IMPLICIT EQUATIONS

EXAM QUESTIONS

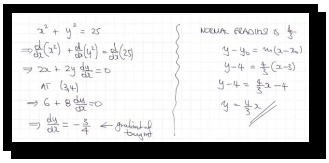
Question 1 (**)

A circle has equation

$$x^2+y^2=25\,.$$

Use implicit differentiation to find an equation of the normal to the circle at the point with coordinates $(3,4)$.

$$\boxed{y=\frac{4}{3}x}$$



Question 2 (**)

A circle has equation

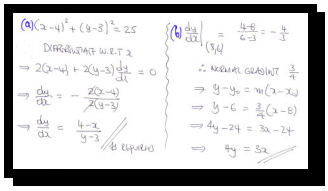
$$(x-4)^2+(y-3)^2=25\,.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{4-x}{y-3}\,.$$

b) Find an equation of the normal to the circle at the point $(8,6)$.

$$\boxed{4y=3x}$$



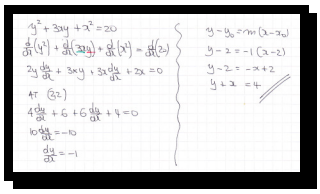
Question 3 (**)

A curve is given implicitly by the equation

$$y^2+3xy+x^2=20\,.$$

Find an equation for the tangent to the curve at the point $P(2,2)$.

$\boxed{}$, $\boxed{x+y=4}$



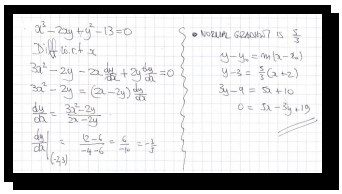
Question 4 (**)

A curve C has equation

$$x^3-2xy+y^2-13=0\,.$$

Find an equation for the normal to C at the point $P(-2,3)$.

$\boxed{5x-3y+19=0}$



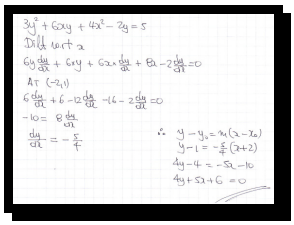
Question 5 (**)

A curve is given implicitly by the equation

$$3y^2+6xy+4x^2-2y=5\,.$$

Find an equation for the tangent to the curve at the point $P(-2,1)$.

$$\boxed{4y+5x+6=0}$$



Question 6 (+)**

A curve has implicit equation

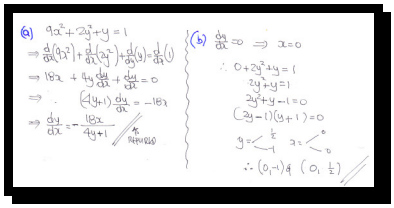
$$9x^2+2y^2+y=1\,.$$

a) Show clearly that

$$\frac{dy}{dx}=-\frac{18x}{4y+1}\,.$$

b) Hence find the coordinates of the points on the curve where $\dfrac{dy}{dx}=0$.

$$\boxed{(0,-1),\left(0,\frac{1}{2}\right)}$$



Question 7 (+)**

A curve is given by

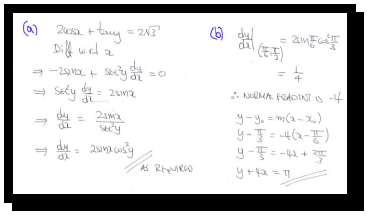
$$2\cos x+\tan y=2\sqrt{3}\,.$$

a) Show clearly that

$$\frac{dy}{dx}=2\sin x\cos^2 y\,.$$

b) Find an equation of the normal to the curve at the point $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$, giving the answer in the form $ax+by=\pi$, where a and b are integers.

$\boxed{}$, $\boxed{4x+y=\pi}$



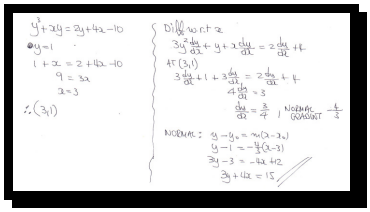
Question 8 (+)**

A curve is described by the implicit relationship

$$y^3+xy=2y+4x-10\,.$$

Find an equation of the normal to the curve at the point where $y=1$.

$\boxed{3y+4x=15}$



Question 9 (+)**

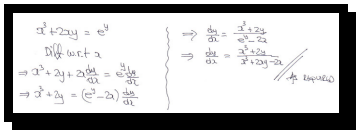
A curve C has implicit equation

$$x^3+2xy=\mathrm{e}^y.$$

Show **clearly** that

$$\frac{dy}{dx}=\frac{x^3+2y}{x^3+2xy-2x}.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 10 (+)**

A curve has equation

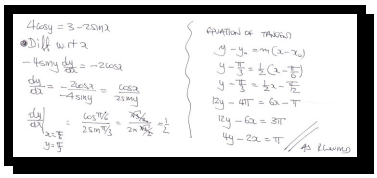
$$4\cos y=3-2\sin x,\ x\in\mathbb{R},\ y\in\mathbb{R}.$$

Show that the straight line with equation

$$4y-2x=\pi$$

is the tangent to the curve at the point with coordinates $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$.

$\boxed{\text{proof}}$



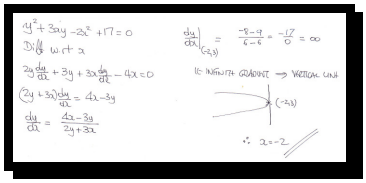
Question 11 (***)

A curve has implicit equation

$$y^2+3xy-2x^2+17=0\,.$$

Find an equation of the tangent to the curve at the point $(-2,3)$.

$\boxed{x=-2}$



Question 12 (***)

The curve C has equation

$$yx(2x-y)+1=0.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{y^2-4xy}{2x^2-2xy}.$$

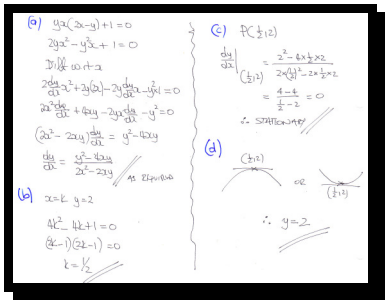
The point $P(k,2)$ lies on C.

b) Find the value of k.

c) Show that P is a stationary point of C.

d) Hence, state an equation of the tangent to C at P

$\square$, $\boxed{k=\frac{1}{2}}$, $\boxed{y=2}$



Question 13 (***)

The curve C has equation

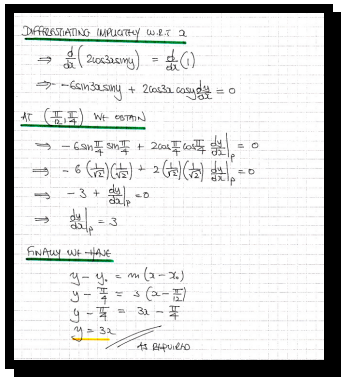
$$2\cos 3x \sin y = 1, \quad 0 \le x, y \le \pi .$$

The point $P\left(\frac{\pi}{12}, \frac{\pi}{4}\right)$ lies on C.

Show that an equation of the tangent to C at P is

$$y = 3x .$$

[ ], proof



Question 14 (***)

A curve has equation

$$3x^2 - xy + y^2 + 2x - 4y = 1 .$$

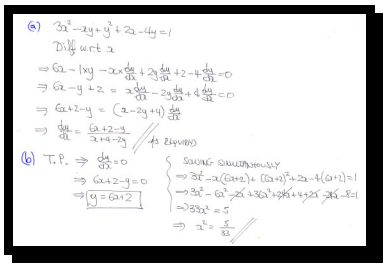
a) Show clearly that

$$\frac{dy}{dx} = \frac{2 + 6x - y}{4 - 2y + x} .$$

b) Hence show further that the value of x at the stationary points of the curve satisfies the equation

$$x^2 = \frac{5}{33} .$$

proof



Question 15 (***)

A curve has equation

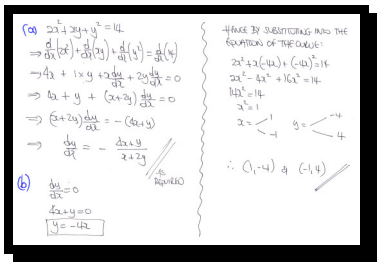
$$2x^2+xy+y^2=14.$$

a) Show clearly that

$$\frac{dy}{dx}=-\frac{4x+y}{x+2y}.$$

b) Hence, find the coordinates of the stationary points of the curve.

$$\boxed{(1,-4),\ (-1,4)}$$



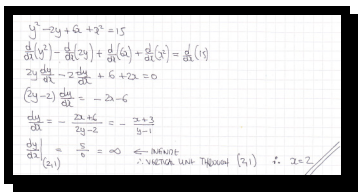
Question 16 (***)

A curve is described by the implicit relationship

$$y^2-2y+6x+x^2=15.$$

Find an equation for the tangent to the curve at the point $P(2,1)$.

$$\boxed{x=2}$$



Question 17 (***)

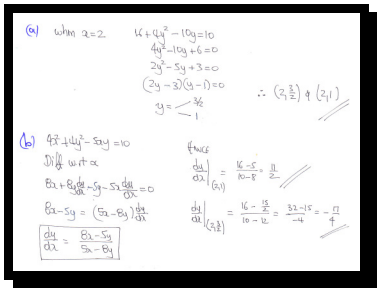
The equation of a curve is given by

$$4x^2+4y^2-5xy=10\,.$$

a) Find the y coordinates of the points on the curve where $x=2$.

b) Find the gradient at these points.

$$\boxed{(2,1)\ \&\ \left(2,\frac{3}{2}\right)},\quad \boxed{\left.\frac{dy}{dx}\right|_{(2,1)}=\frac{11}{2}},\quad \boxed{\left.\frac{dy}{dx}\right|_{\left(2,\frac{3}{2}\right)}=-\frac{17}{4}}$$



Question 18 (***)

A curve C has implicit equation

$$x^2-4xy+y^2=13\,.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{x-2y}{2x-y}\,.$$

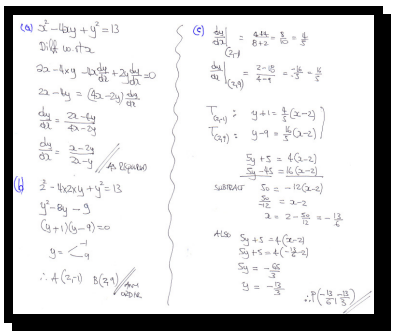
The points A and B are the two points on C whose x coordinate is 2.

b) Find the y coordinates of A and B.

The tangents to C at A and B, meet at the point P.

c) Find the exact coordinates of P.

$\square$, $\boxed{A(2,9),\,B(2,-1)}$, $\boxed{P\left(-\frac{13}{6},-\frac{13}{3}\right)}$



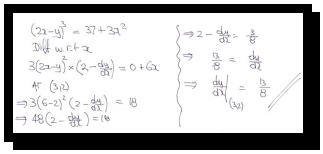
Question 19 (***)

A curve C is defined implicitly by the equation

$$(2x-y)^3=37+3x^2.$$

Find the value of the gradient at the point on C with coordinates $(3,2)$.

$\square$, $\boxed{\frac{13}{8}}$



Question 20 (*+)**

The point $P(2,3)$ lies on the curve with equation

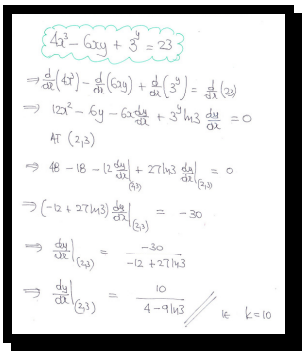
$$4x^3-6xy+3^y=23.$$

Show that the gradient of the curve at P is

$$\frac{k}{4-9\ln 3},$$

where k is a positive integer to be found.

$\square$, $\boxed{k=10}$



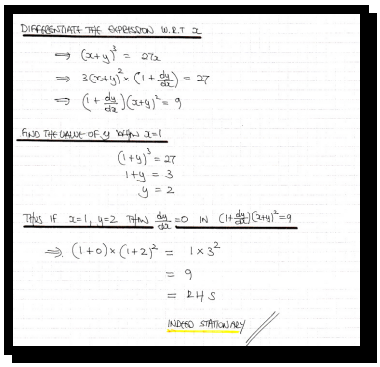
Question 21 (*+)**

A curve C is defined implicitly by

$$(x+y)^3 = 27x, \quad x, y \in \mathbb{R}.$$

Verify that the point on C where $x=1$ is a stationary point.

[ ], [proof]



Question 22 (*+)**

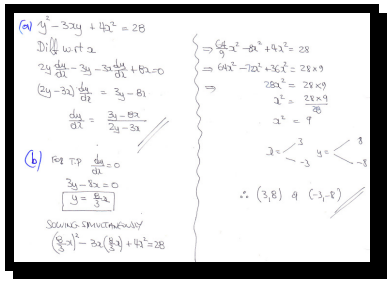
A curve C is defined implicitly by

$$y^2-3xy+4x^2=28\,,\quad x\in\mathbb{R}\,,\ y\in\mathbb{R}\,.$$

a) Find, in terms of x and y, a simplified expression for $\dfrac{dy}{dx}$.

b) Determine the coordinates of the stationary points of C.

$$\boxed{}\,,\quad \boxed{\frac{dy}{dx}=\frac{3y-8x}{2y-3x}}\,,\quad \boxed{(-3,-8)\,,\ (3,8)}$$



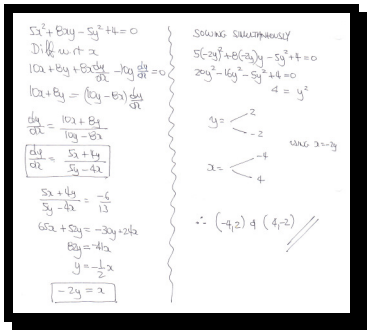
Question 23 (*+)**

A curve has equation

$$5x^2+8xy-5y^2+4=0\,.$$

Find the coordinates of the two points on the curve at which $\dfrac{dy}{dx}=-\dfrac{6}{13}$

$$\boxed{(-4,2)\,,\,(4,-2)}$$



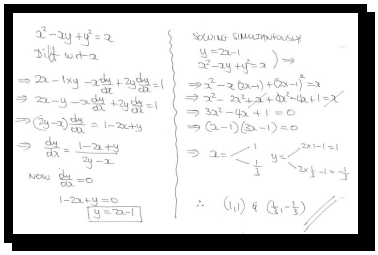
Question 24 (*+)**

A curve C is given implicitly by

$$x^2 - xy + y^2 = x .$$

Find the coordinates of the points on C at which the gradient is zero.

$$\boxed{(1,1) \,\&\, \left(\frac{1}{3}, -\frac{1}{3}\right)}$$



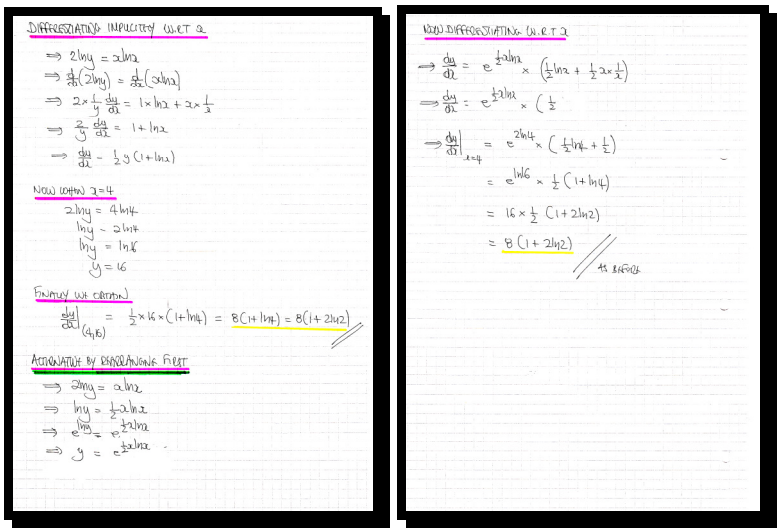
Question 25 (*+)**

The equation of a curve is given implicitly by

$$2\ln y = x\ln x , \quad x, y \in \mathbb{R} \quad x, y > 0 .$$

Find the exact value of the gradient at the point on the curve where $x = 4$.

$$\boxed{}, \quad \boxed{8(1+\ln 4)}$$



Question 26 (*+)**

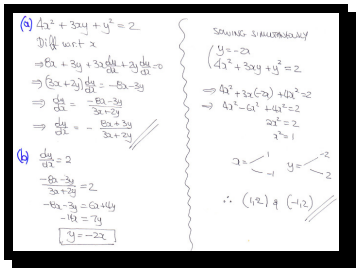
A curve C is given implicitly by the equation

$$4x^2+3xy+y^2=2, \quad x\in\mathbb{R}, \quad y\in\mathbb{R}.$$

a) Find, in terms of x and y, an expression for $\dfrac{dy}{dx}$.

b) Find the coordinates of the points on C, where the gradient is 2.

$$\boxed{\frac{dy}{dx}=-\frac{8x+3y}{3x+2y}}, \quad \boxed{(-1,2),\ (1,-2)}$$



Question 27 (*+)**

A curve C has implicit equation

$$x^2+4xy+2y^2=7\,.$$

a) Show clearly that …

i. … $\dfrac{dy}{dx}=-\dfrac{x+2y}{2x+2y}$.

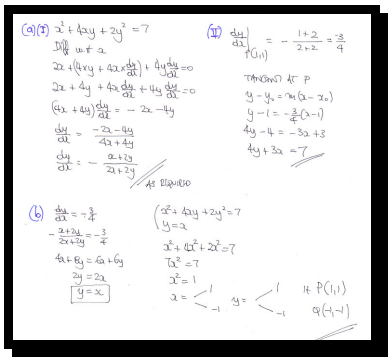
ii. … the equation of the tangent to the curve at $P(1,1)$ is

$$3x+4y=7\,.$$

The tangent to the curve at the point Q is parallel to the tangent to the curve at P.

b) Find the coordinates of Q.

$\square$, $\boxed{Q(-1,-1)}$



Question 28 (*+)**

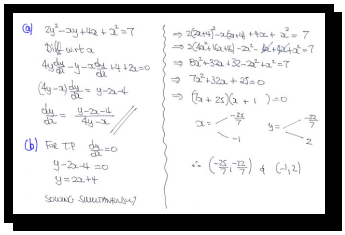
A curve C is defined implicitly by

$$2y^2 - xy + 4x + x^2 = 7, \quad x, y \in \mathbb{R}.$$

a) Find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

b) Show that $(-1,2)$ is one of the stationary points of C and determine the exact coordinates of the other stationary point.

$\square$, $\boxed{\dfrac{dy}{dx} = \dfrac{y-2x-4}{4y-x}}$, $\boxed{\left(-\dfrac{25}{7}, -\dfrac{22}{7}\right)}$



Question 29 **(***+)**

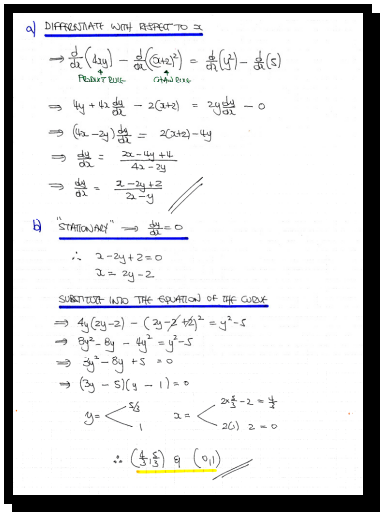
A curve C is defined implicitly by

$$4xy-(x+2)^2=y^2-5\,.$$

a) Find a simplified expression for $\dfrac{dy}{dx}$, in terms of x and y.

b) Hence determine the coordinates of the two stationary points of C.

$\square$, $\boxed{\dfrac{dy}{dx}=\dfrac{x-2y+2}{2x-y}}$, $\boxed{(0,1),\ \left(\dfrac{4}{3},\dfrac{5}{3}\right)}$



Question 30 (*+)**

A curve C is defined implicitly by

$$6^x + 6xy + y^2 = 9 .$$

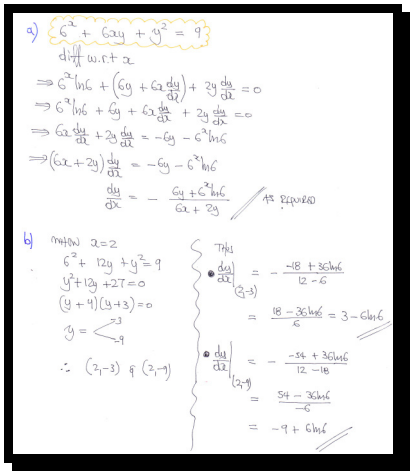
a) Show clearly that

$$\frac{dy}{dx} = -\frac{6y + 6^x \ln 6}{6x + 2y} .$$

b) Find the gradient at each of the two points on C, where $x = 2$.

Give the answers in the form $a + b\ln 6$, where a and b are integers.

$$\square , \quad \boxed{\left.\frac{dy}{dx}\right|_{(2,-3)} = 3 - 6\ln 6} , \quad \boxed{\left.\frac{dy}{dx}\right|_{(2,-9)} = -9 + 6\ln 6}$$



Question 31 **(***+)**

A curve C has implicit equation

$$2xy = 2^x + y^2 .$$

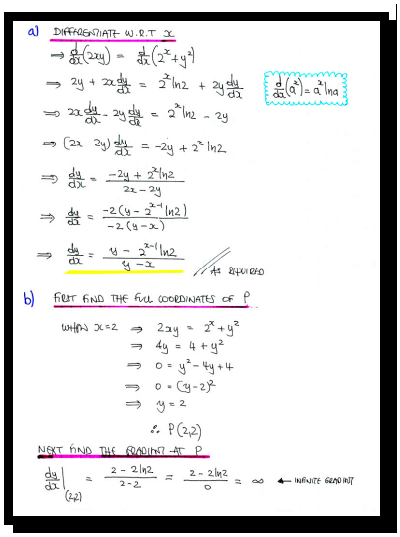
a) Show clearly that

$$\frac{dy}{dx} = \frac{y - 2^{x-1}\ln 2}{y - x} .$$

The point P lies on C, where $x = 2$.

b) Find an equation of the tangent to C at P.

$\boxed{}$, $\boxed{x = 2}$


a) DIFFERENTIATE W.R.T x

$\Rightarrow \frac{d}{dx}(2xy) = \frac{d}{dx}(2^x + y^2)$

$\Rightarrow 2y + 2x\frac{dy}{dx} = 2^x \ln 2 + 2y\frac{dy}{dx}$ $\quad \frac{d}{dx}(a^x) = a^x \ln a$

$\Rightarrow 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2^x \ln 2 - 2y$

$\Rightarrow (2x - 2y)\frac{dy}{dx} = -2y + 2^x \ln 2$

$\Rightarrow \frac{dy}{dx} = \frac{-2y + 2^x \ln 2}{2x - 2y}$

$\Rightarrow \frac{dy}{dx} = \frac{-2(y - 2^{x-1}\ln 2)}{-2(y - x)}$

$\Rightarrow \frac{dy}{dx} = \frac{y - 2^{x-1}\ln 2}{y - x}$ AS REQUIRED

b) FIRST FIND THE FULL COORDINATES OF P

WHEN $x = 2$ $\Rightarrow 2xy = 2^x + y^2$

$\Rightarrow 4y = 4 + y^2$

$\Rightarrow 0 = y^2 - 4y + 4$

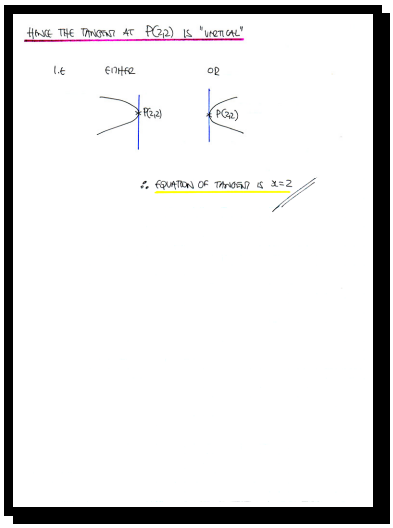
$\Rightarrow 0 = (y - 2)^2$

$\Rightarrow y = 2$

$\therefore P(2,2)$

NEXT FIND THE GRADIENT AT P

$\left.\frac{dy}{dx}\right|_{(2,2)} = \frac{2 - 2\ln 2}{2 - 2} = \frac{2 - 2\ln 2}{0} = \infty$ ← INFINITE GRADIENT


HENCE THE TANGENT AT $P(2,2)$ IS "VERTICAL"

I.E. EITHER OR

$\therefore$ EQUATION OF TANGENT IS $x = 2$

Question 32 (*+)**

A curve C has implicit equation

$$\sin 3x+\sin 2y=\sqrt{2}\,,\quad 0\le x,y\le\frac{\pi}{3}\,.$$

The point P lies on C and its x coordinate is $\dfrac{\pi}{12}$.

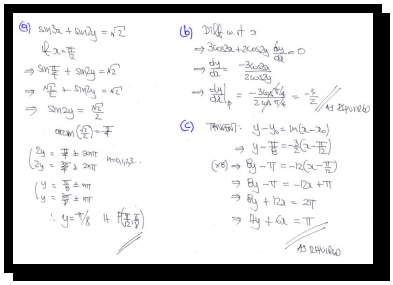
a) Find the y coordinate of P.

b) Show that the gradient at P is $-\dfrac{3}{2}$.

c) Show further that the equation of the tangent to C at P is

$$4y+6x=\pi\,.$$

$$\boxed{P\left(\frac{\pi}{12},\frac{\pi}{8}\right)}$$



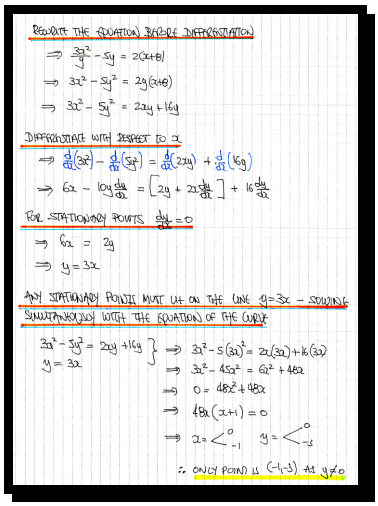
Question 33 (*+)**

A curve has implicit equation

$$\frac{3x^2}{y}-5y=2(x+8), \qquad x\in\mathbb{R},\ y\in\mathbb{R},\ y\neq 0.$$

Find the coordinates of the stationary point of the curve.

$\boxed{}$, $\boxed{(-1,-3)}$



Question 34 (*+)**

A curve C has implicit equation

$$xy(x-y)+16=0\,,\quad x\neq y\,,\quad x,y\neq 0\,.$$

Find the coordinates of the stationary point of C.

$\square$, $\boxed{(2,4)}$

EXPANDING & DIFFERENTIATING IMPLICITLY

$\Rightarrow xy(x-y)+16=0$

$\Rightarrow x^2y-xy^2+16=0$

$\Rightarrow \frac{d}{dx}(x^2y)-\frac{d}{dx}(xy^2)=\frac{d}{dx}(-16)$

$\Rightarrow 2xy+x^2\frac{dy}{dx}-y^2-2\left(y\frac{dy}{dx}\right)=0$

$\Rightarrow 2xy+x^2\frac{dy}{dx}-y^2-2y\frac{dy}{dx}=0$

FOR STATIONARY POINTS $\frac{dy}{dx}=0$

$\Rightarrow 2xy-y^2=0$

$\Rightarrow 2x-y=0 \qquad (y\neq 0)$

$\Rightarrow y=2x$

SUBSTITUTE INTO THE EQUATION

$\Rightarrow x(2x)(x-2x)+16=0$

$\Rightarrow -2x^3+16=0$

$\Rightarrow 16=2x^3$

$\Rightarrow 8=x^3$

$\Rightarrow x=2$

$\therefore (2,4)$

Question 35 (*+)**

A curve C has implicit equation

$$ax^2 + xy - 2y^2 + b = 0,$$

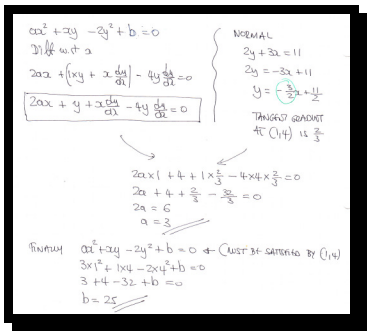
where a and b are constants.

The normal to the curve at the point $P(1,4)$ has equation

$$2y + 3x = 11.$$

Determine the value of a and the value of b.

$\square$, $\boxed{a = 3}$, $\boxed{b = 25}$



Question 36 (*+)**

The equation of a curve is given by

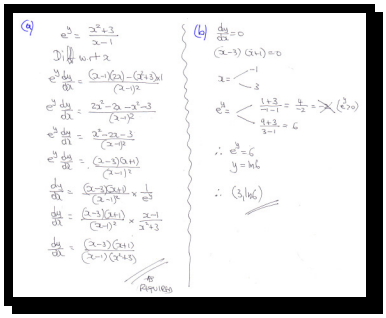
$$\mathrm{e}^{y}=\frac{x^2+3}{x-1}.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{(x-3)(x+1)}{(x^2+3)(x-1)}.$$

b) Find the exact coordinates of the turning point of the curve.

$\boxed{}$, $\boxed{(3,\ln 6)}$



Question 37 (*+)**

The curve C is given implicitly by

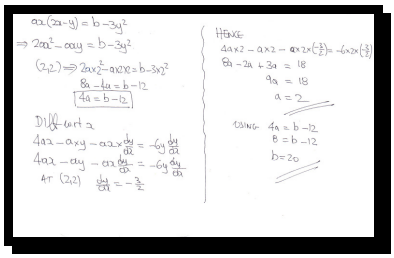
$$ax(2x-y)=b-3y^2,$$

where a and b are non zero constants.

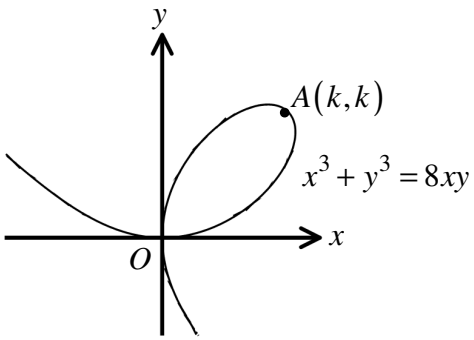
The point $(2,2)$ lies on C and the gradient at that point is $-\frac{3}{2}$.

Find the value of a and the value of b.

$\square$, $\boxed{a=2,\ \ b=20}$



Question 38 (****)



The figure above shows a curve known as "the folium of Descartes", with equation

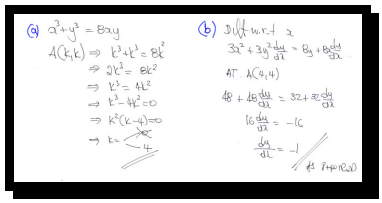
$$x^3 + y^3 = 8xy \, .$$

The point $A(k,k)$, where k is a non zero constant, lies on the curve.

a) Find the value of k.

b) Show that the gradient at A is -1.

$\boxed{}$, $\boxed{k=4}$



Question 39 (****)

A curve C has implicit equation

$$ax^3-3xy+by^2=224,$$

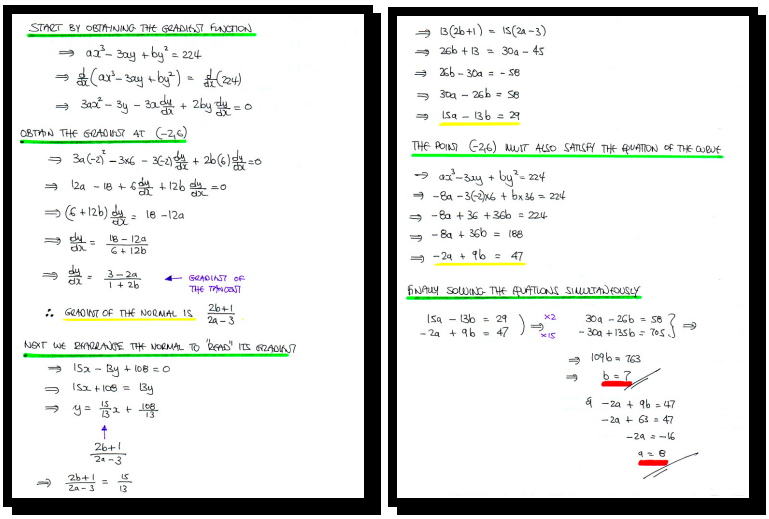
where a and b are non zero constants.

The normal to the curve at the point $P(-2,6)$ has equation

$$15x-13y+108=0.$$

Determine the value of a and the value of b.

$\boxed{}$, $\boxed{a=8}$, $\boxed{b=7}$



Question 40 (****)

The equation of a curve is given implicitly by

$$4y + y^2 \mathrm{e}^{3x} = x^3 + C ,$$

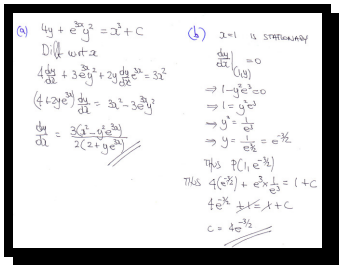
where C is a non zero constant.

a) Find a simplified expression for $\dfrac{dy}{dx}$.

The point $P(1,k)$, where $k > 0$, is a stationary point of the curve.

b) Find an exact value for C.

$$\square , \quad \boxed{\frac{dy}{dx} = \frac{3\left(x^2 - y^2 \mathrm{e}^{3x}\right)}{2\left(2 + y\mathrm{e}^{3x}\right)}} , \quad \boxed{C = 4\mathrm{e}^{-\frac{3}{2}}}$$



Question 41 (****)

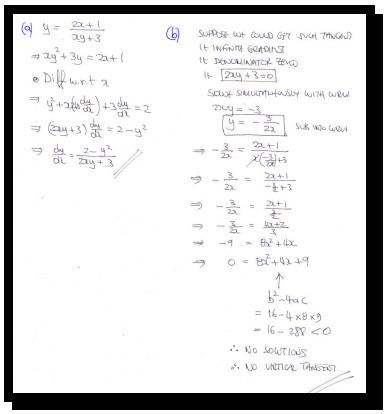
A curve C has implicit equation

$$y=\frac{2x+1}{xy+3}.$$

a) Find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

b) Show that there is **no** point on C, where the tangent is parallel to the y axis.

$\square$, $\boxed{\dfrac{dy}{dx}=\dfrac{2-y^2}{2xy+3}}$



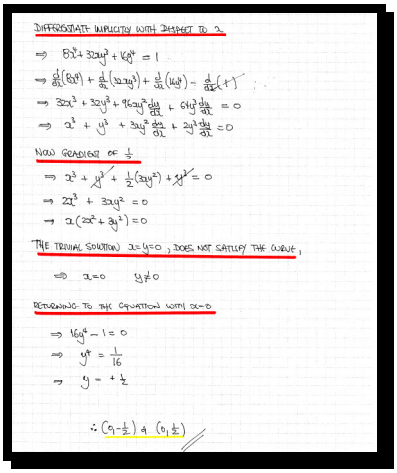
Question 42 (****)

A curve has implicit equation

$$8x^4+32xy^3+16y^4=1.$$

Find the coordinates of any points on the curve whose gradient is $\frac{1}{2}$.

$\square$, $\boxed{\left(0,\frac{1}{2}\right)\cup\left(0,-\frac{1}{2}\right)}$



Question 43 (****)

A curve C has implicit equation

$$(xy-2)(y+5)=10.$$

The curve crosses the y axis at the point A.

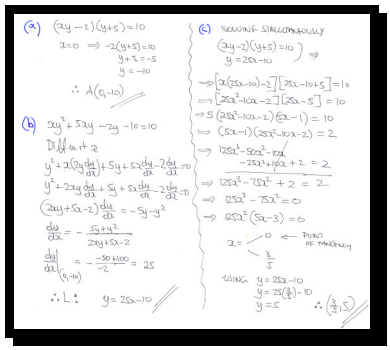
The straight line L is the tangent to C at A.

a) State the coordinates of A.

b) Find an equation for L.

c) Determine the coordinates of the point where L meets C again.

$\square$, $\boxed{A(0,-10)}$, $\boxed{y=25x-10}$, $\boxed{\left(\frac{3}{5},5\right)}$



Question 44 (****)

A curve C has implicit equation

$$4^x + 2xy + y^2 = 13 .$$

a) Show clearly that

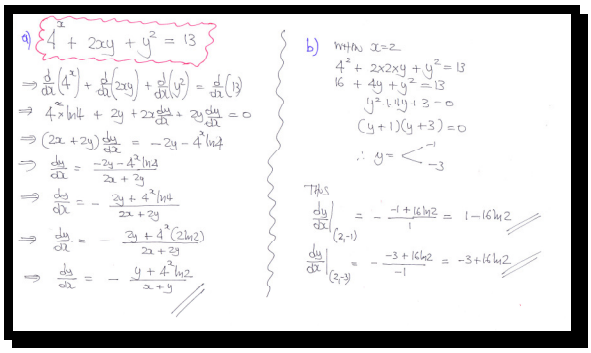
$$\frac{dy}{dx} = -\frac{y + 4^x \ln 2}{x + y} .$$

There are two points on C whose x coordinate is 2.

b) Find the gradient at each of these two points.

Give the answers in the form $a + b\ln 2$, where a and b are integers.

$$\left.\frac{dy}{dx}\right|_{(2,-1)} = 1 - 16\ln 2 \quad , \quad \left.\frac{dy}{dx}\right|_{(2,-3)} = -3 + 16\ln 2$$



Question 45 (****)

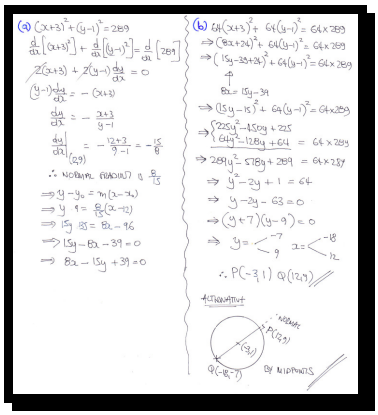
A circle has equation

$$(x+3)^2+(y-1)^2=289\,.$$

a) Find an equation for the normal to the curve at the point $P(12,9)$.

b) Find the coordinates of the point Q, where the normal to the circle at P intersects the circle again.

$\boxed{8x-15y+39=0}$, $\boxed{Q(-18,-7)}$



Question 46 (****)

The point $P(4,2)$ lies on the curve with equation

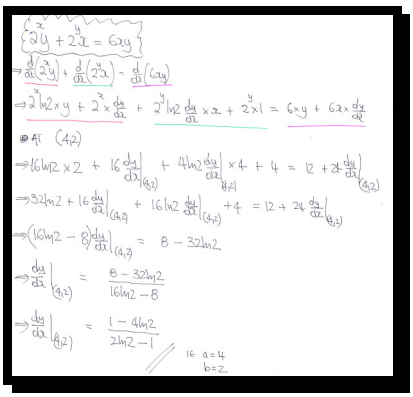
$$2^x y + 2^y x = 6xy .$$

Show that the gradient of the curve at P is

$$\frac{1-a\ln 2}{b\ln 2-1},$$

where a and b are positive integers to be found.

$\boxed{}$, $\boxed{a=4,\ b=2}$



Question 47 **(****)**

A curve C is defined implicitly by

$$\sin 2x \cot y = 1, \quad x \in \mathbb{R}, \quad y \in \mathbb{R}, \quad 0 < x < \frac{\pi}{2}, \quad 0 < y < \frac{\pi}{2}.$$

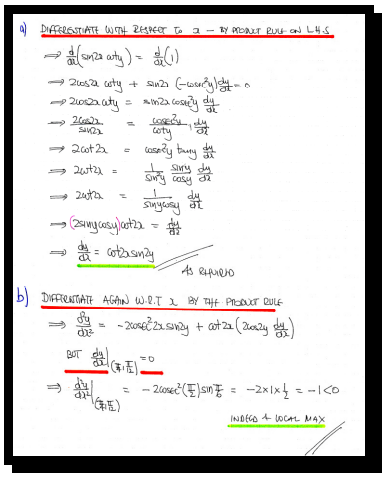
a) Show clearly that

$$\frac{dy}{dx} = \cot 2x \sin 2y .$$

The point $A\left(\frac{\pi}{4}, \frac{\pi}{12}\right)$ is a turning point of C.

b) Use $\frac{d^2y}{dx^2}$ to show that A is a local maximum.

[ ], [proof]



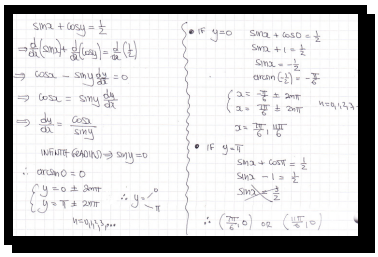
Question 48 (****)

A curve has equation

$$\sin x+\cos y=\frac{1}{2},\ \ 0\le x<2\pi,\ 0\le y<2\pi.$$

Find the coordinates of the points on the curve, where the tangent to the curve is parallel to the y axis.

$$\boxed{\left(\frac{7\pi}{6},0\right),\left(\frac{11\pi}{6},0\right)}$$



Question 49 (****)

The equation of a curve is given by

$$x^2-2y^2-xy-x+5y+34=0\,.$$

a) Show clearly that

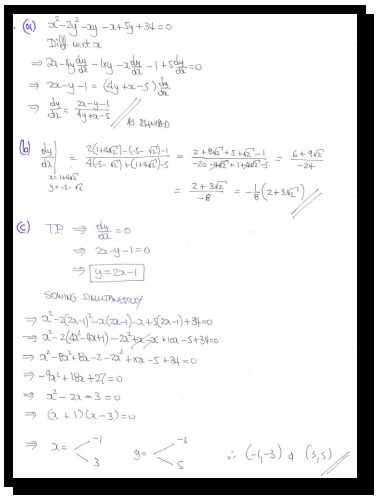
$$\frac{dy}{dx}=\frac{2x-y-1}{x+4y-5}\,.$$

b) Find the exact value of gradient at the point on the curve with coordinates

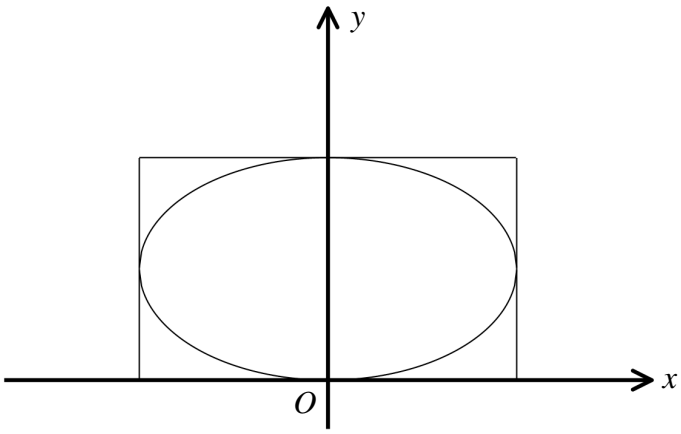
$$\left(1+4\sqrt{2},-5-\sqrt{2}\right).$$

c) Determine the coordinates of the turning points of the curve.

$$\boxed{-\frac{1}{8}\left(2+3\sqrt{2}\right)},\ \boxed{(3,5),\ (-1,-3)}$$



Question 50 (****)



The figure above shows the curve with equation

$$x^2-8y+4y^2=0\,.$$

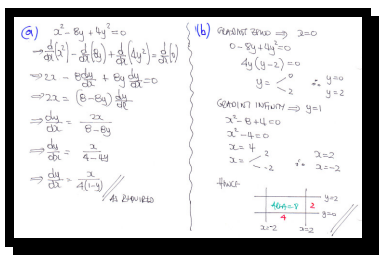
a) Show that

$$\frac{dy}{dx}=\frac{x}{4(1-y)}\,.$$

The curve fits perfectly inside a rectangle whose sides are parallel to the coordinate axes, so they are tangents to the curve.

b) Show further that the area of the rectangle is 8 square units.

$\boxed{}$, $\boxed{\text{proof}}$



Question 51 (****)

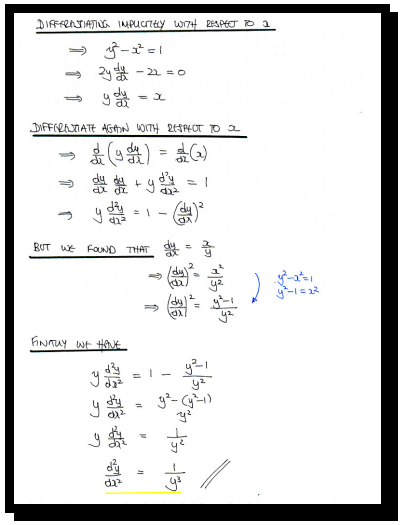
The equation of a curve is given implicitly by

$$y^2 - x^2 = 1, \quad |y| \geq 1 \ .$$

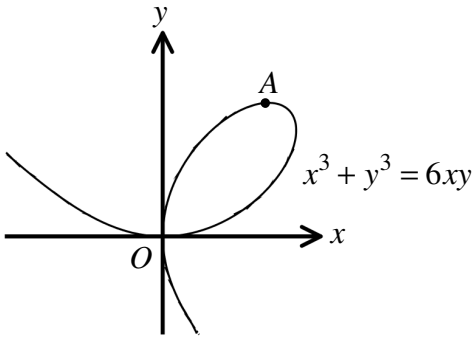
Show clearly that

$$\frac{d^2y}{dx^2} = \frac{1}{y^3} .$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 52 (****)



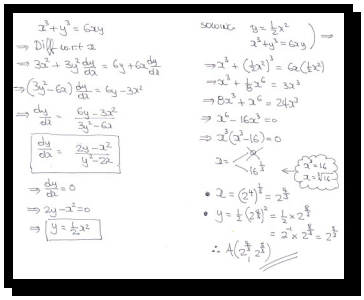
The diagram above shows a curve known as "the folium of Descartes", with equation

$$x^3 + y^3 = 6xy \, .$$

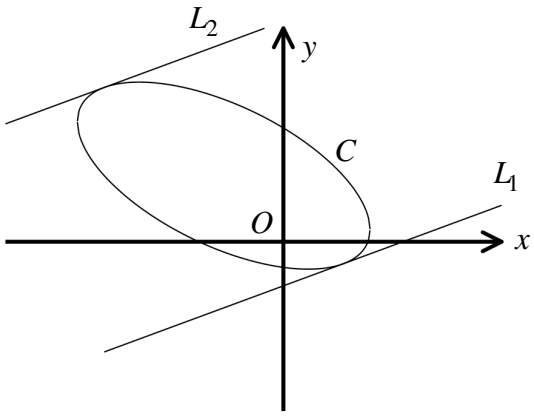
The curve is stationary at the origin O and at the point A.

Find the exact coordinates of A in the form $\left(2^n, 2^m\right)$, where n and m are fractions to be found.

$\square$, $\boxed{A\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)}$



Question 53 (****)



The figure above shows the curve C with the equation

$$4y-2xy+6=y^2+3x^2 .$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{y+3x}{2-x-y}.$$

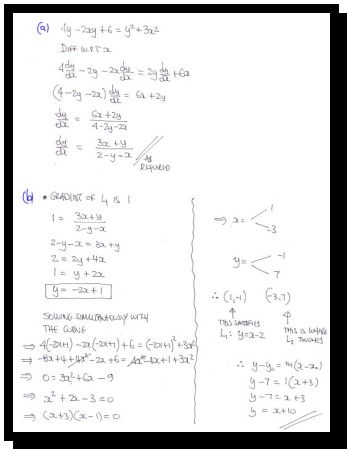
The straight lines L_1 and L_2 are parallel to each other and are both tangents to C.

The equation of L_1 is

$$y=x-2 .$$

b) Find an equation of L_2

$\boxed{}$, $\boxed{y=x+10}$



Question 54 (****)

A curve C is given implicitly by

$$x^2+4y^2-8x-16y+28=0\,.$$

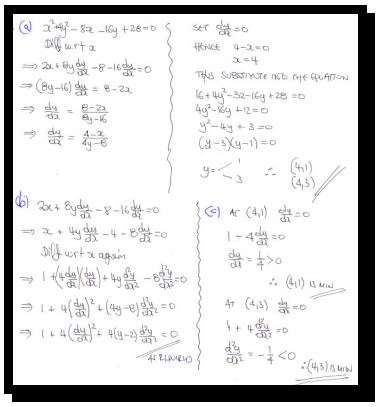
a) Find the coordinates of the turning points of C.

b) Show clearly that

$$1+4\left(\frac{dy}{dx}\right)^2+4(y-2)\frac{d^2y}{dx^2}=0\,.$$

c) Hence determine the nature of these turning points.

$\boxed{(4,3)\,\&\,(4,1)}$, $\boxed{\text{max at } (4,3) \text{ \& min at} (4,1)}$



Question 55 (****)

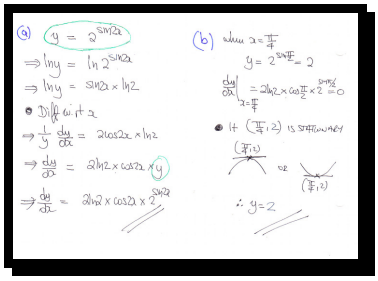
A curve C has equation

$$y = 2^{\sin 2x}, \ x \in \mathbb{R}.$$

a) By taking logarithms on both sides of this equation, or otherwise, find an expression for $\dfrac{dy}{dx}$ in terms of x.

b) Find an equation of the tangent to the curve at the point where $x = \dfrac{\pi}{4}$.

$\square$, $\boxed{\dfrac{dy}{dx} = 2^{\sin 2x} \times 2 \ln 2 \times \cos 2x = 2^{1+\sin 2x} \times \ln 2 \times \cos 2x}$, $\boxed{y = 2}$



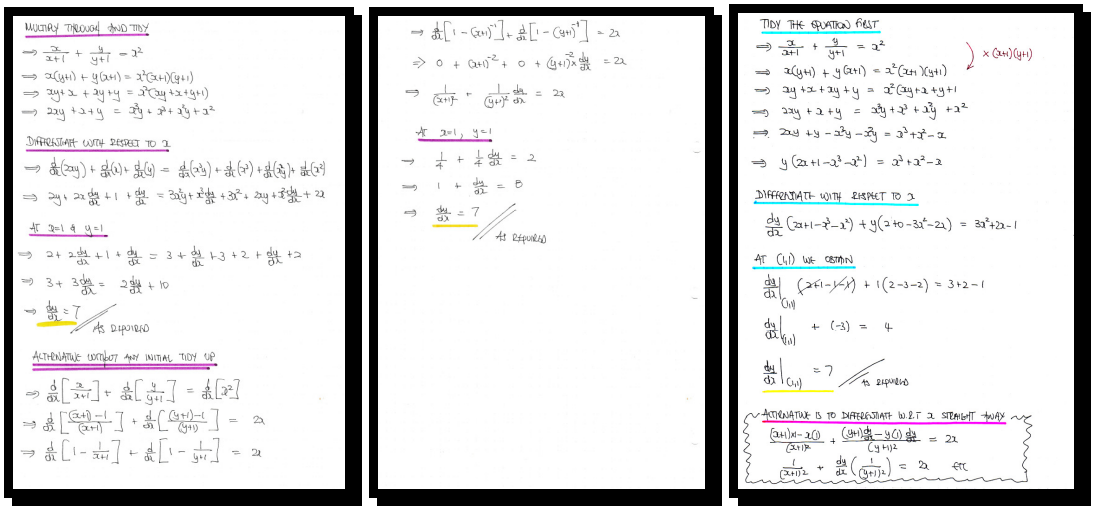
Question 56 (****)

The equation of a curve is given by the implicit relationship

$$\frac{x}{x+1}+\frac{y}{y+1}=x^2.$$

Show that at the point on the curve with coordinates $(1,1)$, the gradient is 7.

$\boxed{}$, $\boxed{\text{proof}}$



Question 57 (****)

A curve C has implicit equation

$$\frac{(x+2y)^2}{4x-y}+y=3x+2\,.$$

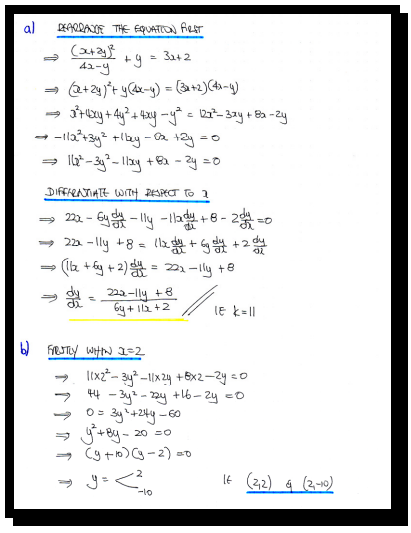
a) Show clearly that

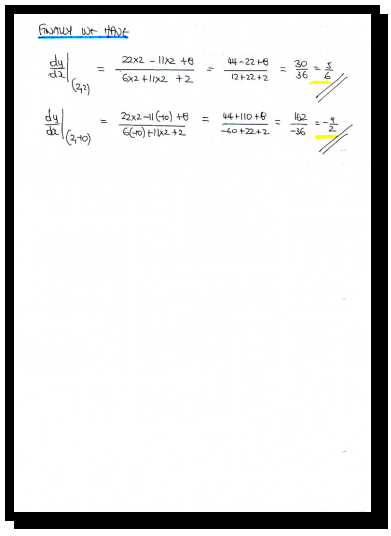
$$\frac{dy}{dx}=\frac{2kx-ky+8}{6y+kx+2},$$

where k is a constant to be found.

b) Find the gradient at each of the points on C, where $x=2$.

$\boxed{}$, $\boxed{\text{gradient}=-\frac{9}{2},\ \frac{5}{6}}$





Question 58 (****)

A curve C is given by the implicit equation

$$xy+y^2=x^2+5\,.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{2x-y}{x+2y}\,.$$

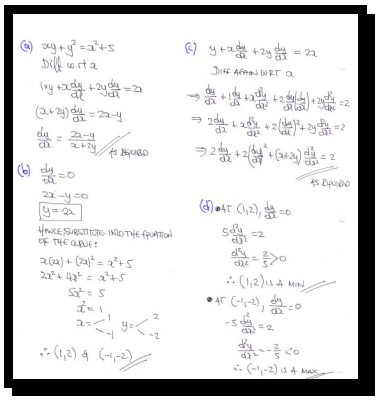
b) Find the coordinates of the turning points of C.

c) Show further that

$$2\frac{dy}{dx}+2\left(\frac{dy}{dx}\right)^2+(x+2y)\frac{d^2y}{dx^2}=2\,.$$

d) Hence determine the nature of these turning points.

$\boxed{(1,2)\ \&\ (-1,-2)}$, $\boxed{\text{max at } (-1,-2)\ \&\ \text{min at } (1,2)}$



Question 59 (****)

$$y = \arcsin x\,, \quad -1 \le y \le 1\,.$$

a) Show clearly that

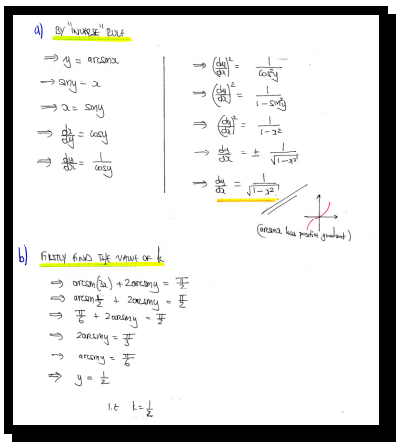
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}\,.$$

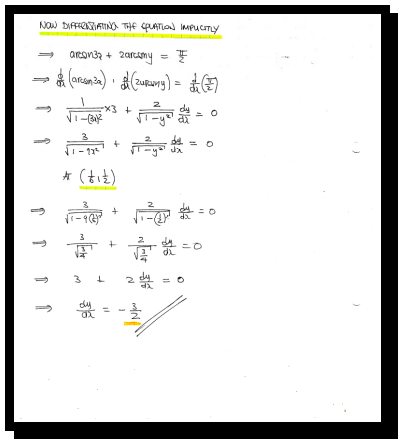
The point $P\left(\frac{1}{6}, k\right)$, where k is a constant, lies on the curve with equation

$$\arcsin 3x + 2\arcsin y = \frac{\pi}{2}\,, \ |x| \le \frac{1}{3}\,, \ |y| \le 1\,.$$

b) Find the value of the gradient at P.

$\square$, $\boxed{-\frac{3}{2}}$





Question 60 (****)

A curve C is given implicitly by

$$x^2+3xy-2y^2+17=0\,.$$

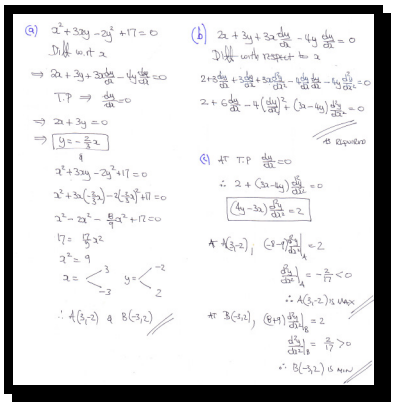
a) Find the coordinates of the turning points of C.

b) Show further that

$$2+6\frac{dy}{dx}-4\left(\frac{dy}{dx}\right)^2+(3x-4y)\frac{d^2y}{dx^2}=0\,.$$

c) Hence determine the nature of these turning points.

$\boxed{(3,-2)\,\&\,(-3,2)}$, $\boxed{\text{max at } (3,-2) \text{ \& min at} (-3,2)}$



Question 61 (****)

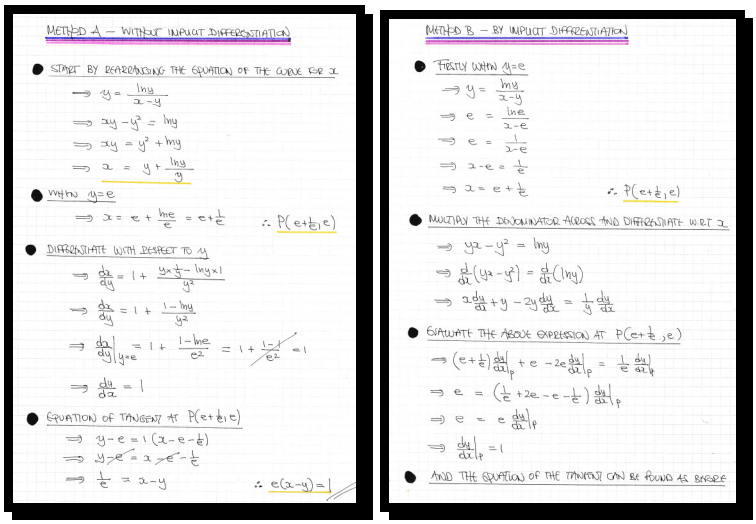
The curve C has equation

$$y=\frac{\ln y}{x-y},\ y>0.$$

Show that the equation of the tangent to C at the point where $y=\mathrm{e}$ can be written as

$$\mathrm{e}(x-y)=1.$$

[ ], [proof]



Question 62 **(****)**

A curve C is given by the implicit equation

$$x^2+2xy-3y^2=4x+4y-20\,.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{x+y-2}{3y-x+2}\,.$$

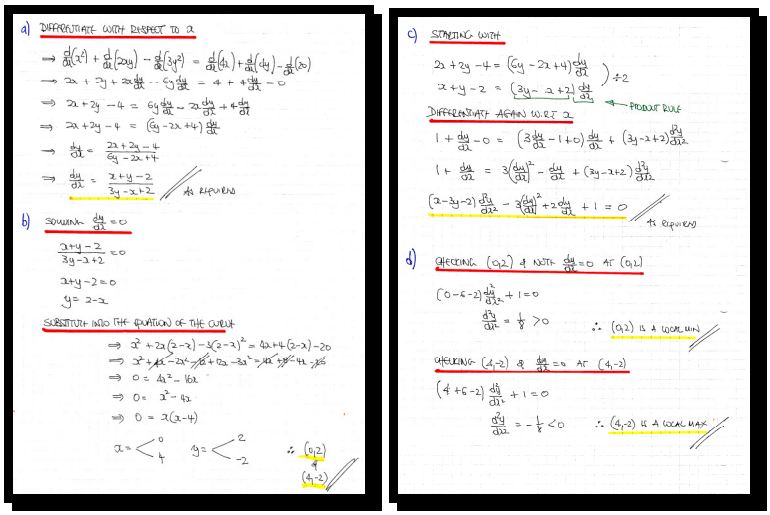
b) Find the coordinates of the turning points of C.

c) Show further that

$$(x-3y-2)\frac{d^2y}{dx^2}-3\left(\frac{dy}{dx}\right)^2+2\frac{dy}{dx}+1=0\,.$$

d) Hence determine the nature of these turning points.

$\boxed{}$, $\boxed{(0,2)\ \&\ (4,-2)}$, $\boxed{\text{max at } (4,-2) \text{ \& min at } (0,2)}$



Question 63 **(****)**

A curve C is given by the implicit equation

$$x^2+4xy+2y^2+18=0\,.$$

a) Show clearly that

$$\frac{dy}{dx}=-\frac{x+2y}{2x+2y}\,.$$

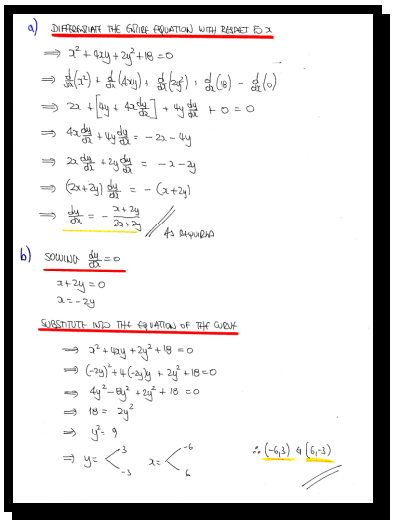
b) Find the coordinates of the turning points of C.

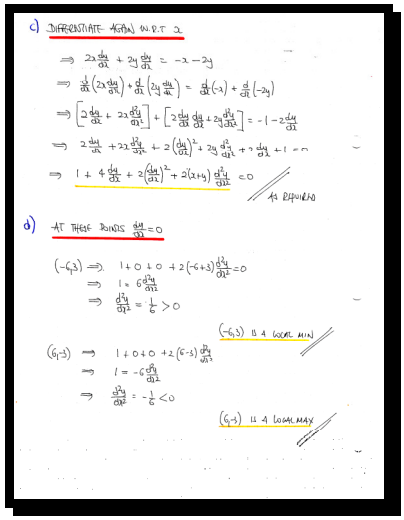
c) Show further that

$$1+4\frac{dy}{dx}+2\left(\frac{dy}{dx}\right)^2+2(x+y)\frac{d^2y}{dx^2}=0\,.$$

d) Hence determine the nature of these turning points.

$\boxed{}$, $\boxed{(-6,3)\,\&\,(6,-3)}$, $\boxed{\text{max at } (6,-3) \text{ \& min at } (-6,3)}$





Question 64 (****)

It is given that

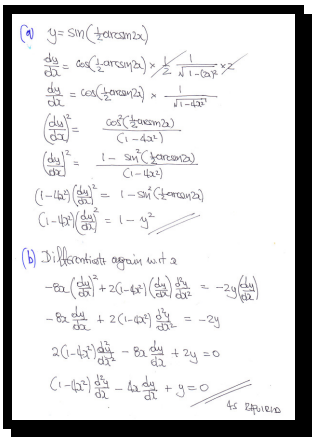
$$\frac{d}{du}\left(\arcsin u\right)=\frac{1}{\sqrt{1-u^2}}, \quad |u|\le 1.$$

Hence show that if $y=\sin\left(\frac{1}{2}\arcsin 2x\right)$, then …

a) … $\left(1-4x^2\right)\left(\frac{dy}{dx}\right)^2=1-y^2$.

b) … $\left(1-4x^2\right)\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+y=0$.

proof



Question 65 (****)

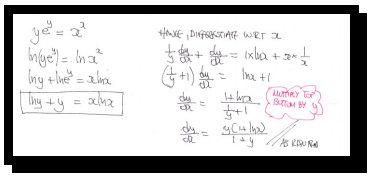
A curve C has implicit equation

$$y\mathrm{e}^{y}=x^{x},\ x>0$$

Show clearly that

$$\frac{dy}{dx}=\frac{y\left(1+\ln x\right)}{1+y}.$$

$\boxed{}$, proof



Question 66 (**+)**

A curve C is given implicitly by

$$4y^2+3xy-2x^2=2x-2y-12.$$

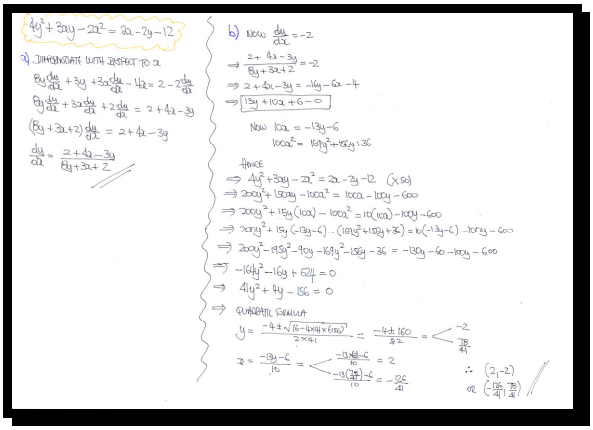
a) Show clearly that

$$\frac{dy}{dx}=\frac{2+4x-3y}{8y+3y+2}.$$

The tangent to C at two distinct points has gradient -2.

b) Find the coordinates of these two points.

$$\boxed{(2,-2)\ \ \&\ \ \left(-\frac{126}{41},\frac{78}{41}\right)}$$



Question 67 **(****+)**

A curve C is given implicitly by

$$2x^2+xy-y^2-4x-y+20=0.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{4x+y-4}{2y-x+1}.$$

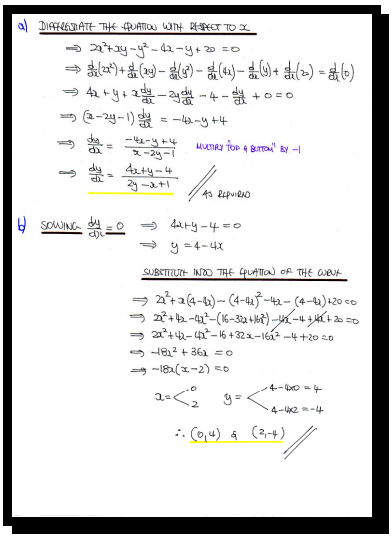
b) Find the coordinates of the stationary points of C.

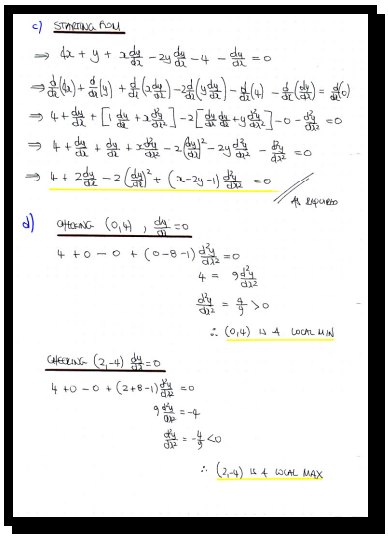
c) Show further that

$$4+2\frac{dy}{dx}-2\left(\frac{dy}{dx}\right)^2+(x-2y-1)\frac{d^2y}{dx^2}=0.$$

d) Hence determine the nature of the stationary points of part **(b)**.

$\boxed{}$, $\boxed{(2,-4)\ \&\ (0,4)}$, $\boxed{\text{max at } (2,-4) \text{ \& min at } (0,4)}$





Question 68 **(****+)**

A curve C has implicit equation

$$x^2-2y^2+4xy-4x-6y+4=0.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{2y+x+a}{2y-2x+b},$$

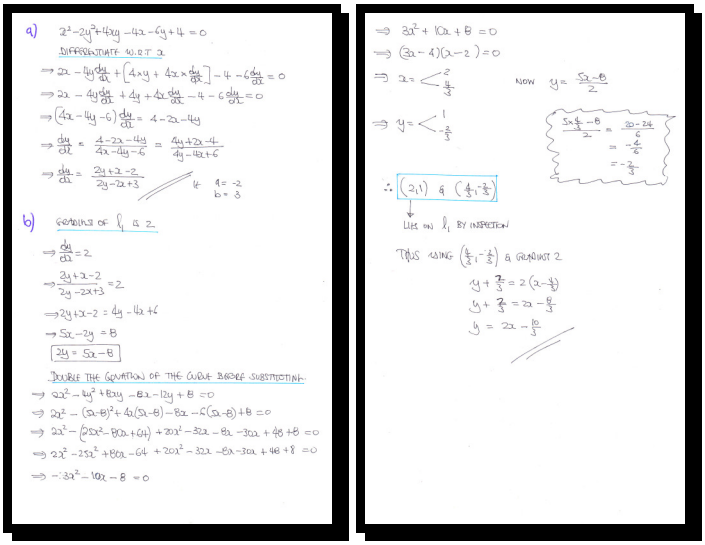
where a and b are integers to be found.

The straight line l_1 with equation $y=2x-3$ is a tangent to C at the point P.

The straight line l_2 is parallel to l_1 and is also a tangent to C at a different point Q.

b) Find an equation of l_2.

$\boxed{}$, $\boxed{a=-2,\ b=3}$, $\boxed{y=2x-\frac{10}{3}}$



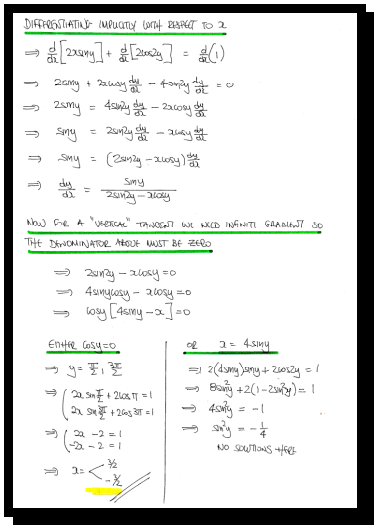
Question 69 (**+)**

A curve has implicit equation

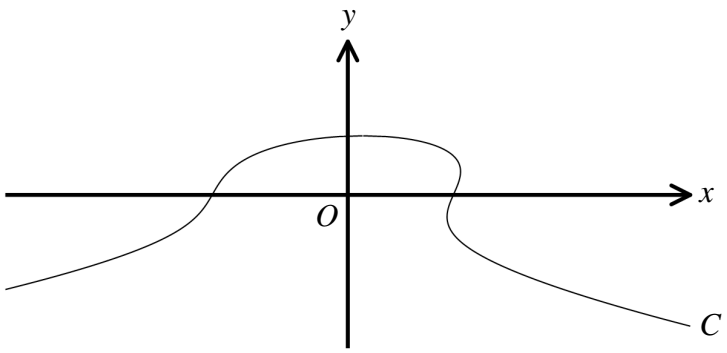
$$2x\sin y+2\cos 2y=1,\quad 0\le y\le 2\pi .$$

Determine the equations of the two straight lines, which are parallel to the y axis, and are tangents to the above curve.

$\boxed{}$, $\boxed{x=\pm\frac{3}{2}}$



Question 70 (**+)**



The figure above shows part of the curve C with equation

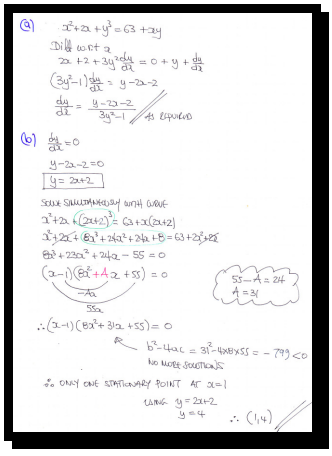
$$x^2+2x+y^3=63+xy\,.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{y-2x-2}{3y^2-x}$$

b) Show further that C has only one stationary point at $(1,4)$.

$\boxed{}$, $\boxed{\text{proof}}$

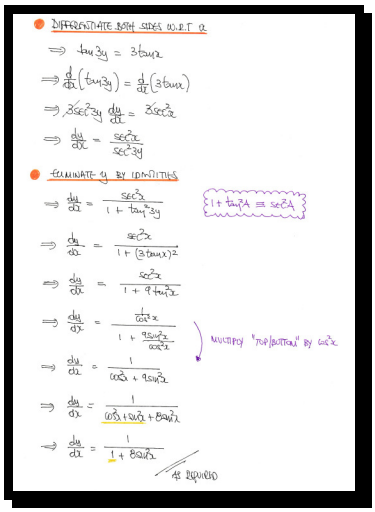


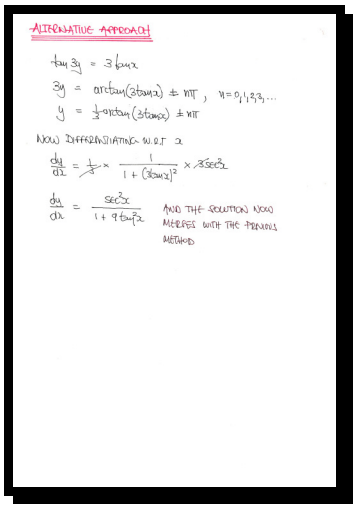
Question 71 **(****+)**

If $\tan 3y = 3\tan x$ show clearly that

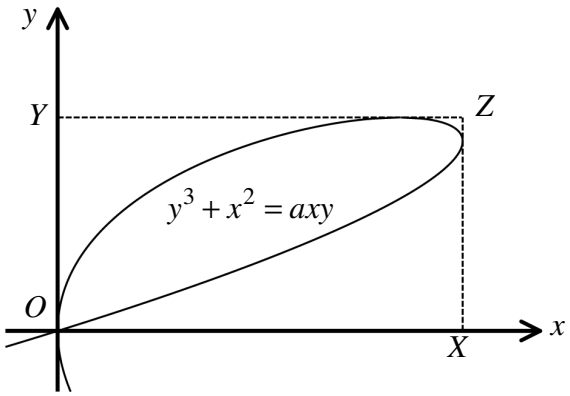
$$\frac{dy}{dx} = \frac{1}{1+8\sin^2 x}.$$

$\boxed{}$, $\boxed{\text{proof}}$





Question 72 **(****+)**



The figure above shows the curve with equation

$$y^3 + x^2 = axy\,,$$

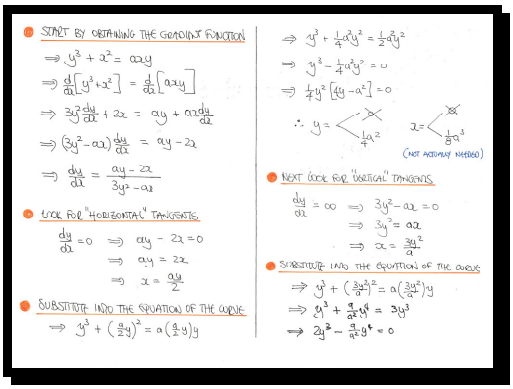
where a is a positive constant.

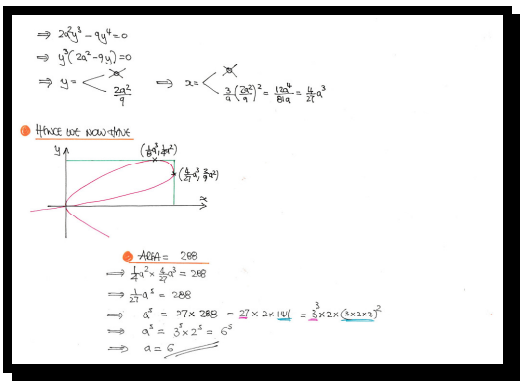
The point Y lies on the y axis so that the straight line segment YZ is a tangent to the curve parallel to the x axis. Similarly the point X lies on the x axis so that the straight line segment XZ is a tangent to the curve parallel to the y axis.

The area of the rectangle $OYZX$, where O is the origin, is 288 square units.

Determine the value of a.

$\boxed{}$, $\boxed{a=6}$





Question 73 (*****)

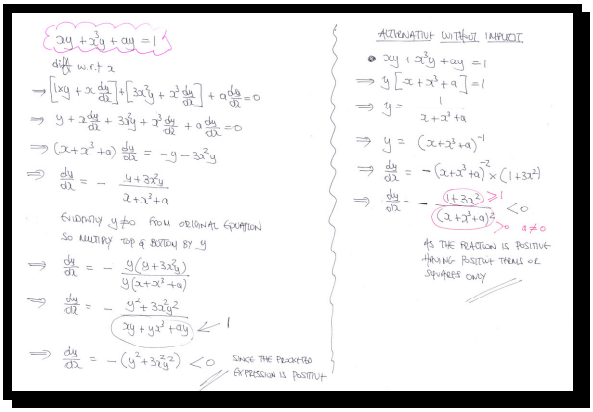
The curve C has implicit equation

$$xy + x^3 y + ay = 1,$$

where a is a positive constant.

Use implicit differentiation to show that the gradient at every point on C is negative.

$\boxed{}$, $\boxed{\text{proof}}$



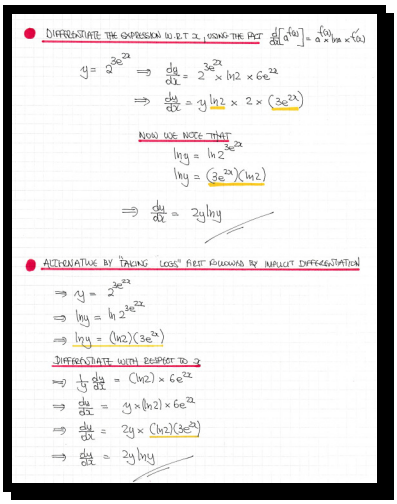
Question 74 **(****+)**

A curve has equation

$$y = 2^{3e^{2x}}, \quad x \in \mathbb{R}.$$

Express $\dfrac{dy}{dx}$ in terms of y .

$\boxed{}$, $\boxed{\dfrac{dy}{dx} = 2y\ln y}$



Question 75 **(*****)**

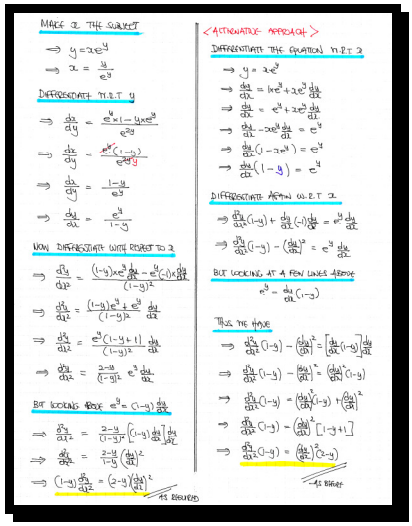
The curve C has implicit equation

$$y = x\mathrm{e}^{y}\,,\ x \neq 0\,,\ y \neq 1\,,\ y \neq 2\,.$$

Show clearly that

$$(1-y)\frac{d^2y}{dx^2} = (2-y)\left(\frac{dy}{dx}\right)^2\,.$$

[ ] , [proof]



Question 76 (*****)

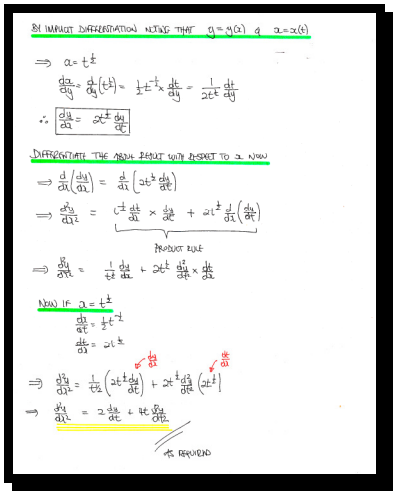
It is given that

$$x = t^{\frac{1}{2}}, \ t > 0.$$

Given further that y is a function of x, show clearly that

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}.$$

[ ], proof



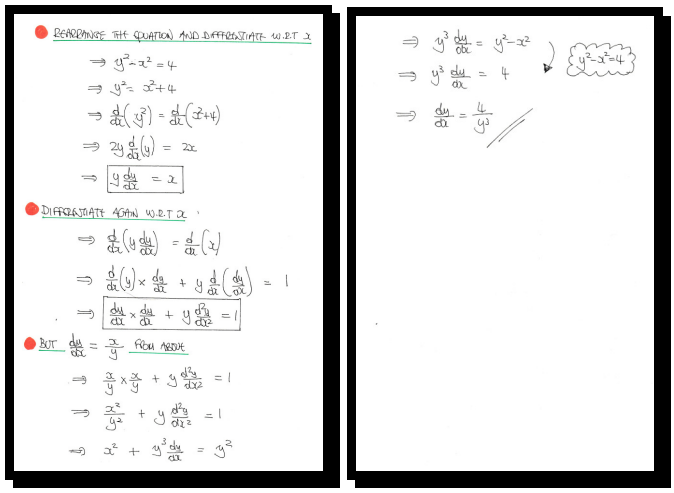
Question 77 (*****)

$$y^2 - x^2 = 4\,,\ |y| \geq 2\,.$$

Use differentiation to show that

$$\frac{d^2y}{dx^2} = \frac{4}{y^3}\,.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 78 (*****)

A curve is defined implicitly as

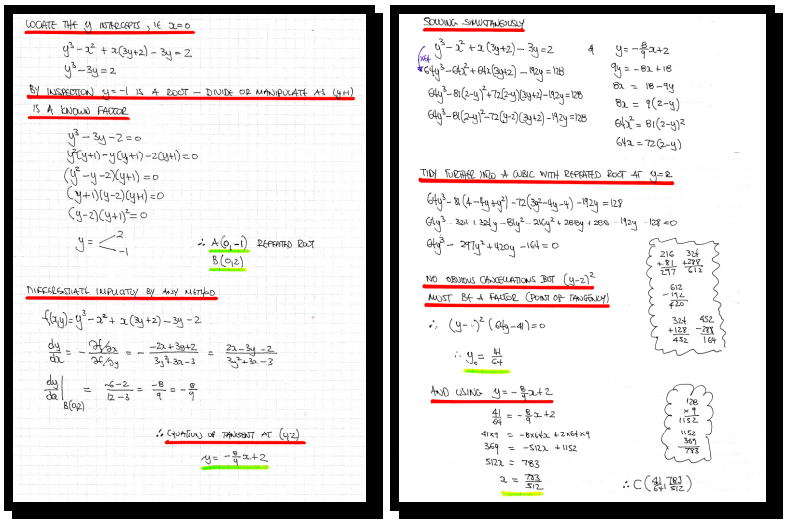
$$y^3-x^2+x(3y+2)-3y=2\,.$$

The y axis is a tangent to the curve at the point A and the point B is another intercept of the curve with the y axis.

The tangent to the curve at the point B meets the curve again at the point C.

Determine the exact coordinate of C.

$\square$, $\boxed{C\left(\frac{41}{64},\frac{783}{512}\right)}$



Question 79 (*****)

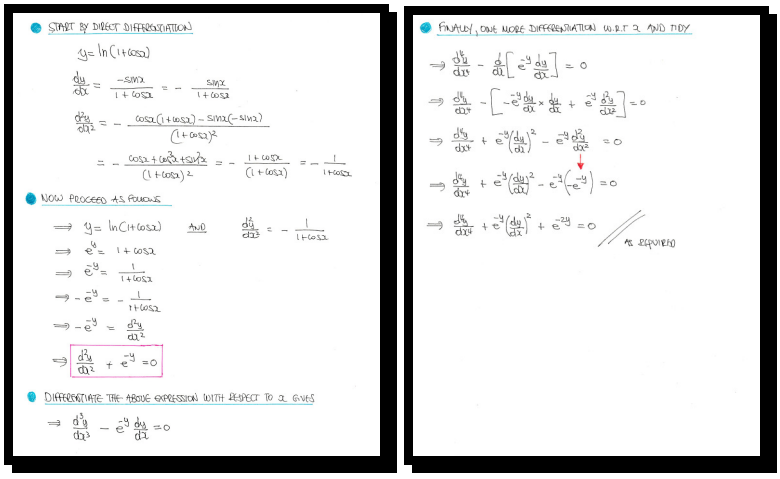
The curve C has equation

$$y=\ln\left(1+\cos x\right),\ x\in\mathbb{R},\ -\pi<x<\pi.$$

Show clearly that

$$\frac{d^4y}{dx^4}+\mathrm{e}^{-y}\left(\frac{dy}{dx}\right)^2+2\mathrm{e}^{-2y}=0.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 80 **(*****)**

A curve is defined implicitly by the equation.

$$\sqrt{x+y}-\sqrt{x-y}=\sqrt{k}\,,$$

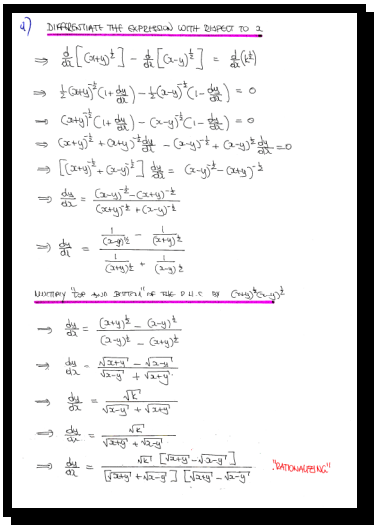
where k is a positive constant.

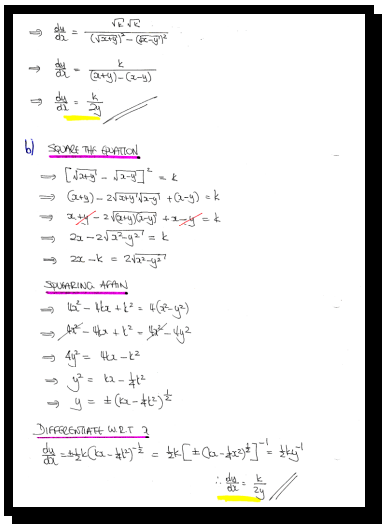
a) Use implicit differentiation, directly onto the above equation, to show that

$$\frac{dy}{dx}=\frac{k}{2y}.$$

b) Verify the result of part **(a)** by differentiating the equation of the curve, having first made y the subject of the equation.

[ ] , [proof]





Question 81 **(*****)**

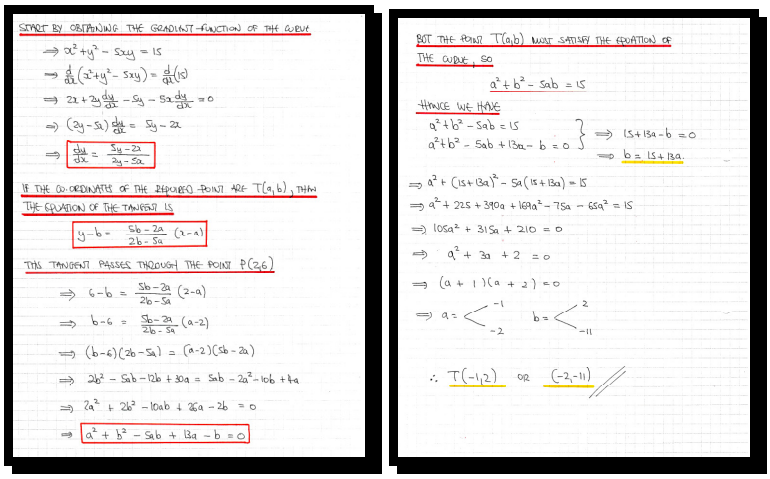
The point T lies on the curve with equation

$$x^2+y^2-5xy=15\,.$$

The tangent to the curve at T passes through the point with coordinates $(2,6)$.

Determine the two possible sets of coordinates for T .

$\boxed{}$, $\boxed{T(-1,2)\ \cup\ T(-2,11)}$



Question 82 (*****)

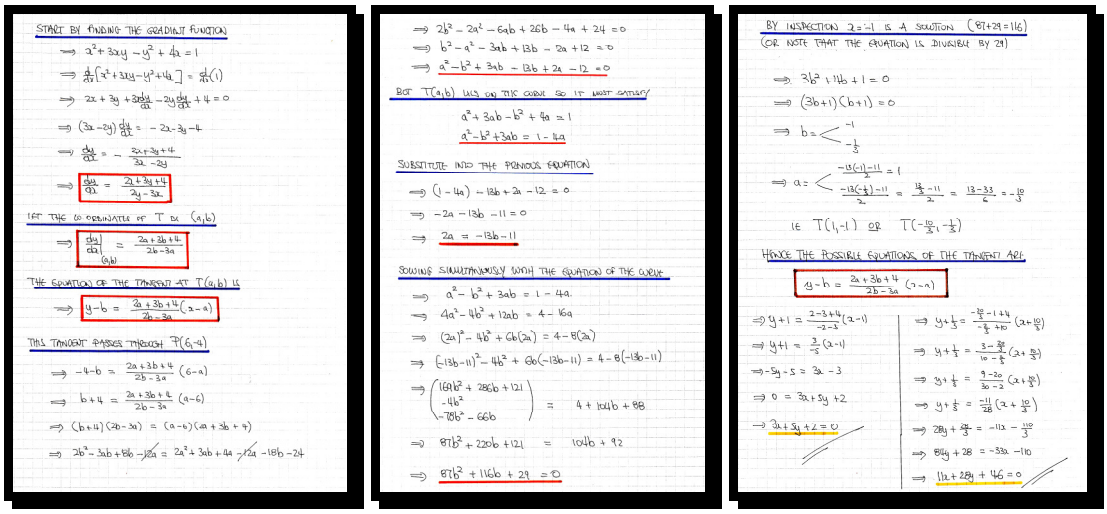
A curve has the following implicit equation

$$x^2+3xy-y^2+4x=1.$$

Two tangents to the curve, at some points on the curve, both pass through the point with coordinates $(6,-4)$.

Determine the equations of these two tangents.

$\boxed{}$, $\boxed{(3x+5y+2)(11x+28y+46)=0}$



Question 83 (*****)

A curve is defined implicitly by the equation

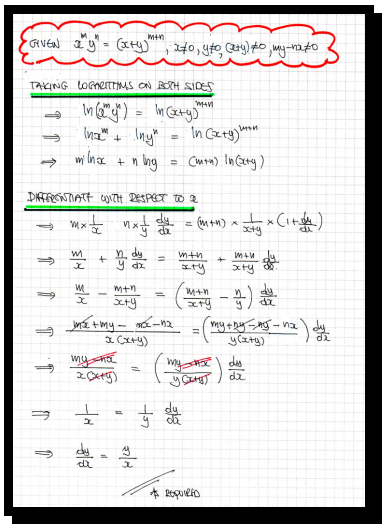
$$x^m y^n = (x+y)^{m+n},$$

where m and n are rational constants, and $x \neq 0$, $y \neq 0$, $x+y \neq 0$, $my - nx \neq 0$.

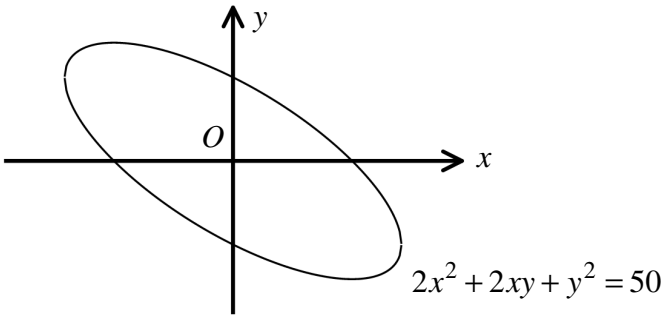
Show that

$$\frac{dy}{dx} = \frac{y}{x}.$$

, proof



Question 84 (*****)



The figure above shows the curve with equation

$$2x^2+2xy+y^2=50\,.$$

Determine the area of the finite region bounded by the x axis and the part of the curve for which $y\geq 0$.

$\boxed{}$, $\boxed{25\pi}$

