INTEGRATION

BY REVERSE CHAIN RULE

(WITHOUT ANSWERS)

Question 1

Carry out each of the following integrations.

1. $\int 3x\left(x^2-1\right)^4 \, dx$

2. $\int x^2\left(1-4x^3\right)^{-\frac{1}{2}} \, dx$

3. $\int 4\sin^3 x\cos x \, dx$

4. $\int \sin x\cos^2 x \, dx$

5. $\int \frac{10x}{\sqrt{x^2-7}} \, dx$

6. $\int 6x\mathrm{e}^{x^2} \, dx$

7. $\int \tan^4 x\sec^2 x \, dx$

8. $\int \sec^4 x\tan x \, dx$

9. $\int \mathrm{e}^{\sin 2x}\cos 2x \, dx$

10. $\int \frac{\ln x}{x} \, dx$

11. $\int \sqrt{\cos x\sin 2x} \, dx$

12. $\int \frac{1}{\sqrt{x^{\frac{3}{2}}+5x}} \, dx$

Question 2

Carry out each of the following integrations.

1. $\displaystyle\int \frac{x^3}{x^4+2}\,dx$

2. $\displaystyle\int \frac{x^2}{4-x^3}\,dx$

3. $\displaystyle\int \frac{4x}{x^2-1}\,dx$

4. $\displaystyle\int \frac{3x^2}{1+x^3}\,dx$

5. $\displaystyle\int \frac{3\mathrm{e}^{2x}}{\mathrm{e}^{2x}-1}\,dx$

6. $\displaystyle\int \frac{4\sec^2 x}{\tan x}\,dx$

7. $\displaystyle\int \frac{x}{9x^2+1}\,dx$

8. $\displaystyle\int \frac{\operatorname{cosec}^2 x}{1+\cot x}\,dx$

9. $\displaystyle\int \frac{4x}{x^2-10}\,dx$

10. $\displaystyle\int \frac{2^x}{2^x+1}\,dx$

Question 3

Carry out each of the following integrations.

1. $\displaystyle\int \frac{x}{x^2-9}\,dx$

2. $\displaystyle\int \frac{10x}{x^2-9}\,dx$

3. $\displaystyle\int \frac{3x}{4-2x^2}\,dx$

4. $\displaystyle\int \frac{x^2}{x^3+1}\,dx$

5. $\displaystyle\int \frac{2x+6}{x^2+6x+1}\,dx$

6. $\displaystyle\int \frac{4\mathrm{e}^{3x}}{1-\mathrm{e}^{3x}}\,dx$

7. $\displaystyle\int \frac{3^x}{3^x+1}\,dx$

8. $\displaystyle\int \frac{5^{2x}}{5^{2x}+3}\,dx$

9. $\displaystyle\int \frac{x-2}{x^2-4x-2}\,dx$

10. $\displaystyle\int \frac{\sin x-\cos x}{\sin x+\cos x}\,dx$

Question 4

Carry out each of the following integrations.

1. $\displaystyle\int \frac{x}{\left(x^2-1\right)^3}\,dx$

2. $\displaystyle\int \cos x \sin x\,dx$

3. $\displaystyle\int \frac{4x}{\sqrt{1-2x^2}}\,dx$

4. $\displaystyle\int \sec^2 x\left(1+\tan^2 x\right)\,dx$

5. $\displaystyle\int \sec^2 x\left(1+\tan x\right)\,dx$

6. $\displaystyle\int \sec x \tan x\sqrt{\sec x+1}\,dx$

7. $\displaystyle\int \tan^2 x \sec^2 x\,dx$

8. $\displaystyle\int \mathrm{e}^{\sin x}\cos x\,dx$

9. $\displaystyle\int \sqrt{\sin x \cos^2 x}\,dx$

10. $\displaystyle\int \left(2x+1\right)\left(x^2+x+1\right)\,dx$

Question 5

Carry out each of the following integrations.

1. $\displaystyle\int (2x+1)\sin\left(x^2+x+1\right)\,dx$

2. $\displaystyle\int (x+1)\cos\left(x^2+2x+1\right)\,dx$

3. $\displaystyle\int \frac{1}{x(1+\ln x)^3}\,dx$

4. $\displaystyle\int 4-\cos^4 x\sin x\,dx$

5. $\displaystyle\int \frac{\cos x}{\sin^3 x}\,dx$

6. $\displaystyle\int \frac{\sqrt{1+2\tan x}}{\cos^2 x}\,dx$

7. $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\,dx$

8. $\displaystyle\int \frac{1}{x\ln x}\,dx$

9. $\displaystyle\int \frac{1}{\cos^2 x\tan^4 x}\,dx$

10. $\displaystyle\int \sin^3 2x\cos 2x\,dx$

Question 6

Carry out each of the following integrations.

1. $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$

2. $\displaystyle\int \frac{3x}{\sqrt{4-2x^2}}\,dx$

3. $\displaystyle\int \frac{\sin x}{\cos^4 x}\,dx$

4. $\displaystyle\int \cos x \sin^3 x\,dx$

5. $\displaystyle\int \frac{\sin^2 x}{\cos^4 x}\,dx$

6. $\displaystyle\int \mathrm{e}^x \sin\left(\mathrm{e}^x\right)\,dx$

7. $\displaystyle\int \sin 2x \cos^4 2x\,dx$

8. $\displaystyle\int 3x^2\left(4-2x^3\right)^{\frac{3}{2}}\,dx$

9. $\displaystyle\int \frac{x+1}{\sqrt[3]{x^2+2x+3}}\,dx$

10. $\displaystyle\int \sin 2x \cos 2x\,dx$

Question 7

Carry out each of the following integrations.

1. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

2. $\displaystyle\int (x+1)\left(x^2+2x+1\right)^4\,dx$

3. $\displaystyle\int \sin x \cos^4 x\,dx$

4. $\displaystyle\int \sec^3 x \tan x\,dx$

5. $\displaystyle\int x\left(3+x^2\right)^4\,dx$

6. $\displaystyle\int \frac{\cos x}{\sqrt{\sin^3 x}}\,dx$

7. $\displaystyle\int \cos x\sqrt{\sin x}\,dx$

8. $\displaystyle\int \frac{\sec^2 x}{(1+\tan x)^3}\,dx$

9. $\displaystyle\int \frac{\sin x \cos x}{\sqrt{\cos 2x+1}}\,dx$

10. $\displaystyle\int \frac{\ln x^2}{x}\,dx$

Question 8

Carry out each of the following integrations.

1. $\displaystyle\int 3x^2\left(4-2x^3\right)^{\frac{3}{2}}\,dx$

2. $\displaystyle\int \frac{\mathrm{e}^{\sqrt{x}}}{\sqrt{x}}\,dx$

3. $\displaystyle\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}}\,dx$

4. $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx$

5. $\displaystyle\int \frac{1}{\sqrt{x}\cos^2\sqrt{x}}\,dx$

6. $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx$

7. $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1}}\,dx$

8. $\displaystyle\int \frac{\sin x}{\cos^5 x}\,dx$

9. $\displaystyle\int x\sqrt{1-x^2}\,dx$

10. $\displaystyle\int \sqrt{\frac{2\sqrt{x}+3}{4x}}\,dx$

Question 9

Carry out each of the following integrations.

1. $\displaystyle\int_0^2 \frac{2x}{\sqrt{x^2+4}}\,dx = 4\left(\sqrt{2}-1\right)$

2. $\displaystyle\int_0^{36} \frac{1}{\sqrt{x}\left(\sqrt{x}+2\right)}\,dx = \ln 16$

3. $\displaystyle\int_0^3 \frac{x}{x^2+9}\,dx = \frac{1}{2}\ln 2$