INTEGRATION BASICS

Question 1

Integrate:

1. $\displaystyle\int (3x+1)^2\,dx=\frac{1}{9}(3x+1)^3+C$

2. $\displaystyle\int 4(2x+1)^5\,dx=\frac{1}{3}(2x+1)^6+C$

3. $\displaystyle\int \frac{6}{(2x-1)^2}\,dx=-\frac{3}{2x-1}+C$

4. $\displaystyle\int 6(4x-3)^{\frac{1}{2}}\,dx=(4x-3)^{\frac{3}{2}}+C$

5. $\displaystyle\int \frac{6}{\sqrt{3x+1}}\,dx=4\sqrt{3x+1}+C$

6. $\displaystyle\int 10(1-4x)^{\frac{3}{2}}\,dx=-(1-4x)^{\frac{5}{2}}+C$

7. $\displaystyle\int 20(1-3x)^4\,dx=-\frac{4}{3}(1-3x)^5+C$

8. $\displaystyle\int \sqrt[3]{8x-1}\,dx=\frac{3}{32}(8x-1)^{\frac{4}{3}}+C$

9. $\displaystyle\int \frac{60}{(1-4x)^{\frac{7}{2}}}dx=6(1-4x)^{-\frac{5}{2}}+C$

10. $\displaystyle\int 12\left(3-\tfrac{1}{2}x\right)^{\frac{1}{2}}\,dx=-16\left(3-\tfrac{1}{2}x\right)^{\frac{3}{2}}+C$

Question 2

Integrate:

1. $\displaystyle\int (5x+1)^3\,dx=\frac{1}{20}(5x+1)^4+C$

2. $\displaystyle\int 3(4x+1)^3\,dx=\frac{3}{16}(4x+1)^4+C$

3. $\displaystyle\int \frac{4}{(3x-1)^2}\,dx=-\frac{4}{3(3x-1)}+C$

4. $\displaystyle\int 18(3x-2)^{\frac{1}{2}}\,dx=4(3x-2)^{\frac{3}{2}}+C$

5. $\displaystyle\int \frac{12}{\sqrt{4x+1}}\,dx=6\sqrt{4x+1}+C$

6. $\displaystyle\int 15(1-2x)^{\frac{3}{2}}\,dx=-3(1-2x)^{\frac{5}{2}}+C$

7. $\displaystyle\int 15(1-6x)^3\,dx=-\frac{5}{8}(1-6x)^4+C$

8. $\displaystyle\int \sqrt[3]{6x-1}\,dx=\frac{1}{8}(6x-1)^{\frac{4}{3}}+C$

9. $\displaystyle\int \frac{30}{(1-2x)^{\frac{9}{2}}}dx=\frac{30}{7}(1-2x)^{-\frac{7}{2}}+C$

10. $\displaystyle\int 30\left(1-\tfrac{1}{3}x\right)^{\frac{3}{2}}\,dx=-36\left(1-\tfrac{1}{3}x\right)^{\frac{5}{2}}+C$

Question 3

Integrate:

1. $\int (5x+1)^6 \, dx = \frac{1}{35}(5x+1)^7 + C$

2. $\int 3(3x+1)^4 \, dx = \frac{1}{5}(3x+1)^5 + C$

3. $\int \frac{12}{(6x-1)^3} \, dx = -\frac{1}{(6x-1)^2} + C$

4. $\int 15(2x-3)^{\frac{3}{2}} \, dx = 3(2x-3)^{\frac{5}{2}} + C$

5. $\int \frac{2}{\sqrt{8x+3}} \, dx = \frac{1}{2}\sqrt{8x+3} + C$

6. $\int \frac{27}{2}(1-2x)^{\frac{7}{2}} \, dx = -\frac{3}{2}(1-2x)^{\frac{9}{2}} + C$

7. $\int 15(1-5x)^7 \, dx = -\frac{3}{8}(1-5x)^8 + C$

8. $\int \sqrt[3]{2x-1} \, dx = \frac{3}{8}(2x-1)^{\frac{4}{3}} + C$

9. $\int \frac{100}{(2-5x)^{\frac{7}{2}}} dx = 8(2-5x)^{-\frac{5}{2}} + C$

10. $\int 15\sqrt[4]{1-\frac{1}{4}x} \, dx = -48\left(1-\frac{1}{4}x\right)^{\frac{5}{4}} + C$

Question 4

Integrate:

1. $\displaystyle\int (4x+1)^3\,dx=\frac{1}{16}(4x+1)^4+C$

2. $\displaystyle\int 2(3x+1)^4\,dx=\frac{2}{15}(3x+1)^5+C$

3. $\displaystyle\int \frac{6}{(3x+1)^3}\,dx=-\frac{1}{(3x+1)^2}+C$

4. $\displaystyle\int 4(3x-2)^{\frac{1}{2}}\,dx=\frac{8}{9}(3x-2)^{\frac{3}{2}}+C$

5. $\displaystyle\int \frac{9}{\sqrt{6x+3}}\,dx=3\sqrt{6x+3}+C$

6. $\displaystyle\int 15(2-3x)^{\frac{3}{2}}\,dx=-2(2-3x)^{\frac{5}{2}}+C$

7. $\displaystyle\int 40(3-2x)^5\,dx=-\frac{10}{3}(3-2x)^6+C$

8. $\displaystyle\int \sqrt[3]{4x-3}\,dx=\frac{3}{16}(4x-3)^{\frac{4}{3}}+C$

9. $\displaystyle\int \frac{30}{(1-3x)^{\frac{7}{2}}}dx=4(1-3x)^{-\frac{5}{2}}+C$

10. $\displaystyle\int \frac{3}{4}\left(3-\frac{5x}{3}\right)^{\frac{1}{2}}\,dx=-\frac{3}{10}\left(3-\frac{5x}{3}\right)^{\frac{3}{2}}+C$

Question 5

Integrate:

1. $\int 10\sin 2x\ dx = -5\cos 2x + C$

2. $\int 4\cos 3x\, dx = \frac{4}{3}\sin 3x + C$

3. $\int 8\sin x - 5\cos x\ dx = -8\cos x - 5\sin x + C$

4. $\int 3\cos x - 2\sin x\ dx = 3\sin x + 2\cos x + C$

5. $\int 6\cos 2x - 6\sin 3x\ dx = 3\sin 2x + 2\cos 3x + C$

6. $\int \sin 2x - \cos 4x\ dx = -\frac{1}{2}\cos 2x - \frac{1}{4}\sin 4x + C$

7. $\int \cos \frac{1}{2}x + \sin \frac{1}{3}x\ dx = 2\sin \frac{1}{2}x - 3\cos \frac{1}{3}x + C$

8. $\int 3\cos 4x - 4\cos 3x\, dx = \frac{3}{4}\sin 4x - \frac{4}{3}\sin 3x + C$

9. $\int 6\sin 4x - 4\sin 6x\ dx = -\frac{3}{2}\cos 4x + \frac{2}{3}\cos 6x + C$

10. $\int 2\cos 2x - \sin \frac{x}{2} + 6\sin \frac{2x}{3}\, dx = \sin 2x + 2\cos \frac{x}{2} - 9\cos \frac{2x}{3} + C$

Question 6

Integrate:

1. $\int 4\sin 2x \, dx = -2\cos 2x + C$

2. $\int 6\cos 2x \, dx = 3\sin 2x + C$

3. $\int 7\sin x - 2\cos x \, dx = -7\cos x - 2\sin x + C$

4. $\int 8\cos x - 5\sin x \, dx = 8\sin x + 5\cos x + C$

5. $\int 8\cos 2x - 12\sin 3x \, dx = 4\sin 2x + 4\cos 3x + C$

6. $\int \sin 3x - \cos 6x \, dx = -\frac{1}{3}\cos 3x - \frac{1}{6}\sin 6x + C$

7. $\int \cos \frac{1}{4}x + \sin \frac{1}{2}x \, dx = 4\sin \frac{1}{4}x - 2\cos \frac{1}{2}x + C$

8. $\int 5\cos 2x - 2\cos 5x \, dx = \frac{5}{2}\sin 2x - \frac{2}{5}\sin 5x + C$

9. $\int 3\sin 2x - 2\sin 3x \, dx = -\frac{3}{2}\cos 2x + \frac{2}{3}\cos 3x + C$

10. $\int 4\cos 2x - \frac{1}{2}\sin \frac{x}{4} + 9\sin \frac{3x}{2} \, dx = 2\sin 2x + 2\cos \frac{x}{4} - 6\cos \frac{3x}{2} + C$

Question 7

Integrate:

1. $\int 5\sin 2x \, dx = -\frac{5}{2}\cos 2x + C$

2. $\int 3\cos 6x \, dx = \frac{1}{2}\sin 6x + C$

3. $\int 5\sin x - 4\cos 2x \, dx = -5\cos x - 2\sin 2x + C$

4. $\int 5\cos 2x - 3\sin 5x \, dx = \frac{5}{2}\sin 2x + \frac{3}{5}\cos 5x + C$

5. $\int 15\cos 3x - 15\sin 5x \, dx = 5\sin 3x + 3\cos 5x + C$

6. $\int \sin 8x - \frac{1}{2}\cos 3x \, dx = -\frac{1}{8}\cos 8x - \frac{1}{6}\sin 3x + C$

7. $\int 2\cos \frac{1}{3}x + 3\sin \frac{1}{2}x \, dx = 6\sin \frac{1}{3}x - 6\cos \frac{1}{2}x + C$

8. $\int 7\cos 3x - 3\cos 7x \, dx = \frac{7}{3}\sin 3x - \frac{3}{7}\sin 7x + C$

9. $\int \frac{1}{2}\sin 5x - \frac{1}{2}\sin \frac{1}{4}x \, dx = -\frac{1}{10}\cos 5x + 2\cos \frac{1}{4}x + C$

10. $\int 10\cos 2x - \sin \frac{x}{4} + 9\sin \frac{3x}{2} \, dx = 5\sin 2x + 4\cos \frac{x}{4} - 6\cos \frac{3x}{2} + C$

Question 8

Integrate:

1. $\displaystyle\int \mathrm{e}^{x}+\mathrm{e}^{2x}+\mathrm{e}^{-x}\,dx=\mathrm{e}^{x}+\tfrac{1}{2}\mathrm{e}^{2x}-\mathrm{e}^{-x}+C$

2. $\displaystyle\int 4\mathrm{e}^{2x}-\mathrm{e}^{-2x}+3\mathrm{e}^{3x}\,dx=2\mathrm{e}^{2x}+\tfrac{1}{2}\mathrm{e}^{-2x}+\mathrm{e}^{3x}+C$

3. $\displaystyle\int 2\mathrm{e}^{4x}-\mathrm{e}^{-3x}+\tfrac{1}{2}\mathrm{e}^{2x}\,dx=\tfrac{1}{2}\mathrm{e}^{4x}+\tfrac{1}{3}\mathrm{e}^{-3x}+\tfrac{1}{4}\mathrm{e}^{2x}+C$

4. $\displaystyle\int 4\mathrm{e}^{-2x}-2\mathrm{e}^{-4x}+\tfrac{1}{2}\mathrm{e}^{3x}\,dx=-2\mathrm{e}^{-2x}+\tfrac{1}{2}\mathrm{e}^{-4x}+\tfrac{1}{6}\mathrm{e}^{3x}+C$

5. $\displaystyle\int 5\mathrm{e}^{\frac{1}{2}x}-\tfrac{1}{2}\mathrm{e}^{-\frac{1}{2}x}+\tfrac{3}{4}\mathrm{e}^{\frac{1}{4}x}\,dx=10\mathrm{e}^{\frac{1}{2}x}+\mathrm{e}^{-\frac{1}{2}x}+3\mathrm{e}^{\frac{1}{4}x}+C$

6. $\displaystyle\int \frac{1}{x+1}+\frac{1}{2x-1}+\frac{1}{2-x}\,dx=\ln|x+1|+\tfrac{1}{2}\ln|2x-1|-\ln|2-x|+C$

7. $\displaystyle\int \frac{4}{2x+1}+\frac{2}{2x-1}+\frac{1}{1-3x}\,dx=2\ln|2x+1|+\ln|2x-1|-\tfrac{1}{3}\ln|1-3x|+C$

8. $\displaystyle\int \frac{6}{2x-1}+\frac{4}{3x-1}-\frac{2}{1-4x}+\frac{1}{2x}\,dx=3\ln|2x-1|+\tfrac{4}{3}\ln|3x-1|+\tfrac{1}{2}\ln|1-4x|+\tfrac{1}{2}\ln|x|+C$

9. $\displaystyle\int \frac{2}{3x-2}+\frac{2}{5x-1}-\frac{2}{(1-x)^{2}}+\frac{4}{3x}\,dx=\tfrac{2}{3}\ln|3x-2|+\tfrac{2}{5}\ln|5x-1|-\frac{2}{1-x}+\tfrac{4}{3}\ln|x|+C$

10. $\displaystyle\int \frac{4}{2x-3}-\frac{2}{1-2x}-\frac{12}{(1+2x)^{3}}+\frac{1}{\frac{1}{2}x}\,dx=2\ln|2x-3|+\ln|1-2x|+\frac{3}{(1+2x)^{2}}+2\ln|x|+C$

Question 9

Integrate:

1. $\displaystyle\int \mathrm{e}^{x}+\mathrm{e}^{2x}+\mathrm{e}^{3x}\,dx=\mathrm{e}^{x}+\tfrac{1}{2}\mathrm{e}^{2x}+\tfrac{1}{3}\mathrm{e}^{3x}+C$

2. $\displaystyle\int 6\mathrm{e}^{2x}+\mathrm{e}^{-2x}-3\mathrm{e}^{-x}\,dx=3\mathrm{e}^{2x}-\tfrac{1}{2}\mathrm{e}^{-2x}+3\mathrm{e}^{-x}+C$

3. $\displaystyle\int 3\mathrm{e}^{2x}-2\mathrm{e}^{-2x}+\tfrac{1}{2}\mathrm{e}^{4x}\,dx=\tfrac{3}{2}\mathrm{e}^{2x}+\mathrm{e}^{-2x}+\tfrac{1}{8}\mathrm{e}^{4x}+C$

4. $\displaystyle\int 6\mathrm{e}^{-3x}-2\mathrm{e}^{-2x}+\tfrac{1}{3}\mathrm{e}^{2x}\,dx=-2\mathrm{e}^{-3x}+\mathrm{e}^{-2x}+\tfrac{1}{6}\mathrm{e}^{2x}+C$

5. $\displaystyle\int 3\mathrm{e}^{\frac{1}{2}x}-\tfrac{1}{2}\mathrm{e}^{-\frac{1}{4}x}+3\mathrm{e}^{\frac{3}{2}x}\,dx=6\mathrm{e}^{\frac{1}{2}x}+2\mathrm{e}^{-\frac{1}{4}x}+2\mathrm{e}^{\frac{3}{2}x}+C$

6. $\displaystyle\int \frac{1}{x+2}+\frac{1}{3x-1}+\frac{1}{1-x}\,dx=\ln|x+2|+\tfrac{1}{3}\ln|3x-1|-\ln|1-x|+C$

7. $\displaystyle\int \frac{6}{3x+1}+\frac{4}{2x-1}+\frac{1}{1-4x}\,dx=2\ln|3x+1|+2\ln|2x-1|-\tfrac{1}{4}\ln|1-4x|+C$

8. $\displaystyle\int \frac{8}{4x-1}+\frac{5}{2x-1}-\frac{2}{1-3x}+\frac{4}{x}\,dx=2\ln|4x-1|+\tfrac{5}{2}\ln|2x-1|+\tfrac{2}{3}\ln|1-3x|+4\ln|x|+C$

9. $\displaystyle\int \frac{9}{3x-1}+\frac{2}{6x-1}-\frac{2}{(1-2x)^{2}}+\frac{1}{2x}\,dx=3\ln|3x-1|+\tfrac{1}{3}\ln|6x-1|-\frac{1}{1-2x}+\tfrac{1}{2}\ln|x|+C$

10. $\displaystyle\int \frac{3}{5x-3}-\frac{2}{x^{2}}-\frac{12}{(1+3x)^{3}}+\frac{9}{2x}\,dx=\tfrac{3}{5}\ln|5x-3|+\frac{2}{x}+\frac{2}{(1+3x)^{2}}+\tfrac{9}{2}\ln|x|+C$

Question 10

Integrate:

1. $\displaystyle\int 2\mathrm{e}^{x}+2\mathrm{e}^{2x}+3\mathrm{e}^{-x}\,dx=2\mathrm{e}^{x}+\mathrm{e}^{2x}-3\mathrm{e}^{-x}+C$

2. $\displaystyle\int 8\mathrm{e}^{2x}-3\mathrm{e}^{-2x}+5\mathrm{e}^{3x}\,dx=4\mathrm{e}^{2x}+\tfrac{3}{2}\mathrm{e}^{-2x}+\tfrac{5}{3}\mathrm{e}^{3x}+C$

3. $\displaystyle\int \tfrac{1}{2}\mathrm{e}^{4x}-\mathrm{e}^{-4x}+2\mathrm{e}^{\frac{1}{2}x}\,dx=\tfrac{1}{8}\mathrm{e}^{4x}+\tfrac{1}{4}\mathrm{e}^{-4x}+4\mathrm{e}^{\frac{1}{2}x}+C$

4. $\displaystyle\int 2\mathrm{e}^{-3x+1}-3\mathrm{e}^{1-x}+\tfrac{1}{3}\mathrm{e}^{3x}\,dx=-\tfrac{2}{3}\mathrm{e}^{-3x+1}+3\mathrm{e}^{1-x}+\tfrac{1}{9}\mathrm{e}^{3x}+C$

5. $\displaystyle\int 2\mathrm{e}^{1-\frac{1}{2}x}-\tfrac{1}{3}\mathrm{e}^{-\frac{1}{6}x}+\tfrac{3}{2}\mathrm{e}^{\frac{1}{2}x}\,dx=-4\mathrm{e}^{1-\frac{1}{2}x}+2\mathrm{e}^{-\frac{1}{6}x}+3\mathrm{e}^{\frac{1}{2}x}+C$

6. $\displaystyle\int \frac{1}{4x+1}+\frac{3}{3x-1}+\frac{4}{1-x}\,dx=\tfrac{1}{4}\ln|4x+1|+\ln|3x-1|-4\ln|1-x|+C$

7. $\displaystyle\int \frac{6}{3x+1}+\frac{3}{3x-1}+\frac{1}{1-4x}\,dx=2\ln|3x+1|+\ln|3x-1|-\tfrac{1}{4}\ln|1-4x|+C$

8. $\displaystyle\int \frac{8}{5x-1}+\frac{6}{4x-1}-\frac{3}{(3x-1)^2}+\frac{1}{5x}\,dx=\tfrac{8}{5}\ln|5x-1|+\tfrac{3}{2}\ln|4x-1|+\frac{1}{3x-1}+\tfrac{1}{5}\ln|x|+C$

9. $\displaystyle\int \frac{7}{5x-3}+\frac{7}{5x-2}+\frac{7}{(5x-1)^2}+\frac{7}{3x}\,dx=\tfrac{7}{5}\ln|5x-3|+\tfrac{7}{5}\ln|5x-2|-\frac{7}{5(5x-1)}+\tfrac{7}{3}\ln|x|+C$

10. $\displaystyle\int \frac{4}{4x-1}-\frac{6}{1-5x}-\frac{30}{(1+3x)^3}+\frac{1}{\frac{1}{3}x}\,dx=\ln|4x-1|+\tfrac{6}{5}\ln|1-5x|+\frac{5}{(1+3x)^2}+3\ln|x|+C$

Question 11

Integrate:

1. $\int 4\sin 2x \, dx = -2\cos 2x + C$

2. $\int 2\cos(3x+1)\, dx = \frac{2}{3}\sin(3x+1) + C$

3. $\int 2\sin x - 3\cos x \, dx = -2\cos x - 3\sin x + C$

4. $\int 4\sin\left(\frac{x}{2}\right) dx = -8\cos\left(\frac{x}{2}\right) + C$

5. $\int 2\cos 3x - 3\sin 2x \, dx = \frac{2}{3}\sin 3x + \frac{3}{2}\cos 2x + C$

6. $\int \frac{1}{2}\cos(2-3x) \, dx = -\frac{1}{6}\sin(2-3x) + C$

7. $\int 4\sin(1-2x) \, dx = 2\cos(1-2x) + C$

8. $\int \frac{1}{2}\mathrm{e}^{2x+3} \, dx = \frac{1}{4}\mathrm{e}^{2x+3} + C$

9. $\int 3\mathrm{e}^{\frac{1}{2}x+1} \, dx = 6\mathrm{e}^{\frac{1}{2}x+1} + C$

10. $\int \frac{15}{2}\mathrm{e}^{1-3x} \, dx = -\frac{5}{2}\mathrm{e}^{1-3x} + C$

Question 12

Integrate:

1. $\displaystyle\int 6(4x+3)^{\frac{1}{2}}\,dx=(4x+3)^{\frac{3}{2}}+C$

2. $\displaystyle\int \frac{3}{2x-1}\,dx=\frac{3}{2}\ln|2x-1|+C$

3. $\displaystyle\int \frac{10}{(2x+1)^6}\,dx=-\frac{1}{(2x+1)^5}+C$

4. $\displaystyle\int 5(2x-3)^{\frac{1}{4}}\,dx=2(2x-3)^{\frac{5}{4}}+C$

5. $\displaystyle\int \frac{\mathrm{e}^{4x}+3}{\mathrm{e}^{3x}}\,dx=\mathrm{e}^{x}-\mathrm{e}^{-3x}+C$

6. $\displaystyle\int \frac{3}{4x+1}\,dx=\frac{3}{4}\ln|4x+1|+C$

7. $\displaystyle\int\left(1+\frac{1}{x}\right)^2\,dx=x+2\ln|x|-\frac{1}{x}+C$

8. $\displaystyle\int \cos x-\sin x\,\,dx=\sin x+\cos x+C$

9. $\displaystyle\int \sin x-\cos x\,\,dx=-\cos x-\sin x+C$

10. $\displaystyle\int \sin(4x+3)\,\,dx=-\frac{1}{4}\cos(4x+3)+C$

Question 13

Integrate:

1. $\displaystyle\int \cos(5-2x)\,dx = -\frac{1}{2}\sin(5-2x)+C$

2. $\displaystyle\int 3\sin 2x\,dx = -\frac{3}{2}\cos 2x+C$

3. $\displaystyle\int \sec^2 x\left(1+\cot^2 x\right)dx = \tan x-\cot x+C$

4. $\displaystyle\int (3x+1)^4\,dx = \frac{1}{15}(3x+1)^5+C$

5. $\displaystyle\int 3(2x+1)^{\frac{1}{2}}\,dx = (2x+1)^{\frac{3}{2}}+C$

6. $\displaystyle\int \frac{2}{\cos^2 x}\,dx = 2\tan x+C$

7. $\displaystyle\int (4-5x)^{-1}\,dx = -\frac{1}{5}\ln|4-5x|+C$

8. $\displaystyle\int \frac{1}{4x}\,dx = \frac{1}{4}\ln|x|+C$

9. $\displaystyle\int \frac{x+1}{x}\,dx = x+\ln|x|+C$

10. $\displaystyle\int \frac{4}{(2x-7)^2}\,dx = -\frac{2}{2x-7}+C$

Question 14

Integrate:

1. $\displaystyle\int \operatorname{cosec}^2(3x+1)\,dx = -\frac{1}{3}\cot(3x+1)+C$

2. $\displaystyle\int 12\sec^2(2x+3)\,dx = 6\tan(2x+3)+C$

3. $\displaystyle\int 6\mathrm{e}^{2x+2}\,dx = 3\mathrm{e}^{2x+2}+C$

4. $\displaystyle\int \sec^2 x\left(1-\cot^2 x\right)dx = \tan x+\cot x+C$

5. $\displaystyle\int \tan 2x\sec 2x\,dx = \frac{1}{2}\sec 2x+C$

6. $\displaystyle\int 7(2x-3)^{\frac{5}{2}}\,dx = (2x-3)^{\frac{7}{2}}+C$

7. $\displaystyle\int \frac{3}{\sqrt{4x+1}}\,dx = \frac{3}{2}\sqrt{4x+1}+C$

8. $\displaystyle\int \frac{1}{3(x-2)^{\frac{1}{2}}}\,dx = \frac{2}{3}\sqrt{x-2}+C$

9. $\displaystyle\int \frac{6x+3}{2x}\,dx = 3x+\frac{3}{2}\ln|x|+C$

10. $\displaystyle\int 4(3x-2)^3\,dx = \frac{1}{3}(3x-2)^4+C$

Question 15

Integrate:

1. $\displaystyle\int \sqrt{x\sqrt{x}}\ dx = \frac{4}{7}x^{\frac{7}{4}} + C$

2. $\displaystyle\int \frac{1}{x^2\sqrt[3]{x^2}}\ dx = -\frac{3}{5}x^{-\frac{5}{3}} + C = -\frac{3}{5\sqrt[3]{x^5}} + C$

3. $\displaystyle\int \frac{3}{\sqrt{2-4x}}\ dx = -\frac{3}{2}\sqrt{2-4x} + C$

4. $\displaystyle\int \frac{4}{\sqrt{6x-1}}\ dx = \frac{4}{3}\sqrt{6x-1} + C$

5. $\displaystyle\int \operatorname{cosec} 2x \cot 2x\ dx = -\frac{1}{2}\operatorname{cosec} 2x + C$

6. $\displaystyle\int (2x+1)^3\ dx = \frac{1}{8}(2x+1)^4 + C$

7. $\displaystyle\int \frac{10}{(3x+1)^{\frac{3}{2}}}\ dx = -\frac{20}{3\sqrt{3x+1}} + C$

8. $\displaystyle\int 2^x\ dx = \frac{2^x}{\ln 2} + C$

9. $\displaystyle\int 2^x 3^x\ dx = \frac{6^x}{\ln 6} + C$

10. $\displaystyle\int 3^{2x+1}\ dx = \frac{3^{2x+1}}{\ln 9} + C$

Question 16

Integrate:

1. $\displaystyle\int \frac{12}{(1-2x)^5}\,dx = \frac{3}{2(1-2x)^4} + C$

2. $\displaystyle\int (2-3x)^{-2}\,dx = \frac{1}{3(2-3x)} + C$

3. $\displaystyle\int 2\sec^2 x + \frac{1}{2}\sin 2x\,dx = 2\tan x - \frac{1}{4}\cos 2x + C$

4. $\displaystyle\int \frac{3}{x} + \frac{4}{x^2} - \frac{2}{x^3}\,dx = 3\ln|x| - \frac{4}{x} + \frac{1}{x^2} + C$

5. $\displaystyle\int 4\cos 3x + \frac{1}{2}\sin 3x\,dx = \frac{4}{3}\sin 3x - \frac{1}{6}\cos 3x + C$

6. $\displaystyle\int \frac{4}{2x-1} + \frac{1}{3-4x}\,dx = 2\ln|2x-1| - \frac{1}{4}\ln|3-4x| + C$

7. $\displaystyle\int \operatorname{cosec}^2 2x\,dx = -\frac{1}{2}\cot 2x + C$

8. $\displaystyle\int \frac{7}{3x}\,dx = \frac{7}{3}\ln|x| + C$

9. $\displaystyle\int 4(3-2x)^5\,dx = -\frac{1}{3}(3-2x)^6 + C$

10. $\displaystyle\int \frac{1+\sin x}{\cos x}\,dx = \ln\left|\sec^2 x + \sec x\tan x\right| + C = -\ln|1-\sin x| + C$

Question 17

Integrate:

1. $\displaystyle\int \frac{\mathrm{e}^{4x}+\mathrm{e}^{-x}}{\mathrm{e}^{2x}}\,dx=\frac{1}{2}\mathrm{e}^{2x}-\frac{1}{3}\mathrm{e}^{-3x}+C$

2. $\displaystyle\int \frac{1}{2(3x+1)^4}\,dx=-\frac{1}{18(3x+1)^3}+C$

3. $\displaystyle\int \frac{4}{3(2x+1)}\,dx=\frac{2}{3}\ln|2x+1|+C$

4. $\displaystyle\int \frac{1}{3}\sin 2x-\frac{1}{2}\cos 3x\,dx=-\frac{1}{6}\cos 2x-\frac{1}{6}\sin 3x+C$

5. $\displaystyle\int \frac{1}{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)}\,dx=\ln|x-4|+C$

6. $\displaystyle\int \sqrt{x}\left(1+\frac{1}{x}\right)\,dx=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$

7. $\displaystyle\int \frac{(x+2)^2}{3x}\,dx=\frac{1}{6}x^2+\frac{4}{3}x+\frac{4}{3}\ln|x|+C$

8. $\displaystyle\int 4\mathrm{e}^{-2x}-\frac{1}{3}\sin 3x\,dx=-2\mathrm{e}^{-2x}+\frac{1}{9}\cos 3x+C$

9. $\displaystyle\int \tan 3x\,dx=\frac{1}{3}\ln|\sec 3x|+C$

10. $\displaystyle\int \frac{(4x-1)^{-1}}{4}\,dx=\frac{1}{16}\ln|4x-1|+C$

Question 18

Integrate:

1. $\displaystyle\int \left(\mathrm{e}^{x}+2\mathrm{e}^{-x}\right)^{2}\,dx=\frac{1}{2}\mathrm{e}^{2x}+4x-2\mathrm{e}^{-2x}+C$

2. $\displaystyle\int x\mathrm{e}^{2}\,dx=\frac{1}{2}\mathrm{e}^{2}\,x^{2}+C$

3. $\displaystyle\int \sqrt[3]{x\sqrt{\frac{1}{x}}}\,dx=\frac{6}{7}x^{\frac{7}{6}}+C$

4. $\displaystyle\int \left(1-x^{-2}\right)^{2}\,dx=x+\frac{2}{x}-\frac{1}{3x^{3}}+C$

5. $\displaystyle\int \cot 2x\,dx=\frac{1}{2}\ln\left|\sin 2x\right|+C$

Question 19

Integrate:

1. $\displaystyle\int_0^2 \frac{1}{\sqrt{4x+1}}\ dx = 1$

2. $\displaystyle\int_0^1 \frac{1}{(2x+1)^4}\ dx = \frac{13}{81}$

3. $\displaystyle\int_0^{\frac{\pi}{4}} \sin\left(2x+\frac{\pi}{4}\right)\ dx = \frac{\sqrt{2}}{2}$

4. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos 3x\ \ dx = -\frac{1}{3}$

5. $\displaystyle\int_{\ln 2}^{\ln 4} \left(\mathrm{e}^{2x}-2\right)^2\ \ dx = 4\left(9+\ln 2\right)$

6. $\displaystyle\int_0^2 \frac{6}{3x+2}\ \ dx = \ln 16$

7. $\displaystyle\int_0^{\frac{\pi}{4}} \cos\left(3x+\frac{\pi}{4}\right)\ dx = -\frac{\sqrt{2}}{6}$

8. $\displaystyle\int_0^{\frac{\pi}{3}} \cos\left(3x+\frac{\pi}{3}\right)\ dx = -\frac{\sqrt{3}}{3}$

9. $\displaystyle\int_0^{\frac{\pi}{6}} \cos^3 x\ \ dx = \frac{11}{12}$

10. $\displaystyle\int_1^{\mathrm{e}} \left(x^2+1\right)\ln x\ \ dx = \frac{2}{9}\left(\mathrm{e}^3+5\right)$

Question 20

Integrate:

1. $\displaystyle\int_0^1 \frac{9}{(2x+1)^2}\ dx = 3$

2. $\displaystyle\int_0^{\frac{\pi}{6}} \sin\left(4x+\frac{\pi}{6}\right)\ dx = \frac{\sqrt{3}}{16}$

3. $\displaystyle\int_0^1 \frac{4}{2x+3}\ dx = \ln 9$

4. $\displaystyle\int_0^4 \mathrm{e}^{\frac{1}{2}x}\ dx = 2\left(\mathrm{e}^2-1\right)$

5. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec x\ dx = \ln\left|\frac{2}{3}\sqrt{3}+1\right|$

6. $\displaystyle\int_2^4 \frac{8}{(3x-4)^3}\ dx = \frac{5}{16}$

7. $\displaystyle\int_0^{\frac{\pi}{2}} \sin 2x\ dx = 1$