INEQUALITIES
FURTHER PRACTICE

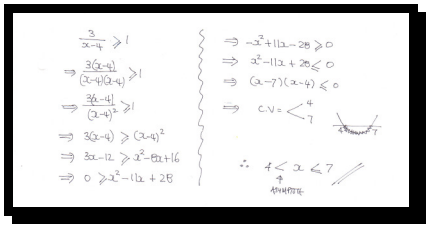
RATIONAL INEQUALITIES

Question 1 (**)

Solve the following inequality.

$$\frac{3}{x-4}\geq 1.$$

$\boxed{4<x\leq 7}$

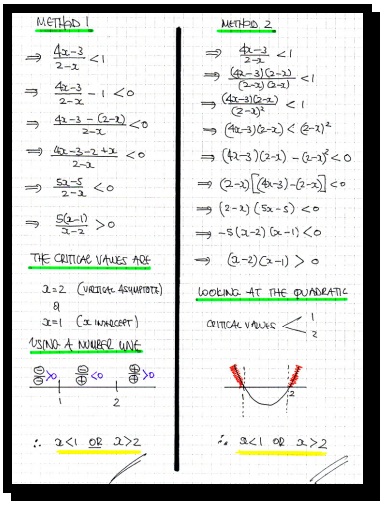


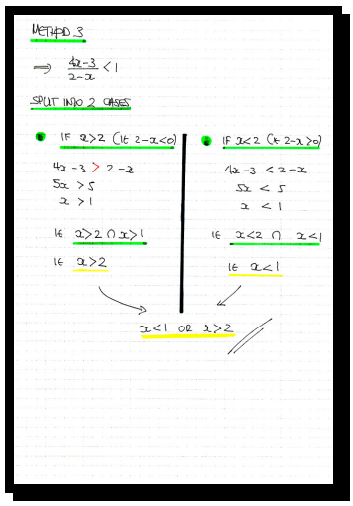
Question 2 (**)

Solve the following rational inequality.

$$\frac{4x-3}{2-x}<1.$$

$\boxed{\quad}$, $\boxed{x<1 \quad \cup \quad x>2}$



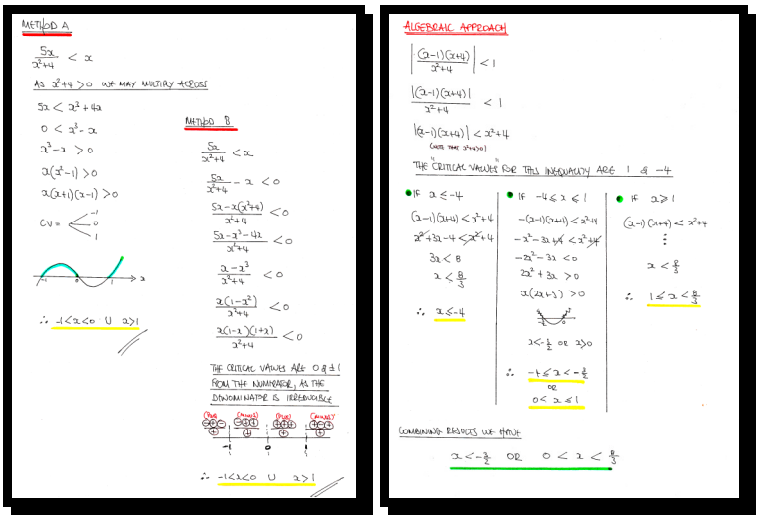


Question 3 **(**)**

Find the set of values of x, that satisfy the following inequality.

$$\frac{5x}{x^2+4}<x.$$

$$\boxed{\quad}, \quad \boxed{-1<x<0}, \quad \boxed{x>1}$$

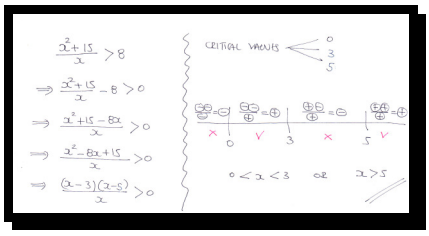


Question 4 **(**)**

Find the set of values of x, that satisfy the following inequality.

$$\frac{x^2+15}{x}>8.$$

$$\boxed{0<x<3 \quad \cup \quad x>5}$$

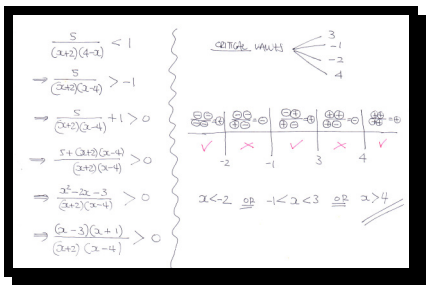


Question 5 (**)

Solve the following rational inequality.

$$\frac{5}{(x+2)(4-x)}<1.$$

$$\boxed{x<-2 \quad \cup \quad -1<x<3 \quad \cup \quad x>4}$$

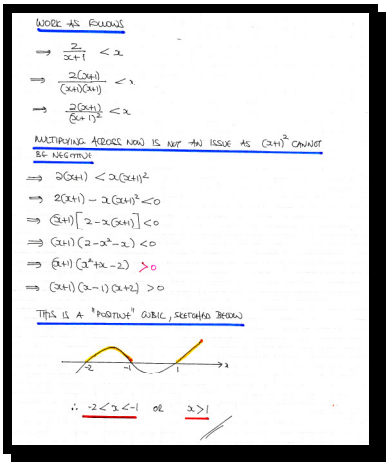


Question 6 (**)

Solve the following inequality.

$$\frac{2}{x+1}<x$$

$$\boxed{}, \quad \boxed{-2<x<-1 \quad \cup \quad x>1}$$

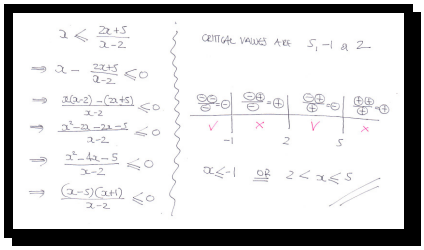


Question 7 (+)**

Find the set of values of x, that satisfy the following inequality.

$$x \leq \frac{2x+5}{x-2}.$$

$$\boxed{x \leq -1 \quad \cup \quad 2 < x \leq 5}$$

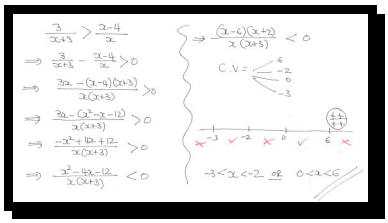


Question 8 (+)**

Find the set of values of x, that satisfy the following inequality.

$$\frac{3}{x+3} > \frac{x-4}{x}.$$

$$\boxed{-3 < x < -2 \quad \cup \quad 0 < x < 6}$$

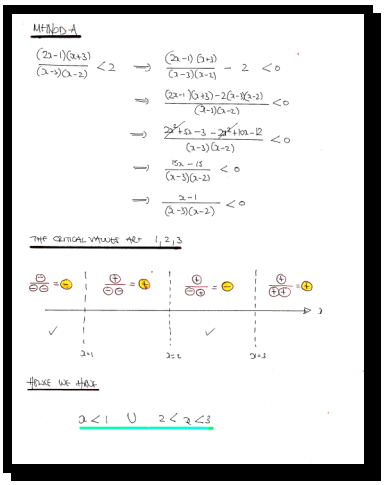


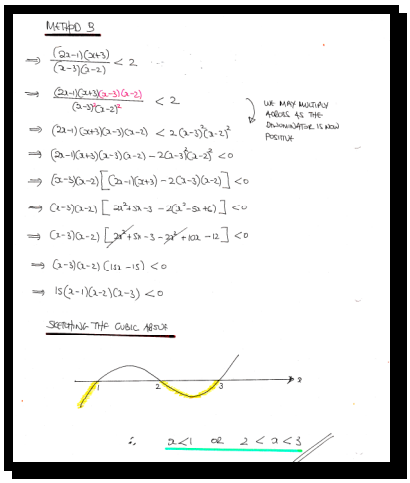
Question 9 (+)**

Solve the following inequality.

$$\frac{(2x-1)(x+3)}{(x-3)(x-2)}<2.$$

$\boxed{}$, $\boxed{x<1 \cup 2<x<3}$



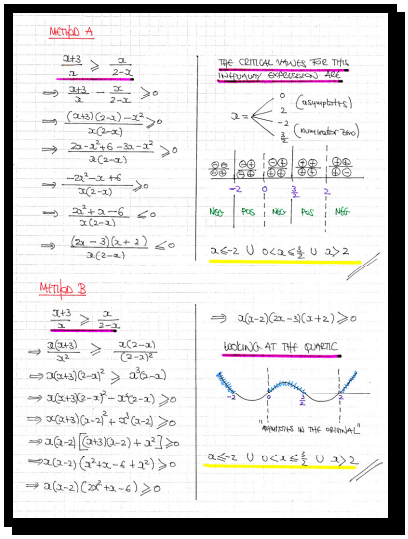


Question 10 (***)

Determine the range of values of x that satisfy the following inequality.

$$\frac{x+3}{x}\geq\frac{x}{2-x}.$$

$\square$, $x\leq -2\ \cup\ 0<x\leq\frac{3}{2}\ \cup\ x>2$

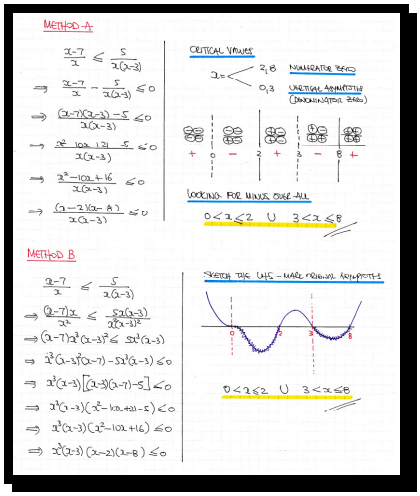


Question 11 (***)

Determine the solution interval of the following inequality.

$$\frac{x-7}{x} \leq \frac{5}{x(x-3)}.$$

$\boxed{}$, $\boxed{0<x\leq 2 \ \cup \ 3<x\leq 8}$

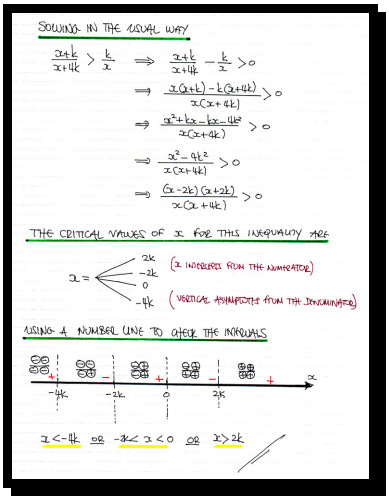


Question 12 **(***)**

Find, in terms of the positive constant k, the solution set of the following inequality.

$$\frac{x+k}{x+4k}>\frac{k}{x}.$$

$\square$, $\boxed{x<-4k \quad \cup \quad -2k<x<0 \quad \cup \quad x>2k}$

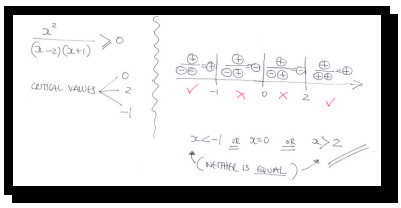


Question 13 **(***+)**

Solve the following rational inequality.

$$\frac{x^2}{(x-2)(x+1)}\geq 0.$$

$\boxed{x<-1 \quad \cup \quad x=0 \quad \cup \quad x>2}$

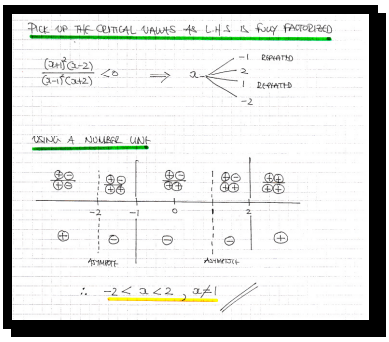


Question 14 (*+)**

Solve the following rational inequality.

$$\frac{(x+1)^2(x-2)}{(x-1)^2(x+2)}<0.$$

$\boxed{\quad}$, $\boxed{-2<x<2, \quad x\neq 1}$

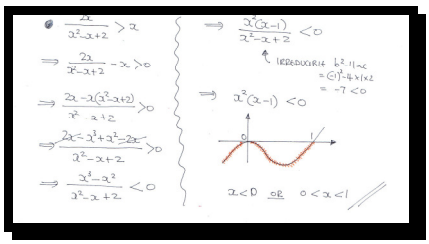


Question 15 (*+)**

Solve the following rational inequality.

$$\frac{2x}{x^2-x+2}>x.$$

$\boxed{x<0 \quad \cup \quad 0<x<1}$

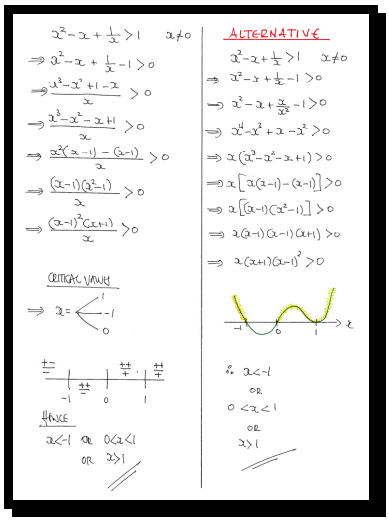


Question 16 (**+)**

Solve the following inequality.

$$x^2 - x + \frac{1}{x} > 1 .$$

$\square$, $\boxed{x < -1 \ \cup \ 0 < x < 1 \ \cup \ x > 1}$



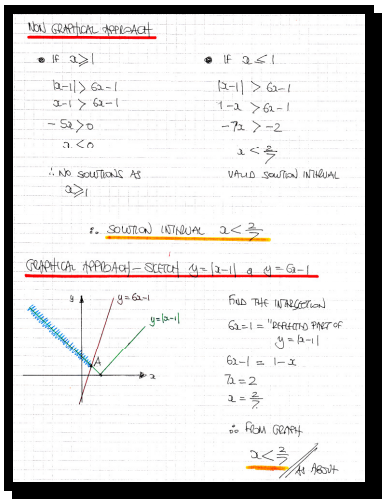
MODULUS INEQUALTIES

Question 1 (**)

Find the set of values of x that satisfy the inequality

$$|x-1|>6x-1$$

$\square$, $\boxed{x<\frac{2}{7}}$

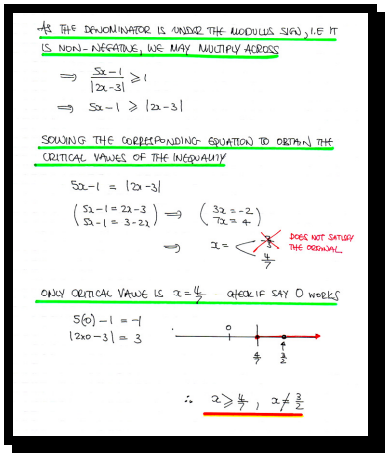


Question 2 (***)

Find the set of values of x that satisfy the inequality

$$\frac{5x-1}{|2x-3|}\geq 1.$$

$\square$, $\boxed{x\geq\frac{4}{7},\quad x\neq\frac{3}{2}}$

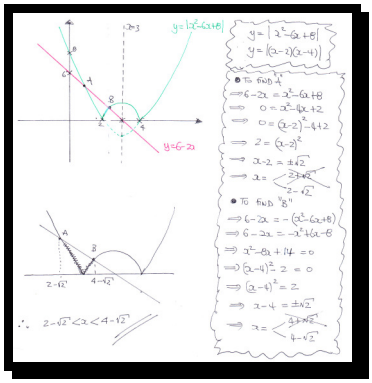


Question 3 (***)

Find the set of values of x that satisfy the inequality

$$\left|x^2-6x+8\right|<6-2x\,.$$

$$\boxed{2-\sqrt{2}<x<4-\sqrt{2}}$$

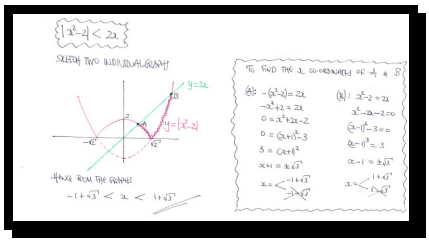


Question 4 (+)**

Find the set of values of x that satisfy the inequality

$$\left|x^2-2\right|<2x\,.$$

$$\boxed{-1+\sqrt{3}<x<1+\sqrt{3}}$$



Question 5 (****)

$$f(x) \equiv \frac{3x-1}{x+2}, \ x \in \mathbb{R}, \ x \neq -2.$$

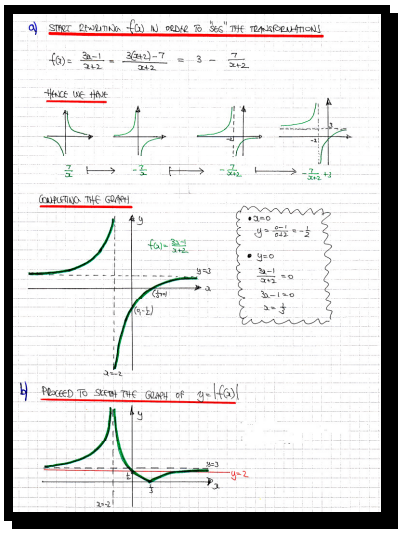
a) Sketch the graph of $f(x)$.

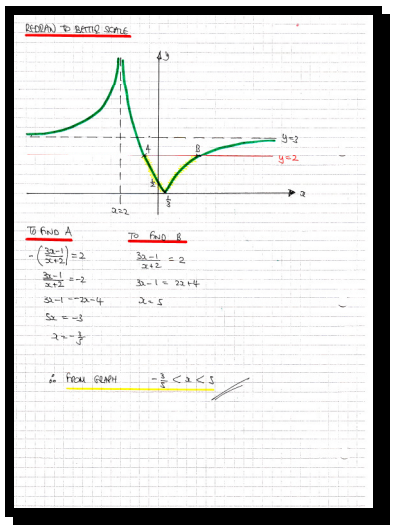
The sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes.

b) Hence, or otherwise, solve the inequality

$$\left|\frac{3x-1}{x+2}\right| < 2.$$

$\square$, $\boxed{-\frac{3}{5} < x < 5}$



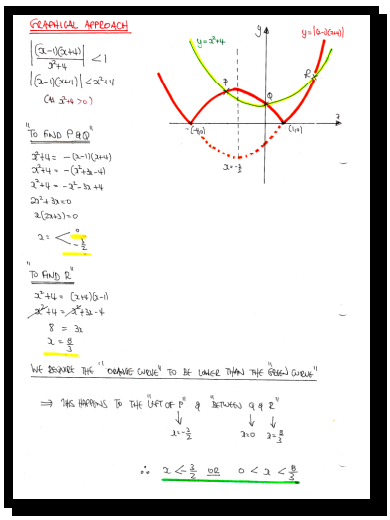


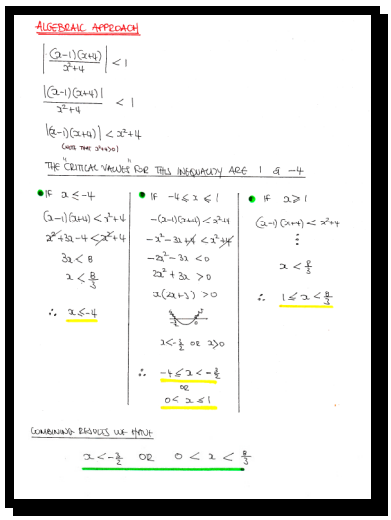
Question 6 (**+)**

Find the set of values of x, that satisfy the following inequality.

$$\left|\frac{(x-1)(x+4)}{x^2+4}\right|<1.$$

$\square$, $\boxed{x<-\frac{3}{2} \text{ or } 0<x<\frac{8}{3}}$



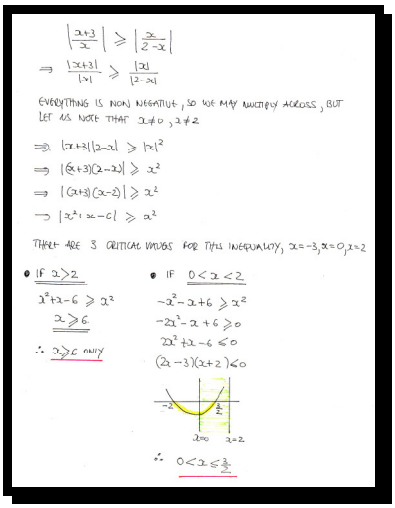


Question 7 (*****)

Determine the range of values of x that satisfy the inequality

$$\left|\frac{x+3}{x}\right| \geq \left|\frac{x}{2-x}\right|.$$

$\square$, $\boxed{-2 \leq x < 0 \quad \cup \quad 0 < x \leq \frac{3}{2} \quad \cup \quad x \geq 6}$



$\left|\frac{x+3}{x}\right| \geqslant \left|\frac{x}{2-x}\right|$

$\Rightarrow \frac{|x+3|}{|x|} \geqslant \frac{|x|}{|2-x|}$

EVERYTHING IS NON NEGATIVE, SO WE MAY MULTIPLY ACROSS, BUT LET US NOTE THAT $x \neq 0$, $x \neq 2$

$\Rightarrow |x+3||2-x| \geqslant |x|^2$

$\Rightarrow |(x+3)(2-x)| \geqslant x^2$

$\Rightarrow |(x+3)(x-2)| \leqslant x^2$

$\Rightarrow |x^2+x-6| \leqslant x^2$

THERE ARE 3 CRITICAL VALUES FOR THIS INEQUALITY, $x=-3$, $x=0$, $x=2$

• IF $x \geqslant 2$

$x^2+x-6 \leqslant x^2$

$x \leqslant 6$

$\therefore 2 \leqslant x \leqslant 6$

• IF $0 < x < 2$

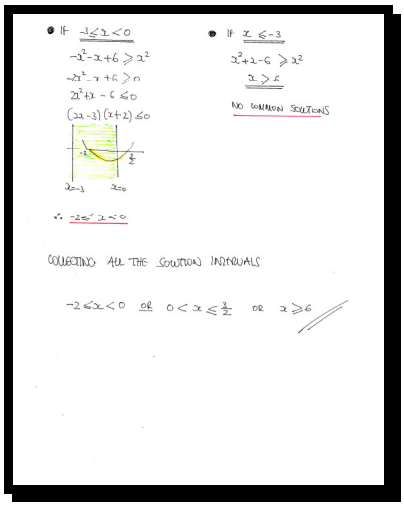
$-x^2-x+6 \leqslant x^2$

$-2x^2-x+6 \leqslant 0$

$2x^2+x-6 \geqslant 0$

$(2x-3)(x+2) \geqslant 0$

$\therefore 0 < x \leqslant \frac{3}{2}$



• IF $-3 \leqslant x < 0$

$-x^2-x+6 \leqslant x^2$

$-2x^2-x+6 \leqslant 0$

$2x^2+x-6 \geqslant 0$

$(2x-3)(x+2) \geqslant 0$

$\therefore -2 \geqslant x \geqslant 0$

• IF $x \leqslant -3$

$x^2+x-6 \leqslant x^2$

$x \leqslant 6$

NO COMMON SOLUTIONS

COLLECTING ALL THE SOLUTION INTERVALS

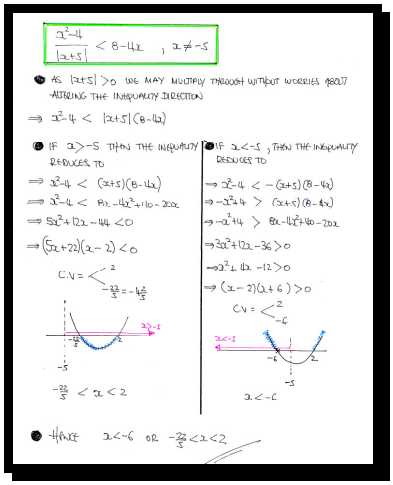
$-2 \leqslant x < 0$ OR $0 < x \leqslant \frac{3}{2}$ OR $x \geqslant 6$

Question 8 (*****)

Find the set of values of x that satisfy the inequality

$$\frac{x^2-4}{|x+5|}<8-4x\,.$$

$\square$, $x<-6 \quad \cup \quad -\frac{22}{5}<x<2$

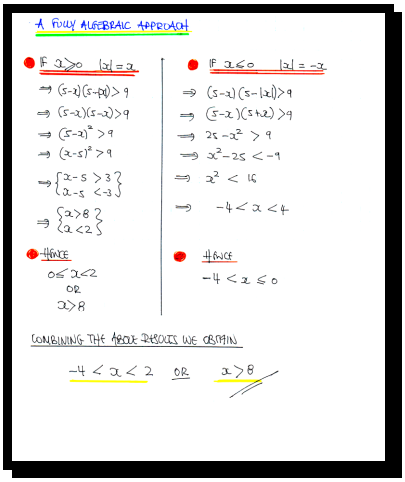


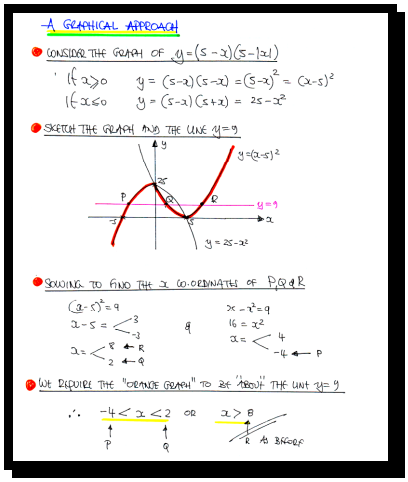
Question 9 **(*****)**

Solve the following inequality.

$$(5-x)(5-|x|)>9\,,\quad x\in\mathbb{R}$$

$\square$, $\boxed{-4<x<2,\quad\cup\quad x>8}$



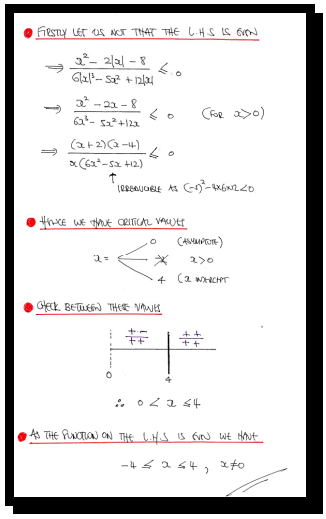


Question 10 (*****)

Solve the following inequality in the largest real domain.

$$\frac{x^2-2|x|-8}{6|x|^3-5x^2+12|x|}\leq 0\,.$$

$\boxed{}$, $\boxed{-4\leq x\leq 4,\ \ x\neq 0}$

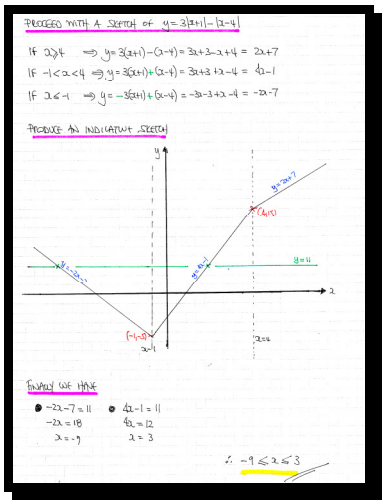


Question 11 (*****)

Solve the following modulus inequality.

$$3|x+1|-|x-4|\leq 11,\ x\in\mathbb{R}.$$

$\boxed{}$, $\boxed{}$, $\boxed{-9\leq x\leq 3}$

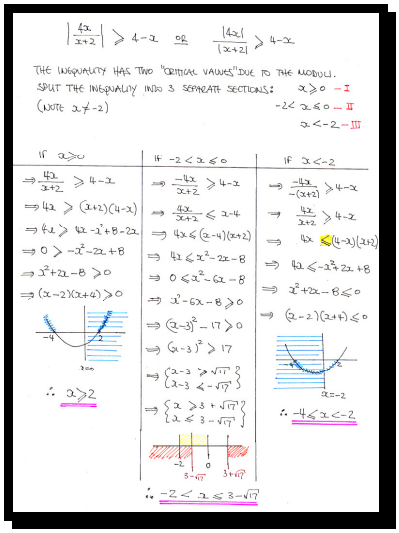


Question 12 (*****)

Find the set of values of x that satisfy the inequality

$$\left|\frac{4x}{x+2}\right| \geq 4-x\,.$$

$\square$, $\boxed{-4 \leq x < -2 \quad \cup \quad -2 < x \leq 3-\sqrt{17} \quad \cup \quad x \geq 2}$



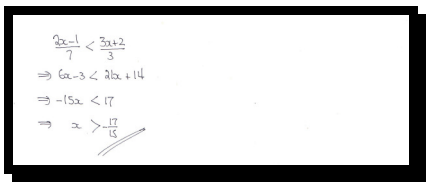
VARIOUS INEQUALITIES

Question 1 (**)

Solve the inequality

$$\frac{2x-1}{7}<\frac{3x+2}{3}.$$

$$\boxed{x>-\frac{17}{15}}$$

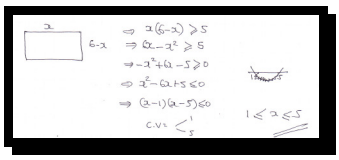


Question 2 (+)**

An advertising sign has a rectangular design so the its length is x metres and its width is $(6-x)$ metres.

Given that the area of the advertising sign must be at least 5 square metres, determine the range of possible values of x.

$$\boxed{1\leq x\leq 5}$$



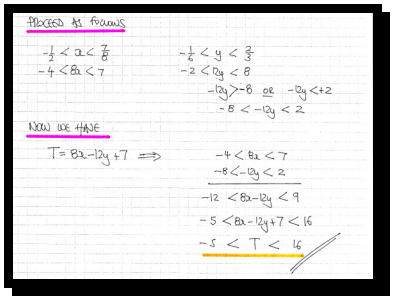
Question 3 (+)**

$$T = 8x - 12y + 7 .$$

It is further given that $-\frac{1}{2} < x < \frac{7}{8}$ and $-\frac{1}{6} < y < \frac{2}{3}$.

Determine the range of possible values of T.

$\boxed{}$, $\boxed{-5 < T < 16}$

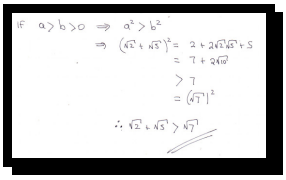


Question 4 (***)

Show clearly, without approximating and without using any calculating aid that

$$\sqrt{2} + \sqrt{5} > \sqrt{7} .$$

proof

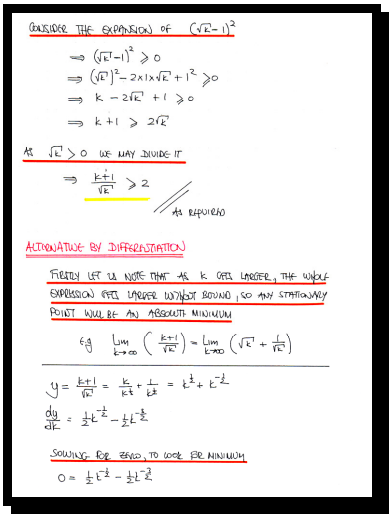


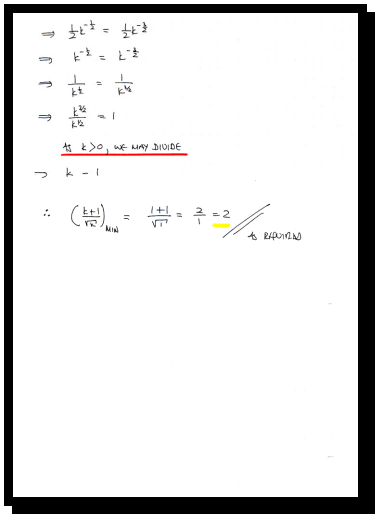
Question 5 (*+)**

Given that $k>0$ show clearly that

$$\frac{k+1}{\sqrt{k}}\geq 2\,.$$

[ ], [proof]





Question 6 (**+)**

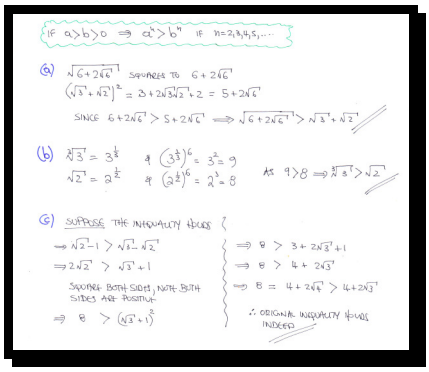
Show clearly, without approximating and without using any calculating aid that

a) $\sqrt{6+2\sqrt{6}}>\sqrt{3}+\sqrt{2}$.

b) $\sqrt[3]{3}>\sqrt{2}$.

c) $\sqrt{2}-1>\sqrt{3}-\sqrt{2}$.

proof



Question 7 (**+)**

Show clearly that for all real numbers α, β and γ

$$\alpha^2+\beta^2+\gamma^2 \geq \alpha\beta+\beta\gamma+\gamma\alpha \, .$$

[ ], [proof]

