IMPLICIT DIFFERENTIATION

BASIC DIFFERENTIATION

Question 1

For each of the following implicit relationships, find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

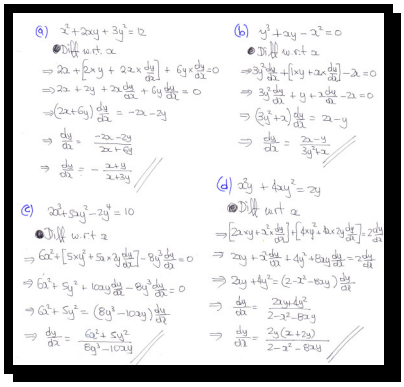
a) $x^2+2xy+3y^2=12$

b) $y^3+xy-x^2=0$

c) $2x^3+5xy^2-2y^4=10$

d) $x^2y+4xy^2=2y$

$$\boxed{\frac{dy}{dx}=-\frac{x+y}{x+3y}},\quad \boxed{\frac{dy}{dx}=\frac{2x-y}{3y^2+x}},\quad \boxed{\frac{dy}{dx}=\frac{6x^2+5y^2}{8y^3-10xy}},\quad \boxed{\frac{dy}{dx}=\frac{2y(x+2y)}{2-x^2-8xy}}$$



Question 2

For each of the following implicit relationships, find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

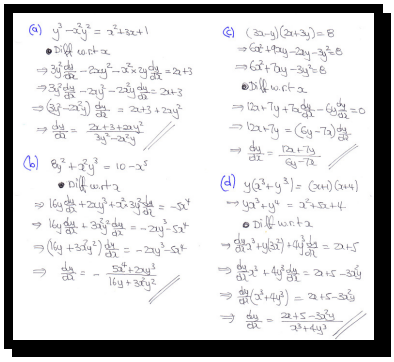
a) $y^3 - x^2y^2 = x^2 + 3x + 1$

b) $8y^2 + x^2y^3 = 10 - x^5$

c) $(3x - y)(2x + 3y) = 8$

d) $y\left(x^3 + y^3\right) = (x+1)(x+4)$

$$\boxed{\frac{dy}{dx} = \frac{2xy^2 + 2x + 3}{3y^2 - 2yx^2}}, \quad \boxed{\frac{dy}{dx} = -\frac{2xy^3 + 5x^4}{16y + 3x^2y^2}}, \quad \boxed{\frac{dy}{dx} = \frac{12x + 7y}{6y - 7x}}, \quad \boxed{\frac{dy}{dx} = \frac{2x + 5 - 3yx^2}{4y^3 + x^3}}$$



Question 3

For each of the following implicit relationships, find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

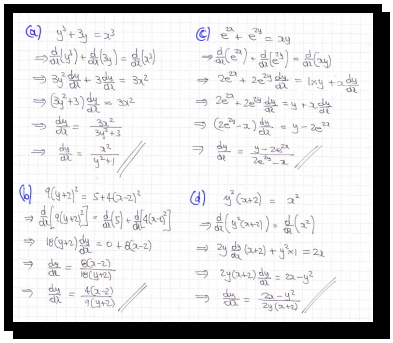
a) $y^3+3y=x^3$

b) $9(y+2)^2=5+4(x-2)^2$

c) $\mathrm{e}^{2x}+\mathrm{e}^{2y}=xy$

d) $y^2(x+2)=x^2$

$$\boxed{\frac{dy}{dx}=\frac{x^2}{y^2+1}},\quad \boxed{\frac{dy}{dx}=\frac{4(x-2)}{9(y+2)}},\quad \boxed{\frac{dy}{dx}=\frac{y-2\mathrm{e}^{2x}}{2\mathrm{e}^{2y}-x}},\quad \boxed{\frac{dy}{dx}=\frac{2x-y^2}{2xy+4y}}$$



Question 4

For each of the following implicit relationships, find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

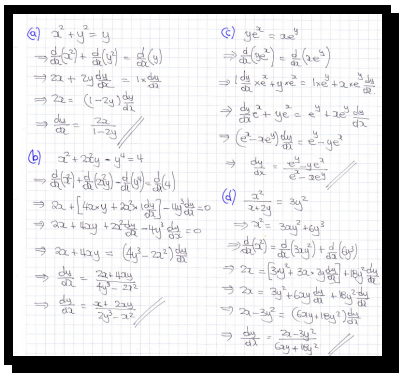
a) $x^2 + y^2 = y$

b) $x^2 + 2x^2 y - y^4 = 4$

c) $y\mathrm{e}^x = x\mathrm{e}^y$

d) $\dfrac{x^2}{x+2y} = 3y^2$

$$\boxed{\frac{dy}{dx} = \frac{2x}{1-2y}}, \quad \boxed{\frac{dy}{dx} = \frac{x+2xy}{2y^3 - x^2}}, \quad \boxed{\frac{dy}{dx} = \frac{\mathrm{e}^y - y\mathrm{e}^x}{\mathrm{e}^x - x\mathrm{e}^y}}, \quad \boxed{\frac{dy}{dx} = \frac{2x - 3y^2}{6y(x+3y)}}$$



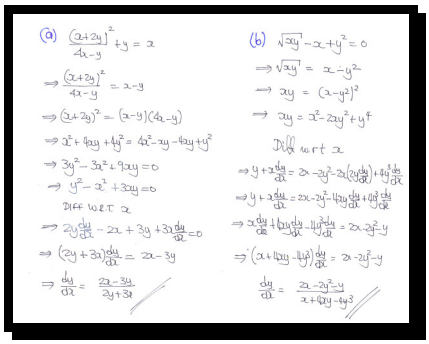
Question 5

For each of the following implicit relationships, find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

a) $\dfrac{(x+2y)^2}{4x-y}+y=x$

b) $\sqrt{xy}-x+y^2=0$

$$\boxed{\frac{dy}{dx}=\frac{2x-3y}{2y+3x}}\,,\qquad \boxed{\frac{dy}{dx}=\frac{2x-2y^2-y}{x+4xy-4y^3}}$$



(a) $\dfrac{(x+2y)^2}{4x-y}+y=x$

$\Rightarrow \dfrac{(x+2y)^2}{4x-y}=x-y$

$\Rightarrow (x+2y)^2=(x-y)(4x-y)$

$\Rightarrow x^2+4xy+4y^2=4x^2-xy-4xy+y^2$

$\Rightarrow 3y^2-3x^2+9xy=0$

$\Rightarrow y^2-x^2+3xy=0$

Diff wrt x

$\Rightarrow 2y\dfrac{dy}{dx}-2x+3y+3x\dfrac{dy}{dx}=0$

$\Rightarrow (2y+3x)\dfrac{dy}{dx}=2x-3y$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{2x-3y}{2y+3x}$

(b) $\sqrt{xy}-x+y^2=0$

$\Rightarrow \sqrt{xy}=x-y^2$

$\Rightarrow xy=(x-y^2)^2$

$\Rightarrow xy=x^2-2xy^2+y^4$

Diff wrt x

$\Rightarrow y+x\dfrac{dy}{dx}=2x-2y^2-2x\left(2y\dfrac{dy}{dx}\right)+4y^3\dfrac{dy}{dx}$

$\Rightarrow y+x\dfrac{dy}{dx}=2x-2y^2-4xy\dfrac{dy}{dx}+4y^3\dfrac{dy}{dx}$

$\Rightarrow x\dfrac{dy}{dx}+4xy\dfrac{dy}{dx}-4y^3\dfrac{dy}{dx}=2x-2y^2-y$

$\Rightarrow (x+4xy-4y^3)\dfrac{dy}{dx}=2x-2y^2-y$

$\dfrac{dy}{dx}=\dfrac{2x-2y^2-y}{x+4xy-4y^3}$

TANGENTS AND NORMALS

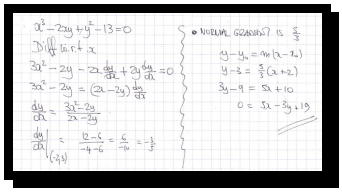
Question 1

A curve C has equation

$$x^3 - 2xy + y^2 - 13 = 0 .$$

Find an equation for the normal to C at the point $P(-2,3)$.

$$\boxed{5x - 3y + 19 = 0}$$



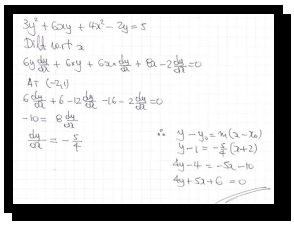
Question 2

A curve is given implicitly by the equation

$$3y^2 + 6xy + 4x^2 - 2y = 5 .$$

Find an equation for the tangent to the curve at the point $P(-2,1)$.

$$\boxed{4y + 5x + 6 = 0}$$



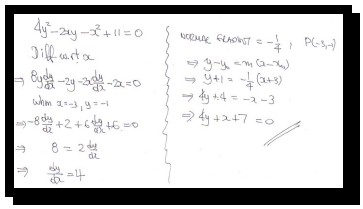
Question 3

A curve has implicit equation

$$4y^2 - 2xy - x^2 + 11 = 0 .$$

Find an equation of the normal to the curve at the point $P(-3,-1)$.

$$\boxed{4y + x + 7 = 0}$$



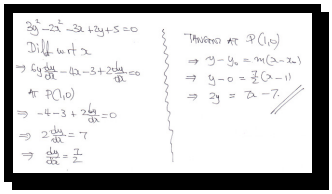
Question 4

A curve is given implicitly by the equation

$$3y^2 - 2x^2 - 3x + 2y + 5 = 0 .$$

Find an equation for the tangent to the curve at the point $P(1,0)$.

$$\boxed{2y = 7x - 7}$$



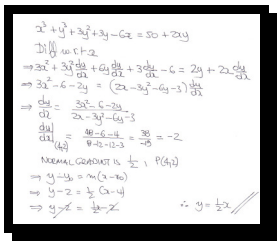
Question 5

A curve has implicit equation

$$x^3 + y^3 + 3y^2 + 3y - 6x = 50 + 2xy\,.$$

Find an equation of the normal to the curve at the point $P(4,2)$.

$\boxed{x = 2y}$



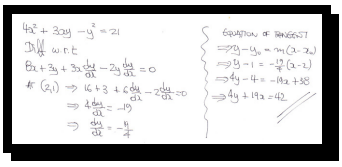
Question 6

A curve is described by the implicit relationship

$$4x^2 + 3xy - y^2 = 21\,.$$

Find an equation of the tangent to the curve at the point $(2,1)$.

$\boxed{4y + 19x = 42}$



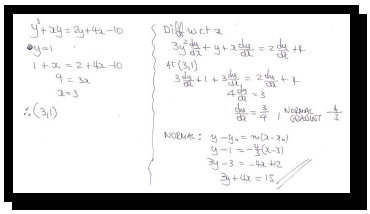
Question 7

A curve is described by the implicit relationship

$$y^3 + xy = 2y + 4x - 10 .$$

Find an equation of the normal to the curve at the point where $y = 1$.

$\boxed{3y + 4x = 15}$



Question 8

A curve has equation

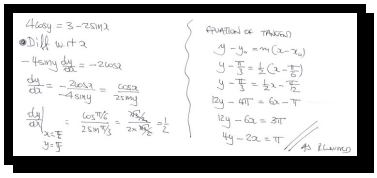
$$4\cos y = 3 - 2\sin x , \ x \in \mathbb{R} , \ y \in \mathbb{R} .$$

Show clearly that

$$4y - 2x = \pi$$

is the equation of the tangent to the curve at the point with coordinates $\left(\frac{\pi}{6}, \frac{\pi}{3} \right)$.

$\boxed{\text{proof}}$



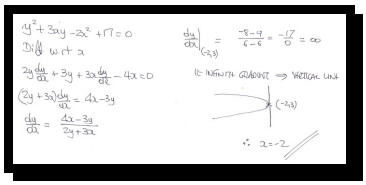
Question 9

A curve has implicit equation

$$y^2+3xy-2x^2+17=0.$$

Find an equation of the tangent to the curve at the point $(-2,3)$.

$\boxed{x=-2}$



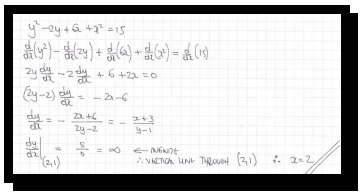
Question 10

A curve is described by the implicit relationship

$$y^2-2y+6x+x^2=15.$$

Find an equation for the tangent to the curve at the point $P(2,1)$.

$\boxed{x=2}$



TURNING POINTS

Question 1

A curve has implicit equation

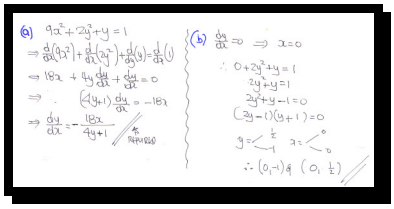
$$9x^2 + 2y^2 + y = 1 .$$

a) Show clearly that

$$\frac{dy}{dx} = -\frac{18x}{4y+1} .$$

b) Hence find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

$$\boxed{(0,-1),\left(0,\frac{1}{2}\right)}$$



Question 2

A curve has equation

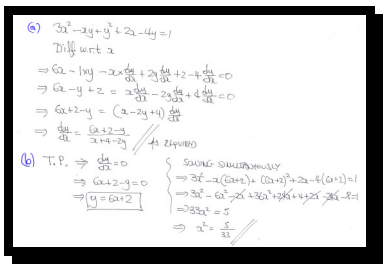
$$3x^2 - xy + y^2 + 2x - 4y = 1.$$

a) Show that

$$\frac{dy}{dx} = \frac{2 + 6x - y}{4 - 2y + x}.$$

b) Hence show that at the turning points of the curve, $x^2 = \frac{5}{33}$.

proof



Question 3

A curve has equation

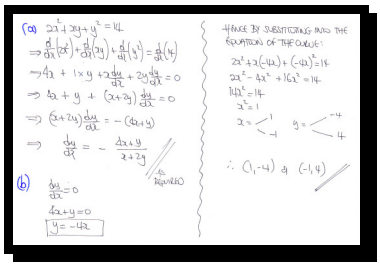
$$2x^2 + xy + y^2 = 14 .$$

a) Show clearly that

$$\frac{dy}{dx} = -\frac{4x + y}{x + 2y} .$$

b) Hence, find the coordinates of the turning points of the curve.

$$\boxed{(1,-4),(-1,4)}$$



Question 4

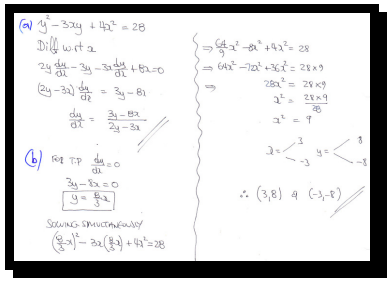
A curve C is defined implicitly by

$$y^2 - 3xy + 4x^2 = 28, \quad x, y \in \mathbb{R}.$$

a) Find an expression for $\dfrac{dy}{dx}$, in terms of x and y.

b) Determine the coordinates of the turning points of C.

$$\boxed{\frac{dy}{dx} = \frac{3y - 8x}{2y - 3x}}, \quad \boxed{(-3,-8),(3,8)}$$



Question 5

The equation of a curve is given by

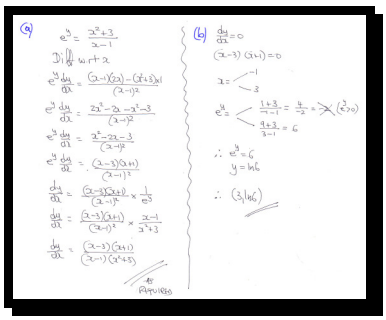
$$\mathrm{e}^{y}=\frac{x^2+3}{x-1},\ x>1.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{(x-3)(x+1)}{(x^2+3)(x-1)}.$$

b) Find the exact coordinates of the turning point of the curve.

$\boxed{(3,\ln 6)}$



Question 6

The equation of a curve is given by

$$x^2-2y^2-xy-x+5y+34=0.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{2x-y-1}{x+4y-5}.$$

b) Find the exact value of gradient at the point on the curve with coordinates

$$\left(1+4\sqrt{2},-5-\sqrt{2}\right).$$

c) Determine the coordinates of the turning point of the curve.

$$\boxed{-\frac{1}{8}\left(2+3\sqrt{2}\right)},\ \boxed{(3,5),(-1,-3)}$$

