INTEGRATION

BY REVERSE CHAIN RULE

Question 1

Carry out each of the following integrations.

1. $\displaystyle\int 3x\left(x^2-1\right)^4\,dx=\frac{3}{10}\left(x^2-1\right)^5+C$

2. $\displaystyle\int x^2\left(1-4x^3\right)^{-\frac{1}{2}}\,dx=-\frac{1}{6}\left(1-4x^3\right)^{\frac{1}{2}}+C$

3. $\displaystyle\int 4\sin^3 x\cos x\,dx=\sin^4 x+C$

4. $\displaystyle\int \sin x\cos^2 x\,dx=-\frac{1}{3}\cos^3 x+C$

5. $\displaystyle\int \frac{10x}{\sqrt{x^2-7}}\,dx=10\sqrt{x^2-7}+C$

6. $\displaystyle\int 6x\mathrm{e}^{x^2}\,dx=3\mathrm{e}^{x^2}+C$

7. $\displaystyle\int \tan^4 x\sec^2 x\,dx=\frac{1}{5}\tan^5 x+C$

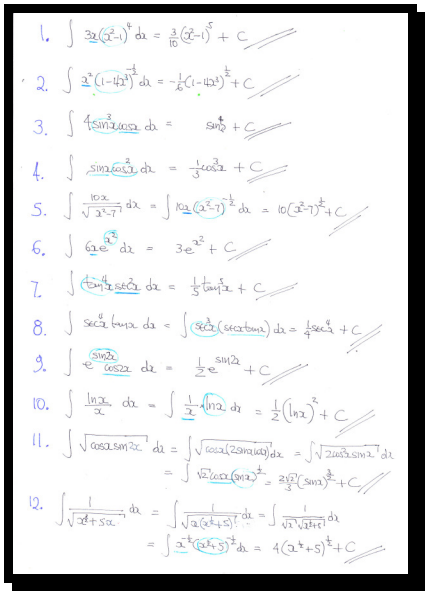
8. $\displaystyle\int \sec^4 x\tan x\,dx=\frac{1}{4}\sec^4 x+C$

9. $\displaystyle\int \mathrm{e}^{\sin 2x}\cos 2x\,dx=\frac{1}{2}\mathrm{e}^{\sin 2x}+C$

10. $\displaystyle\int \frac{\ln x}{x}\,dx=\frac{1}{2}\left(\ln x\right)^2+C$

11. $\displaystyle\int \sqrt{\cos x\sin 2x}\,dx=\frac{2\sqrt{2}}{3}\left(\sin x\right)^{\frac{3}{2}}+C$

12. $\displaystyle\int \frac{1}{\sqrt{x^{\frac{3}{2}}+5x}}\,dx=4\sqrt{x^{\frac{3}{2}}+5x}+C$



1. $\int 3x\left(x^2-1\right)^4 dx = \frac{3}{10}\left(x^2-1\right)^5 + C$
2. $\int x^3\left(1-4x^4\right)^{-\frac{1}{2}} dx = -\frac{1}{8}\left(1-4x^4\right)^{\frac{1}{2}} + C$
3. $\int 4\sin^3 x\cos x\, dx = \sin^4 x + C$
4. $\int \sin 2x\cos^2 2x\, dx = \frac{1}{3}\cos^3 2x + C$
5. $\int \frac{10x}{\sqrt{x^2-7}} dx = \int 10x\left(x^2-7\right)^{-\frac{1}{2}} dx = 10\left(x^2-7\right)^{\frac{1}{2}} + C$
6. $\int 6xe^{x^2} dx = 3e^{x^2} + C$
7. $\int \tan^4 x\sec^2 x\, dx = \frac{1}{5}\tan^5 x + C$
8. $\int \sec^4 x\tan x\, dx = \int \sec^3 x\left(\sec x\tan x\right) dx = \frac{1}{4}\sec^4 x + C$
9. $\int e^{\sin 2x}\cos 2x\, dx = \frac{1}{2}e^{\sin 2x} + C$
10. $\int \frac{\ln x}{x} dx = \int \frac{1}{x}\ln x\, dx = \frac{1}{2}\left(\ln x\right)^2 + C$
11. $\int \sqrt{\cos x\sin^3 x}\, dx = \int \sqrt{\cos x\left(2\sin x\cos x\right)} dx = \int \sqrt{2\cos^2 x\sin x}\, dx$
$= \int \sqrt{2}\cos x\left(\sin x\right)^{\frac{1}{2}} dx = \frac{2\sqrt{2}}{3}\left(\sin x\right)^{\frac{3}{2}} + C$
12. $\int \frac{1}{\sqrt{x^4+5x}} dx = \int \frac{1}{\sqrt{x}\left(x^3+5\right)^{\frac{1}{2}}} dx = \int \frac{1}{\sqrt{x}\sqrt{x^3+5}} dx$
$= \int x^{-\frac{1}{2}}\left(x^3+5\right)^{-\frac{1}{2}} dx = 4\left(x^3+5\right)^{\frac{1}{2}} + C$

Question 2

Carry out each of the following integrations.

1. $\displaystyle\int \frac{x^3}{x^4+2}\,dx = \frac{1}{4}\ln\left(x^4+2\right)+C$

2. $\displaystyle\int \frac{x^2}{4-x^3}\,dx = -\frac{1}{3}\ln\left|4-x^3\right|+C$

3. $\displaystyle\int \frac{4x}{x^2-1}\,dx = 2\ln\left|x^2-1\right|+C$

4. $\displaystyle\int \frac{3x^2}{1+x^3}\,dx = \ln\left|1+x^3\right|+C$

5. $\displaystyle\int \frac{3\mathrm{e}^{2x}}{\mathrm{e}^{2x}-1}\,dx = \frac{3}{2}\ln\left|\mathrm{e}^{2x}-1\right|+C$

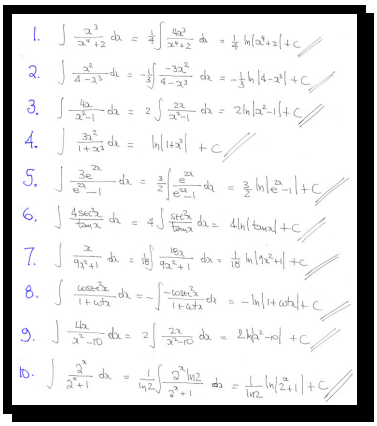
6. $\displaystyle\int \frac{4\sec^2 x}{\tan x}\,dx = 4\ln\left|\tan x\right|+C = 4\ln\sec x+4\ln\sin x+C$

7. $\displaystyle\int \frac{x}{9x^2+1}\,dx = \frac{1}{18}\ln\left(9x^2+1\right)+C$

8. $\displaystyle\int \frac{\operatorname{cosec}^2 x}{1+\cot x}\,dx = -\ln\left|1+\cot x\right|+C$

9. $\displaystyle\int \frac{4x}{x^2-10}\,dx = 2\ln\left|x^2-10\right|+C$

10. $\displaystyle\int \frac{2^x}{2^x+1}\,dx = \frac{\ln\left(2^x+1\right)}{\ln 2}+C$



1. $\int \frac{x^3}{x^2+2}\,dx = \frac{1}{2}\int \frac{4x^3}{x^4+2}\,dx = \frac{1}{4}\ln\left|x^4+2\right|+C$
2. $\int \frac{x^2}{4-x^3}\,dx = -\frac{1}{3}\int \frac{-3x^2}{4-x^3}\,dx = -\frac{1}{3}\ln\left|4-x^3\right|+C$
3. $\int \frac{4x}{x^2-1}\,dx = 2\int \frac{2x}{x^2-1}\,dx = 2\ln\left|x^2-1\right|+C$
4. $\int \frac{3x^2}{1+x^3}\,dx = \ln\left|1+x^3\right|+C$
5. $\int \frac{3e^{2x}}{e^{2x}-1}\,dx = \frac{3}{2}\int \frac{2e^{2x}}{e^{2x}-1}\,dx = \frac{3}{2}\ln\left|e^{2x}-1\right|+C$
6. $\int \frac{4\sec^2 x}{\tan x}\,dx = 4\int \frac{\sec^2 x}{\tan x}\,dx = 4\ln\left|\tan x\right|+C$
7. $\int \frac{x}{9x^2+1}\,dx = \frac{1}{18}\int \frac{18x}{9x^2+1}\,dx = \frac{1}{18}\ln\left|9x^2+1\right|+C$
8. $\int \frac{\cos ec^2 x}{1+\cot x}\,dx = -\int \frac{-\cos ec^2 x}{1+\cot x}\,dx = -\ln\left|1+\cot x\right|+C$
9. $\int \frac{4x}{x^2-10}\,dx = 2\int \frac{2x}{x^2-10}\,dx = 2\ln\left|x^2-10\right|+C$
10. $\int \frac{2^x}{2^x+1}\,dx = \frac{1}{\ln 2}\int \frac{2^x \ln 2}{2^x+1}\,dx = \frac{1}{\ln 2}\ln\left|2^x+1\right|+C$

Question 3

Carry out each of the following integrations.

1. $\displaystyle\int \frac{x}{x^2-9}\,dx=\frac{1}{2}\ln\left|x^2-9\right|+C$

2. $\displaystyle\int \frac{10x}{x^2-9}\,dx=5\ln\left|x^2-9\right|+C$

3. $\displaystyle\int \frac{3x}{4-2x^2}\,dx=-\frac{3}{4}\ln\left|4-2x^2\right|+C$

4. $\displaystyle\int \frac{x^2}{x^3+1}\,dx=\frac{1}{3}\ln\left|x^3+1\right|+C$

5. $\displaystyle\int \frac{2x+6}{x^2+6x+1}\,dx=\ln\left|x^2+6x+1\right|+C$

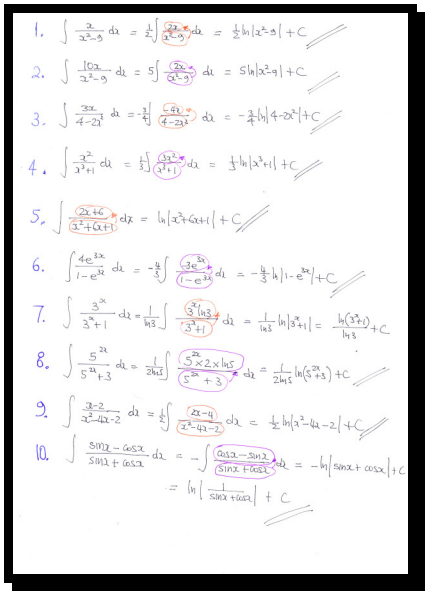
6. $\displaystyle\int \frac{4\mathrm{e}^{3x}}{1-\mathrm{e}^{3x}}\,dx=-\frac{4}{3}\ln\left|1-\mathrm{e}^{3x}\right|+C$

7. $\displaystyle\int \frac{3^x}{3^x+1}\,dx=\frac{\ln\left(3^x+1\right)}{\ln 3}+C$

8. $\displaystyle\int \frac{5^{2x}}{5^{2x}+3}\,dx=\frac{\ln\left(5^{2x}+3\right)}{\ln 25}+C$

9. $\displaystyle\int \frac{x-2}{x^2-4x-2}\,dx=\frac{1}{2}\ln\left|x^2-4x-2\right|+C$

10. $\displaystyle\int \frac{\sin x-\cos x}{\sin x+\cos x}\,dx=-\ln\left|\sin x+\cos x\right|+C$



1. $\int \frac{x}{x^2-9}\,dx = \frac{1}{2}\int \frac{2x}{x^2-9}\,dx = \frac{1}{2}\ln|x^2-9|+C$
2. $\int \frac{10x}{x^2-9}\,dx = 5\int \frac{2x}{x^2-9}\,dx = 5\ln|x^2-9|+C$
3. $\int \frac{3x}{4-2x^2}\,dx = -\frac{3}{4}\int \frac{-4x}{4-2x^2}\,dx = -\frac{3}{4}\ln|4-2x^2|+C$
4. $\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx = \frac{1}{3}\ln|x^3+1|+C$
5. $\int \frac{2x+6}{x^2+6x+1}\,dx = \ln|x^2+6x+1|+C$
6. $\int \frac{4e^{3x}}{1-e^{3x}}\,dx = -\frac{4}{3}\int \frac{-3e^{3x}}{1-e^{3x}}\,dx = -\frac{4}{3}\ln|1-e^{3x}|+C$
7. $\int \frac{3^x}{3^x+1}\,dx = \frac{1}{\ln 3}\int \frac{3^x \ln 3}{3^x+1}\,dx = \frac{1}{\ln 3}\ln|3^x+1| = \frac{\ln(3^x+1)}{\ln 3}+C$
8. $\int \frac{5^{2x}}{5^{2x}+3}\,dx = \frac{1}{2\ln 5}\int \frac{5^{2x}\times 2\times\ln 5}{5^{2x}+3}\,dx = \frac{1}{2\ln 5}\ln(5^{2x}+3)+C$
9. $\int \frac{2x-2}{x^2-4x-2}\,dx = \frac{1}{2}\int \frac{2x-4}{x^2-4x-2}\,dx = \frac{1}{2}\ln|x^2-4x-2|+C$
10. $\int \frac{\sin x-\cos x}{\sin x+\cos x}\,dx = -\int \frac{\cos x-\sin x}{\sin x+\cos x}\,dx = -\ln|\sin x+\cos x|+C$
$= \ln\left|\frac{1}{\sin x+\cos x}\right|+C$

Question 4

Carry out each of the following integrations.

1. $\displaystyle\int \frac{x}{\left(x^2-1\right)^3}\,dx=-\frac{1}{4}\left(x^2-1\right)^{-2}+C$

2. $\displaystyle\int \cos x\sin x\,dx=\frac{1}{2}\sin^2 x+C=-\frac{1}{2}\cos^2 x+C=-\frac{1}{4}\cos 2x+C$

3. $\displaystyle\int \frac{4x}{\sqrt{1-2x^2}}\,dx=-2\sqrt{1-2x^2}+C$

4. $\displaystyle\int \sec^2 x\left(1+\tan^2 x\right)\,dx=\tan x+\frac{1}{3}\tan^3 x+C$

5. $\displaystyle\int \sec^2 x\left(1+\tan x\right)\,dx=\frac{1}{2}\left(1+\tan x\right)^2+C$

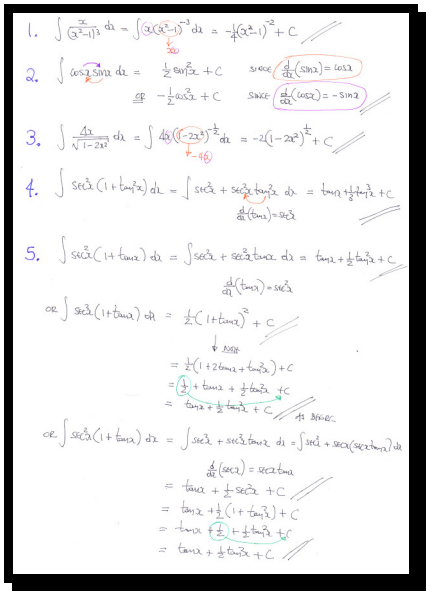
6. $\displaystyle\int \sec x\tan x\sqrt{\sec x+1}\,dx=\frac{2}{3}\left(\sec x+1\right)^{\frac{3}{2}}+C$

7. $\displaystyle\int \tan^2 x\sec^2 x\,dx=\frac{1}{3}\tan^3 x+C$

8. $\displaystyle\int \mathrm{e}^{\sin x}\cos x\,dx=\mathrm{e}^{\sin x}+C$

9. $\displaystyle\int \sqrt{\sin x\cos^2 x}\,dx=\frac{2}{3}\left(\sin x\right)^{\frac{3}{2}}+C$

10. $\displaystyle\int \left(2x+1\right)\left(x^2+x+1\right)\,dx=\frac{1}{2}\left(x^2+x+1\right)^2+C=\frac{1}{2}x^4+x^3+\frac{3}{2}x^2+x+C$



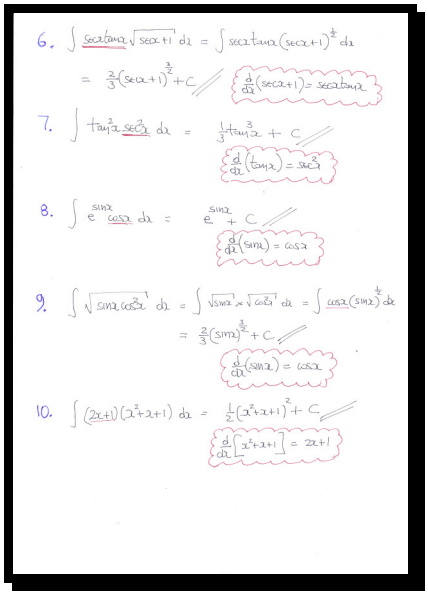
1. $\int \frac{x}{(x^2-1)^3}\,dx = \int x(x^2-1)^{-3}\,dx = -\frac{1}{4}(x^2-1)^{-2}+C$

2. $\int \cos x \sin x\,dx = \frac{1}{2}\sin^2 x + C$ since $\frac{d}{dx}(\sin x)=\cos x$
   or $-\frac{1}{2}\cos^2 x + C$ since $\frac{d}{dx}(\cos x)=-\sin x$

3. $\int \frac{dx}{\sqrt{1-2x^2}}\,dx = \int 4\left(1-2x^2\right)^{-\frac{1}{2}}\,dx = -2\left(1-2x^2\right)^{\frac{1}{2}}+C$

4. $\int \sec^2 x\left(1+\tan^2 x\right)dx = \int \sec^2 x + \sec^2 x\tan^2 x\,dx = \tan x + \frac{1}{3}\tan^3 x + C$
   $\frac{d}{dx}(\tan x)=\sec^2 x$

5. $\int \sec^2 x(1+\tan x)\,dx = \int \sec^2 x + \sec^2 x\tan x\,dx = \tan x + \frac{1}{2}\tan^2 x + C$
   $\frac{d}{dx}(\tan x)=\sec^2 x$
   or $\int \sec^2 x(1+\tan x)\,dx = \frac{1}{2}(1+\tan x)^2 + C$
   NOW
   $= \frac{1}{2}(1+2\tan x+\tan^2 x)+C$
   $= \frac{1}{2}+\tan x+\frac{1}{2}\tan^2 x+C$
   $= \tan x+\frac{1}{2}\tan^2 x+C$ AS ABOVE
   or $\int \sec^2 x(1+\tan x)\,dx = \int \sec^2 x + \sec^2 x\tan x\,dx = \int \sec^2 x + \sec x(\sec x\tan x)\,dx$
   $\frac{d}{dx}(\sec x)=\sec x\tan x$
   $= \tan x+\frac{1}{2}\sec^2 x+C$
   $= \tan x+\frac{1}{2}(1+\tan^2 x)+C$
   $= \tan x+\frac{1}{2}+\frac{1}{2}\tan^2 x+C$
   $= \tan x+\frac{1}{2}\tan^2 x+C$



6. $\int \sec x\tan x\sqrt{\sec x+1}\,dx = \int \sec x\tan x(\sec x+1)^{\frac{1}{2}}\,dx$
   $= \frac{2}{3}(\sec x+1)^{\frac{3}{2}}+C$
   $\frac{d}{dx}(\sec x+1)=\sec x\tan x$

7. $\int \tan^2 x\sec^2 x\,dx = \frac{1}{3}\tan^3 x + C$
   $\frac{d}{dx}(\tan x)=\sec^2 x$

8. $\int e^{\sin x}\cos x\,dx = e^{\sin x}+C$
   $\frac{d}{dx}(\sin x)=\cos x$

9. $\int \sqrt{\sin x}\cos^3 x\,dx = \int \sqrt{\sin x}\times\sqrt{\cos^3 x}\,dx = \int \cos x(\sin x)^{\frac{1}{2}}\,dx$
   $= \frac{2}{3}(\sin x)^{\frac{3}{2}}+C$
   $\frac{d}{dx}(\sin x)=\cos x$

10. $\int (2x+1)(x^2+x+1)^1\,dx = \frac{1}{2}(x^2+x+1)^2+C$
    $\frac{d}{dx}(x^2+x+1)=2x+1$

Question 5

Carry out each of the following integrations.

1. $\displaystyle\int (2x+1)\sin\left(x^2+x+1\right)\,dx = -\cos\left(x^2+x+1\right)+C$

2. $\displaystyle\int (x+1)\cos\left(x^2+2x+1\right)\,dx = \frac{1}{2}\sin\left(x^2+2x+1\right)+C$

3. $\displaystyle\int \frac{1}{x(1+\ln x)^3}\,dx = -\frac{1}{2(1+\ln x)^2}+C$

4. $\displaystyle\int 4-\cos^4 x\sin x\,dx = 4x+\frac{1}{5}\cos^5 x+C$

5. $\displaystyle\int \frac{\cos x}{\sin^3 x}\,dx = -\frac{1}{2}\operatorname{cosec}^2 x+C = -\frac{1}{2}\cot^2 x+C$

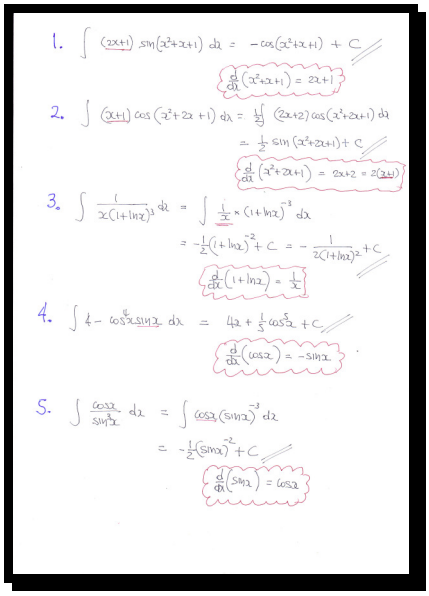
6. $\displaystyle\int \frac{\sqrt{1+2\tan x}}{\cos^2 x}\,dx = \frac{1}{3}(1+2\tan x)^{\frac{3}{2}}+C$

7. $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\,dx = 2\sqrt{\sin x}+C$

8. $\displaystyle\int \frac{1}{x\ln x}\,dx = \ln\left|\ln|x|\right|+C$

9. $\displaystyle\int \frac{1}{\cos^2 x\tan^4 x}\,dx = -\frac{1}{3\tan^3 x}+C$

10. $\displaystyle\int \sin^3 2x\cos 2x\,dx = \frac{1}{8}\sin^4 2x+C$



1. $\int (2x+1)\sin(x^2+x+1)\,dx = -\cos(x^2+x+1)+C$

$\frac{d}{dx}(x^2+x+1) = 2x+1$

2. $\int (2x+1)\cos(x^2+2x+1)\,dx = \frac{1}{2}\int (2x+2)\cos(x^2+2x+1)\,dx$

$= \frac{1}{2}\sin(x^2+2x+1)+C$

$\frac{d}{dx}(x^2+2x+1) = 2x+2 = 2(2x)$

3. $\int \frac{1}{x(1+\ln x)^3}\,dx = \int \frac{1}{x}\times(1+\ln x)^{-3}\,dx$

$= -\frac{1}{2}(1+\ln x)^{-2}+C = -\frac{1}{2(1+\ln x)^2}+C$

$\frac{d}{dx}(1+\ln x) = \frac{1}{x}$

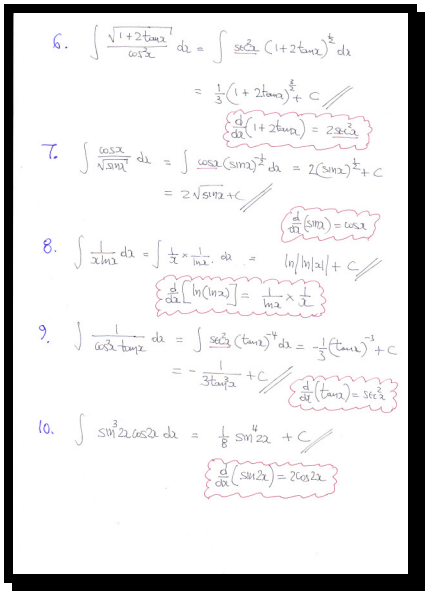
4. $\int 4-\cos^4 x\sin x\,dx = 4x+\frac{1}{5}\cos^5 x+C$

$\frac{d}{dx}(\cos x) = -\sin x$

5. $\int \frac{\cos x}{\sin^3 x}\,dx = \int \cos x(\sin x)^{-3}\,dx$

$= -\frac{1}{2}(\sin x)^{-2}+C$

$\frac{d}{dx}(\sin x) = \cos x$



6. $\int \frac{\sqrt{1+2\tan x}}{\cos^2 x}\,dx = \int \sec^2 x(1+2\tan x)^{\frac{1}{2}}\,dx$

$= \frac{1}{3}(1+2\tan x)^{\frac{3}{2}}+C$

$\frac{d}{dx}(1+2\tan x) = 2\sec^2 x$

7. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx = \int \cos x(\sin x)^{-\frac{1}{2}}\,dx = 2(\sin x)^{\frac{1}{2}}+C$

$= 2\sqrt{\sin x}+C$

$\frac{d}{dx}(\sin x) = \cos x$

8. $\int \frac{1}{x\ln x}\,dx = \int \frac{1}{x}\times\frac{1}{\ln x}\,dx = \ln|\ln|x||+C$

$\frac{d}{dx}\left[\ln(\ln x)\right] = \frac{1}{\ln x}\times\frac{1}{x}$

9. $\int \frac{1}{\cos^2 x\tan^4 x}\,dx = \int \sec^2 x(\tan x)^{-4}\,dx = -\frac{1}{3}(\tan x)^{-3}+C$

$= -\frac{1}{3\tan^3 x}+C$

$\frac{d}{dx}\left(\frac{1}{\tan x}\right) = \sec^2 x$

10. $\int \sin^3 2x\cos 2x\,dx = \frac{1}{8}\sin^4 2x+C$

$\frac{d}{dx}(\sin 2x) = 2\cos 2x$

Question 6

Carry out each of the following integrations.

1. $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx = \sin(\ln x) + C$

2. $\displaystyle\int \frac{3x}{\sqrt{4-2x^2}}\,dx = -\frac{3}{2}\sqrt{4-2x^2} + C$

3. $\displaystyle\int \frac{\sin x}{\cos^4 x}\,dx = \frac{1}{3}\sec^3 x + C$

4. $\displaystyle\int \cos x \sin^3 x\,dx = \frac{1}{4}\sin^4 x + C$

5. $\displaystyle\int \frac{\sin^2 x}{\cos^4 x}\,dx = \frac{1}{3}\tan^3 x + C$

6. $\displaystyle\int \mathrm{e}^x \sin\left(\mathrm{e}^x\right)\,dx = -\cos\left(\mathrm{e}^x\right) + C$

7. $\displaystyle\int \sin 2x \cos^4 2x\,dx = -\frac{1}{10}\cos^5 2x + C$

8. $\displaystyle\int 3x^2\left(4-2x^3\right)^{\frac{3}{2}}\,dx = -\frac{1}{5}\left(4-2x^3\right)^{\frac{5}{2}} + C$

9. $\displaystyle\int \frac{x+1}{\sqrt[3]{x^2+2x+3}}\,dx = \frac{3}{4}\sqrt[3]{\left(x^2+2x+3\right)^2} + C$

10. $\displaystyle\int \sin 2x \cos 2x\,dx = \frac{1}{4}\sin^2 2x + C$ or $-\frac{1}{8}\cos 4x + C$

Question 7

Carry out each of the following integrations.

1. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx = \frac{1}{3}(\ln x)^3 + C$

2. $\displaystyle\int (x+1)\left(x^2+2x+1\right)^4\,dx = \frac{1}{10}\left(x^2+2x+1\right)^5 + C$

3. $\displaystyle\int \sin x \cos^4 x\,dx = -\frac{1}{5}\cos^5 x + C$

4. $\displaystyle\int \sec^3 x \tan x\,dx = \frac{1}{3}\sec^3 x + C$

5. $\displaystyle\int x\left(3+x^2\right)^4\,dx = \frac{1}{10}\left(3+x^2\right)^5 + C$

6. $\displaystyle\int \frac{\cos x}{\sqrt{\sin^3 x}}\,dx = -\frac{2}{\sqrt{\sin x}} + C$

7. $\displaystyle\int \cos x\sqrt{\sin x}\,dx = \frac{2}{3}\sqrt{\sin^3 x} + C$

8. $\displaystyle\int \frac{\sec^2 x}{(1+\tan x)^3}\,dx = -\frac{1}{2(1+\tan x)^2} + C$

9. $\displaystyle\int \frac{\sin x \cos x}{\sqrt{\cos 2x+1}}\,dx = -\frac{1}{2}\sqrt{\cos 2x+1} + C = -\frac{\sqrt{2}}{2}\cos x$

10. $\displaystyle\int \frac{\ln x^2}{x}\,dx = (\ln x)^2 + C$

Question 8

Carry out each of the following integrations.

1. $\displaystyle\int 3x^2\left(4-2x^3\right)^{\frac{3}{2}}\,dx=-\frac{1}{5}\left(4-2x^3\right)^{\frac{5}{2}}+C$

2. $\displaystyle\int \frac{\mathrm{e}^{\sqrt{x}}}{\sqrt{x}}\,dx=2\,\mathrm{e}^{\sqrt{x}}+C$

3. $\displaystyle\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}}\,dx=\frac{4}{3}\left(x^{\frac{1}{2}}+1\right)^{\frac{3}{2}}+C$

4. $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx=\sqrt{x^2+1}+C$

5. $\displaystyle\int \frac{1}{\sqrt{x}\cos^2\sqrt{x}}\,dx=2\tan\sqrt{x}+C$

6. $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx=\sqrt{x^2+1}+C$

7. $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1}}\,dx=4\sqrt{\sqrt{x}+1}+C$

8. $\displaystyle\int \frac{\sin x}{\cos^5 x}\,dx=\frac{1}{4}\sec^4 x+C$

9. $\displaystyle\int x\sqrt{1-x^2}\,dx=-\frac{1}{3}\left(1-x^2\right)^{\frac{3}{2}}+C$

10. $\displaystyle\int \sqrt{\frac{2\sqrt{x}+3}{4x}}\,dx=\frac{2}{3}\left(2x^{\frac{1}{2}}+3\right)^{\frac{3}{2}}+C$

Question 9

Carry out each of the following integrations.

1. $\displaystyle\int_0^2 \frac{2x}{\sqrt{x^2+4}}\,dx = 4\left(\sqrt{2}-1\right)$

2. $\displaystyle\int_0^{36} \frac{1}{\sqrt{x}\left(\sqrt{x}+2\right)}\,dx = \ln 16$

3. $\displaystyle\int_0^3 \frac{x}{x^2+9}\,dx = \frac{1}{2}\ln 2$