INTEGRATION BY PARTS

Question 1

Carry out the following integrations:

1. $\displaystyle\int x\mathrm{e}^{2x}\,dx=\frac{1}{2}x\mathrm{e}^{2x}-\frac{1}{4}\mathrm{e}^{2x}+C$

2. $\displaystyle\int 3x\cos 2x\,dx=\frac{3}{2}x\sin 2x+\frac{3}{4}\cos 2x+C$

3. $\displaystyle\int x\sin 4x\,dx=-\frac{1}{4}x\cos 4x+\frac{1}{16}\sin 4x+C$

4. $\displaystyle\int -2x\sin 5x\,dx=\frac{2}{5}x\cos 5x-\frac{2}{25}\sin 5x+C$

5. $\displaystyle\int (1-2x)\mathrm{e}^{-x}\,dx=(2x-1)\mathrm{e}^{-x}+2\mathrm{e}^{-x}+C$

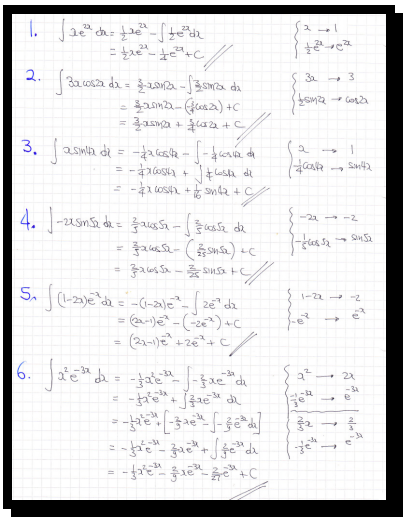
6. $\displaystyle\int x^2\mathrm{e}^{-3x}\,dx=-\frac{1}{3}x^2\mathrm{e}^{-3x}-\frac{2}{9}x\mathrm{e}^{-3x}-\frac{2}{27}\mathrm{e}^{-3x}+C$

7. $\displaystyle\int 16x^3\ln x\,dx=4x^4\ln|x|-x^4+C$

8. $\displaystyle\int \ln x\,dx=x\ln x-x+C$

9. $\displaystyle\int x\cos\left(\tfrac{1}{2}x\right)dx=2x\sin\left(\tfrac{1}{2}x\right)+4\cos\left(\tfrac{1}{2}x\right)+C$

10. $\displaystyle\int (3x-1)\sin(3x-1)\,dx=-\frac{1}{3}(3x-1)\cos(3x-1)+\frac{1}{3}\sin(3x-1)+C$



1. $\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$
$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$
$\begin{cases} x \to 1 \\ \frac{1}{2}e^{2x} \to e^{2x} \end{cases}$

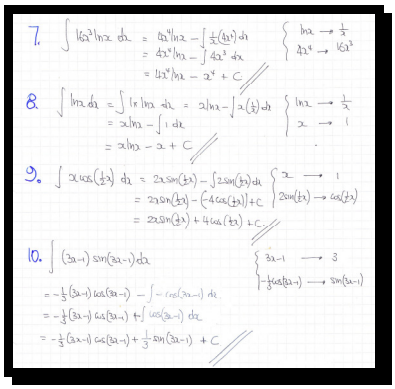
2. $\int 3x\cos 2x\,dx = \frac{3}{2}x\sin 2x - \int \frac{3}{2}\sin 2x\,dx$
$= \frac{3}{2}x\sin 2x - \left(-\frac{3}{4}\cos 2x\right) + C$
$= \frac{3}{2}x\sin 2x + \frac{3}{4}\cos 2x + C$
$\begin{cases} 3x \to 3 \\ \frac{1}{2}\sin 2x \to \cos 2x \end{cases}$

3. $\int x\sin 4x\,dx = -\frac{1}{4}x\cos 4x - \int -\frac{1}{4}\cos 4x\,dx$
$= -\frac{1}{4}x\cos 4x + \int \frac{1}{4}\cos 4x\,dx$
$= -\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x + C$
$\begin{cases} x \to 1 \\ -\frac{1}{4}\cos 4x \to \sin 4x \end{cases}$

4. $\int -2x\sin 5x\,dx = \frac{2}{5}x\cos 5x - \int \frac{2}{5}\cos 5x\,dx$
$= \frac{2}{5}x\cos 5x - \left(\frac{2}{25}\sin 5x\right) + C$
$= \frac{2}{5}x\cos 5x - \frac{2}{25}\sin 5x + C$
$\begin{cases} -2x \to -2 \\ -\frac{1}{5}\cos 5x \to \sin 5x \end{cases}$

5. $\int (1-2x)e^{-x}\,dx = -(1-2x)e^{-x} - \int 2e^{-x}\,dx$
$= (2x-1)e^{-x} - \left(-2e^{-x}\right) + C$
$= (2x-1)e^{-x} + 2e^{-x} + C$
$\begin{cases} 1-2x \to -2 \\ -e^{-x} \to e^{-x} \end{cases}$

6. $\int x^2e^{-3x}\,dx = -\frac{1}{3}x^2e^{-3x} - \int -\frac{2}{3}xe^{-3x}\,dx$
$= -\frac{1}{3}x^2e^{-3x} + \int \frac{2}{3}xe^{-3x}\,dx$
$= -\frac{1}{3}x^2e^{-3x} + \left[-\frac{2}{9}xe^{-3x} - \int -\frac{2}{9}e^{-3x}\,dx\right]$
$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} + \int \frac{2}{9}e^{-3x}\,dx$
$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C$
$\begin{cases} x^2 \to 2x \\ \frac{1}{3}e^{-3x} \to e^{-3x} \end{cases}$
$\begin{cases} \frac{2}{3}x \to \frac{2}{3} \\ -\frac{1}{3}e^{-3x} \to e^{-3x} \end{cases}$



7. $\int 16x^3\ln x\,dx = 4x^4\ln x - \int \frac{1}{x}\left(4x^4\right)dx$
$= 4x^4\ln x - \int 4x^3\,dx$
$= 4x^4\ln x - x^4 + C$
$\begin{cases} \ln x \to \frac{1}{x} \\ 4x^4 \to 16x^3 \end{cases}$

8. $\int \ln x\,dx = \int 1\times\ln x\,dx = x\ln x - \int x\left(\frac{1}{x}\right)dx$
$= x\ln x - \int 1\,dx$
$= x\ln x - x + C$
$\begin{cases} \ln x \to \frac{1}{x} \\ x \to 1 \end{cases}$

9. $\int x\cos\left(\frac{1}{2}x\right)dx = 2x\sin\left(\frac{1}{2}x\right) - \int 2\sin\left(\frac{1}{2}x\right)dx$
$= 2x\sin\left(\frac{1}{2}x\right) - \left(-4\cos\left(\frac{1}{2}x\right)\right) + C$
$= 2x\sin\left(\frac{1}{2}x\right) + 4\cos\left(\frac{1}{2}x\right) + C$
$\begin{cases} x \to 1 \\ 2\sin\left(\frac{1}{2}x\right) \to \cos\left(\frac{1}{2}x\right) \end{cases}$

10. $\int (3x-1)\sin(3x-1)\,dx$
$= -\frac{1}{3}(3x-1)\cos(3x-1) - \int -\cos(3x-1)\,dx$
$= -\frac{1}{3}(3x-1)\cos(3x-1) + \int \cos(3x-1)\,dx$
$= -\frac{1}{3}(3x-1)\cos(3x-1) + \frac{1}{3}\sin(3x-1) + C$
$\begin{cases} 3x-1 \to 3 \\ -\frac{1}{3}\cos(3x-1) \to \sin(3x-1) \end{cases}$

Question 2

Carry out the following integrations:

1. $\displaystyle\int 6x\mathrm{e}^{3x}\,dx = 2x\mathrm{e}^{3x} - \frac{2}{3}\mathrm{e}^{3x} + C$

2. $\displaystyle\int 12x\cos 3x\,dx = 4x\sin 3x + \frac{4}{3}\cos 3x + C$

3. $\displaystyle\int x\sin 6x\,dx = -\frac{1}{6}x\cos 6x + \frac{1}{36}\sin 6x + C$

4. $\displaystyle\int -x\sin 2x\,dx = \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + C$

5. $\displaystyle\int (2-x)\mathrm{e}^{-3x}\,dx = -\frac{1}{3}(2-x)\mathrm{e}^{-3x} + \frac{1}{9}\mathrm{e}^{-3x} + C$

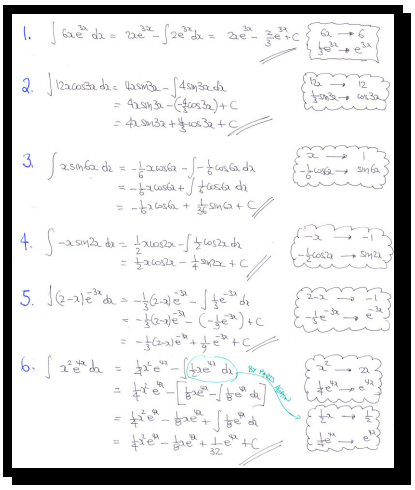
6. $\displaystyle\int x^2\mathrm{e}^{4x}\,dx = \frac{1}{4}x^2\mathrm{e}^{4x} - \frac{1}{8}x\mathrm{e}^{4x} + \frac{1}{32}\mathrm{e}^{4x} + C$

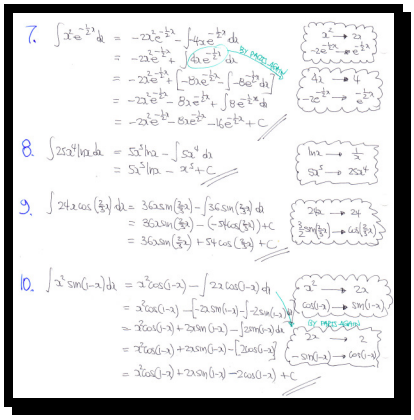
7. $\displaystyle\int x^2\mathrm{e}^{-\frac{1}{2}x}\,dx = -2x^2\mathrm{e}^{-\frac{1}{2}x} - 8x\mathrm{e}^{-\frac{1}{2}x} - 16\mathrm{e}^{-\frac{1}{2}x} + C$

8. $\displaystyle\int 25x^4\ln x\,dx = 5x^5\ln x - x^5 + C$

9. $\displaystyle\int 24x\cos\left(\tfrac{2}{3}x\right)dx = 36x\sin\left(\tfrac{2}{3}x\right) + 54\cos\left(\tfrac{2}{3}x\right) + C$

10. $\displaystyle\int x^2\sin(1-x)\,dx = x^2\cos(1-x) + 2x\sin(1-x) - 2\cos(1-x) + C$





Question 3

Carry out the following integrations:

1. $\displaystyle\int \tfrac{1}{2}x\mathrm{e}^{4x}\,dx=\frac{1}{8}x\mathrm{e}^{4x}-\frac{1}{32}\mathrm{e}^{4x}+C$

2. $\displaystyle\int 5x\sin 4x\,dx=-\frac{5}{4}x\cos 4x+\frac{5}{16}\sin 4x+C$

3. $\displaystyle\int (2x+1)\cos 2x\,dx=\frac{1}{2}(2x+1)\sin 2x+\frac{1}{2}\cos 2x+C$

4. $\displaystyle\int -3x\cos 4x\,dx=-\frac{3}{4}x\sin 4x-\frac{3}{16}\cos 4x+C$

5. $\displaystyle\int x^2\mathrm{e}^{-2x}\,dx=-\frac{1}{2}x^2\mathrm{e}^{-2x}-\frac{1}{2}x\mathrm{e}^{-2x}-\frac{1}{4}\mathrm{e}^{-2x}+C$

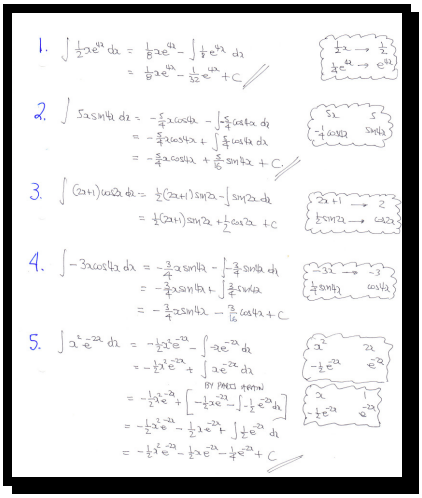
6. $\displaystyle\int x^2\sin 5x\,dx=-\frac{1}{5}x^2\cos 5x+\frac{2}{25}x\sin 5x+\frac{2}{125}\cos 5x+C$

7. $\displaystyle\int x^2\cos\tfrac{1}{3}x\,dx=3x^2\sin\tfrac{1}{3}x+18x\cos\tfrac{1}{3}x-54\sin\tfrac{1}{3}x+C$

8. $\displaystyle\int \tfrac{1}{2}x^3\ln x\,dx=\frac{1}{8}x^4\ln x-\frac{1}{32}x^4+C$

9. $\displaystyle\int x\ln 3x\,dx=\frac{1}{2}x^2\ln 3x-\frac{1}{4}x^2+C$

10. $\displaystyle\int \frac{\ln x}{x^3}\,dx=-\frac{\ln x}{2x^2}-\frac{1}{4x^2}+C$



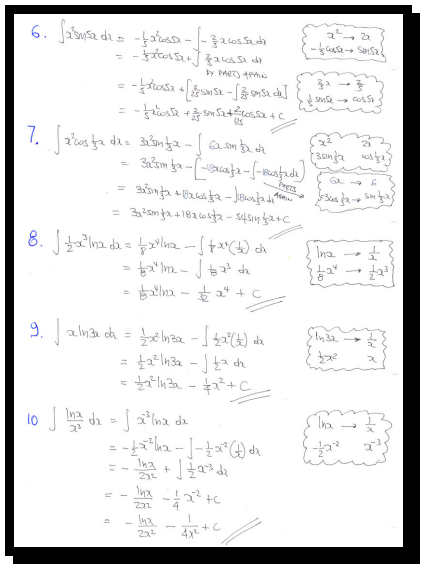
1. $\int \frac{1}{2}xe^{4x}\,dx = \frac{1}{8}xe^{4x} - \int \frac{1}{8}e^{4x}\,dx$
$= \frac{1}{8}xe^{4x} - \frac{1}{32}e^{4x} + C$

2. $\int 5x\sin 4x\,dx = -\frac{5}{4}x\cos 4x - \int -\frac{5}{4}\cos 4x\,dx$
$= -\frac{5}{4}x\cos 4x + \int \frac{5}{4}\cos 4x\,dx$
$= -\frac{5}{4}x\cos 4x + \frac{5}{16}\sin 4x + C$

3. $\int (2x+1)\cos 2x\,dx = \frac{1}{2}(2x+1)\sin 2x - \int \sin 2x\,dx$
$= \frac{1}{2}(2x+1)\sin 2x + \frac{1}{2}\cos 2x + C$

4. $\int -3x\cos 4x\,dx = -\frac{3}{4}x\sin 4x - \int -\frac{3}{4}\sin 4x\,dx$
$= -\frac{3}{4}x\sin 4x + \int \frac{3}{4}\sin 4x\,dx$
$= -\frac{3}{4}x\sin 4x - \frac{3}{16}\cos 4x + C$

5. $\int x^2 e^{-2x}\,dx = -\frac{1}{2}x^2e^{-2x} - \int -xe^{-2x}\,dx$
$= -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\,dx$
BY PARTS AGAIN
$= -\frac{1}{2}x^2e^{-2x} + \left[-\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx\right]$
$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\,dx$
$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$



6. $\int x^2\sin 5x\,dx = -\frac{1}{5}x^2\cos 5x - \int -\frac{2}{5}x\cos 5x\,dx$
$= -\frac{1}{5}x^2\cos 5x + \int \frac{2}{5}x\cos 5x\,dx$
BY PARTS AGAIN
$= -\frac{1}{5}x^2\cos 5x + \left[\frac{2}{25}x\sin 5x - \int \frac{2}{25}\sin 5x\,dx\right]$
$= -\frac{1}{5}x^2\cos 5x + \frac{2}{25}x\sin 5x + \frac{2}{125}\cos 5x + C$

7. $\int x^2\cos\frac{1}{3}x\,dx = 3x^2\sin\frac{1}{3}x - \int 6x\sin\frac{1}{3}x\,dx$
$= 3x^2\sin\frac{1}{3}x - \left[-18x\cos\frac{1}{3}x - \int -18\cos\frac{1}{3}x\,dx\right]$ PARTS AGAIN
$= 3x^2\sin\frac{1}{3}x + 18x\cos\frac{1}{3}x - 18\int \cos\frac{1}{3}x\,dx$
$= 3x^2\sin\frac{1}{3}x + 18x\cos\frac{1}{3}x - 54\sin\frac{1}{3}x + C$

8. $\int \frac{1}{2}x^3\ln x\,dx = \frac{1}{8}x^4\ln x - \int \frac{1}{8}x^4\left(\frac{1}{x}\right)dx$
$= \frac{1}{8}x^4\ln x - \int \frac{1}{8}x^3\,dx$
$= \frac{1}{8}x^4\ln x - \frac{1}{32}x^4 + C$

9. $\int x\ln 3x\,dx = \frac{1}{2}x^2\ln 3x - \int \frac{1}{2}x^2\left(\frac{1}{x}\right)dx$
$= \frac{1}{2}x^2\ln 3x - \int \frac{1}{2}x\,dx$
$= \frac{1}{2}x^2\ln 3x - \frac{1}{4}x^2 + C$

10. $\int \frac{\ln x}{x^3}\,dx = \int x^{-3}\ln x\,dx$
$= -\frac{1}{2}x^{-2}\ln x - \int -\frac{1}{2}x^{-2}\left(\frac{1}{x}\right)dx$
$= -\frac{\ln x}{2x^2} + \int \frac{1}{2}x^{-3}\,dx$
$= -\frac{\ln x}{2x^2} - \frac{1}{4}x^{-2} + C$
$= -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C$

Question 4

Carry out the following integrations:

1. $\displaystyle\int x\mathrm{e}^{5x}\,dx=\frac{1}{5}x\mathrm{e}^{5x}-\frac{1}{25}\mathrm{e}^{5x}+C$

2. $\displaystyle\int 2x\cos 3x\,dx=\frac{2}{3}x\sin 3x+\frac{2}{9}\cos 3x+C$

3. $\displaystyle\int x\sin 3x\,dx=-\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x+C$

4. $\displaystyle\int x\sin 4x\,dx=\frac{1}{16}\sin 4x-\frac{1}{4}x\cos 4x+C$

5. $\displaystyle\int 2\ln x\,dx=2x\ln x-2x+C$

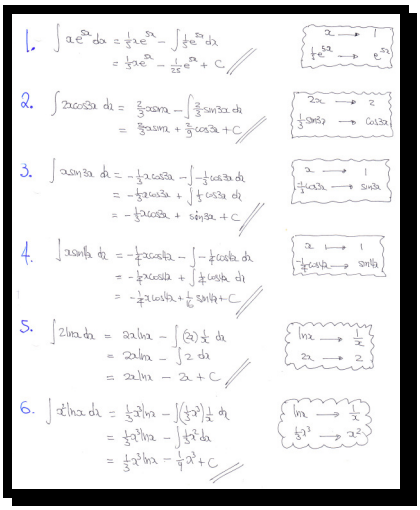
6. $\displaystyle\int x^2\ln x\,dx=\frac{1}{3}x^3\ln|x|-\frac{1}{9}x^3+C$

7. $\displaystyle\int x\sin\left(\tfrac{1}{2}x\right)dx=4\sin\left(\tfrac{1}{2}x\right)-2x\cos\left(\tfrac{1}{2}x\right)+C$

8. $\displaystyle\int x\sin\left(2x-1\right)dx=-\frac{1}{2}x\cos\left(2x-1\right)+\frac{1}{4}\sin\left(2x-1\right)+C$

9. $\displaystyle\int\frac{\ln x}{x^2}\,dx=-\frac{\ln x}{x}-\frac{1}{x}+C$

10. $\displaystyle\int x\sec^2 x\,dx=x\tan x-\ln|\sec x|+C$



1. $\int xe^{5x}\,dx = \frac{1}{5}xe^{5x} - \int \frac{1}{5}e^{5x}\,dx$
$= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$

$x \longrightarrow 1$
$\frac{1}{5}e^{5x} \longrightarrow e^{5x}$

2. $\int x\cos 3x\,dx = \frac{x}{3}\sin 3x - \int \frac{1}{3}\sin 3x\,dx$
$= \frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + C$

$2x \longrightarrow 2$
$\frac{1}{3}\sin 3x \longrightarrow \cos 3x$

3. $\int x\sin 3x\,dx = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,dx$
$= -\frac{1}{3}x\cos 3x + \int \frac{1}{3}\cos 3x\,dx$
$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$

$x \longrightarrow 1$
$-\frac{1}{3}\cos 3x \longrightarrow \sin 3x$

4. $\int x\sin\frac{x}{4}\,dx = -\frac{1}{4}x\cos\frac{x}{4} - \int -\frac{1}{4}\cos\frac{x}{4}\,dx$
$= -\frac{1}{4}x\cos\frac{x}{4} + \int \frac{1}{4}\cos\frac{x}{4}\,dx$
$= -\frac{1}{4}x\cos\frac{x}{4} + \frac{1}{16}\sin\frac{x}{4} + C$

$x \longrightarrow 1$
$-\frac{1}{4}\cos\frac{x}{4} \longrightarrow \sin\frac{x}{4}$

5. $\int 2\ln x\,dx = 2x\ln x - \int \left(\frac{2}{x}\right)\frac{1}{x}\,dx$
$= 2x\ln x - \int 2\,dx$
$= 2x\ln x - 2x + C$

$\ln x \longrightarrow \frac{1}{x}$
$2x \longrightarrow 2$

6. $\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \int \left(\frac{1}{3}x^3\right)\frac{1}{x}\,dx$
$= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2\,dx$
$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

$\ln x \longrightarrow \frac{1}{x}$
$\frac{1}{3}x^3 \longrightarrow x^2$

7. $\int x\sin\left(\frac{1}{2}x\right)dx = -2x\cos\left(\frac{1}{2}x\right) - \int -2\cos\left(\frac{1}{2}x\right)dx$
$= -2x\cos\left(\frac{1}{2}x\right) + \int 2\cos\left(\frac{1}{2}x\right)dx$
$= -2x\cos\left(\frac{1}{2}x\right) + 4\sin\left(\frac{1}{2}x\right) + C$

$x \longrightarrow 1$
$2\cos\left(\frac{1}{2}x\right) \longrightarrow \sin\left(\frac{1}{2}x\right)$

8. $\int x\sin(2x-1)\,dx = -\frac{1}{2}x\cos(2x-1) - \int -\frac{1}{2}\cos(2x-1)\,dx$
$= -\frac{1}{2}x\cos(2x-1) + \int \frac{1}{2}\cos(2x-1)\,dx$
$= -\frac{1}{2}x\cos(2x-1) + \frac{1}{4}\sin(2x-1) + C$

$x \longrightarrow 1$
$-\frac{1}{2}\cos(2x-1) \longrightarrow \sin(2x-1)$

9. $\int \frac{\ln x}{x^2}\,dx = \int (\ln x)\times(x^{-2})\,dx$
$= -x^{-1}\ln x - \int -x^{-1}\times\frac{1}{x}\,dx$
$= -\frac{\ln x}{x} + \int x^{-2}\,dx$
$= -\frac{\ln x}{x} - x^{-1} + C$
$= -\frac{\ln x}{x} - \frac{1}{x} + C$

$\ln x \longrightarrow \frac{1}{x}$
$-x^{-1} \longrightarrow x^{-2}$

10. $\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$
$= x\tan x - \ln|\sec x| + C$

Formula Booklet

$x \longrightarrow 1$
$\tan x \longrightarrow \sec^2 x$

Question 5

Carry out the following integrations:

1. $\displaystyle\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

2. $\displaystyle\int x^3 \ln x \, dx = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

3. $\displaystyle\int \sin x \ln(\sec x) \, dx = -\cos x\left(1 + \ln|\sec x|\right) + C$

4. $\displaystyle\int x\cos 5x \, dx = \frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$

5. $\displaystyle\int x^2 \sin 3x \, dx = -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$

6. $\displaystyle\int 4x\,\mathrm{e}^{-\frac{2}{3}x} \, dx = -3(2x+3)\mathrm{e}^{-\frac{2}{3}x} + C$

7. $\displaystyle\int x^2 \cos\left(\frac{1}{3}x\right) dx = 3x^2 \sin\left(\frac{1}{3}x\right) + 18x\cos\left(\frac{1}{3}x\right) - 54\sin\left(\frac{1}{3}x\right) + C$

8. $\displaystyle\int 2x^2 \sec^2 x \tan x \, dx = x^2 \sec^2 x - 2x\tan x + 2\ln|\sec x| + C$

9. $\displaystyle\int x^2\,\mathrm{e}^{\frac{1}{2}x} \, dx = \left(2x^2 - 8x + 16\right)\mathrm{e}^{\frac{1}{2}x} + C$

10. $\displaystyle\int x\sec x\tan x \, dx = x\sec x - \ln|\sec x + \tan x| + C$

Question 6

Carry out the following integrations:

1. $\displaystyle\int x^2 \mathrm{e}^{-\frac{1}{4}x}\, dx = -4\mathrm{e}^{-\frac{1}{4}x}\left(x^2+8x+32\right)+C$

2. $\displaystyle\int x^2 \mathrm{e}^{-x}\, dx = -\mathrm{e}^{-x}\left(x^2+2x+2\right)+C$

3. $\displaystyle\int \mathrm{e}^{x}\cos x\, dx = \frac{1}{2}\mathrm{e}^{x}\left(\cos x+\sin x\right)+C$

4. $\displaystyle\int \left(\ln x\right)^2\, dx = x\left(\ln x\right)^2-2x\ln x+2x+C$

5. $\displaystyle\int \mathrm{e}^{x}\sin x\, dx = \frac{1}{2}\mathrm{e}^{x}\left(\sin x-\cos x\right)+C$

6. $\displaystyle\int \left(x^3+5x^2-2\right)\mathrm{e}^{2x}\, dx = \frac{1}{8}\mathrm{e}^{2x}\left(4x^3+14x^2-14x-1\right)+C$

7. $\displaystyle\int x\cos^2 x\, dx = \frac{1}{8}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x+C$

8. $\displaystyle\int x\ln 2x^3\, dx = \frac{3}{4}x^2\ln 2\left(2\ln x-1\right)+C$

Question 7

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\ln 2} x\mathrm{e}^{2x}\ dx = \ln 4 - \frac{3}{4}$

2. $\displaystyle\int_0^{\frac{\pi}{3}} 6x\sin 3x\ dx = \frac{2\pi}{3}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x^2\cos x\ dx = \frac{1}{4}\left(\pi^2 - 8\right)$

4. $\displaystyle\int_1^{\mathrm{e}} x\ln x\ dx = \frac{1}{4}\left(\mathrm{e}^2 + 1\right)$

5. $\displaystyle\int_0^{1} 4x\mathrm{e}^{3x}\ dx = \frac{4}{9}\left(2\mathrm{e}^3 + 1\right)$

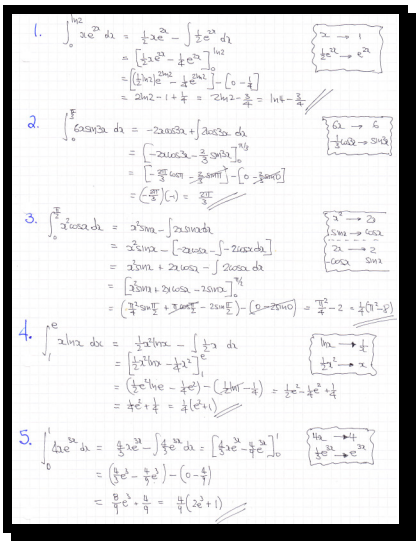
6. $\displaystyle\int_0^{\frac{\pi}{4}} x\sin 4x\ dx = \frac{\pi}{16}$

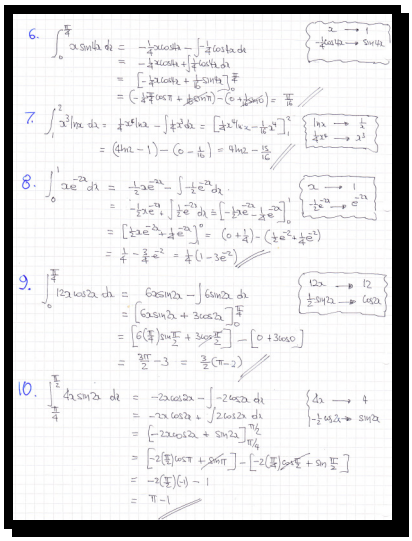
7. $\displaystyle\int_1^{2} x^3\ln x\ dx = 4\ln 2 - \frac{15}{16}$

8. $\displaystyle\int_0^{1} x\mathrm{e}^{-2x}\ dx = \frac{1}{4}\left(1 - 3\mathrm{e}^{-2}\right)$

9. $\displaystyle\int_0^{\frac{\pi}{4}} 12x\cos 2x\ dx = \frac{3}{2}\left(\pi - 2\right)$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x\sin 2x\ dx = \pi - 1$





Question 8

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{3}} x\sin 3x \ dx = \frac{\pi}{9}$

2. $\displaystyle\int_0^{\frac{\pi}{4}} 2x\cos 4x \ dx = -\frac{1}{4}$

3. $\displaystyle\int_0^{\ln 2} 4x\mathrm{e}^{-x} \ dx = 2-\ln 4$

4. $\displaystyle\int_1^{\mathrm{e}} \ln x \ dx = 1$

5. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin 2x \ dx = \frac{\pi}{4}$

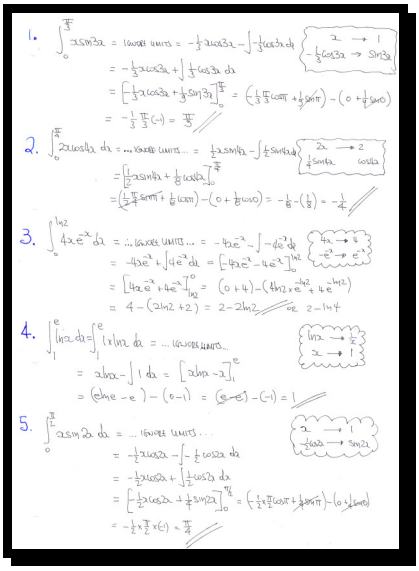
6. $\displaystyle\int_0^{\ln 4} x\mathrm{e}^{\frac{1}{2}x} \ dx = 8\ln 2-4$

7. $\displaystyle\int_0^{\pi} x\cos\left(\frac{1}{4}x\right) \ dx = 2\sqrt{2}\left(\pi+4\right)-16$

8. $\displaystyle\int_0^{1} \left(2x+1\right)\mathrm{e}^{2x} \ dx = \mathrm{e}^2$

9. $\displaystyle\int_{\frac{1}{\mathrm{e}}}^{1} x\ln x \ dx = \frac{1}{4}\left(\frac{3}{\mathrm{e}^2}-1\right)$

10. $\displaystyle\int_{-1}^{0} 3\ln\left(2x+3\right) \ dx = \tfrac{3}{2}\left(\ln 27-2\right)$ **REQUIRES ADDITIONAL TECHNIQUES**



1. $\int_0^{\frac{\pi}{3}} x\sin 3x\,dx = \ldots$ (known until) $= -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,dx$ — $x \to 1$, $-\frac{1}{3}\cos 3x \to \sin 3x$
$= -\frac{1}{3}x\cos 3x + \int \frac{1}{3}\cos 3x\,dx$
$= \left[-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{3}} = \left(-\frac{1}{3}\cdot\frac{\pi}{3}\cos\pi + \frac{1}{9}\sin\pi\right) - \left(0 + \frac{1}{9}\sin 0\right)$
$= -\frac{1}{3}\cdot\frac{\pi}{3}(-1) = \frac{\pi}{9}$

2. $\int_0^{\frac{\pi}{4}} 2x\cos 2x\,dx = \ldots$ (known until) $= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$ — $2x \to 2$, $\frac{1}{2}\sin 2x \to \cos 2x$
$= \left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}}$
$= \left(\frac{1}{2}\cdot\frac{\pi}{4}\sin\frac{\pi}{2} + \frac{1}{4}\cos\frac{\pi}{2}\right) - \left(0 + \frac{1}{4}\cos 0\right) = \frac{1}{8}\pi - \frac{1}{4} = \frac{1}{8}(\pi - 2)$

3. $\int_0^{\ln 2} 4xe^{-2x}\,dx = \ldots$ (known until) $= -4xe^{-2} - \int -4e^{-2}\,dx$ — $4x \to 4$, $-e^{-2} \to e^{-2}$
$= -4xe^{-2} + \int 4e^{-2}\,dx = \left[-4xe^{-2} - 4e^{-2}\right]_0^{\ln 2}$
$= \left[4xe^{-2} + 4e^{-2}\right]_{\ln 2}^{0} = (0+4) - (4\ln 2\times e^{-2\ln 2} + 4e^{-2\ln 2})$
$= 4 - (2\ln 2 + 2) = 2 - 2\ln 2$ or $2 - \ln 4$

4. $\int_1^e \ln x\,dx = \int_1^e 1\times\ln x\,dx = \ldots$ (known until) — $\ln x \to \frac{1}{x}$, $x \to 1$
$= x\ln x - \int 1\,dx = \left[x\ln x - x\right]_1^e$
$= (e\ln e - e) - (0 - 1) = (e - e) - (-1) = 1$

5. $\int_0^{\frac{\pi}{4}} x\sin 2x\,dx = \ldots$ (known until) $= -\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\,dx$ — $x \to 1$, $\frac{1}{2}\cos 2x \to \sin 2x$
$= -\frac{1}{2}x\cos 2x + \int \frac{1}{2}\cos 2x\,dx$
$= \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{4}} = \left(-\frac{1}{2}\times\frac{\pi}{4}\cos\frac{\pi}{2} + \frac{1}{4}\sin\frac{\pi}{2}\right) - \left(0 + \frac{1}{4}\sin 0\right)$
$= -\frac{1}{2}\times\frac{\pi}{4}\times(0) + \frac{1}{4} = \frac{1}{4}$

6. $\int_0^{\ln 4} xe^{\frac{1}{2}x}\,dx = \ldots$ (known until) $= 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\,dx$ — $x \to 1$, $\frac{1}{2}e^{\frac{1}{2}x} \to e^{\frac{1}{2}x}$
$= \left[2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right]_0^{\ln 4}$
$= \left(2\ln 4\times e^{\frac{1}{2}\ln 4} - 4e^{\frac{1}{2}\ln 4}\right) - (0 - 4) = 4\ln 4 - 8 + 4$
$= 4\ln 4 - 4 = 8\ln 2 - 4$

7. $\int_0^{\pi} x\cos\left(\frac{1}{2}x\right)dx = \ldots$ (known until) $= 4x\sin\left(\frac{1}{2}x\right) - \int 4\sin\left(\frac{1}{2}x\right)dx$ — $x \to 1$, $\sin\left(\frac{1}{2}x\right) \to \cos\left(\frac{1}{2}x\right)$
$= \left[4x\sin\left(\frac{1}{2}x\right) + 16\cos\left(\frac{1}{2}x\right)\right]_0^{\pi}$
$= \left(4\pi\sin\frac{\pi}{4} + 16\cos\frac{\pi}{4}\right) - (0 + 16\cos 0) = 2\pi\sqrt{2} + 8\sqrt{2} - 16$
$= 2\sqrt{2}(\pi + 4) - 16$

8. $\int_0^1 (2x+1)e^{2x}\,dx = \ldots$ (known until) $= \frac{1}{2}(2x+1)e^{2x} - \int e^{2x}\,dx$ — $2x+1 \to 2$, $\frac{1}{2}e^{2x} \to e^{2x}$
$= \left[\frac{1}{2}(2x+1)e^{2x} - \frac{1}{2}e^{2x}\right]_0^1 = \left(\frac{3}{2}e^2 - \frac{1}{2}e^2\right) - \left(\frac{1}{2}e^0\right)$
$= e^2 - \frac{1}{2}$

9. $\int_{\frac{1}{e}}^1 x\ln x\,dx = \ldots$ (known until) $= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx$ — $\ln x \to \frac{1}{x}$, $\frac{1}{2}x^2 \to x$
$= \left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_{\frac{1}{e}}^1$
$= \left(\frac{1}{2}\ln 1 - \frac{1}{4}\right) - \left[\frac{1}{2}\left(\frac{1}{e}\right)^2\ln\left(\frac{1}{e}\right) - \frac{1}{4}\left(\frac{1}{e}\right)^2\right] = -\frac{1}{4} - \left[-\frac{1}{2e^2} - \frac{1}{4e^2}\right]$
$= -\frac{1}{4} + \frac{1}{2e^2} + \frac{1}{4e^2} = \frac{1}{4}\left[-1 + \frac{2}{e^2} + \frac{1}{e^2}\right] = \frac{1}{4}\left[\frac{3}{e^2} - 1\right]$

10. $\int_{-1}^{0} 3\ln(2x+3)\,dx = \ldots$ (known until) $= \int 3\times\ln(2x+3)\,dx$ — $\ln(2x+3) \to \frac{2}{2x+3}$, $3x \to 3$
$= 3x\ln(2x+3) - \int \frac{6x}{2x+3}\,dx$
$= 3x\ln(2x+3) - \int \frac{3(2x+3)-9}{2x+3}\,dx$
$= 3x\ln(2x+3) - \int 3 - \frac{9}{2x+3}\,dx$
$= \left[3x\ln(2x+3) - 3x + \frac{9}{2}\ln(2x+3)\right]_{-1}^{0}$
$= \left(0 + \frac{9}{2}\ln 3\right) - \left(-3\ln 1 + 3 + \frac{9}{2}\ln 1\right) = \frac{9}{2}\ln 3 - 3$
or $\frac{3}{2}(3\ln 3 - 2) = \frac{3}{2}(\ln 27 - 2)$

Question 9

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{4}} x\sec^2 x \ dx = \frac{1}{4}\left(\pi - \ln 4\right)$

2. $\displaystyle\int_1^2 \frac{\ln x}{x} \ dx = \frac{1}{2}\left(\ln 2\right)^2$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin^2 x \ dx = \frac{1}{16}\left(\pi^2 + 4\right)$