FUNCTION PRACTICE

FUNCTION INTRODUCTION

Question 1

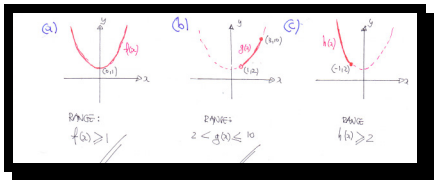
Find the range for each of the following functions.

a) $f(x)=x^2+1,\ \ x\in\mathbb{R}$.

b) $g(x)=x^2+1,\ \ x\in\mathbb{R},\ 1<x\leq 3$.

c) $h(x)=x^2+1,\ \ x\in\mathbb{R},\ x\leq -1$.

$$\boxed{f(x)\in\mathbb{R},\ f(x)\geq 1},\ \ \boxed{g(x)\in\mathbb{R},\ 2<g(x)\leq 10},\ \ \boxed{h(x)\in\mathbb{R},\ h(x)\geq 2}$$



Question 2

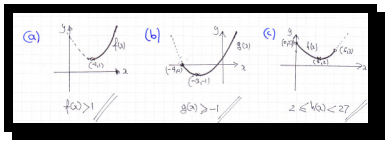
Find the range for each of the following functions.

a) $f(x)=(x-4)^2+1,\ \ x\in\mathbb{R},\ x>4$.

b) $g(x)=(x+3)^2-1,\ \ x\in\mathbb{R},\ x\geq -4$.

c) $h(x)=(x-5)^2+2,\ \ x\in\mathbb{R},\ 0<x<6$.

$$\boxed{f(x)\in\mathbb{R},\ f(x)>1},\ \ \boxed{g(x)\in\mathbb{R},\ g(x)\geq -1},\ \ \boxed{h(x)\in\mathbb{R},\ 2\leq h(x)<27}$$



Question 3

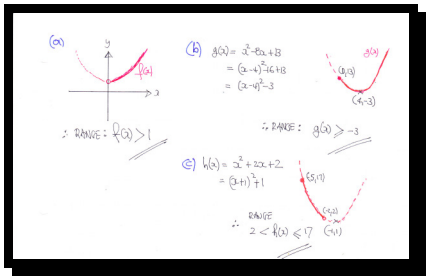
Find the range for each of the following functions.

a) $f(x)=x^2+1,\quad x\in\mathbb{R},\ x>0$.

b) $g(x)=x^2-8x+13,\quad x\in\mathbb{R},\ x\geq 0$.

c) $h(x)=x^2+2x+2,\quad x\in\mathbb{R},\ -5\leq x<-2$.

$\boxed{f(x)\in\mathbb{R},\ f(x)>1}$, $\boxed{g(x)\in\mathbb{R},\ g(x)\geq -3}$, $\boxed{h(x)\in\mathbb{R},\quad 2<h(x)\leq 17}$



Question 4

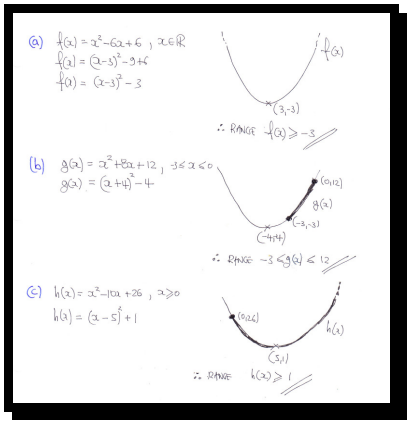
Find the range for each of the following functions.

a) $f(x)=x^2-6x+6,\quad x\in\mathbb{R}$.

b) $g(x)=x^2+8x+12,\quad x\in\mathbb{R},\ -3\le x\le 0$.

c) $h(x)=x^2-10x+26,\quad x\in\mathbb{R},\ x\ge 0$.

$\boxed{f(x)\in\mathbb{R},\ f(x)\ge -3}$, $\boxed{g(x)\in\mathbb{R},\ -3\le g(x)\le 12}$, $\boxed{h(x)\in\mathbb{R},\ h(x)\ge 1}$



Question 5

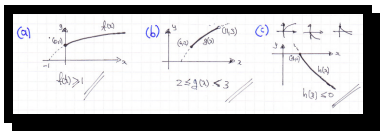
Find the range for each of the following functions.

a) $f(x)=\sqrt{x+1},\ \ x\in\mathbb{R},\ x\geq 0$.

b) $g(x)=\sqrt{x-2},\ \ x\in\mathbb{R},\ 6\leq x<11$.

c) $h(x)=2-\sqrt{x},\ \ x\in\mathbb{R},\ x\geq 4$.

$$\boxed{f(x)\in\mathbb{R},\ f(x)\geq 1},\ \boxed{g(x)\in\mathbb{R},\ 2\leq g(x)<3},\ \boxed{h(x)\in\mathbb{R},\ h(x)\leq 0}$$



Question 6

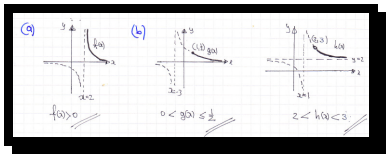
Find the range for each of the following functions.

a) $f(x)=\dfrac{1}{x-2},\ \ x\in\mathbb{R},\ x>2$.

b) $g(x)=\dfrac{2}{x+3},\ \ x\in\mathbb{R},\ x\geq 1$.

c) $h(x)=\dfrac{1}{x-1}+2,\ \ x\in\mathbb{R},\ x>2$.

$$\boxed{f(x)\in\mathbb{R},\ f(x)>0},\ \boxed{g(x)\in\mathbb{R},\ 0<g(x)\leq\frac{1}{2}},\ \boxed{h(x)\in\mathbb{R},\ 2<h(x)<3}$$



Question 7

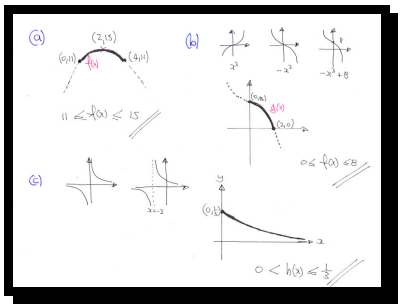
Find the range for each of the following functions.

a) $f(x)=15-(x-2)^2,\ \ x\in\mathbb{R},\ 0\le x\le 4$.

b) $g(x)=8-x^3,\ \ x\in\mathbb{R},\ 0\le x\le 2$.

c) $h(x)=\dfrac{1}{x+3},\ \ x\in\mathbb{R},\ x\ge 0$.

$\boxed{f(x)\in\mathbb{R},\ 11\le f(x)\le 15}$, $\boxed{g(x)\in\mathbb{R},\ 0\le g(x)\le 8}$, $\boxed{h(x)\in\mathbb{R},\ 0<h(x)\le\frac{1}{3}}$



Question 8

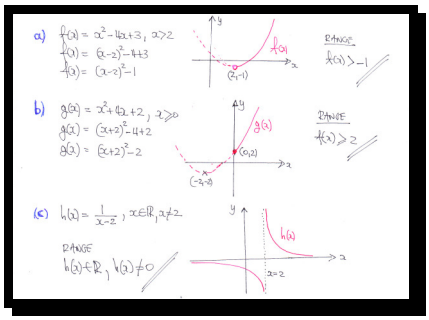
Find the range for each of the following functions.

a) $f(x)=x^2-4x+3,\quad x\in\mathbb{R},\ x>2$.

b) $g(x)=x^2+4x+2,\quad x\in\mathbb{R},\ x\geq 0$.

c) $h(x)=\dfrac{1}{x-2},\quad x\in\mathbb{R},\ x\neq 2$.

$\boxed{f(x)\in\mathbb{R},\ f(x)>-1}$, $\boxed{g(x)\in\mathbb{R},\ g(x)\geq 2}$, $\boxed{h(x)\in\mathbb{R},\ h(x)\neq 0}$



Question 9

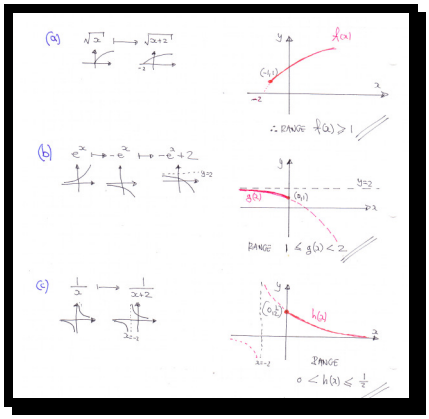
Find the range for each of the following functions.

a) $f(x)=\sqrt{x+2},\ \ x\in\mathbb{R},\ x\geq -1$.

b) $g(x)=2-\mathrm{e}^{x},\ \ x\in\mathbb{R},\ x\leq 0$.

c) $h(x)=\dfrac{1}{x+2},\ \ x\in\mathbb{R},\ x\geq 0$.

$$\boxed{f(x)\in\mathbb{R},\ f(x)\geq 1},\quad \boxed{g(x)\in\mathbb{R},\ 1\leq g(x)<2},\quad \boxed{h(x)\in\mathbb{R},\ 0<h(x)\leq\frac{1}{2}}$$



Question 10

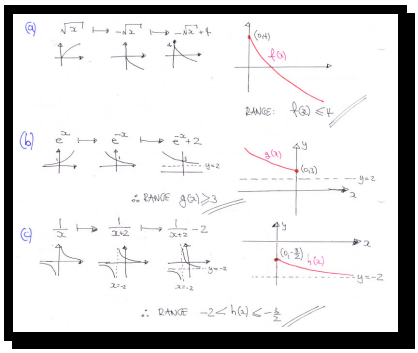
Find the range for each of the following functions.

a) $f(x)=4-\sqrt{x},\ \ x\in\mathbb{R},\ x\geq 0$.

b) $g(x)=2+\mathrm{e}^{-x},\ \ x\in\mathbb{R},\ x\leq 0$.

c) $h(x)=\dfrac{1}{x+2}-2,\ \ x\in\mathbb{R},\ x\geq 0$.

$\boxed{f(x)\in\mathbb{R},\ f(x)\leq 4}$, $\boxed{g(x)\in\mathbb{R},\ \ g(x)\geq 3}$, $\boxed{h(x)\in\mathbb{R},\ -2<h(x)\leq -\frac{3}{2}}$



Question 11

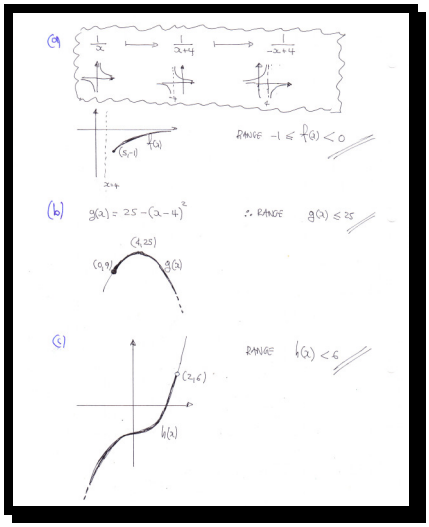
Find the range for each of the following functions.

a) $f(x)=\dfrac{1}{4-x},\ \ x\in\mathbb{R},\ x\geq 5$.

b) $g(x)=25-(x-4)^2,\ \ x\in\mathbb{R},\ \ x\geq 0$.

c) $h(x)=x^3-2,\ \ x\in\mathbb{R},\ x<2$.

$$\boxed{f(x)\in\mathbb{R},\ -1\leq f(x)<0},\quad \boxed{g(x)\in\mathbb{R},\ \ g(x)\leq 25},\quad \boxed{h(x)\in\mathbb{R},\ \ h(x)<6}$$



Question 12

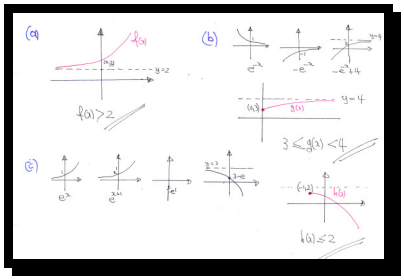
Find the range for each of the following functions.

a) $f(x)=\mathrm{e}^{x}+2,\ \ x\in\mathbb{R}$.

b) $g(x)=4-\mathrm{e}^{-x},\ \ x\in\mathbb{R},\ \ x\geq 0$.

c) $h(x)=3-\mathrm{e}^{x+1},\ \ x\in\mathbb{R},\ \ x\geq -1$.

$\boxed{f(x)\in\mathbb{R},\ f(x)>2}$, $\boxed{g(x)\in\mathbb{R},\ \ g(x)\leq 25}$, $\boxed{h(x)\in\mathbb{R},\ \ h(x)\leq 2}$



Question 13

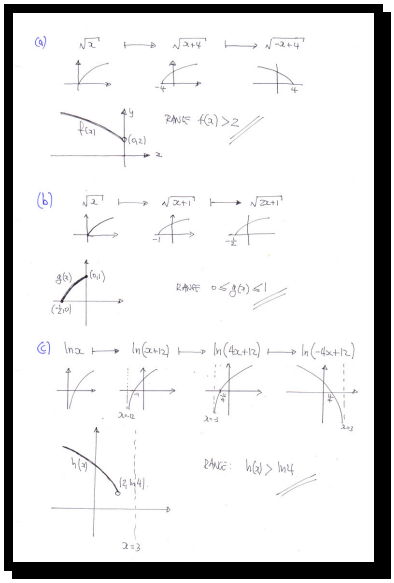
Find the range for each of the following functions.

a) $f(x)=\sqrt{4-x},\ \ x\in\mathbb{R},\ x<0$.

b) $g(x)=\sqrt{2x+1},\ \ x\in\mathbb{R},\ -\frac{1}{2}\leq x\leq 0$.

c) $h(x)=\ln(12-4x),\ \ x\in\mathbb{R},\ x<2$.

$\boxed{f(x)\in\mathbb{R},\ f(x)>2}$, $\boxed{g(x)\in\mathbb{R},\ 0\leq g(x)\leq 1}$, $\boxed{h(x)\in\mathbb{R},\ \ h(x)>\ln 4}$



FUNCTION COMPOSITIONS

Question 1

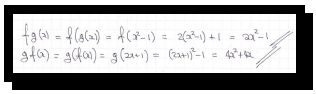
Find $fg(x)$ and $gf(x)$ if

$$f(x)=2x+1, \quad x\in\mathbb{R}$$

$$g(x)=x^2-1, \quad x\in\mathbb{R}.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=2x^2-1}, \quad \boxed{gf(x)=4x^2+4x}$$



Question 2

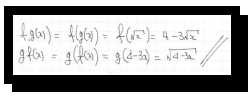
Find $fg(x)$ and $gf(x)$ if

$$f(x)=4-3x, \quad x\in\mathbb{R}$$

$$g(x)=\sqrt{x}, \quad x\in\mathbb{R}, \quad x\geq 0.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=4-3\sqrt{x}}, \quad \boxed{gf(x)=\sqrt{4-3x}}$$



Question 3

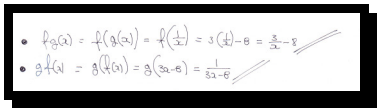
Find $fg(x)$ and $gf(x)$ if

$$f(x)=3x-8,\quad x\in\mathbb{R}$$

$$g(x)=\frac{1}{x},\quad x\in\mathbb{R},\quad x\neq 0.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=\frac{3}{x}-8},\quad \boxed{gf(x)=\frac{1}{3x-8}}$$



Question 4

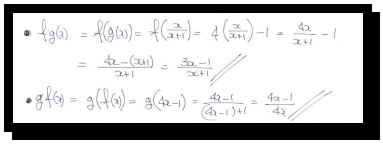
Find $fg(x)$ and $gf(x)$ if

$$f(x)=4x-1,\quad x\in\mathbb{R}$$

$$g(x)=\frac{x}{x+1},\quad x\in\mathbb{R},\quad x\neq -1.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=\frac{3x-1}{x+1}},\quad \boxed{gf(x)=\frac{4x-1}{4x}}$$



Question 5

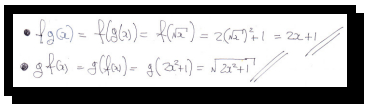
Find $fg(x)$ and $gf(x)$ if

$$f(x)=2x^2+1,\quad x\in\mathbb{R}$$

$$g(x)=\sqrt{x},\quad x\in\mathbb{R},\quad x\geq 0\,.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=2x+1},\quad \boxed{gf(x)=\sqrt{2x^2+1}}$$



Question 6

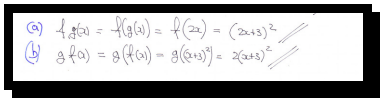
Find $fg(x)$ and $gf(x)$ if

$$f(x)=(x+3)^2,\quad x\in\mathbb{R}$$

$$g(x)=2x,\quad x\in\mathbb{R}\,.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=(2x+3)^2},\quad \boxed{gf(x)=2(x+3)^2}$$



Question 7

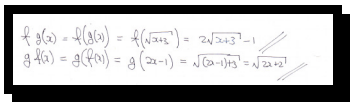
Find $fg(x)$ and $gf(x)$ if

$$f(x)=2x-1, \quad x\in\mathbb{R}$$

$$g(x)=\sqrt{x+3}, \quad x\in\mathbb{R}, \quad x\geq -3.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=2\sqrt{x+3}-1}, \quad \boxed{gf(x)=\sqrt{2x+2}}$$



Question 8

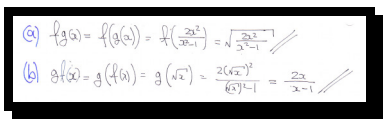
Find $fg(x)$ and $gf(x)$ if

$$f(x)=\sqrt{x}, \quad x\in\mathbb{R}, \quad x\geq 0$$

$$g(x)=\frac{2x^2}{x^2-1}, \quad x\in\mathbb{R}, \ x\neq\pm 1.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=\sqrt{\frac{2x^2}{x^2-1}}}, \quad \boxed{gf(x)=\frac{2x}{x-1}}$$



Question 9

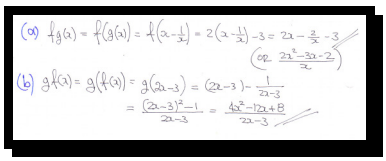
Find $fg(x)$ and $gf(x)$ if

$$f(x)=2x-3,\quad x\in\mathbb{R}$$

$$g(x)=x-\frac{1}{x},\quad x\in\mathbb{R},\ x\neq 0.$$

Simplify the answer as much as possible.

$$\boxed{fg(x)=\frac{2x^2-3x-2}{x}},\quad \boxed{gf(x)=\frac{4x^2-12x+8}{2x-3}}$$



Question 10

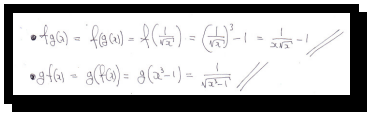
Find $fg(x)$ and $gf(x)$ if

$$f(x)=x^3-1,\quad x\in\mathbb{R}$$

$$g(x)=\frac{1}{\sqrt{x}},\quad x\in\mathbb{R},\ x>0.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=\frac{1}{x\sqrt{x}}-1},\quad \boxed{gf(x)=\frac{1}{\sqrt{x^3-1}}}$$



Question 11

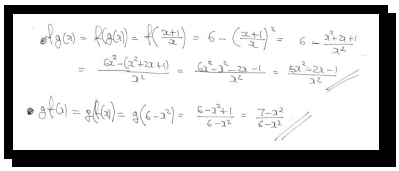
Find $fg(x)$ and $gf(x)$ if

$$f(x)=6-x^2,\ \ x\in\mathbb{R}$$

$$g(x)=\frac{x+1}{x},\quad x\in\mathbb{R},\ x\neq 0.$$

Simplify the answers as much as possible.

$$\boxed{fg(x)=\frac{5x^2-2x-1}{x^2}},\quad \boxed{gf(x)=\frac{7-x^2}{6-x^2}}$$



Question 12

The following functions are defined by

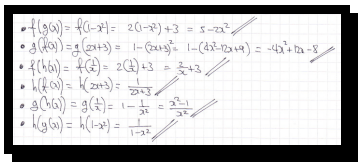
$$f(x)=2x+3,\quad x\in\mathbb{R}.$$

$$g(x)=1-x^2,\quad x\in\mathbb{R}.$$

$$h(x)=\frac{1}{x},\quad x\in\mathbb{R},\ x\neq 0.$$

Find all six possible two-fold compositions for the above functions, simplifying the final answers as much as possible.

$$\boxed{fg(x)=5-2x^2},\quad \boxed{gf(x)=-4x^2-12x-8},\quad \boxed{fh(x)=\frac{2}{x}+3},\quad \boxed{hf(x)=\frac{1}{2x+3}},\quad \boxed{gh(x)=1-\frac{1}{x^2}},\quad \boxed{hg(x)=\frac{1}{1-x^2}}$$



Question 13

The following functions are defined by

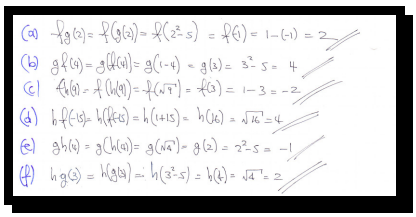
$$f(x)=2x+1,\quad x\in\mathbb{R}.$$

$$g(x)=\mathrm{e}^{x},\quad x\in\mathbb{R}.$$

$$h(x)=\sin x,\quad x\in\mathbb{R}.$$

Find all six possible two-fold compositions for the above functions simplifying the final answers as much as possible.

$$\boxed{\begin{aligned} fg(x)&=2\mathrm{e}^{x}+1\\ gf(x)&=\mathrm{e}^{2x+1}\end{aligned}},\quad \boxed{\begin{aligned} fh(x)&=2\sin x+1\\ hf(x)&=\sin(2x+1)\end{aligned}},\quad \boxed{\begin{aligned} gh(x)&=\mathrm{e}^{\sin x}\\ hg(x)&=\sin\left(\mathrm{e}^{x}\right)\end{aligned}}$$



Question 14

The following functions are defined by

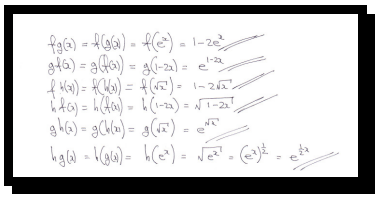
$$f(x)=1-2x,\quad x\in\mathbb{R}\ .$$

$$g(x)=\mathrm{e}^{x},\quad x\in\mathbb{R}\,.$$

$$h(x)=\sqrt{x}\ ,\quad x\in\mathbb{R},\ x\geqslant 0\,.$$

Find all six possible two-fold compositions for the above functions simplifying the final answers as much as possible.

$$\boxed{\begin{aligned} fg(x)&=1-2\mathrm{e}^{x}\\ gf(x)&=\mathrm{e}^{1-2x}\end{aligned}},\quad \boxed{\begin{aligned} fh(x)&=1-2\sqrt{x}\\ hf(x)&=\sqrt{1-2x}\end{aligned}},\quad \boxed{\begin{aligned} gh(x)&=\mathrm{e}^{\sqrt{x}}\\ hg(x)&=\mathrm{e}^{\frac{1}{2}x}\end{aligned}}$$



Question 15

The following functions are defined by

$$f(x)=1-x,\quad x\in\mathbb{R}.$$

$$g(x)=x^2-5,\quad x\in\mathbb{R}.$$

$$h(x)=\sqrt{x},\quad x\in\mathbb{R},\ x\geq 0.$$

Evaluate the following function compositions.

a) $fg(2)$

b) $gf(4)$

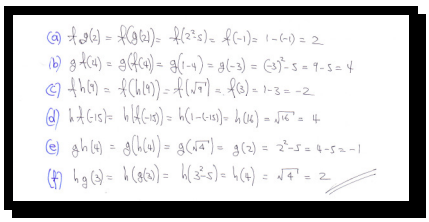
c) $fh(9)$

d) $hf(-15)$

e) $gh(4)$

f) $hg(3)$

$$\boxed{fg(2)=2},\ \boxed{gf(4)=4},\ \boxed{fh(9)=-2},\ \boxed{hf(-15)=4},\ \boxed{gh(4)=-1},\ \boxed{hg(3)=2}$$



Question 16

The following functions are defined by

$$f(x)=2x+5,\quad x\in\mathbb{R}$$

$$g(x)=\frac{4}{x},\quad x\in\mathbb{R},\ x\neq 0$$

$$h(x)=\sqrt{x+2},\quad x\in\mathbb{R},\ x\geq -2.$$

Evaluate the following function compositions.

a) $fg\left(\frac{1}{2}\right)$

b) $gf(-2)$

c) $hf(1)$

d) $fh(2)$

e) $hg\left(\frac{2}{7}\right)$

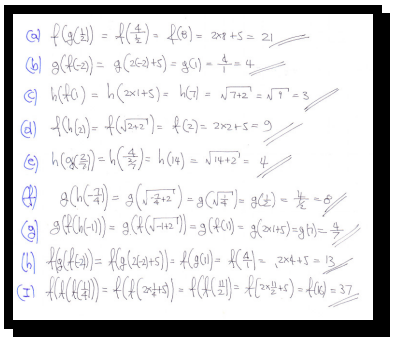
f) $gh\left(-\frac{7}{4}\right)$

g) $gfh(-1)$

h) $fgf(-2)$

i) $fff\left(\frac{1}{4}\right)$

$$\boxed{fg\left(\tfrac{1}{2}\right)=21},\ \boxed{gf(-2)=4},\ \boxed{hf(1)=3},\ \boxed{fh(2)=9},\ \boxed{hg\left(\tfrac{2}{7}\right)=4},\ \boxed{gh\left(-\tfrac{7}{4}\right)=8},$$
$$\boxed{gfh(1)=\tfrac{4}{7}},\ \boxed{fgf(-2)=13},\ \boxed{fff\left(\tfrac{1}{4}\right)=37}$$



Question 17

The following functions are defined by

$$f(x) = x - 2, \quad x \in \mathbb{R}$$

$$g(x) = \ln x, \quad x \in \mathbb{R}, \ x > 0$$

$$h(x) = \mathrm{e}^{2x}, \quad x \in \mathbb{R}.$$

Find simplified expressions the following function compositions, stating in each case the domain and range.

a) $fg(x)$

b) $gf(x)$

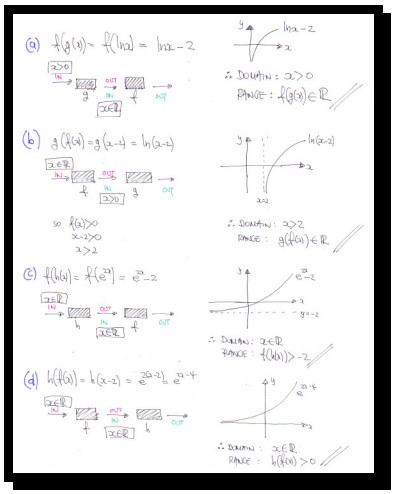
c) $fh(x)$

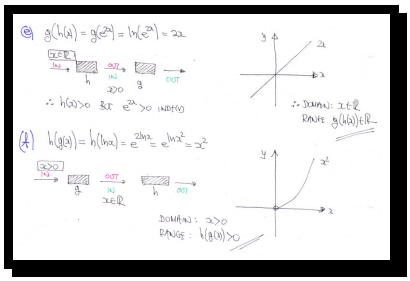
d) $hf(x)$

e) $gh(x)$

f) $hg(x)$

$$\boxed{fg(x) = \ln x - 2, x > 0, fg(x) \in \mathbb{R}}, \boxed{gf(x) = \ln(x-2), \ x > 2, \ gf(x) \in \mathbb{R}},$$
$$\boxed{fh(x) = \mathrm{e}^{2x} - 2, \ x \in \mathbb{R}, \ fh(x) > -2}, \boxed{hf(x) = \mathrm{e}^{2x-4}, \ x \in \mathbb{R}, \ hf(x) \in \mathbb{R}},$$
$$\boxed{gh(x) = 2x, \ x \in \mathbb{R}, \ gh(x) \in \mathbb{R}}, \boxed{hg(x) = x^2, \ x > 0, \ hg(x) > 0}$$





Question 17

The following functions are defined by

$$f(x)=2x-1,\quad x\in\mathbb{R},\quad x\leq 18$$

$$g(x)=x^2+2,\quad x\in\mathbb{R},\quad x\geq 1$$

$$h(x)=\sqrt{x},\quad x\in\mathbb{R},\quad x\geq 0.$$

Find simplified expressions for each of the following function compositions, stating in each case the domain and range.

a) $fg(x)$

b) $gf(x)$

c) $fh(x)$

d) $hf(x)$

e) $gh(x)$

f) $hg(x)$

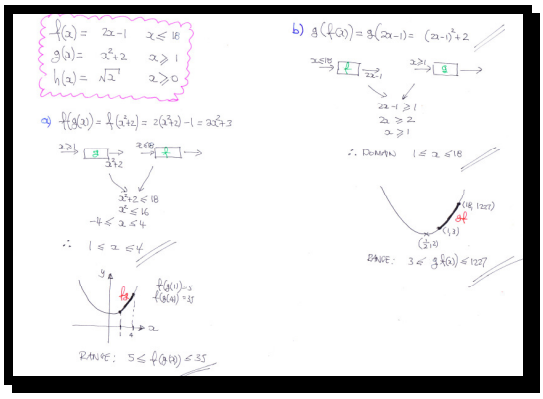
$$\boxed{fg(x)=2x^2+3,\quad 1\leq x\leq 4,\quad 5\leq fg(x)\leq 35},$$

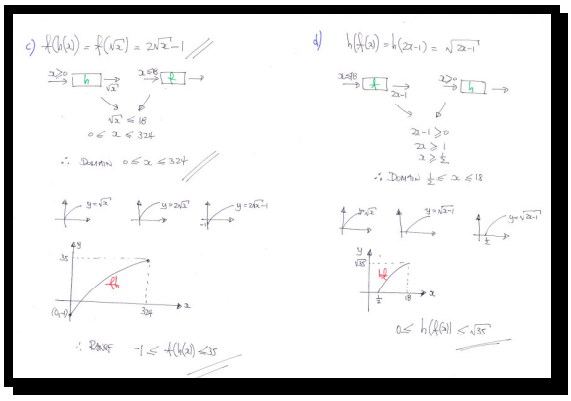
$$\boxed{gf(x)=(2x-1)^2+2,\quad 1\leq x\leq 18,\quad 3\leq gf(x)\leq 1227},$$

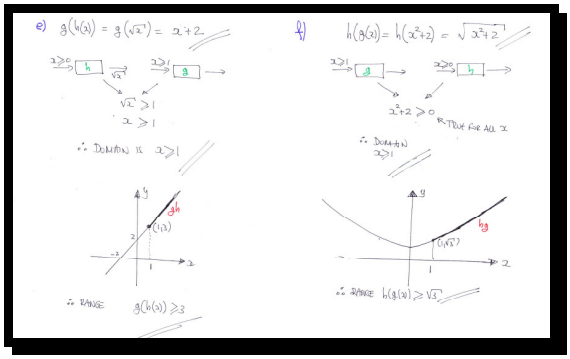
$$\boxed{fh(x)=2\sqrt{x}-1,\quad 0\leq x\leq 324,\quad -1\leq fh(x)\leq 35},$$

$$\boxed{hf(x)=\sqrt{2x-1},\quad \frac{1}{2}\leq x\leq 18,\quad 0\leq hf(x)\leq\sqrt{35}},\ \boxed{gh(x)=x+2,\quad x\geq 1,\quad gh(x)\geq 3},$$

$$\boxed{hg(x)=\sqrt{x^2+2},\quad x\geq 1,\quad hg(x)\geq\sqrt{3}}$$







FUNCTION INVERSES

Question 1

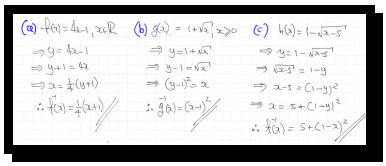
For each of the following functions find an expression for its inverse.

a) $f(x)=4x-1, \quad x\in\mathbb{R}$.

b) $g(x)=1+\sqrt{x}, \quad x\in\mathbb{R},\ x\geqslant 0$.

c) $h(x)=1-\sqrt{x-5}, \quad x\in\mathbb{R},\ x\geqslant 5$.

$$\boxed{f^{-1}(x)=\frac{x+1}{4}}, \boxed{g^{-1}(x)=(x-1)^2}, \boxed{h^{-1}(x)=5+(1-x)^2}$$



Question 2

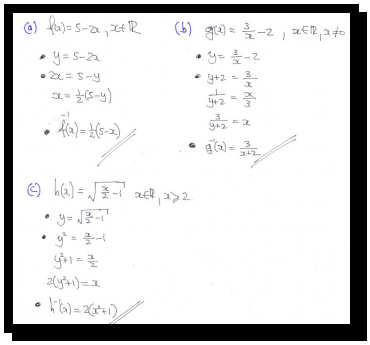
For each of the following functions find an expression for its inverse.

a) $f(x)=5-2x, \quad x\in\mathbb{R}$.

b) $g(x)=\dfrac{3}{x}-2, \quad x\in\mathbb{R},\ x\neq 0$.

c) $h(x)=\sqrt{\dfrac{x}{2}-1}, \quad x\in\mathbb{R},\ x\geq 2$.

$$\boxed{f^{-1}(x)=\frac{5-x}{2}}, \boxed{g^{-1}(x)=\frac{3}{x+2}}, \boxed{h^{-1}(x)=2x^2+2}$$



Question 3

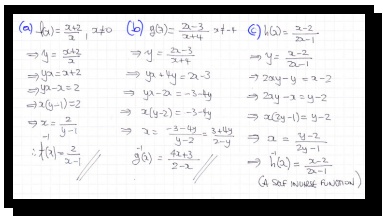
For each of the following functions find an expression for its inverse.

a) $f(x)=\dfrac{x+2}{x},\quad x\in\mathbb{R},\ x\neq 0.$

b) $g(x)=\dfrac{2x-3}{x+4},\quad x\in\mathbb{R},\ x\neq -4.$

c) $h(x)=\dfrac{x-2}{2x-1},\quad x\in\mathbb{R},\ x\neq \dfrac{1}{2}.$

$$\boxed{f^{-1}(x)=\frac{2}{x-1}},\ \boxed{g^{-1}(x)=\frac{4x+3}{2-x}},\ \boxed{h^{-1}(x)=\frac{x-2}{2x-1}}$$



Question 4

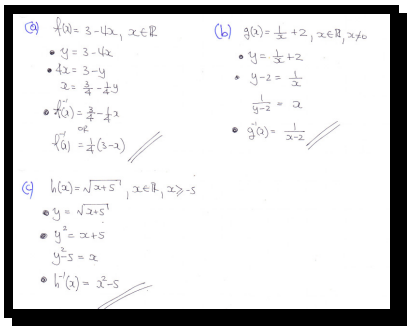
For each of the following functions find an expression for its inverse.

a) $f(x)=3-4x, \quad x\in\mathbb{R}$.

b) $g(x)=\dfrac{1}{x}+2, \quad x\in\mathbb{R},\ x\neq 0$.

c) $h(x)=\sqrt{x+5}, \quad x\in\mathbb{R},\ x\geq -5$.

$$\boxed{f^{-1}(x)=\frac{3-x}{4}}, \quad \boxed{g^{-1}(x)=\frac{1}{x-2}}, \quad \boxed{h^{-1}(x)=x^2-5}$$



Question 5

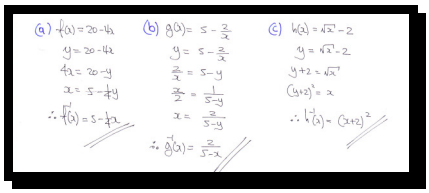
For each of the following functions find an expression for its inverse.

a) $f(x)=20-4x,\ x\in\mathbb{R}$.

b) $g(x)=5-\dfrac{2}{x},\quad x\in\mathbb{R},\ x\neq 0$.

c) $h(x)=\sqrt{x}-2,\quad x\in\mathbb{R},\ x\geq 0$.

$$\boxed{f^{-1}(x)=5-\frac{1}{4}x},\ \boxed{g^{-1}(x)=\frac{2}{5-x}},\ \boxed{h^{-1}(x)=(x+2)^2}$$



Question 6

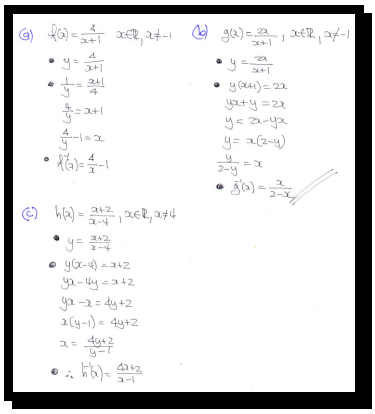
For each of the following functions find an expression for its inverse.

a) $f(x)=\dfrac{4}{x+1},\quad x\in\mathbb{R},\ x\neq -1.$

b) $g(x)=\dfrac{2x}{x+1},\quad x\in\mathbb{R},\ x\neq -1.$

c) $h(x)=\dfrac{x+2}{x-4},\quad x\in\mathbb{R},\ x\neq 4.$

$$\boxed{f^{-1}(x)=\frac{4}{x}-1},\ \boxed{g^{-1}(x)=\frac{x}{2-x}},\ \boxed{h^{-1}(x)=\frac{4x+2}{x-1}}$$



Question 7

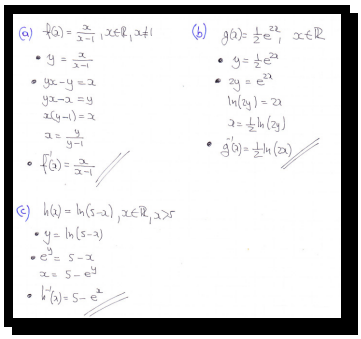
For each of the following functions find an expression for its inverse.

a) $f(x)=\dfrac{x}{x-1}, \quad x\in\mathbb{R},\ x\neq 1.$

b) $g(x)=\dfrac{1}{2}\mathrm{e}^{2x}, \quad x\in\mathbb{R}.$

c) $h(x)=\ln(5-x), \quad x\in\mathbb{R},\ x>5.$

$$\boxed{f^{-1}(x)=\frac{x}{x-1}}, \quad \boxed{g^{-1}(x)=\frac{1}{2}\ln 2x}, \quad \boxed{h^{-1}(x)=5-\mathrm{e}^{x}}$$



Question 8

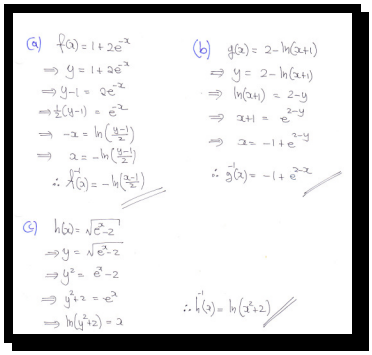
For each of the following functions find an expression for its inverse.

a) $f(x)=1+2\mathrm{e}^{-x},\quad x\in\mathbb{R}$.

b) $g(x)=2-\ln(x+1),\quad x\in\mathbb{R},\ x>-1$.

c) $h(x)=\sqrt{\mathrm{e}^{x}-2},\quad x\in\mathbb{R},\ x\geq\ln 2$.

$$\boxed{f^{-1}(x)=-\ln\left(\frac{x-1}{2}\right)},\ \boxed{g^{-1}(x)=\mathrm{e}^{2-x}-1},\ \boxed{h^{-1}(x)=\ln\left(x^{2}+2\right)}$$



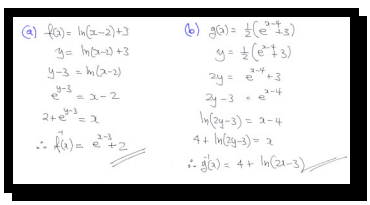
Question 9

For each of the following functions find an expression for its inverse.

a) $f(x)=\ln(x-2)+3,\quad x\in\mathbb{R},\ x>2$.

b) $g(x)=\frac{1}{2}\left(\mathrm{e}^{x-4}+3\right),\quad x\in\mathbb{R}$.

$$\boxed{f^{-1}(x)=\mathrm{e}^{x-3}+2},\ \boxed{g^{-1}(x)=4+\ln(2x-3)}$$



Question 10

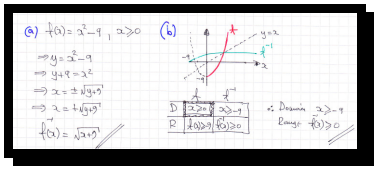
A function f is defined by

$$f(x)=x^2-9,\quad x\in\mathbb{R},\ x\geq 0.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\sqrt{x+9}},\ \boxed{x\geq -9},\ \boxed{f^{-1}(x)\geq 0}$$



Question 11

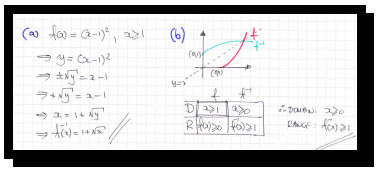
A function f is defined by

$$f(x)=(x-1)^2,\quad x\in\mathbb{R},\ x\geq 1.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=1+\sqrt{x}},\ \boxed{x\geq 0},\ \boxed{f^{-1}(x)\geq 1}$$



Question 12

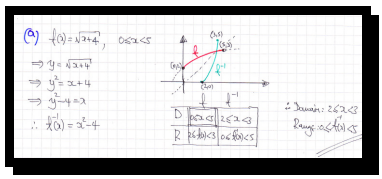
A function f is defined by

$$f(x)=\sqrt{x+4},\quad x\in\mathbb{R},\ 0\leq x<5.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=x^2-4},\ \boxed{2\leq x<3},\ \boxed{0\leq f^{-1}(x)<5}$$



Question 13

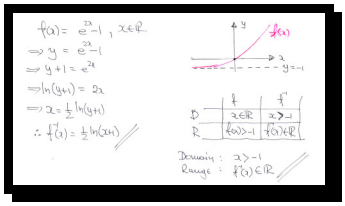
A function f is defined by

$$f(x)=\mathrm{e}^{2x}-1,\quad x\in\mathbb{R}.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\tfrac{1}{2}\ln(x+1)},\ \boxed{x>-1},\ \boxed{f^{-1}(x)\in\mathbb{R}}$$



Question 14

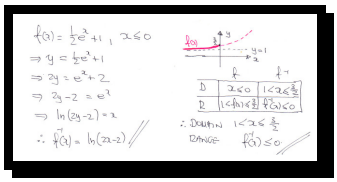
A function f is defined by

$$f(x)=\frac{1}{2}e^{x}+1,\quad x\in\mathbb{R},\ x\leq 0.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\ln(2x-2)},\ \boxed{1<x\leq\frac{3}{2}},\ \boxed{f^{-1}(x)\leq 0}$$



Question 15

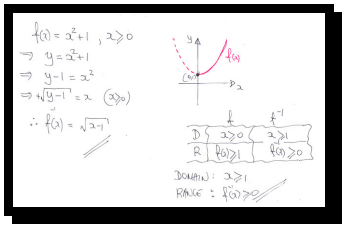
A function f is defined by

$$f(x)=x^{2}+1,\quad x\in\mathbb{R},\ x\geq 0.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\sqrt{x-1}},\ \boxed{x\geq 1},\ \boxed{f^{-1}(x)\geq 0}$$



Question 16

A function f is defined by

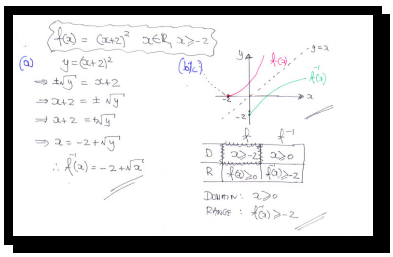
$$f(x)=(x+2)^2,\quad x\in\mathbb{R},\ x\geq -2.$$

a) Find an expression for $f^{-1}(x)$.

b) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x)$.

c) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=-2+\sqrt{x}},\ \boxed{x\geq 0},\ \boxed{f^{-1}(x)\geq -2}$$



Question 17

A function f is defined by

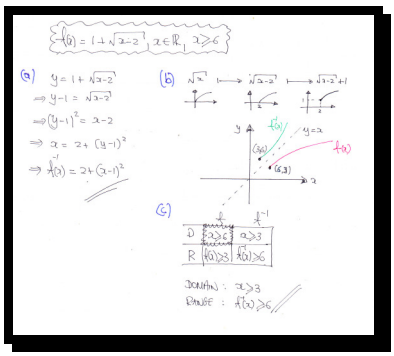
$$f(x)=1+\sqrt{x-2},\quad x\in\mathbb{R},\ x\geq 6.$$

a) Find an expression for $f^{-1}(x)$.

b) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x)$.

c) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=2+(x-1)^2},\ \boxed{x\geq 3},\ \boxed{f^{-1}(x)\geq 0}$$



Question 18

A function f is defined by

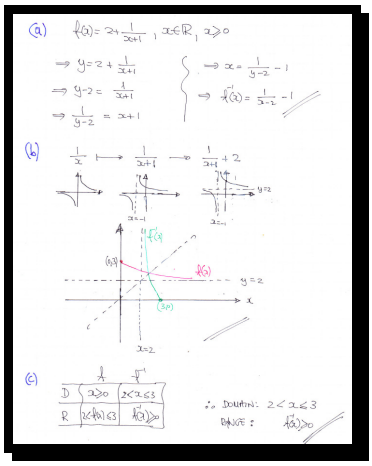
$$f(x)=2+\frac{1}{x+1},\quad x\in\mathbb{R},\ x\geq 0.$$

a) Find an expression for $f^{-1}(x)$.

b) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x)$.

c) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\frac{3-x}{x-2}},\ \boxed{2<x\leq 3},\ \boxed{f^{-1}(x)\geq 0}$$



Question 19

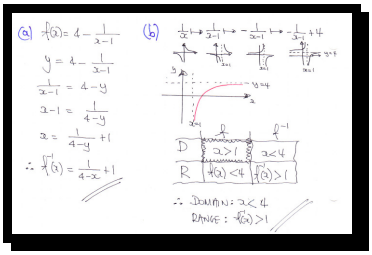
A function f is defined by

$$f(x)=4-\frac{1}{x-1},\quad x\in\mathbb{R},\ x>1.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\frac{x-5}{x-4}},\ \boxed{x<4},\ \boxed{f^{-1}(x)>1}$$



Question 20

A function f is defined by

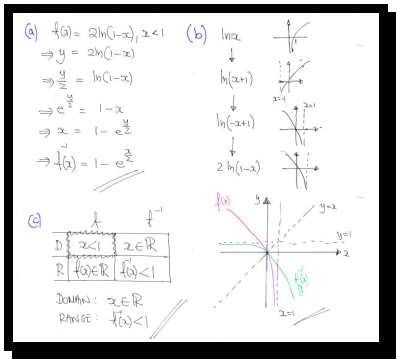
$$f(x)=2\ln(1-x),\quad x<1.$$

a) Find an expression for $f^{-1}(x)$.

b) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x)$.

c) Find the domain and range of $f^{-1}(x)$.

$\boxed{f^{-1}(x)=1-\mathrm{e}^{\frac{1}{2}x}}$, $\boxed{x\in\mathbb{R}}$, $\boxed{f^{-1}(x)<1}$



Question 21

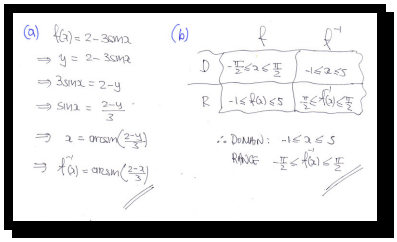
A function f is defined by

$$f(x)=2-3\sin x,\ \ -\frac{\pi}{2}\le x<\frac{\pi}{2}.$$

a) Find an expression for $f^{-1}(x)$.

b) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\arcsin\left(\frac{2-x}{3}\right)},\ \boxed{-1\le x\le 1},\ \boxed{0\le f^{-1}(x)<2\pi}$$



Question 22

A function f is defined by

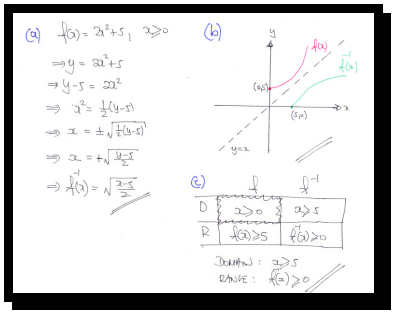
$$f(x)=2x^2+5,\ \ x\in\mathbb{R},\ x\geq 0.$$

a) Find an expression for $f^{-1}(x)$.

b) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x)$.

c) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=\sqrt{\frac{x-5}{2}}},\ \boxed{x\geq 5},\ \boxed{f^{-1}(x)\geq 0}$$



Question 23

A function f is defined by

$$f(x)=x^2-4x-1,\quad x\in\mathbb{R},\ x<0.$$

a) By completing the square, or otherwise, find an expression for $f^{-1}(x)$.

b) Sketch in the same diagram the graphs of $f(x)$ and $f^{-1}(x)$.

c) Find the domain and range of $f^{-1}(x)$.

$$\boxed{f^{-1}(x)=2-\sqrt{x+5}},\ \boxed{x>-1},\ \boxed{f^{-1}(x)<0}$$

