INTEGRATION BY PARTS

(student version)

Question 1

Carry out the following integrations:

1. $\int x\mathrm{e}^{2x}\,dx$

2. $\int 3x\cos 2x\,dx$

3. $\int x\sin 4x\,dx$

4. $\int -2x\sin 5x\,dx$

5. $\int (1-2x)\mathrm{e}^{-x}\,dx$

6. $\int x^2\mathrm{e}^{-3x}\,dx$

7. $\int 16x^3\ln x\,dx$

8. $\int \ln x\,dx$

9. $\int x\cos\left(\frac{1}{2}x\right)\,dx$

10. $\int (3x-1)\sin(3x-1)\,dx$

Question 2

Carry out the following integrations:

1. $\int 6x\mathrm{e}^{3x}\,dx$

2. $\int 12x\cos 3x\,dx$

3. $\int x\sin 6x\,dx$

4. $\int -x\sin 2x\,dx$

5. $\int (2-x)\mathrm{e}^{-3x}\,dx$

6. $\int x^2\mathrm{e}^{4x}\,dx$

7. $\int x^2\mathrm{e}^{-\frac{1}{2}x}\,dx$

8. $\int 25x^4\ln x\,dx$

9. $\int 24x\cos\left(\frac{2}{3}x\right)dx$

10. $\int x^2\sin(1-x)\,dx$

Question 3

Carry out the following integrations:

1. $\int \frac{1}{2} x \mathrm{e}^{4x} \, dx$

2. $\int 5x \sin 4x \, dx$

3. $\int (2x+1) \cos 2x \, dx$

4. $\int -3x \cos 4x \, dx$

5. $\int x^2 \mathrm{e}^{-2x} \, dx$

6. $\int x^2 \sin 5x \, dx$

7. $\int x^2 \cos \frac{1}{3} x \, dx$

8. $\int \frac{1}{2} x^3 \ln x \, dx$

9. $\int x \ln 3x \, dx$

10. $\int \frac{\ln x}{x^3} \, dx$

Question 4

Carry out the following integrations:

1. $\int x\mathrm{e}^{5x}\,dx$

2. $\int 2x\cos 3x\,dx$

3. $\int x\sin 3x\,dx$

4. $\int x\sin 4x\,dx$

5. $\int 2\ln x\,dx$

6. $\int x^2\ln x\,dx$

7. $\int x\sin\left(\frac{1}{2}x\right)dx$

8. $\int x\sin\left(2x-1\right)dx$

9. $\int \frac{\ln x}{x^2}\,dx$

10. $\int x\sec^2 x\,dx$

Question 5

Carry out the following integrations:

1. $\int x^2 \sin x \, dx$

2. $\int x^3 \ln x \, dx$

3. $\int \sin x \ln(\sec x) \, dx$

4. $\int x \cos 5x \, dx$

5. $\int x^2 \sin 3x \, dx$

6. $\int 4x \mathrm{e}^{-\frac{2}{3}x} \, dx$

7. $\int x^2 \cos\left(\frac{1}{3}x\right) dx$

8. $\int 2x^2 \sec^2 x \tan x \, dx$

9. $\int x^2 \mathrm{e}^{\frac{1}{2}x} \, dx$

10. $\int x \sec x \tan x \, dx$

Question 6

Carry out the following integrations:

1. $\int x^2 \mathrm{e}^{-\frac{1}{4}x} \, dx$

2. $\int x^2 \mathrm{e}^{-x} \, dx$

3. $\int \mathrm{e}^x \cos x \, dx$

4. $\int (\ln x)^2 \, dx$

5. $\int \mathrm{e}^x \sin x \, dx$

6. $\int \left(x^3 + 5x^2 - 2\right) \mathrm{e}^{2x} \, dx$

7. $\int x \cos^2 x \, dx$

8. $\int x \ln 2x^3 \, dx$

Question 7

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\ln 2} x\mathrm{e}^{2x}\ dx = \ln 4 - \frac{3}{4}$

2. $\displaystyle\int_0^{\frac{\pi}{3}} 6x\sin 3x\ dx = \frac{2\pi}{3}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x^2\cos x\ dx = \frac{1}{4}\left(\pi^2 - 8\right)$

4. $\displaystyle\int_1^{\mathrm{e}} x\ln x\ dx = \frac{1}{4}\left(\mathrm{e}^2 + 1\right)$

5. $\displaystyle\int_0^{1} 4x\mathrm{e}^{3x}\ dx = \frac{4}{9}\left(2\mathrm{e}^3 + 1\right)$

6. $\displaystyle\int_0^{\frac{\pi}{4}} x\sin 4x\ dx = \frac{\pi}{16}$

7. $\displaystyle\int_1^{2} x^3\ln x\ dx = 4\ln 2 - \frac{15}{16}$

8. $\displaystyle\int_0^{1} x\mathrm{e}^{-2x}\ dx = \frac{1}{4}\left(1 - 3\mathrm{e}^{-2}\right)$

9. $\displaystyle\int_0^{\frac{\pi}{4}} 12x\cos 2x\ dx = \frac{3}{2}\left(\pi - 2\right)$

10. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x\sin 2x\ dx = \pi - 1$

Question 8

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{3}} x\sin 3x \ dx = \frac{\pi}{9}$

2. $\displaystyle\int_0^{\frac{\pi}{4}} 2x\cos 4x \ dx = -\frac{1}{4}$

3. $\displaystyle\int_0^{\ln 2} 4x\mathrm{e}^{-x} \ dx = 2-\ln 4$

4. $\displaystyle\int_1^{\mathrm{e}} \ln x \ dx = 1$

5. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin 2x \ dx = \frac{\pi}{4}$

6. $\displaystyle\int_0^{\ln 4} x\mathrm{e}^{\frac{1}{2}x} \ dx = 8\ln 2-4$

7. $\displaystyle\int_0^{\pi} x\cos\left(\frac{1}{4}x\right) \ dx = 2\sqrt{2}\left(\pi+4\right)-16$

8. $\displaystyle\int_0^{1} \left(2x+1\right)\mathrm{e}^{2x} \ dx = \mathrm{e}^2$

9. $\displaystyle\int_{\frac{1}{\mathrm{e}}}^{1} x\ln x \ dx = \frac{1}{4}\left(\frac{3}{\mathrm{e}^2}-1\right)$

10. $\displaystyle\int_{-1}^{0} 3\ln\left(2x+3\right) \ dx = \tfrac{3}{2}\left(\ln 27-2\right)$ **REQUIRES ADDITIONAL TECHNIQUES**

Question 9

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{4}} x\sec^2 x \ dx = \frac{1}{4}\left(\pi - \ln 4\right)$

2. $\displaystyle\int_1^2 \frac{\ln x}{x} \ dx = \frac{1}{2}\left(\ln 2\right)^2$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin^2 x \ dx = \frac{1}{16}\left(\pi^2 + 4\right)$