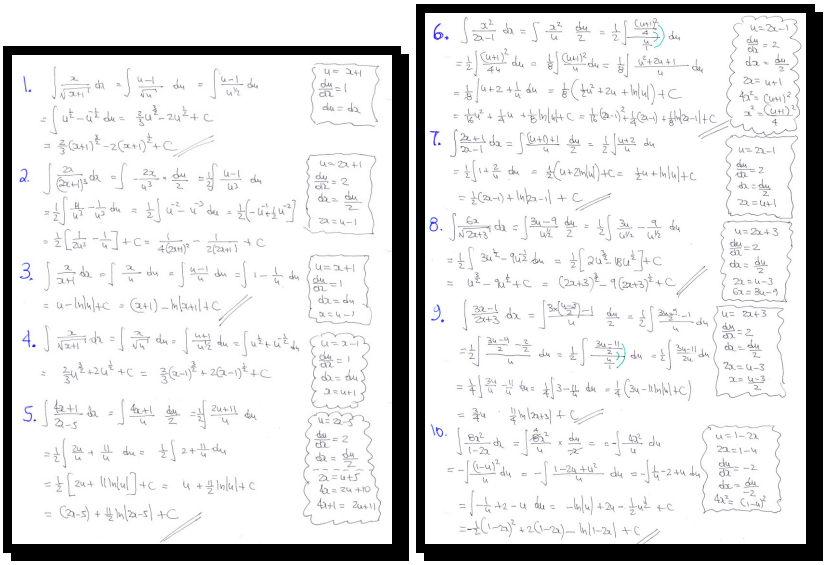
6. $\displaystyle\int \frac{x^2}{2x-1}\,dx=\frac{1}{16}(2x-1)^2+\frac{1}{4}(2x-1)+\frac{1}{8}\ln|2x-1|+C=\frac{1}{4}x^2+\frac{1}{4}x+\frac{1}{8}\ln|2x-1|+C$

7. $\displaystyle\int \frac{2x+1}{2x-1}\,dx=\frac{1}{2}(2x-1)+\ln|2x-1|+C=x+\ln|2x-1|+C$

8. $\displaystyle\int \frac{6x}{\sqrt{2x+3}}\,dx=(2x+3)^{\frac{3}{2}}-9(2x+3)^{\frac{1}{2}}+C=2(x-3)\sqrt{2x+3}+C$

9. $\displaystyle\int \frac{3x-1}{2x+3}\,dx=\frac{3}{4}(2x-3)-\frac{11}{4}\ln|2x+3|+C=\frac{3}{2}x-\frac{11}{4}\ln|2x+3|+C$

10. $\displaystyle\int \frac{8x^2}{1-2x}\,dx=-\frac{1}{2}(1-2x)^2+2(1-2x)-\ln|1-2x|+C=-2x^2-2x-\ln|1-2x|+C$



Question 7

Carry out the following integrations **using the substitutions** given.

1. $\displaystyle\int x\sqrt{1-x}\,dx=\frac{2}{5}(1-x)^{\frac{5}{2}}-\frac{2}{3}(1-x)^{\frac{3}{2}}+C$ Use $u=1-x$, or $u=\sqrt{1-x}$

2. $\displaystyle\int \frac{6x}{\sqrt{2x+1}}\,dx=(2x+1)^{\frac{3}{2}}-3(2x+1)^{\frac{1}{2}}+C$ Use $u=2x+1$, or $u=\sqrt{2x+1}$

3. $\displaystyle\int \cos^3 x\,dx=\sin x-\frac{1}{3}\sin^3 x+C$ Use $u=\sin x$

4. $\displaystyle\int \sec^4 x\,dx=\tan x+\frac{1}{3}\tan^3 x+C$ Use $u=\tan x$

5. $\displaystyle\int \frac{1}{\sqrt{x}(x-4)}\,dx=\frac{1}{2}\ln\left|\frac{\sqrt{x}-2}{\sqrt{x}+2}\right|+C$ Use $u=\sqrt{x}$

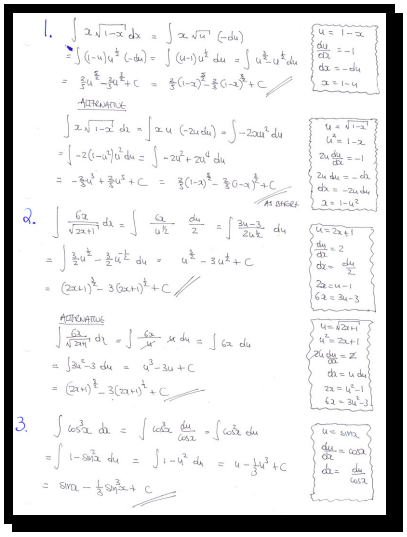
6. $\displaystyle\int \frac{\sqrt{x^2+9}}{x}\,dx=\sqrt{x^2+9}+\frac{3}{2}\ln\left|\frac{\sqrt{x^2+9}-3}{\sqrt{x^2+9}+3}\right|+C$ Use $u=\sqrt{x^2+9}$

7. $\displaystyle\int \frac{1+\cos x}{\sin x}\,dx=\ln|\cos x-1|+C$ Use $u=\cos x$

8. $\displaystyle\int \frac{1}{1+\sqrt{x-2}}\,dx=2\sqrt{x-2}+2\ln\left(1+\sqrt{x-2}\right)+C$ Use $u=\sqrt{x-2}$

9. $\displaystyle\int \sec^2 x\tan x\sqrt{1+\tan x}\,dx=\frac{2}{15}(3\tan x-2)(1+\tan x)^{\frac{3}{2}}+C$ Use $u=\sqrt{1+\tan x}$

10. $\displaystyle\int \frac{9}{\sqrt{x}(9x-1)}\,dx=3\ln\left|\frac{3\sqrt{x}-1}{3\sqrt{x}+1}\right|+C$ Use $u=\sqrt{x}$



1. $\int x\sqrt{1-x}\,dx = \int x\sqrt{u}\,(-du) = \int (1-u)u^{\frac{1}{2}}(-du) = \int (u-1)u^{\frac{1}{2}}\,du = \int u^{\frac{3}{2}}-u^{\frac{1}{2}}\,du = \frac{2}{5}u^{\frac{5}{2}}-\frac{2}{3}u^{\frac{3}{2}}+C = \frac{2}{5}(1-x)^{\frac{5}{2}}-\frac{2}{3}(1-x)^{\frac{3}{2}}+C$

$u=1-x$, $\frac{du}{dx}=-1$, $dx=-du$, $x=1-u$

ALTERNATIVE

$\int x\sqrt{1-x}\,dx = \int x\,u\,(-2u\,du) = \int -2xu^2\,du = \int -2(1-u^2)u^2\,du = \int -2u^2+2u^4\,du = -\frac{2}{3}u^3+\frac{2}{5}u^5+C = \frac{2}{5}(1-x)^{\frac{5}{2}}-\frac{2}{3}(1-x)^{\frac{3}{2}}+C$

AS BEFORE

$u=\sqrt{1-x}$, $u^2=1-x$, $2u\frac{du}{dx}=-1$, $dx=-2u\,du$, $x=1-u^2$

2. $\int \frac{6x}{\sqrt{2x+1}}\,dx = \int \frac{6x}{u^{\frac{1}{2}}}\,\frac{du}{2} = \int \frac{3u-3}{2u^{\frac{1}{2}}}\,du = \int \frac{3}{2}u^{\frac{1}{2}}-\frac{3}{2}u^{-\frac{1}{2}}\,du = u^{\frac{3}{2}}-3u^{\frac{1}{2}}+C = (2x+1)^{\frac{3}{2}}-3(2x+1)^{\frac{1}{2}}+C$

$u=2x+1$, $\frac{du}{dx}=2$, $dx=\frac{du}{2}$, $2x=u-1$, $6x=3u-3$

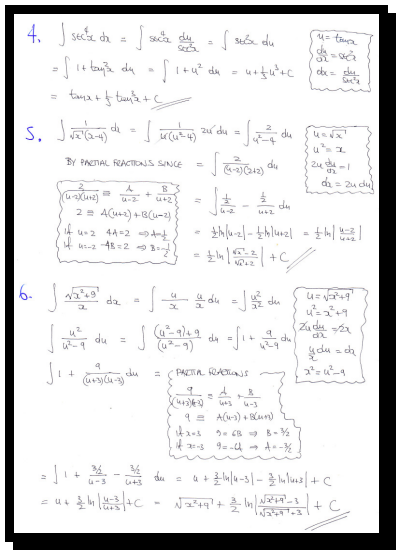
ALTERNATIVE

$\int \frac{6x}{\sqrt{2x+1}}\,dx = \int \frac{6x}{u}\,u\,du = \int 6x\,du = \int 3u^2-3\,du = u^3-3u+C = (2x+1)^{\frac{3}{2}}-3(2x+1)^{\frac{1}{2}}+C$

$u=\sqrt{2x+1}$, $u^2=2x+1$, $2u\frac{du}{dx}=2$, $dx=u\,du$, $2x=u^2-1$, $6x=3u^2-3$

3. $\int \cos^3 x\,dx = \int \cos^2 x\,\frac{du}{\cos x} = \int \cos^2 x\,du = \int 1-\sin^2 x\,du = \int 1-u^2\,du = u-\frac{1}{3}u^3+C = \sin x-\frac{1}{3}\sin^3 x+C$

$u=\sin x$, $\frac{du}{dx}=\cos x$, $dx=\frac{du}{\cos x}$



4. $\int \sec^4 x\,dx = \int \sec^2 x\,\frac{du}{\sec^2 x} = \int \sec^2 x\,du = \int 1+\tan^2 x\,du = \int 1+u^2\,du = u+\frac{1}{3}u^3+C = \tan x+\frac{1}{3}\tan^3 x+C$

$u=\tan x$, $\frac{du}{dx}=\sec^2 x$, $dx=\frac{du}{\sec^2 x}$

5. $\int \frac{1}{\sqrt{x}(x-4)}\,dx = \int \frac{1}{u(u^2-4)}\,2u\,du = \int \frac{2}{u^2-4}\,du$

$u=\sqrt{x}$, $u^2=x$, $2u\frac{du}{dx}=1$, $dx=2u\,du$

BY PARTIAL FRACTIONS SINCE $= \int \frac{2}{(u-2)(u+2)}\,du$

$\frac{2}{(u-2)(u+2)} \equiv \frac{A}{u-2}+\frac{B}{u+2}$

$2 \equiv A(u+2)+B(u-2)$

If $u=2$: $4A=2 \Rightarrow A=\frac{1}{2}$

If $u=-2$: $-4B=2 \Rightarrow B=-\frac{1}{2}$

$= \int \frac{\frac{1}{2}}{u-2}-\frac{\frac{1}{2}}{u+2}\,du = \frac{1}{2}\ln|u-2|-\frac{1}{2}\ln|u+2| = \frac{1}{2}\ln\left|\frac{u-2}{u+2}\right|+C = \frac{1}{2}\ln\left|\frac{\sqrt{x}-2}{\sqrt{x}+2}\right|+C$

6. $\int \frac{\sqrt{x^2+9}}{x}\,dx = \int \frac{u}{x}\,\frac{u}{x}\,du = \int \frac{u^2}{x^2}\,du = \int \frac{u^2}{u^2-9}\,du = \int \frac{(u^2-9)+9}{(u^2-9)}\,du = \int 1+\frac{9}{u^2-9}\,du = \int 1+\frac{9}{(u+3)(u-3)}\,du$

$u=\sqrt{x^2+9}$, $u^2=x^2+9$, $2u\frac{du}{dx}=2x$, $\frac{u}{x}du=dx$, $x^2=u^2-9$

PARTIAL FRACTIONS

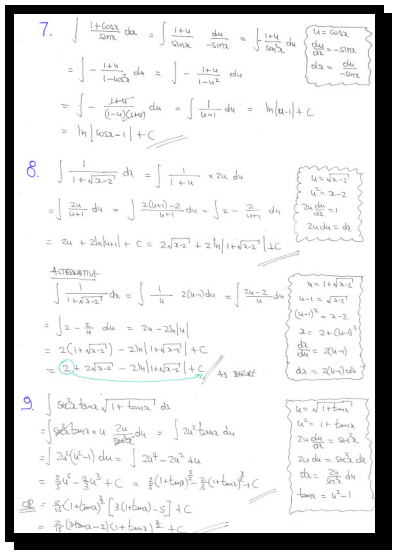
$\frac{9}{(u+3)(u-3)} = \frac{A}{u+3}+\frac{B}{u-3}$

$9 \equiv A(u-3)+B(u+3)$

If $x=3$: $9=6B \Rightarrow B=3/2$

If $x=-3$: $9=-6A \Rightarrow A=-3/2$

$= \int 1+\frac{3/2}{u-3}-\frac{3/2}{u+3}\,du = u+\frac{3}{2}\ln|u-3|-\frac{3}{2}\ln|u+3|+C = u+\frac{3}{2}\ln\left|\frac{u-3}{u+3}\right|+C = \sqrt{x^2+9}+\frac{3}{2}\ln\left|\frac{\sqrt{x^2+9}-3}{\sqrt{x^2+9}+3}\right|+C$



7. $\int \frac{1+\cos x}{\sin x}\,dx = \int \frac{1+u}{\sin x}\,\frac{du}{-\sin x} = \int \frac{1+u}{\sin^2 x}\,du = \int -\frac{1+u}{1-\cos^2 x}\,du = \int -\frac{1+u}{1-u^2}\,du = \int -\frac{1+u}{(1-u)(1+u)}\,du = \int \frac{1}{u-1}\,du = \ln|u-1|+C = \ln|\cos x-1|+C$

$u=\cos x$, $\frac{du}{dx}=-\sin x$, $dx=\frac{du}{-\sin x}$

8. $\int \frac{1}{1+\sqrt{2x-2}}\,dx = \int \frac{1}{1+u}\times 2u\,du = \int \frac{2u}{u+1}\,du = \int \frac{2(u+1)-2}{u+1}\,du = \int 2-\frac{2}{u+1}\,du = 2u+2\ln|u+1|+C = 2\sqrt{2x-2}+2\ln|1+\sqrt{2x-2}|+C$

$u=\sqrt{2x-2}$, $u^2=2x-2$, $2u\frac{du}{dx}=1$, $2u\,du=dx$

ALTERNATIVE

$\int \frac{1}{1+\sqrt{2x-2}}\,dx = \int \frac{1}{u}\,2(u+1)\,du = \int \frac{2u+2}{u}\,du = \int 2+\frac{2}{u}\,du = 2u+2\ln|u| = 2(1+\sqrt{2x-2})+2\ln|1+\sqrt{2x-2}|+C = 2+2\sqrt{2x-2}+2\ln|1+\sqrt{2x-2}|+C$

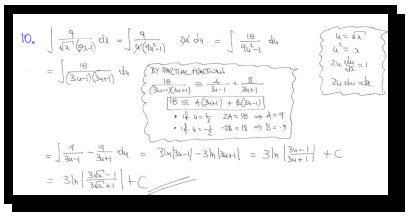
AS BEFORE

$u=1+\sqrt{2x-2}$, $u-1=\sqrt{2x-2}$, $(u-1)^2=2x-2$, $x=2+(u-1)^2$, $\frac{dx}{du}=2(u-1)$, $dx=2(u-1)\,du$

9. $\int \sec^2 x\tan x\sqrt{1+\tan x}\,dx = \int \sec^2 x\tan x\times u\,\frac{2u}{\sec^2 x}\,du = \int 2u^2\tan x\,du = \int 2u^2(u^2-1)\,du = \int 2u^4-2u^2\,du = \frac{2}{5}u^5-\frac{2}{3}u^3+C = \frac{2}{5}(1+\tan x)^{\frac{5}{2}}-\frac{2}{3}(1+\tan x)^{\frac{3}{2}}+C$

OR $= \frac{2}{15}(1+\tan x)^{\frac{3}{2}}\left[3(1+\tan x)-5\right]+C = \frac{2}{15}(3\tan x-2)(1+\tan x)^{\frac{3}{2}}+C$

$u=\sqrt{1+\tan x}$, $u^2=1+\tan x$, $2u\frac{du}{dx}=\sec^2 x$, $2u\,du=\sec^2 x\,dx$, $\frac{2u}{\sec^2 x}\,du=dx$, $\tan x=u^2-1$



10. $\int \frac{9}{\sqrt{x}(9x-1)}\,dx = \int \frac{9}{u(9u^2-1)}\,2u\,du = \int \frac{18}{9u^2-1}\,du = \int \frac{18}{(3u-1)(3u+1)}\,du$

$u=\sqrt{x}$, $u^2=x$, $2u\frac{du}{dx}=1$, $2u\,du=dx$

BY PARTIAL FRACTIONS

$\frac{18}{(3u-1)(3u+1)} \equiv \frac{A}{3u-1}+\frac{B}{3u+1}$

$18 \equiv A(3u+1)+B(3u-1)$

- If $u=\frac{1}{3}$: $2A=18 \Rightarrow A=9$
- If $u=-\frac{1}{3}$: $-2B=18 \Rightarrow B=-9$

$= \int \frac{9}{3u-1}-\frac{9}{3u+1}\,du = 3\ln|3u-1|-3\ln|3u+1| = 3\ln\left|\frac{3u-1}{3u+1}\right|+C = 3\ln\left|\frac{3\sqrt{x}-1}{3\sqrt{x}+1}\right|+C$

Question 8

Carry out the following integrations.

1. $\displaystyle\int \frac{1}{2+\sqrt{x}}\,dx = 2\sqrt{x}-4\ln\left|2+\sqrt{x}\right|+C$

2. $\displaystyle\int \frac{x^2}{1-2x}\,dx = -\frac{1}{4}x(x+1)-\frac{1}{8}\ln\left|1-2x\right| = -\frac{1}{16}(1-2x)^2+\frac{1}{4}(1-2x)-\frac{1}{8}\ln\left|1-2x\right|+C$

3. $\displaystyle\int \frac{3x^2+2}{4x+1}\,dx = \frac{1}{64}\left(24x^2-12x+35\ln\left|4x+1\right|\right)+C = \frac{3}{128}(4x+1)^2-\frac{3}{32}(4x+1)+\frac{35}{64}\ln\left|4x+1\right|$

4. $\displaystyle\int \frac{4-3x}{2x+1}\,dx = -\frac{3}{2}x+\frac{11}{4}\ln\left|2x+1\right|+C$

5. $\displaystyle\int \frac{x+1}{x-5}\,dx = x+6\ln\left|x-5\right|+C$

6. $\displaystyle\int \frac{x^2}{x-2}\,dx = \frac{1}{2}x^2+2x+4\ln\left|x-2\right|+C$

7. $\displaystyle\int \frac{1}{2+\sqrt{x-1}}\,dx = 2\sqrt{x-1}-4\ln\left|\sqrt{x-1}+2\right|+C$

8. $\displaystyle\int \frac{x+4}{x-4}\,dx = x+8\ln\left|x-4\right|+C$

Question 9

Carry out the following integrations.

1. $\displaystyle\int x\sqrt{x+1}\,dx=\frac{2}{15}(3x+2)\sqrt{(x+1)^3}+C$

2. $\displaystyle\int \frac{x+1}{\sqrt[3]{x^2+2x+3}}\,dx=\frac{3}{4}\sqrt[3]{\left(x^2+2x+3\right)^2}+C$

3. $\displaystyle\int \frac{3x^3+5x}{x^2+1}\,dx=\frac{3}{2}x^2+\ln\left|x^2+1\right|+C$

4. $\displaystyle\int \frac{2x+1}{3x-1}\,dx=\frac{2}{3}x+\frac{5}{9}\ln\left|3x-1\right|+C$

5. $\displaystyle\int \frac{1}{(x-1)\sqrt{x^2-1}}\,dx=-\sqrt{\frac{x+1}{x-1}}+C$, use $x-1=\frac{1}{u}$

6. $\displaystyle\int \frac{4x^3\sqrt{x^4+1}}{1+\sqrt{x^4+1}}\,dx=x^4-2\sqrt{x^4+1}+2\ln\left|1+\sqrt{x^4+1}\right|+C$

Question 10

Carry out the following integrations to the answers given.

1. $\displaystyle\int_0^{\frac{1}{2}} \frac{x}{(2-x)^2}\, dx = \frac{1}{3} + \ln\left(\frac{3}{4}\right)$

2. $\displaystyle\int_1^2 \frac{x}{(2x-1)^2}\, dx = \frac{2+\ln 27}{12}$

3. $\displaystyle\int_0^2 \frac{x+2}{\sqrt{4x+1}}\, dx = \frac{17}{6}$

4. $\displaystyle\int_0^{36} \frac{1}{\sqrt{x}\left(\sqrt{x}+2\right)}\, dx = \ln 16$

5. $\displaystyle\int_{-6}^{\frac{3}{2}} \frac{x}{\sqrt{4-2x}}\, dx = -\frac{9}{2}$

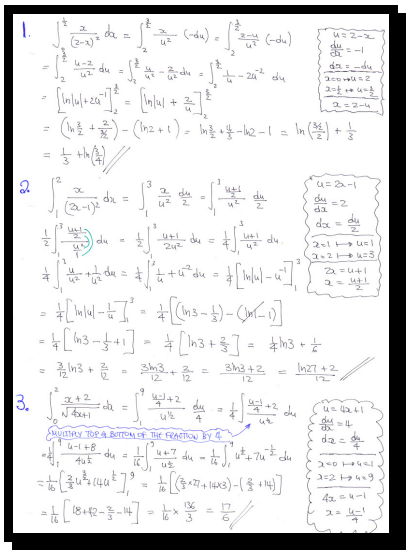
6. $\displaystyle\int_1^5 \frac{x+1}{(2x-1)^{\frac{3}{2}}}\, dx = 2$

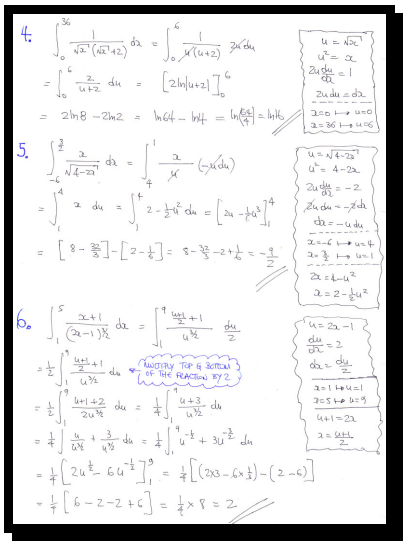
7. $\displaystyle\int_{\frac{2}{3}}^1 \frac{x}{2x-1}\, dx = \frac{1}{6} + \frac{1}{4}\ln 3$

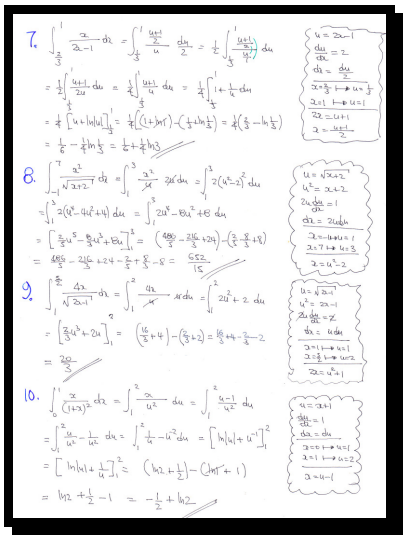
8. $\displaystyle\int_{-1}^7 \frac{x^2}{\sqrt{x+2}}\, dx = \frac{652}{15}$

9. $\displaystyle\int_1^{\frac{5}{2}} \frac{4x}{\sqrt{2x-1}}\, dx = \frac{20}{3}$

10. $\displaystyle\int_0^1 \frac{x}{(1+x)^2}\, dx = -\frac{1}{2} + \ln 2$







Question 11

Carry out the following integrations to the answers given.

1. $\displaystyle\int_0^3 x\sqrt{x+1}\ dx = \frac{116}{15}$

2. $\displaystyle\int_0^2 \frac{6x^3}{\sqrt{x^2+1}}\ dx = 4\left(1+\sqrt{5}\right)$

3. $\displaystyle\int_{-1}^0 \frac{x^2}{1-x}\ dx = -\frac{1}{2}+\ln 2$

4. $\displaystyle\int_0^{100} \frac{1}{20-\sqrt{x}}\ dx = 40\ln 2-20$

5. $\displaystyle\int_0^{\frac{1}{4}} 2x\sqrt{1-4x}\ dx = \frac{1}{30}$

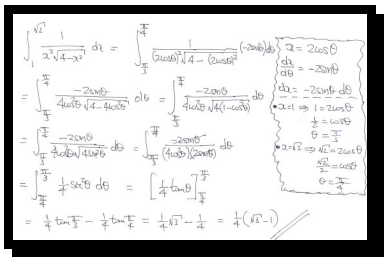
6. $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\cos x\left(1+\sin x\right)^5\ dx = \frac{107}{14}$

7. $\displaystyle\int_2^5 \frac{x^2}{\sqrt{x-1}}\ dx = \frac{356}{15}$

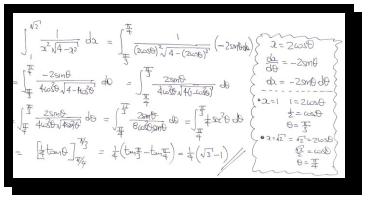
Question 12

Carry out the following integrations to the answers given.

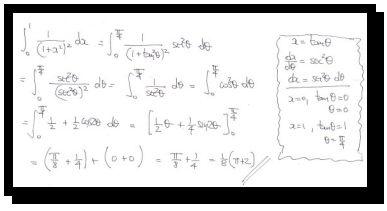
1. $\displaystyle\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\, dx = \frac{\pi}{2}-1$, use $x = 2\sin\theta$



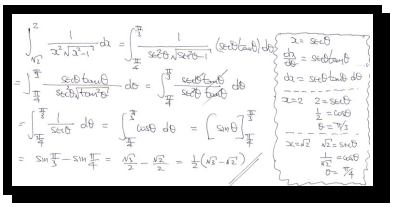
2. $\displaystyle\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\, dx = \frac{1}{4}\left(\sqrt{3}-1\right)$, use $x = 2\cos\theta$



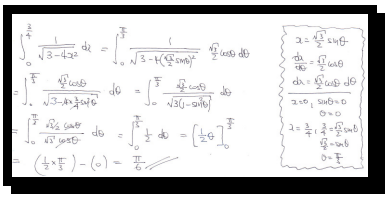
3. $\displaystyle\int_0^{1} \frac{1}{\left(1+x^2\right)^2}\, dx = \frac{1}{8}\left(\pi+2\right)$, use $x = \tan\theta$



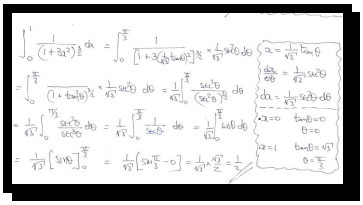
4. $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx = \frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$, use $x=\sec\theta$



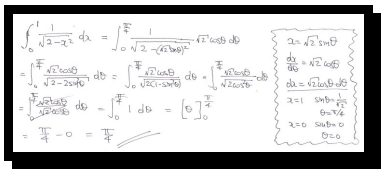
5. $\displaystyle\int_{0}^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx = \frac{\pi}{6}$, use $x=\frac{\sqrt{3}}{2}\sin\theta$



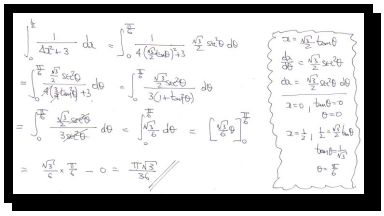
6. $\displaystyle\int_{0}^{1} \frac{1}{\left(1+3x^2\right)^{\frac{3}{2}}}\,dx = \frac{1}{2}$, use $x=\frac{1}{\sqrt{3}}\tan\theta$



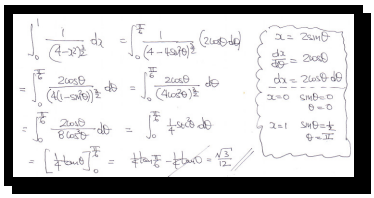
7. $\displaystyle\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx = \frac{\pi}{4}$, use $x = \sqrt{2}\sin\theta$



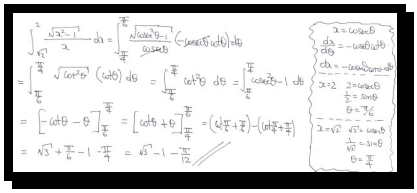
8. $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{4x^2+3}\,dx = \frac{\pi\sqrt{3}}{36}$, use $x = \frac{\sqrt{3}}{2}\tan\theta$



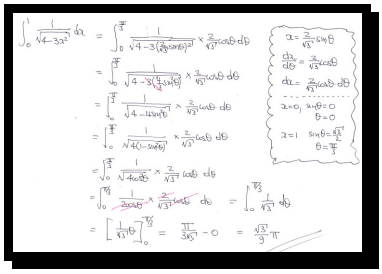
9. $\displaystyle\int_0^1 \frac{1}{\left(4-x^2\right)^{\frac{3}{2}}}\,dx = \frac{\sqrt{3}}{12}$, use $x = 2\sin\theta$



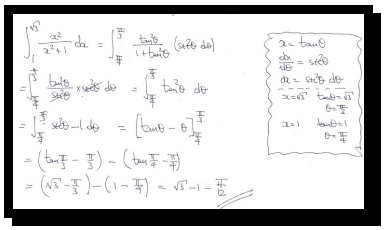
10. $\displaystyle\int_{\sqrt{2}}^{2}\frac{\sqrt{x^2-1}}{x}\,dx=\sqrt{3}-1-\frac{\pi}{12}$, use $x=\operatorname{cosec}\theta$



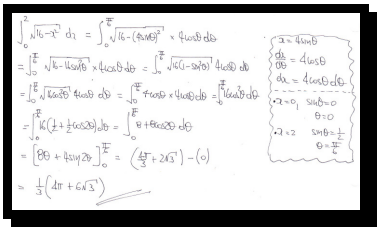
11. $\displaystyle\int_{0}^{1}\frac{1}{\sqrt{4-3x^2}}\,dx=\frac{\pi\sqrt{3}}{9}$, use $x=\frac{2}{\sqrt{3}}\sin\theta$



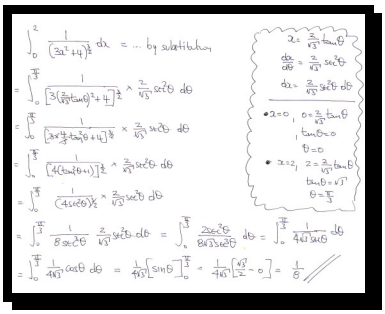
12. $\displaystyle\int_{1}^{\sqrt{3}}\frac{x^2}{x^2+1}\,dx=\sqrt{3}-1-\frac{\pi}{12}$, use $x=\tan\theta$



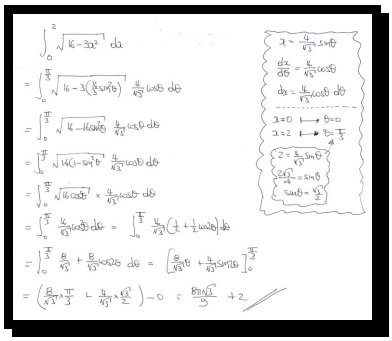
13. $\displaystyle\int_0^2 \sqrt{16-x^2}\ dx = \frac{1}{3}\left(4\pi+6\sqrt{3}\right)$, use $x = 4\sin\theta$



14. $\displaystyle\int_0^2 \frac{1}{\left(3x^2+4\right)^{\frac{3}{2}}}\ dx = \frac{1}{8}$, use $x = \frac{2}{\sqrt{3}}\tan\theta$



15. $\displaystyle\int_0^2 \sqrt{16-3x^2}\ dx = \frac{8\pi\sqrt{3}}{9}+2$, use $x = \frac{4}{\sqrt{3}}\sin\theta$



16. $\displaystyle\int_0^3 \frac{27}{\left(9+x^2\right)^2}\,dx = \frac{\pi}{8}+\frac{1}{4}$, use $x = 3\tan\theta$

