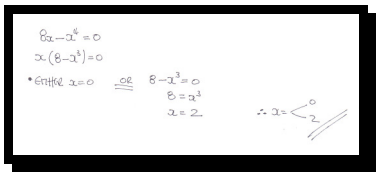
ADVANCED EQUATIONS SOLVING SKILLS

Question 1 (**)

Find the solutions of the equation

$$8x - x^4 = 0 .$$

$\boxed{x = 0, 2}$

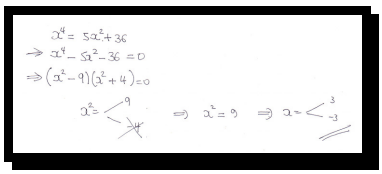


Question 2 (**)

Find the solutions of the equation

$$x^4 = 5x^2 + 36 .$$

$\boxed{x = \pm 3}$

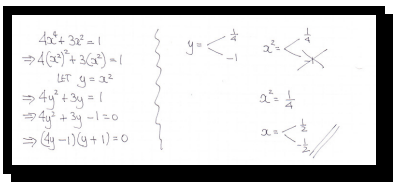


Question 3 (**)

Find the solutions of the equation

$$4x^4 + 3x^2 = 1 .$$

$\boxed{x = \pm \frac{1}{2}}$

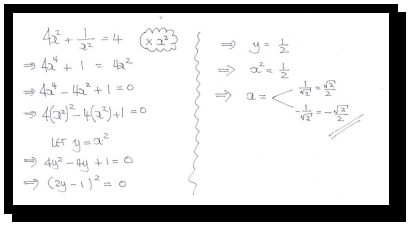


Question 4 (+)**

Find as exact surds the roots of the equation

$$4x^2+\frac{1}{x^2}=4,\ x\neq 0.$$

$$\boxed{x=\pm\frac{\sqrt{2}}{2}}$$

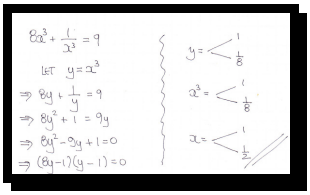


Question 5 (+)**

Solve the equation

$$8x^3+\frac{1}{x^3}=9,\ x\neq 0.$$

$$\boxed{x=1,\ \ x=\frac{1}{8}}$$

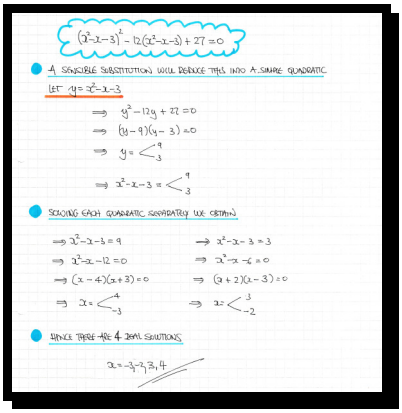


Question 6 **(***)**

Find the real solutions of the following equation

$$\left(x^2-x-3\right)^2-12\left(x^2-x-3\right)+27=0\,.$$

$\boxed{}$, $\boxed{x=-3,\ \ x=-2,\ \ x=3,\ \ x=4}$

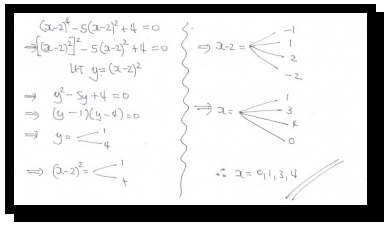


Question 7 **(***)**

Determine the four real roots of the equation

$$(x-2)^4-5(x-2)^2+4=0\,.$$

$\boxed{x=0,1,3,4}$



Question 8 (***)

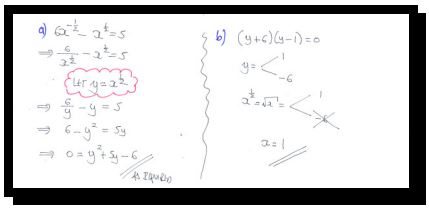
$$6x^{-\frac{1}{2}} - x^{\frac{1}{2}} = 5 .$$

a) Show clearly that the substitution $y = x^{\frac{1}{2}}$ transforms the above irrational equation into the quadratic equation

$$y^2 + 5y - 6 = 0 .$$

b) Solve the quadratic equation and hence find the root of the **irrational** equation.

$$\boxed{x = 1}$$



Question 9 (***)

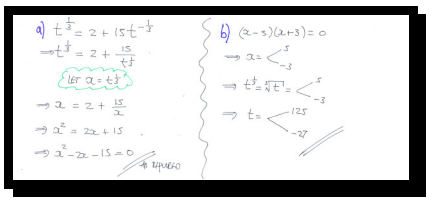
$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}.$$

a) Show that the substitution $x = t^{\frac{1}{3}}$ transforms the above irrational equation into the quadratic equation

$$x^2 - 2x - 15 = 0.$$

b) Solve the quadratic equation and hence find the two solutions of the **irrational** equation.

$$\boxed{t = -27, \ \ t = 125}$$

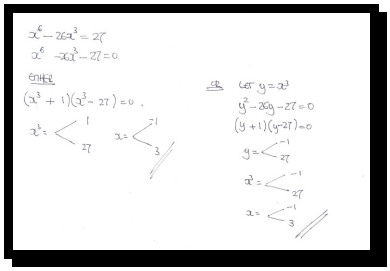


Question 10 (*+)**

Find the solutions of the equation

$$x^6 - 26x^3 = 27.$$

$$\boxed{x = -1, 3}$$

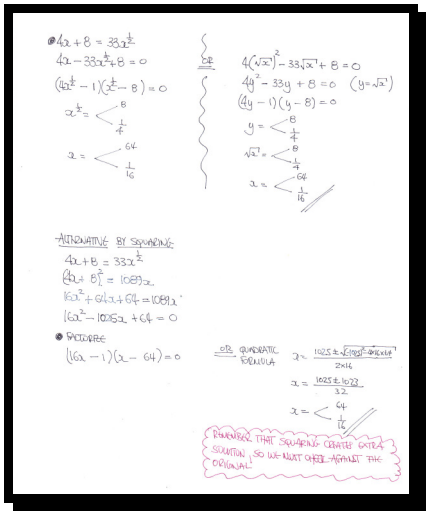


Question 11 (*+)**

Find the roots of the equation

$$4x+8=33x^{\frac{1}{2}},\ x\geq 0.$$

$$\boxed{x=\frac{1}{16},64}$$

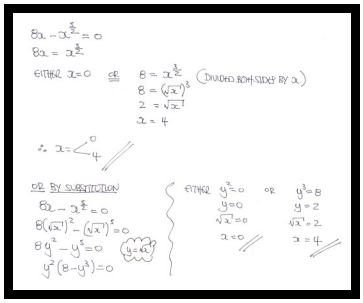


Question 12 (*+)**

Find the solutions of the equation

$$8x-x^{\frac{5}{2}}=0,\ x\geq 0.$$

$$\boxed{x=0,\ 4}$$



Question 13 **(***+)**

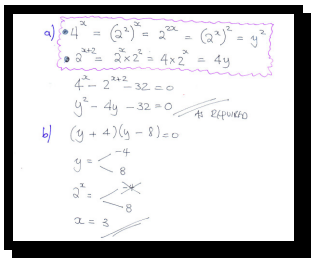
$$4^x - 2^{x+2} = 32 .$$

a) Show that the substitution $y = 2^x$ transforms the above indicial equation into the quadratic equation

$$y^2 - 4y - 32 = 0 .$$

b) Solve the quadratic equation and hence find the root of the **indicial** equation.

$$\boxed{x = 3}$$

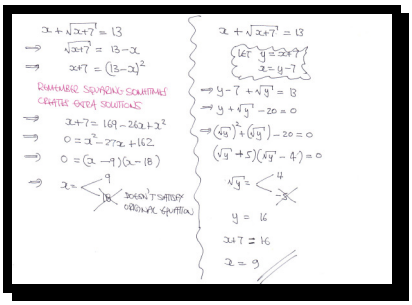


Question 14 **(***+)**

Find the solution of the equation

$$x + \sqrt{x+7} = 13 , \quad x \geq -7 .$$

$$\boxed{x = 9}$$

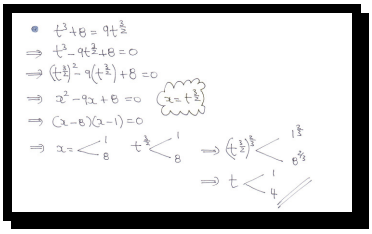


Question 15 (*+)**

Solve the equation

$$t^3+8=9t^{\frac{3}{2}},\ x\geq 0.$$

$$\boxed{t=1,4}$$

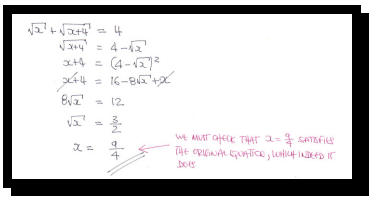


Question 16 (*+)**

Find the solution of the equation

$$\sqrt{x}+\sqrt{x+4}=4,\ x\geq 0.$$

$$\boxed{x=\frac{9}{4}}$$

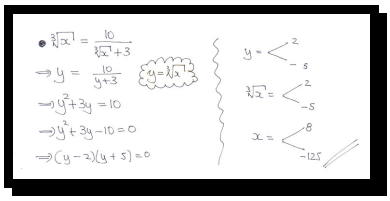


Question 17 (*+)**

Solve the equation

$$\sqrt[3]{x}=\frac{10}{\sqrt[3]{x}+3},\ x\neq-\sqrt[3]{x}\,.$$

$$\boxed{x=-125,8}$$

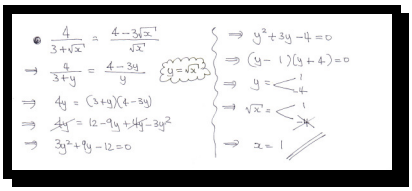


Question 18 (*+)**

Solve the equation

$$\frac{4}{3+\sqrt{x}}=\frac{4-3\sqrt{x}}{\sqrt{x}},\ x>0\,.$$

$$\boxed{x=1}$$

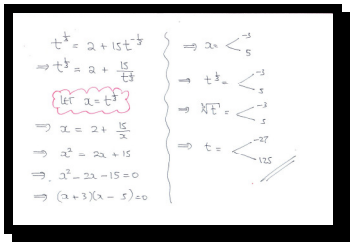


Question 19 (*+)**

$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}, \quad t \neq 0.$$

Use the substitution $x = t^{\frac{1}{3}}$ to solve the above irrational equation.

$$\boxed{t = -27, \ t = 125}$$



Question 20 (****)

The indicial equation

$$2^{x+1} + 2^{3-x} = 17, \ x \in \mathbb{R},$$

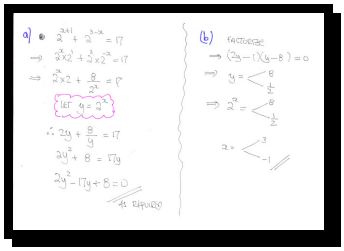
is to be solved by a suitable substitution.

a) Show clearly that the substitution $y = 2^x$ transforms the above indicial equation into the quadratic equation

$$2y^2 - 17y + 8 = 0.$$

b) Solve the quadratic equation by factorization and hence determine the two solutions of the **indicial** equation.

$$\boxed{}, \ \boxed{x = -1, \ 3}$$



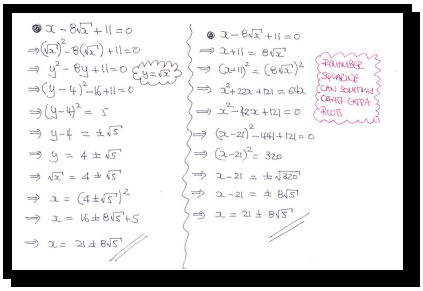
Question 21 (****)

Solve the equation

$$x-8\sqrt{x}+11=0,\ x\geq 0,$$

giving the answers in the form $a+b\sqrt{5}$, where a and b are integers.

$$\boxed{x=21\pm 8\sqrt{5}}$$

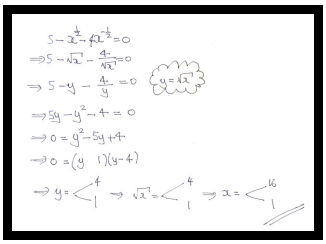


Question 22 (****)

Find the two roots of the equation

$$5-x^{\frac{1}{2}}-4x^{-\frac{1}{2}}=0,\ x>0.$$

$$\boxed{x=1,16}$$

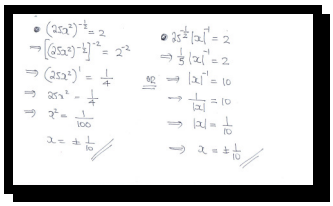


Question 23 (****)

Solve the equation

$$\left(25x^2\right)^{-\frac{1}{2}}=2\,,\quad x\neq 0\,.$$

$$\boxed{x=\pm\frac{1}{10}}$$



Question 24 (****)

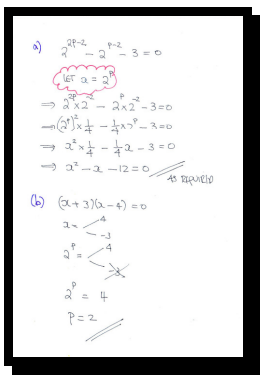
$$2^{2p-2}-2^{p-2}-3=0\,,\ p\in\mathbb{R}\,,$$

a) Show clearly that the substitution $x=2^p$ transforms the above indicial equation into the quadratic equation

$$x^2-x-12=0\,.$$

b) Solve the quadratic equation and hence determine the value of p.

$$\boxed{p=2}$$

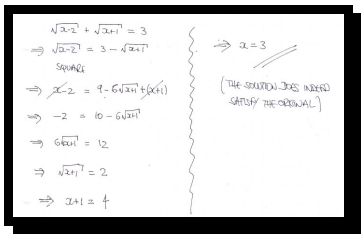


Question 25 (****)

Solve the equation

$$\sqrt{x-2}+\sqrt{x+1}=3,\ x\geq 2.$$

$$\boxed{x=3}$$

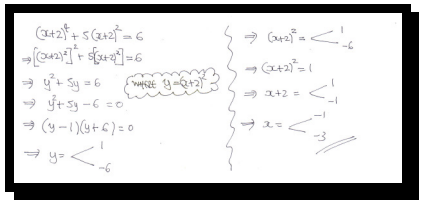


Question 26 (****)

Determine the two real roots of the equation

$$(x+2)^4+5(x+2)^2=6.$$

$$\boxed{x=-1,-3}$$

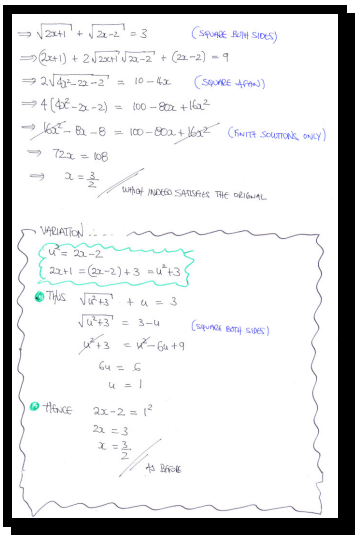


Question 27 (****)

Solve the equation

$$\sqrt{2x+1}+\sqrt{2x-2}=3$$

$$\boxed{x=\frac{3}{2}}$$



Question 28 (****)

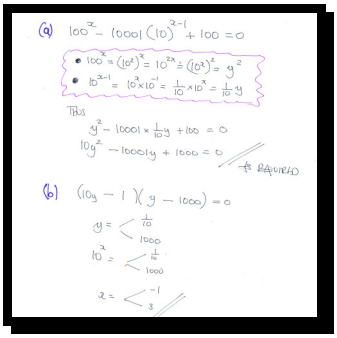
$$100^x - 10001\left(10^{x-1}\right) + 100 = 0 .$$

a) Show that the substitution $y = 2^x$ transforms the above indicial equation into the quadratic equation

$$10y^2 - 10001y + 1000 = 0 .$$

b) Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$$\boxed{x = -1, \quad x = 3}$$

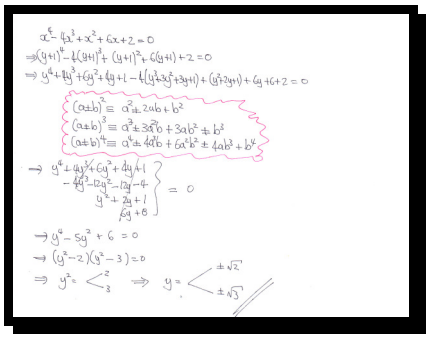


Question 29 (****)

By using the substitution $x = y+1$, or otherwise, find in exact surd form the roots of the equation

$$x^4 - 4x^3 + x^2 + 6x + 2 = 0 .$$

$$\boxed{x = \pm\sqrt{2}, \pm\sqrt{3}}$$



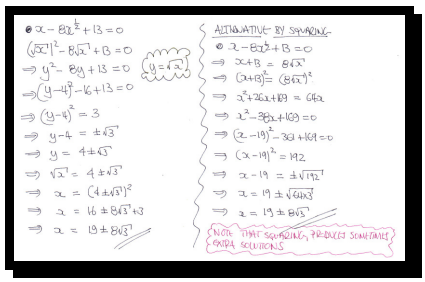
Question 30 (****)

Solve the equation

$$x - 8x^{\frac{1}{2}} + 13 = 0 , \ x \geq 0 ,$$

giving the answers in the form $a + b\sqrt{3}$, where a and b are integers.

$$\boxed{x = 19 \pm 8\sqrt{3}}$$

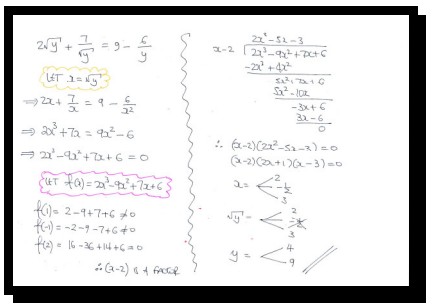


Question 31 (****)

Find the two real solutions of the equation

$$2\sqrt{y}+\frac{7}{\sqrt{y}}=9-\frac{6}{y},\ \ y>0$$

$$\boxed{y=4,\ \ y=9}$$



Question 32 **(****)**

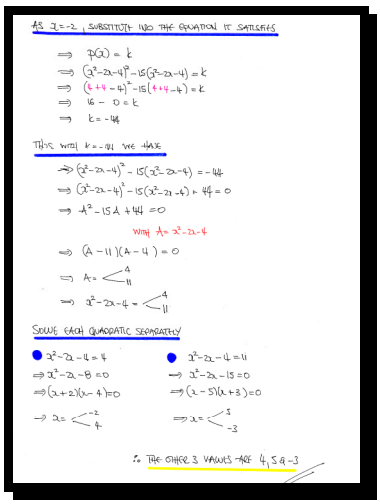
A polynomial $p(x)$ is defined as

$$p(x)\equiv\left(x^2-2x-4\right)^2-15\left(x^2-2x-4\right),\quad x\in\mathbb{R}.$$

The equation $p(x)=k$, where k is a constant, is satisfied by $x=-2$.

Determine the other three values of x that satisfy the equation $p(x)=k$.

$\boxed{}$, $\boxed{x=-3\ \cup\ x=4\ \cup\ x=5}$

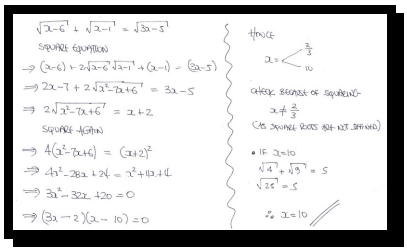


Question 33 (**+)**

Determine the real root of the equation

$$\sqrt{x-6}+\sqrt{x-1}=\sqrt{3x-5}\,.$$

$$\boxed{x=10}$$

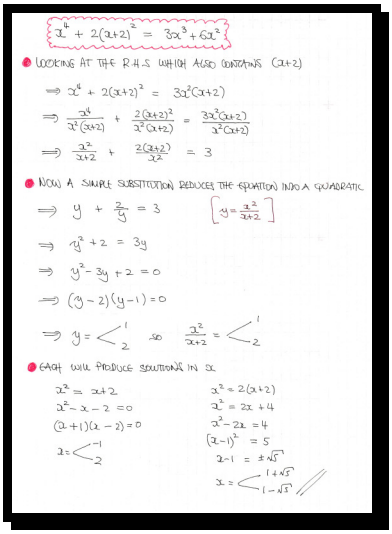


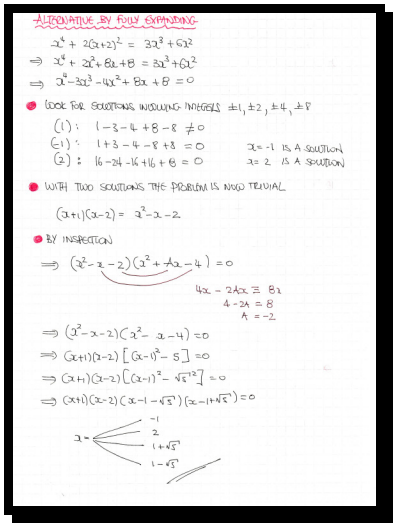
Question 34 (****)

Determine, in exact form where appropriate, the solutions of the following equation.

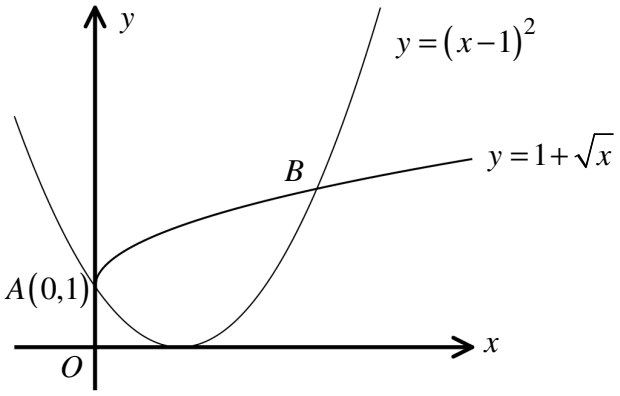
$$x^4+2(x+2)^2=3x^3+6x^2\,.$$

$$\boxed{}\,,\ \boxed{x=-1}\,,\ \boxed{x=2}\,,\ \boxed{x=1+\sqrt{5}}\,,\ \boxed{x=1-\sqrt{5}}$$





Question 35 (**+)**



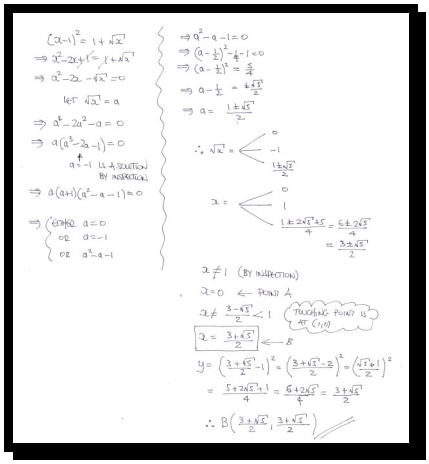
The figure above shows the graphs of the curves with equations

$$y=(x-1)^2 \quad \text{and} \quad y=1+\sqrt{x}\,.$$

The curves meet at the point $A(0,1)$ and at the point B.

Determine the exact coordinates of Q.

$$\boxed{B\left(\frac{3+\sqrt{5}}{2},\frac{3+\sqrt{5}}{2}\right)}$$

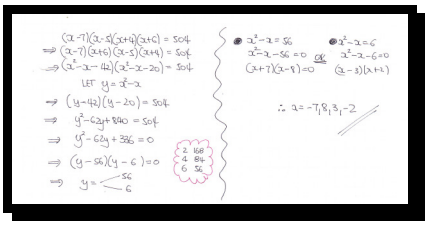


Question 36 (**+)**

By using the substitution $y = x^2 - x$, or otherwise, find the roots of the equation

$$(x-7)(x-5)(x+4)(x+6) = 504.$$

$$\boxed{x = -7, -2, 3, 8}$$

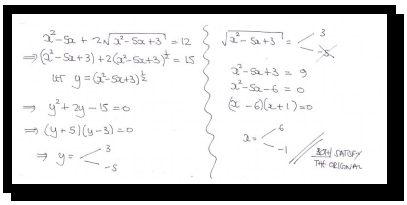


Question 37 (**+)**

Determine the two real roots of the equation

$$x^2 - 5x + 2\sqrt{x^2 - 5x + 3} = 12.$$

$$\boxed{x = -1, 6}$$

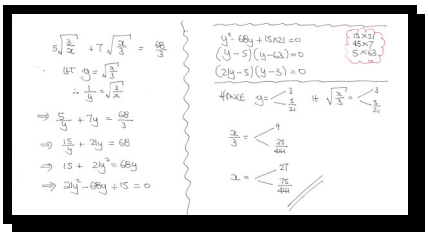


Question 38 (**+)**

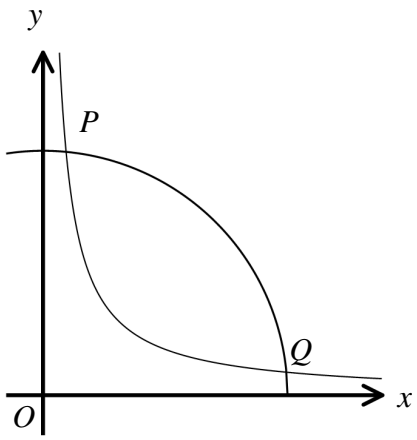
By using a suitable substitution, or otherwise, find as exact fractions where appropriate, the solutions of the equation

$$5\sqrt{\frac{3}{x}}+7\sqrt{\frac{x}{3}}=\frac{68}{3},\ x>0.$$

$$\boxed{x=27,\frac{75}{441}}$$



Question 39 (**+)**



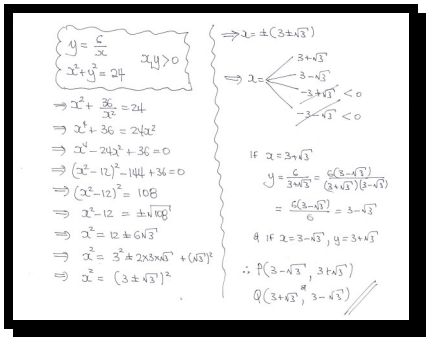
The figure above shows a rectangular hyperbola and a circle with respective Cartesian equations

$$y=\frac{6}{x},\ x>0 \qquad \text{and} \qquad x^2+y^2=8,\ x>0,\ y>0.$$

The points P and Q are the points of intersection between the rectangular hyperbola and the circle.

Find the coordinates of P and Q, in the form $\left(a+\sqrt{a},b+\sqrt{b}\right)$

$$\boxed{P\left(3-\sqrt{3},3+\sqrt{3}\right)},\ \boxed{Q\left(3+\sqrt{3},3-\sqrt{3}\right)}$$



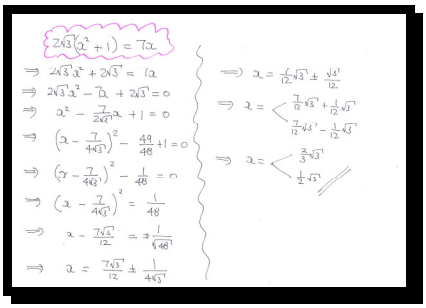
Question 40 (**+)**

Find the solutions of the quadratic equation

$$2\sqrt{3}\left(x^2+1\right)=7x.$$

Give the answers in the form $k\sqrt{3}$, where k is a constant.

$$\boxed{x=\frac{2}{3}\sqrt{3},\ x=\frac{1}{2}\sqrt{3}}$$

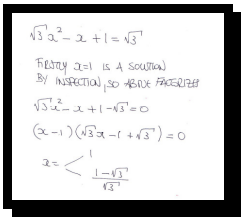


Question 41 (**+)**

Find in exact simplified form where appropriate the solutions of the equation

$$\sqrt{3}x^2-x+1=\sqrt{3}\,.$$

$$\boxed{x=1,\ x=\frac{1-\sqrt{3}}{\sqrt{3}}}$$

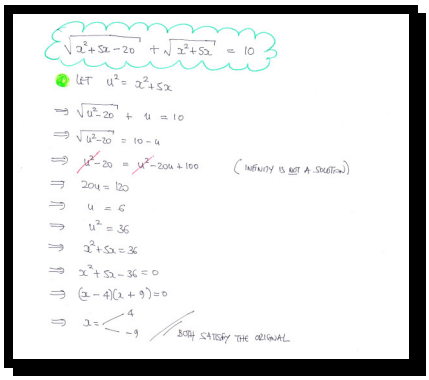


Question 42 (**+)**

Solve the equation

$$\sqrt{x^2+5x-20}+\sqrt{x^2+5x}=10$$

$$\boxed{x=4,\ x=-9}$$

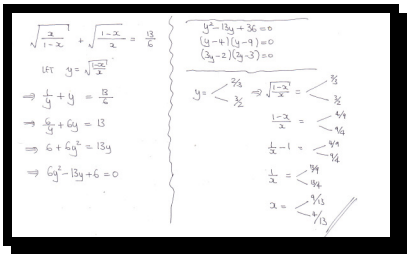


Question 43 (**+)**

By using a suitable substitution, or otherwise, find as exact fractions the solutions of the equation

$$\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6},\ x>0\,.$$

$$\boxed{x=\frac{9}{13},\frac{4}{13}}$$

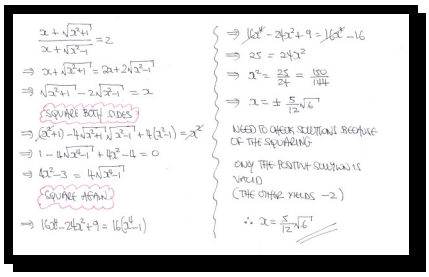


Question 44 (**+)**

Determine, in exact surd form, the solution of the equation

$$\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2-1}}=2\,.$$

$$\boxed{x=\frac{5}{12}\sqrt{6}}$$

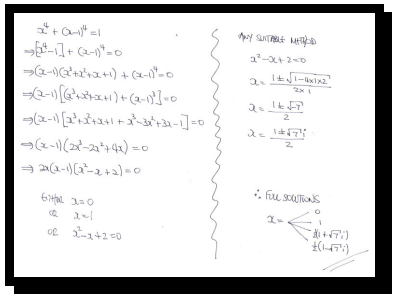


Question 45 (**+)**

$$x^4+(x-1)^4=1\,,\ x\in\mathbb{C}\,.$$

Determine, in exact form where appropriate, the four roots of the above equation.

$$\boxed{x=0,\ 1,\ \frac{1}{2}\left(1\pm\mathrm{i}\sqrt{7}\right)}$$

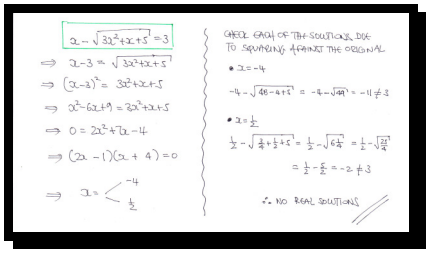


Question 46 (**+)**

Solve the following equation

$$x-\sqrt{3x^2+x+5}=3,\ x\in\mathbb{R}.$$

no solutions

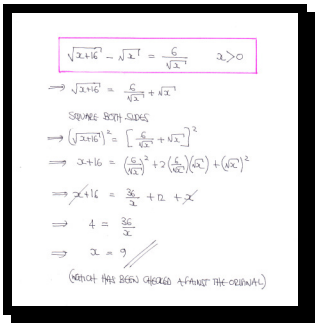


Question 47 (**+)**

Solve the following equation.

$$\sqrt{x+16}-\sqrt{x}=\frac{6}{\sqrt{x}},\quad x\in\mathbb{R},\ x>0.$$

$x=9$

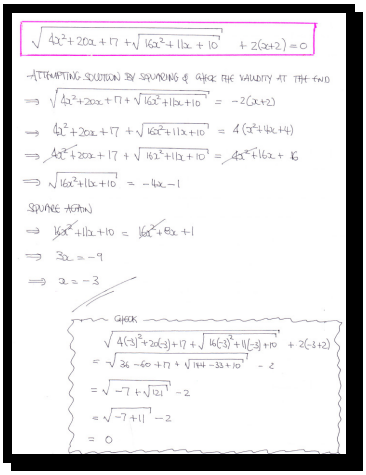


Question 48 (**+)**

Determine the real root of the equation

$$\sqrt{4x^2+20x+17+\sqrt{16x^2+11x+10}}+2(x+2)=0.$$

$\boxed{x=-3}$

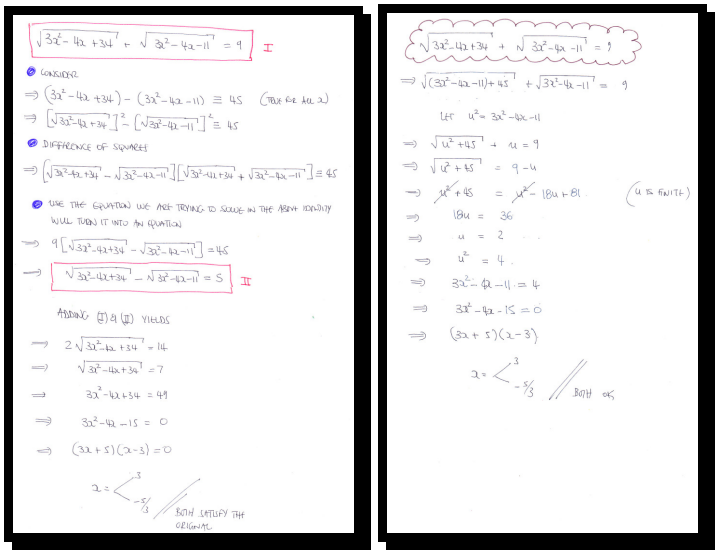


Question 49 (**+)**

Determine the two real roots of the equation

$$\sqrt{3x^2-4x+34}+\sqrt{3x^2-4x-11}=9\,.$$

$$\boxed{x=3,-\frac{5}{3}}$$

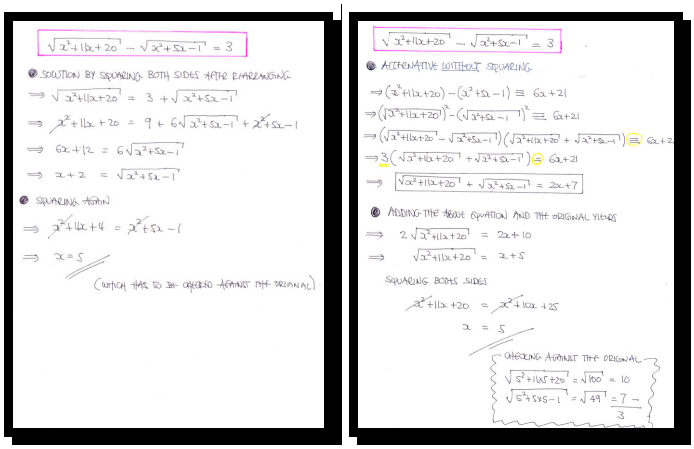


Question 50 (**+)**

Solve the following equation.

$$\sqrt{x^2+11x+20}+\sqrt{x^2+5x-1}=3\,,\quad x\in\mathbb{R}\,.$$

$$\boxed{\quad}\,,\ \boxed{x=5}$$



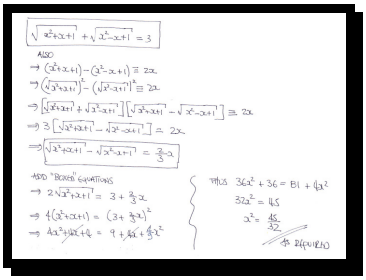
Question 51 **(****+)**

Given the equation

$$\sqrt{x^2+x+1}+\sqrt{x^2-x+1}=3,\ x\in\mathbb{R},$$

show clearly that $x^2=\dfrac{45}{32}$.

proof

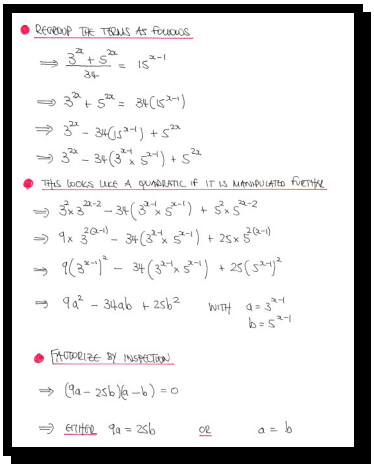


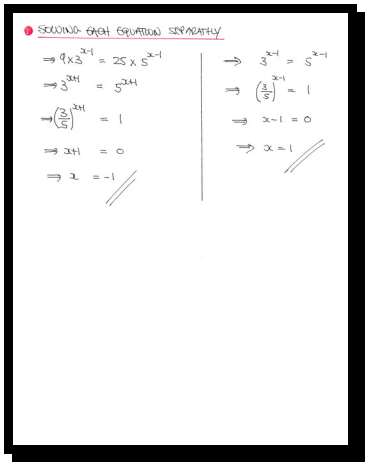
Question 52 **(*****)**

Solve the exponential equation

$$\frac{3^{2x}+5^{2x}}{34}=15^{x-1}.$$

V, [ ], $x=\pm 1$



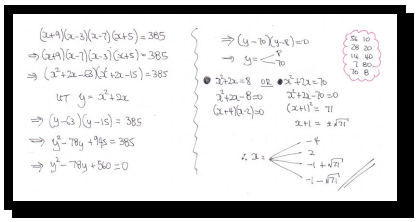


Question 53 (*****)

By using a suitable quadratic substitution, or otherwise, find in exact surd form where appropriate, the four real roots of the equation

$$(x-7)(x-3)(x+5)(x+9)=385.$$

V, $\boxed{x=-4,2,-1\pm\sqrt{71}}$

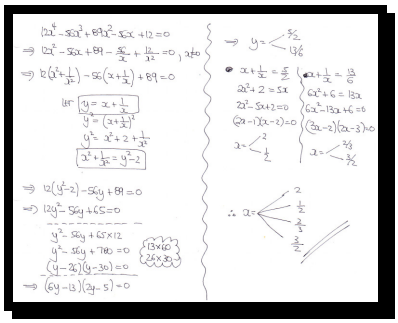


Question 54 (*****)

Solve the equation

$$12x^4-56x^3+89x^2-56x+12=0.$$

$$\boxed{x=\frac{1}{2},\frac{2}{3},\frac{3}{2},2}$$

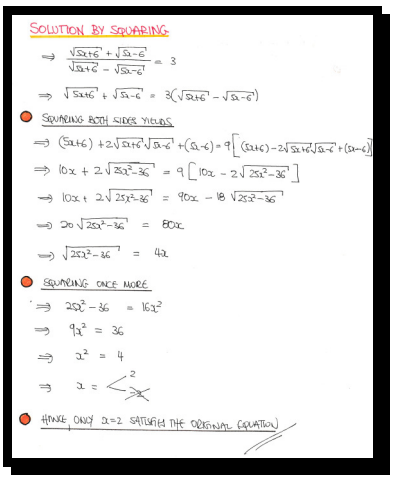


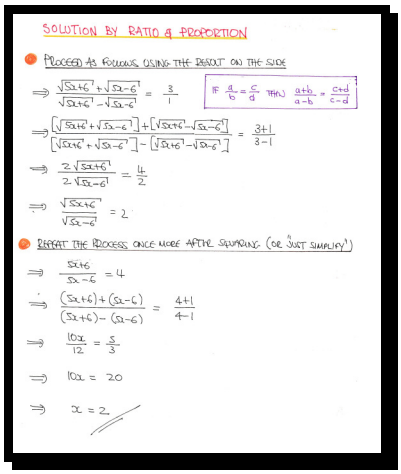
Question 55 (*****)

Solve the equation

$$\frac{\sqrt{5x+6}+\sqrt{5x-6}}{\sqrt{5x+6}-\sqrt{5x-6}}=3, \quad x>\frac{6}{5}.$$

$\boxed{}$, $\boxed{x=2}$





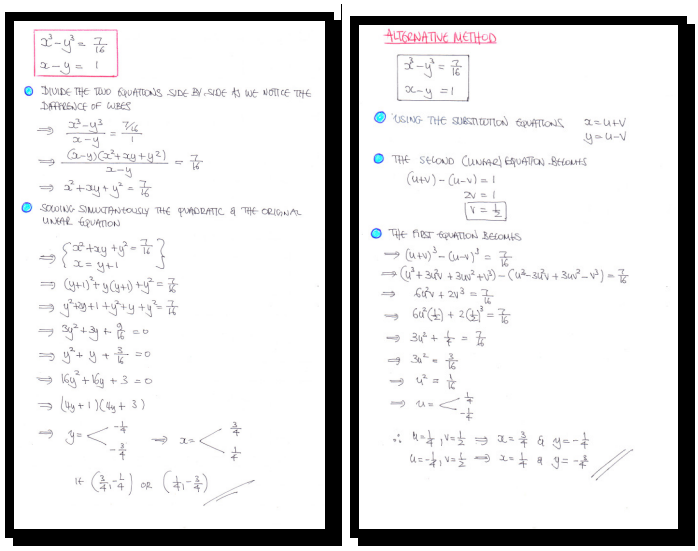
Question 56 (*****)

Use algebra to solve the following simultaneous equations

$$x^3-y^3=\frac{7}{16} \qquad \text{and} \qquad x-y=1,$$

given further that $x\in\mathbb{R}$ and $y\in\mathbb{R}$.

$\square$, $\boxed{(x,y)=\left(\frac{3}{4},-\frac{1}{4}\right)=\left(\frac{1}{4},-\frac{3}{4}\right)}$

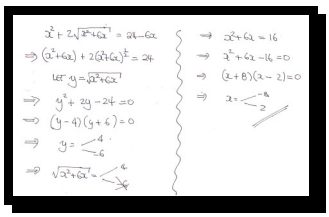


Question 57 (*****)

Determine the two real roots of the equation

$$x^2+2\sqrt{x^2+6x}=24-6x.$$

$\boxed{x=-8,2}$

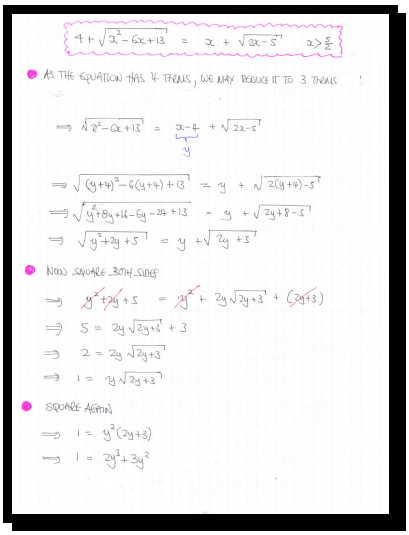


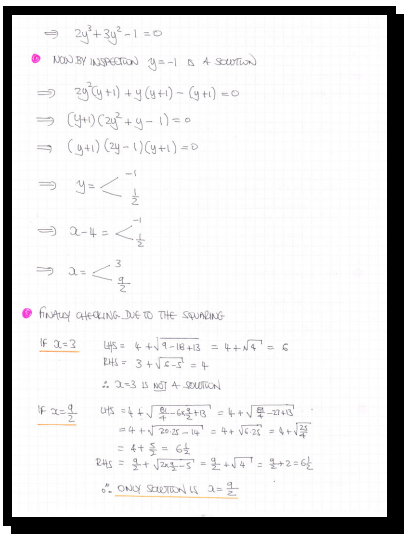
Question 58 (*****)

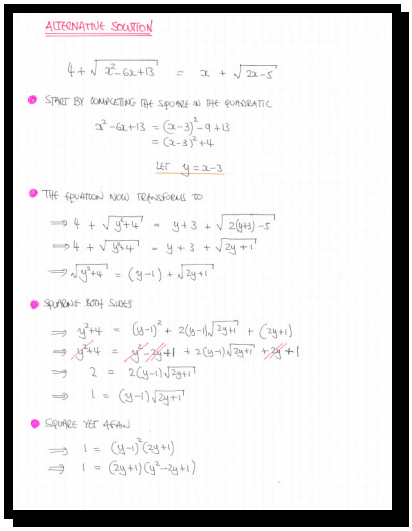
Solve following equation

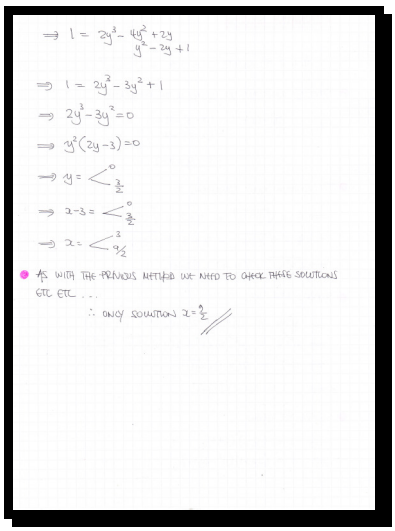
$$4+\sqrt{x^2-6x+13}=x+\sqrt{2x-5}\,,\quad x\in\mathbb{R},\ x\geq\frac{5}{2}\,.$$

$\square$, $\boxed{x=\frac{9}{2}}$









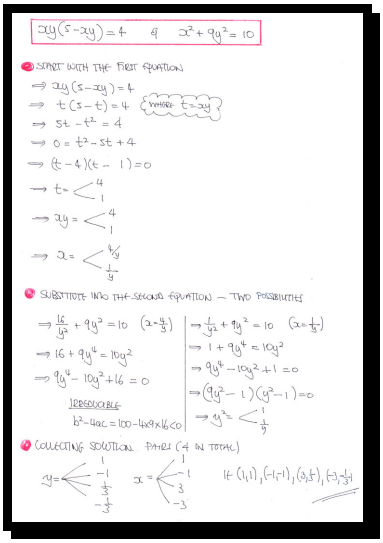
Question 59 (*****)

Use algebra to solve the following simultaneous equations

$$xy(5-xy)=4 \qquad \text{and} \qquad x^2+9y^2=10,$$

given further that $x\in\mathbb{R}$ and $y\in\mathbb{R}$.

$\square$, $\boxed{(x,y)=(1,1),(-1,-1),\left(3,\frac{1}{3}\right),\left(-3,-\frac{1}{3}\right)}$



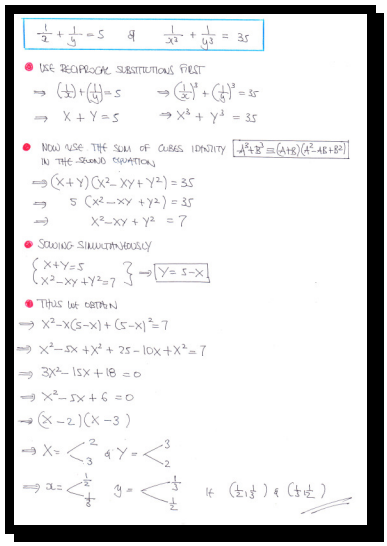
Question 60 (*****)

Use algebra to solve the following simultaneous equations

$$\frac{1}{x}+\frac{1}{y}=5 \qquad \text{and} \qquad \frac{1}{x^3}+\frac{1}{y^3}=35,$$

given further that $x\in\mathbb{R}$ and $y\in\mathbb{R}$.

$\square$, $\boxed{(x,y)=\left(\frac{1}{2},\frac{1}{3}\right)=\left(\frac{1}{3},\frac{1}{2}\right)}$

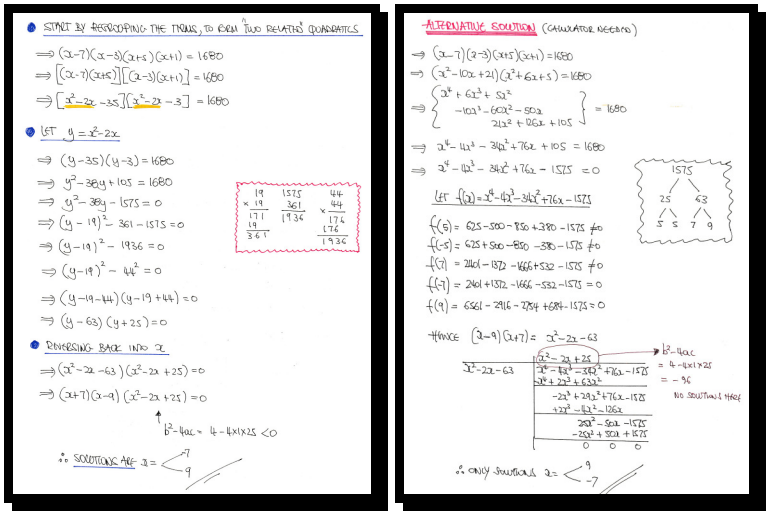


Question 61 **(*****)**

Determine the two real roots of the equation

$$(x-7)(x-3)(x+5)(x+1)=1680\,.$$

$\square$, $\boxed{x=-7,9}$

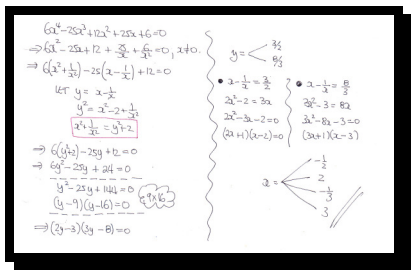


Question 62 **(*****)**

Solve the equation

$$6x^4-25x^3+12x^2+25x+6=0\,.$$

V, $\boxed{x=-\frac{1}{2},-\frac{1}{3},2,3}$

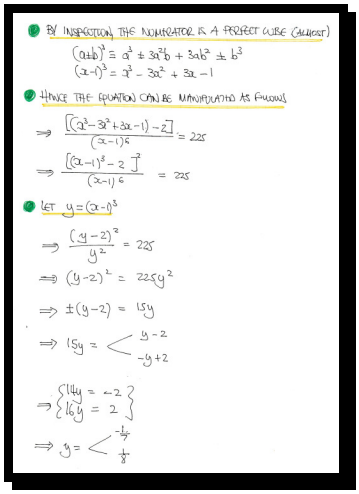


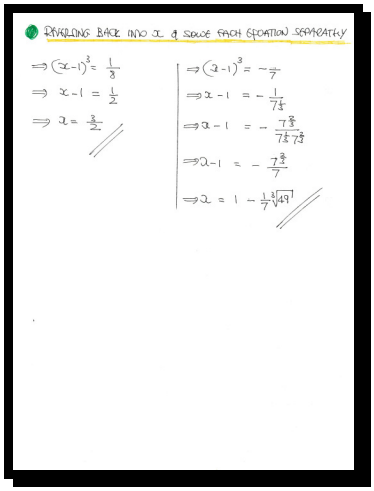
Question 63 (*****)

Find in exact form the two real solutions of the equation

$$\frac{\left(x^3-3x^2+3x-3\right)^2}{\left(x-1\right)^6}=225\,.$$

$\square$, $\boxed{x=\frac{3}{2},\qquad x=1-\frac{1}{7}\sqrt[3]{49}}$



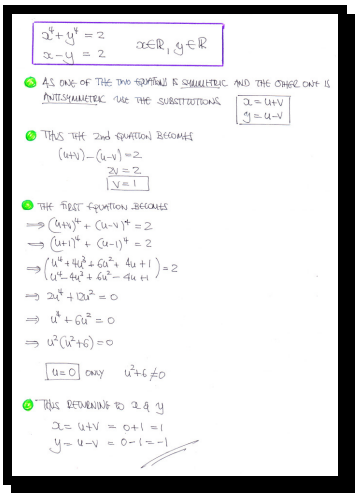


Question 64 (*****)

Use an algebraic method to show that $x=1$ and $y=-1$ is the only real solution pair of the following simultaneous equations

$$x^4+y^4=2 \quad \text{and} \quad x-y=2\,.$$

V, [ ], proof

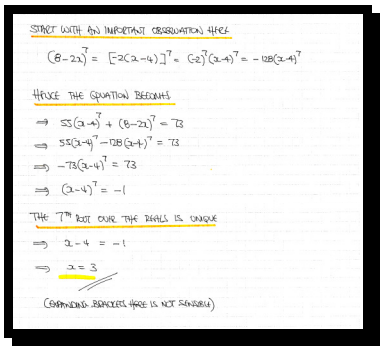


Question 65 **(*****)**

Use algebra to solve the equation

$$55(x-4)^7+(8-2x)^7=73,\quad x\in\mathbb{R}.$$

V, [ ], $x=3$

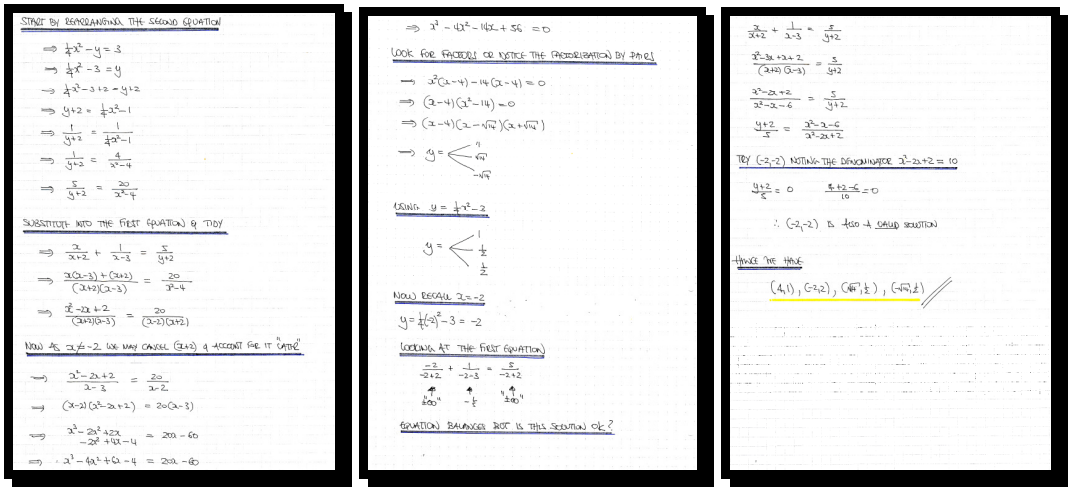


Question 66 (*****)

Use algebra to solve following simultaneous equations over the set of real numbers.

$$\frac{x}{x+2}+\frac{1}{x-3}=\frac{5}{y+2} \quad \text{and} \quad \frac{1}{4}x^2-y=3\,.$$

V, $\square$, $\boxed{(4,1)\cup(-2,-2)\cup\left(\sqrt{14},\frac{1}{2}\right)\cup\left(-\sqrt{14},\frac{1}{2}\right)}$

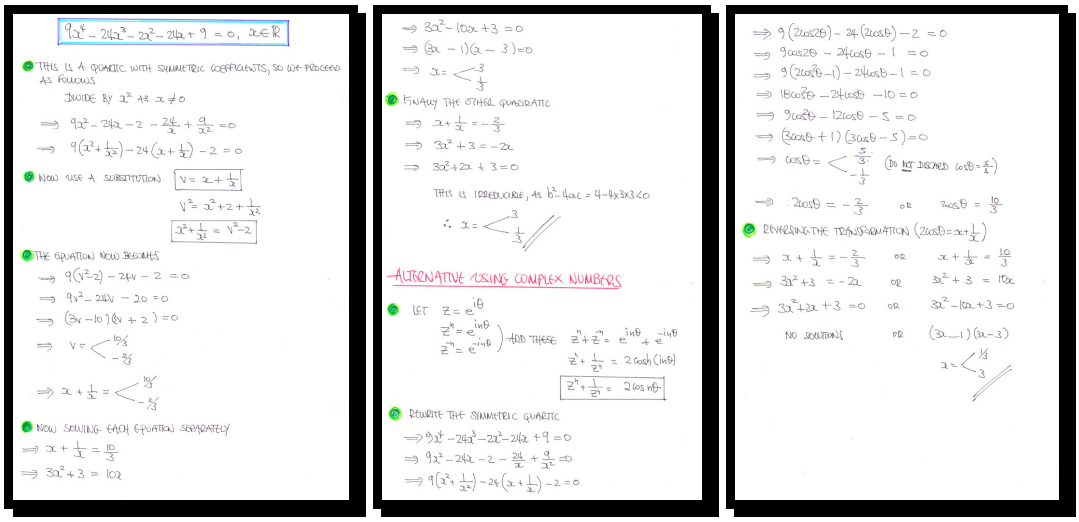


Question 67 (*****)

Solve the equation

$$9x^4-24x^3-2x^2-24x+9=0,\ x\in\mathbb{R}.$$

$\square$, $\boxed{x=\frac{1}{3},3}$



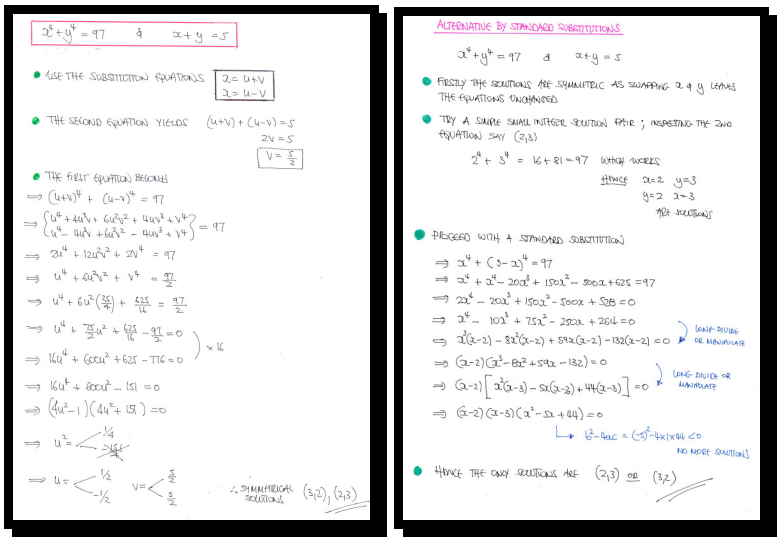
Question 68 (*****)

Use algebra to solve the following simultaneous equations

$$x^4 + y^4 = 97 \qquad \text{and} \qquad x + y = 5 ,$$

given further that $x \in \mathbb{R}$ and $y \in \mathbb{R}$.

$\boxed{}$, $\boxed{(x, y) = (3,2) = (2,3)}$

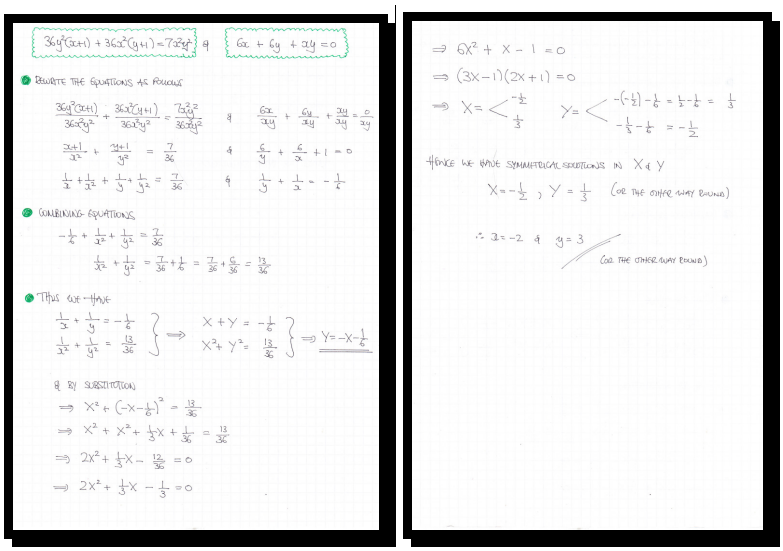


Question 69 (*****)

If $x \in \mathbb{R}$, $y \in \mathbb{R}$, find the non-trivial solution the following simultaneous equations.

$$36y^2(x+1)+36x^2(y+1)=7x^2y^2 \qquad \text{and} \qquad 6x+6y+xy=0 .$$

$\square$, $\boxed{(x,y)=(-2,3)=(3,-2)}$



Question 70 **(*****)**

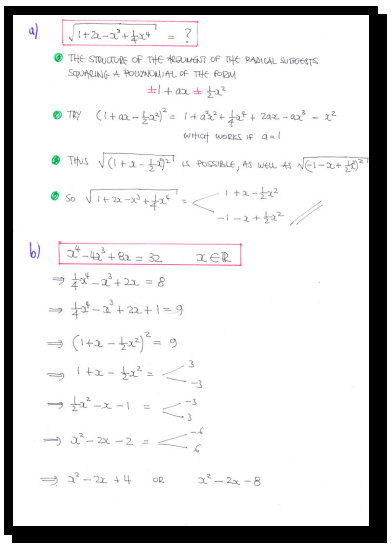
$$f(x)=1+2x-x^3+\frac{1}{4}x^4 \ , \quad x\in\mathbb{R}.$$

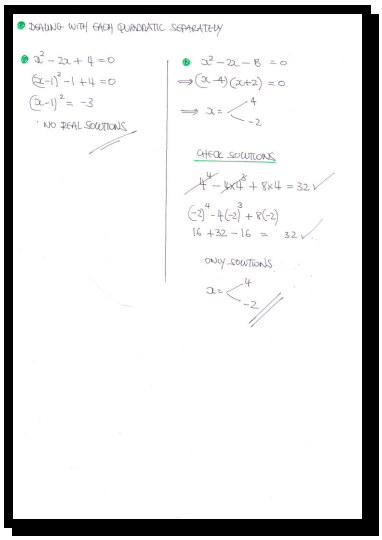
a) Extract the square roots of $f(x)$.

b) Hence, or otherwise, solve the equation

$$x^4-4x^3+8x=32 \ , \ x\in\mathbb{R}.$$

$\square$, $\boxed{\sqrt{f(x)}=\pm\left(1+x-\frac{1}{2}x^2\right)}$, $\boxed{x=4, \ \ x=-2}$





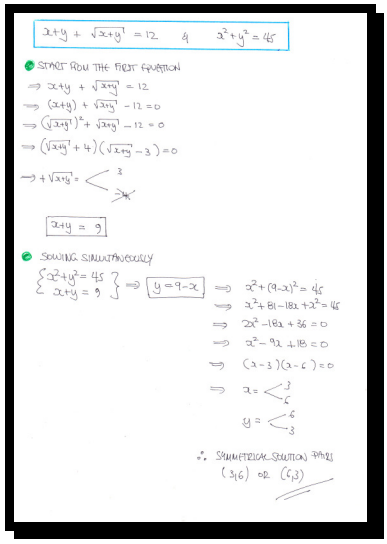
Question 71 (*****)

Use algebra to solve the following simultaneous equations

$$x+y+\sqrt{x+y}=12 \qquad \text{and} \qquad x^2+y^2=45\,,$$

given further that $x\in\mathbb{R}$ and $y\in\mathbb{R}$.

$\boxed{}$, $\boxed{(x,y)=(3,6)=(6,3)}$



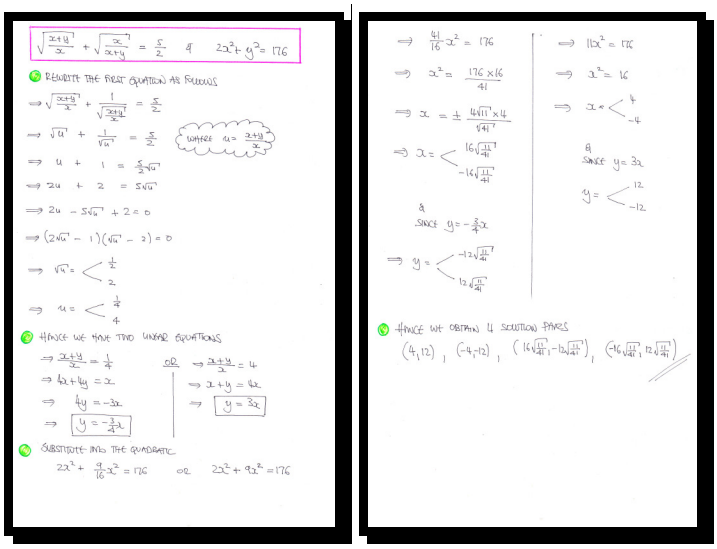
Question 72 **(*****)**

Use algebra to solve the following simultaneous equations

$$\sqrt{\frac{x+y}{x}}+\sqrt{\frac{x}{x+y}}=\frac{5}{2} \qquad \text{and} \qquad 2x^2+y^2=176,$$

given further that $x\in\mathbb{R}$ and $y\in\mathbb{R}$.

$\square$, $\boxed{(x,y)=(4,12),(-4,-12),\left(16\sqrt{\frac{11}{41}},-12\sqrt{\frac{11}{41}}\right),\left(-16\sqrt{\frac{11}{41}},-16\sqrt{\frac{11}{41}}\right)}$



Question 73 (*****)

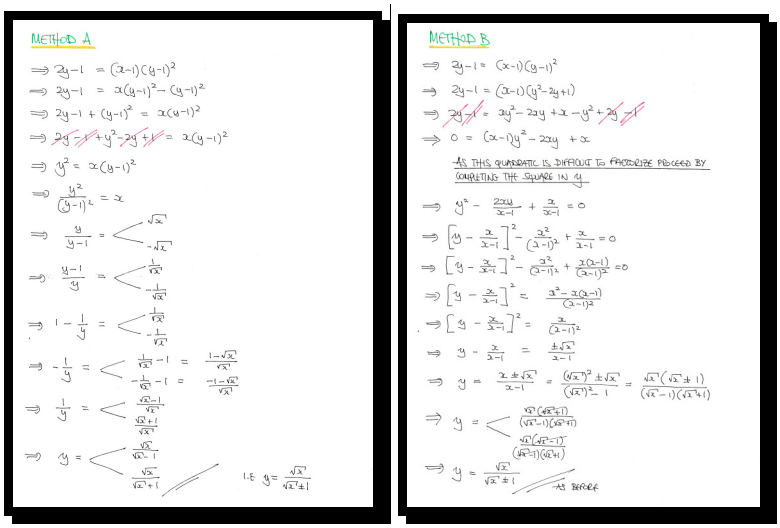
A curve has Cartesian equation

$$2y-1=(x-1)(y-1)^2,\ x\geq 0\,.$$

Make y the subject of the above equation, to show that

$$y=\frac{\sqrt{x}}{\sqrt{x}\pm 1}\,.$$

$\square$, $\boxed{y=\dfrac{\sqrt{x}}{\pm 1+\sqrt{x}}}$

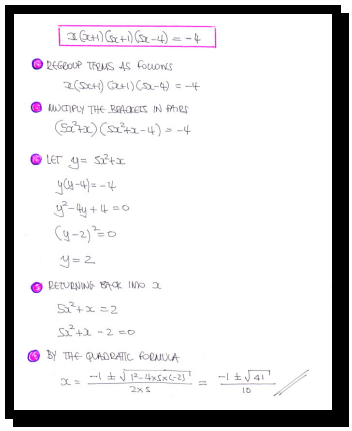


Question 74 **(*****)**

Find as exact surds the solutions of the equation

$$x(x+1)(5x+1)(5x-4)=-4\,,\ x\in\mathbb{R}\,.$$

$\square$, $\boxed{x=\dfrac{-1\pm\sqrt{41}}{10}}$

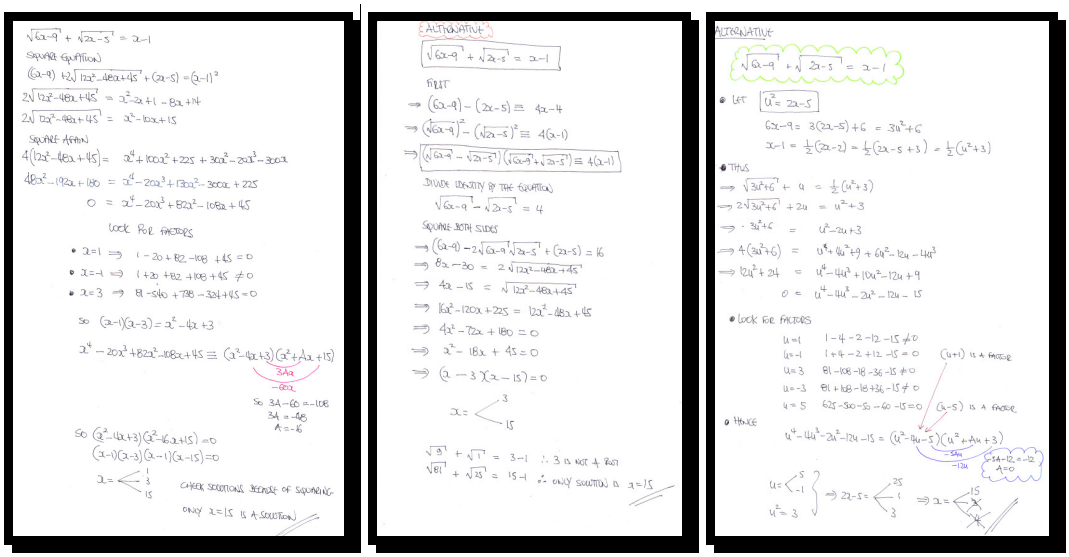


Question 75 **(*****)**

Determine the real root of the equation

$$\sqrt{6x-9}+\sqrt{2x-5}=x-1\,.$$

$\boxed{x=15}$

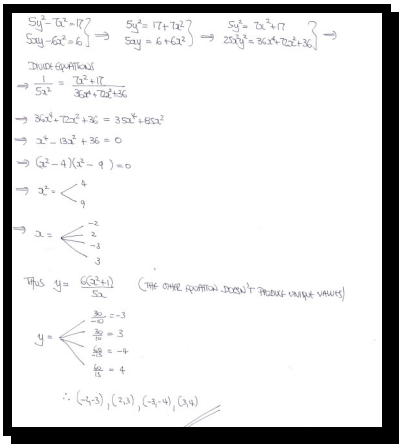


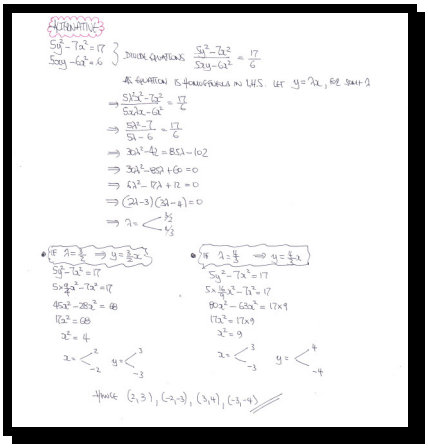
Question 76 (*****)

Find the real solutions for the following system of simultaneous equations

$$5y^2-7x^2=17$$
$$5xy-6x^2=6\,.$$

$$\boxed{(-2,-3),\ (2,3),\ (3,4),\ (-3,-4)}$$



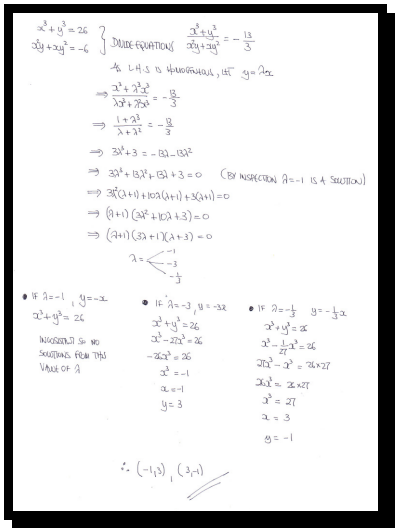


Question 77 (*****)

Find the real solutions for the following system of simultaneous equations

$$x^3+y^3=26$$
$$x^2y+xy^2=-6\,.$$

V, $\boxed{(x,y)=(-1,3)\ \text{ in any order}}$

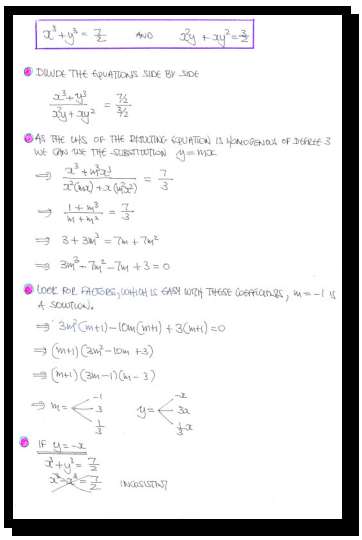


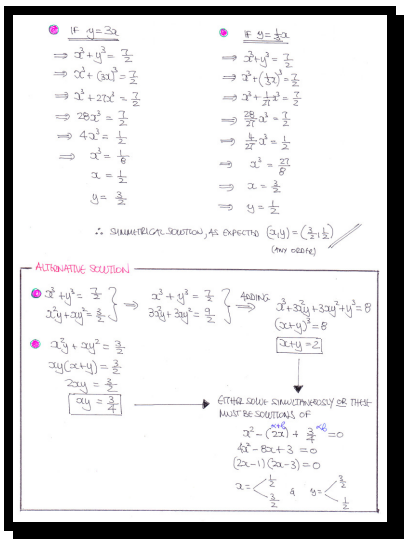
Question 78 (*****)

Find the real solutions for the following system of simultaneous equations

$$x^3+y^3=\frac{7}{2}$$
$$x^2y+xy^2=\frac{3}{2}.$$

$\square$, $\boxed{(x,y)=\left(\frac{1}{2},\frac{3}{2}\right) \text{ in any order}}$





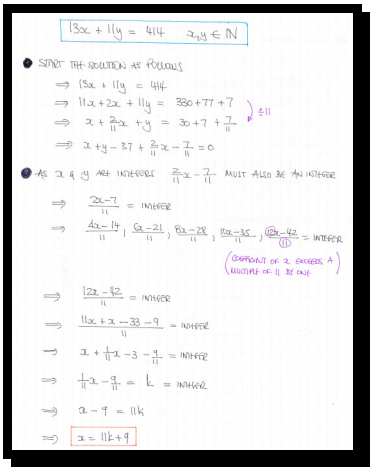
Question 79 (*****)

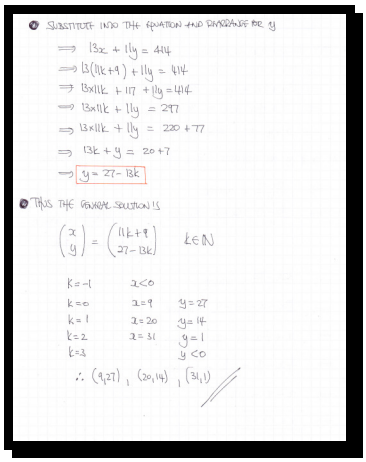
Solve the equation

$$13x+11y=414,$$

given further that $x\in\mathbb{N}$, $y\in\mathbb{N}$.

V, [ ], [ ], $\boxed{(x,y)=(9,27),\ (20,14),\ (31,1)}$





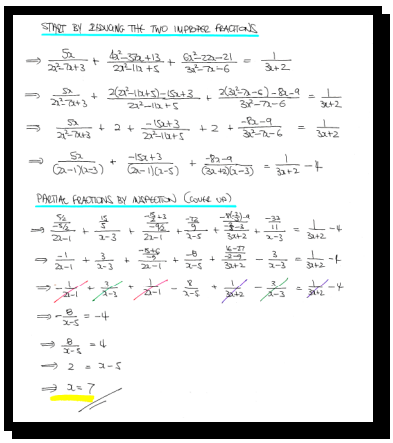
Question 80 (*****)

Solve the following rational equation, over the set of real numbers.

$$\frac{5x}{2x^2-7x+3}+\frac{4x^2-37x+13}{2x^2-11x+5}+\frac{6x^2-22x-21}{3x^2-7x-6}=\frac{1}{3x+2}.$$

You may ignore non finite solutions.

$\boxed{}$, $\boxed{x=7}$

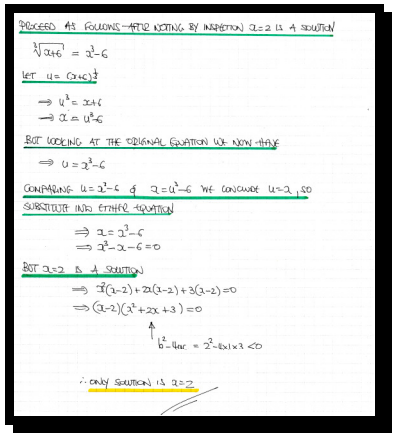


Question 81 (*****)

Show that $x=2$ is the only real solution of the following equation.

$$x^3-6=\sqrt[3]{x+6}\,.$$

V, [ ], proof



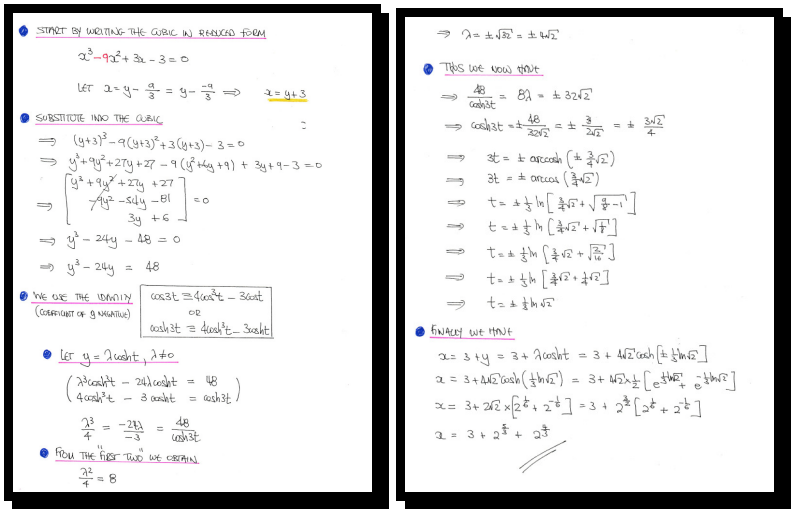
Question 82 (*****)

Solve the cubic equation

$$x^3-9x^2+3x-3=0,\ x\in\mathbb{R}.$$

You may assume that this cubic equation only has one real root.

V, , $x=1+2^{-\frac{2}{3}}-2^{-\frac{4}{3}}$



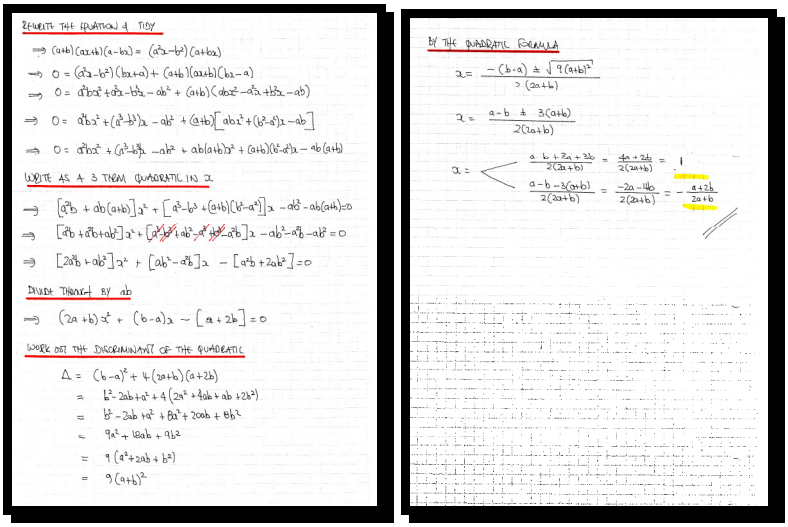
Question 83 **(*****)**

Solve the following equation

$$(a+b)(ax+b)(a-bx)=\left(a^2x-b^2\right)(a+bx),\ x\in\mathbb{R}.$$

Give the solutions in terms of a and b, where appropriate.

$\square$, $\boxed{x=1 \ \cup \ x=-\dfrac{a+2b}{2a+b}}$

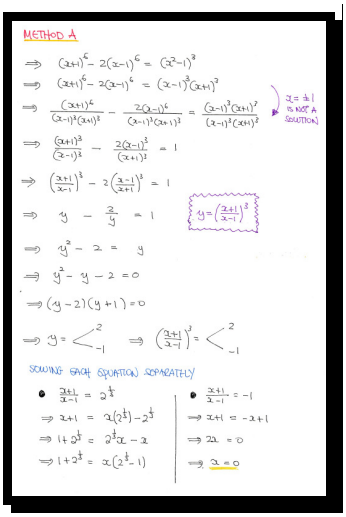


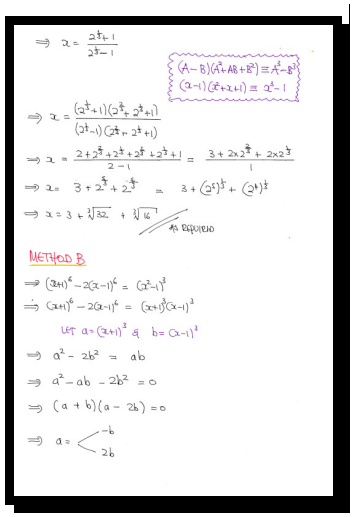
Question 84 (*****)

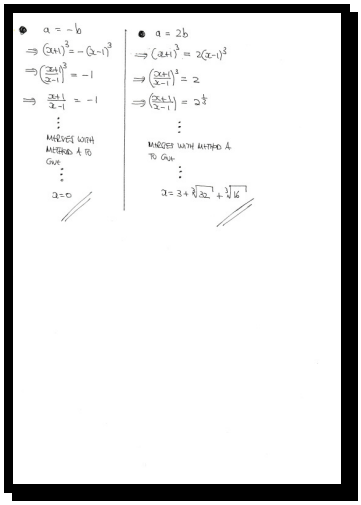
Determine, in exact form where appropriate, the two real roots of the equation

$$(x+1)^6-2(x-1)^6=\left(x^2-1\right)^3.$$

$\square$, $\boxed{x=0}$, $\boxed{x=\dfrac{2^{\frac{1}{3}}-1}{2^{\frac{1}{3}}+1}}$





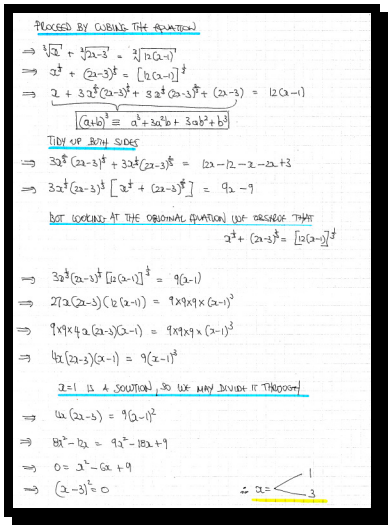


Question 85 (*****)

Solve the following equation

$$\sqrt[3]{x} + \sqrt[3]{2x-3} = \sqrt[3]{12(x-1)}, \ x \in \mathbb{R}.$$

$\boxed{}$, $\boxed{x=1 \cup x=3}$



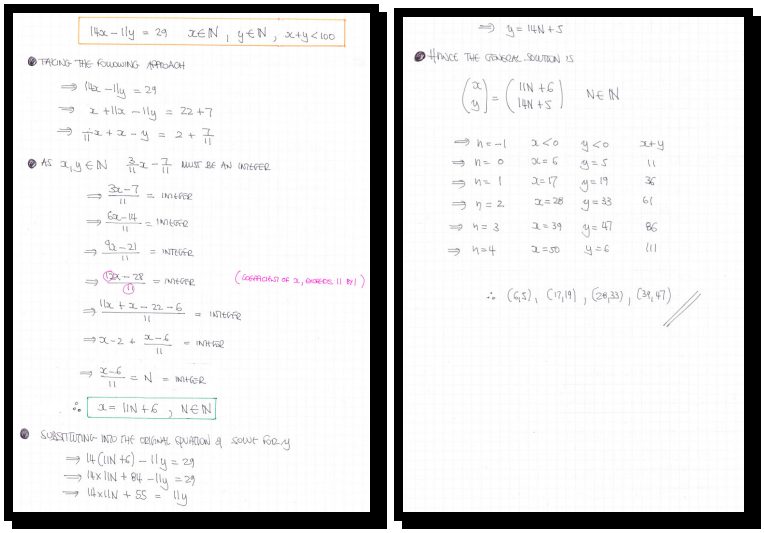
Question 86 (*****)

Solve the equation

$$14x-11y=29,$$

given further that $x\in\mathbb{N}$, $y\in\mathbb{N}$, and $x+y<100$.

V, [ ], $\boxed{(x,y)=(6,5),\ (17,19),\ (28,33),\ (39,47)}$



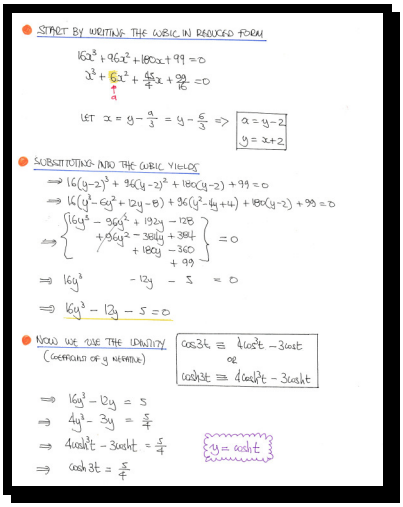
Question 87 **(*****)**

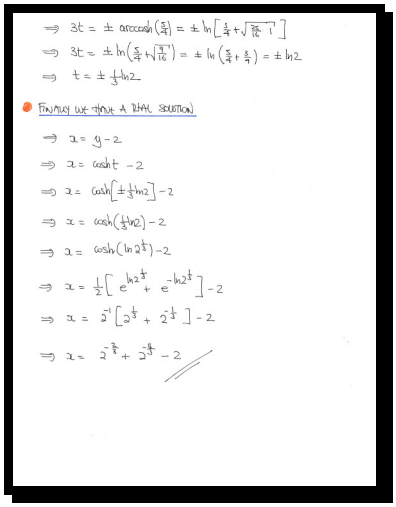
Solve the cubic equation

$$16x^3+96x^2+180x+99=0,\ x\in\mathbb{R}.$$

You may assume that this cubic equation only has one real root.

V, [ ], $x=-2+2^{-\frac{2}{3}}-2^{-\frac{4}{3}}$





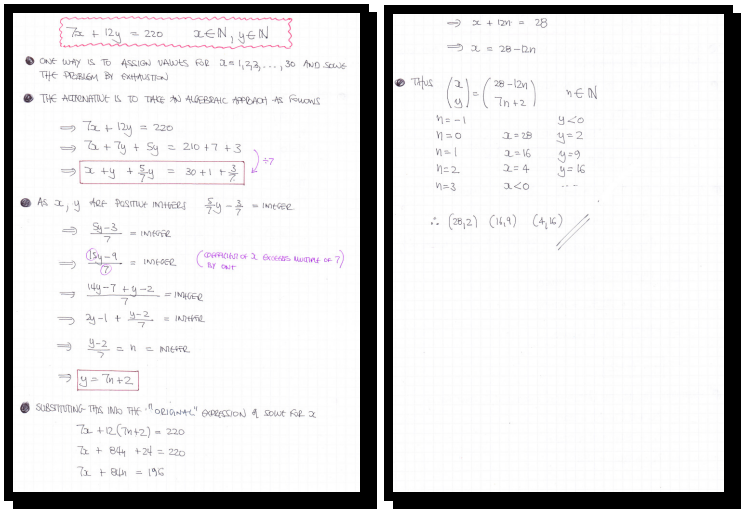
Question 88 (*****)

Solve the equation

$$7x+12y=220\,,$$

given further that $x\in\mathbb{N}$, $y\in\mathbb{N}$.

$\boxed{}$, $\boxed{(x,y)=(28,2),\ (16,9),\ (4,16)}$



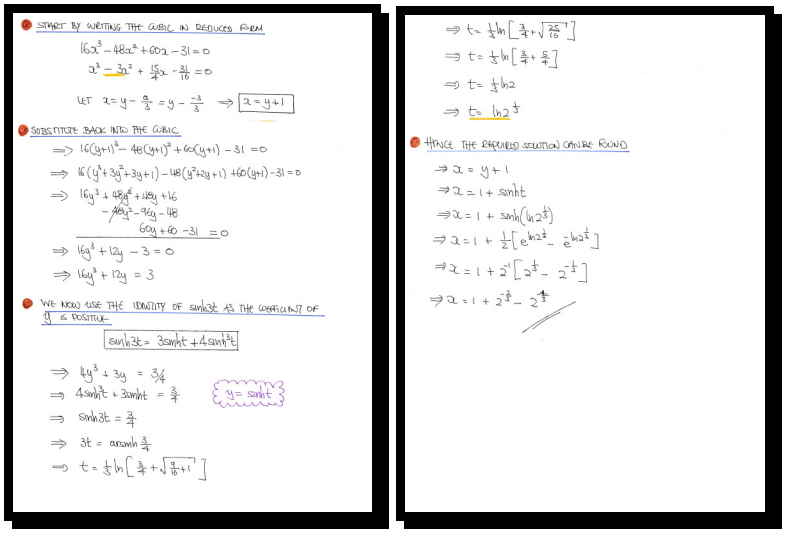
Question 89 **(*****)**

Solve the cubic equation

$$16x^3-48x^2+60x-31=0,\ x\in\mathbb{R}.$$

You may assume that this cubic equation only has one real root.

$\square$, $\boxed{x=3+2^{\frac{5}{3}}+2^{\frac{4}{3}}}$



Question 90 (*****)

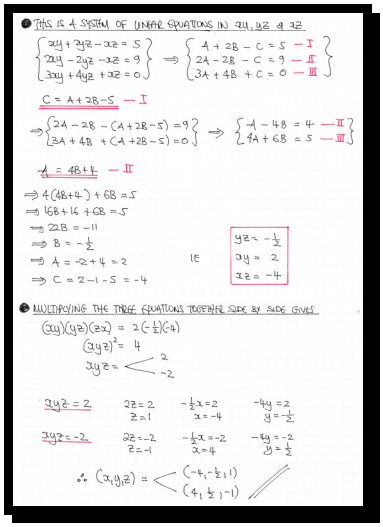
Solve the following system of equations for $x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$.

$$xy + 2yz - xz = 5,$$

$$2xy - 2yz - xz = 9,$$

$$3xy + 4yz + xz = 0.$$

$\square$, $\boxed{(x, y, z) = \left(-4, -\frac{1}{2}, 1\right), \quad (x, y, z) = \left(4, \frac{1}{2}, -1\right)}$



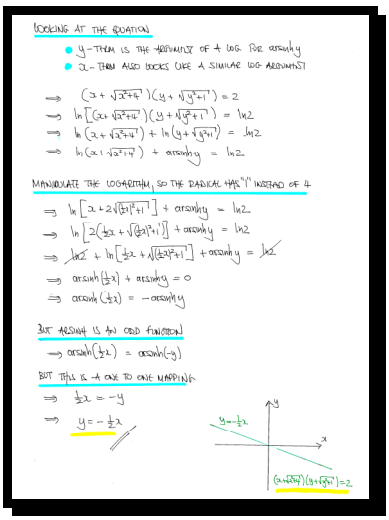
Question 91 (*****)

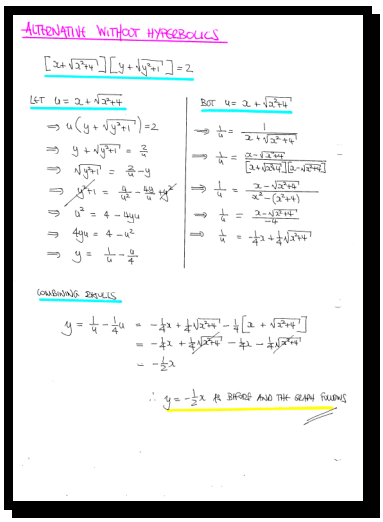
Sketch the graph of

$$\left[x+\sqrt{x^2+4}\right]\left[y+\sqrt{y^2+1}\right]=2,\quad x\in(-\infty,\infty),\ y\in(-\infty,\infty)$$

You must show a detailed method in this question

V, [ ], [proof]



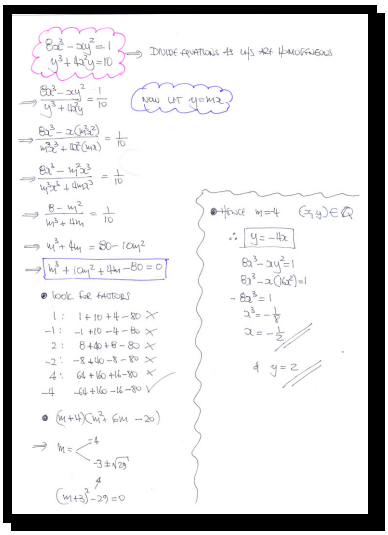


Question 92 (*****)

Given that $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$, find the solution pair for the following system of simultaneous equations

$$8x^3 - xy^2 = 1$$
$$y^3 + 4x^2 y = 10 .$$

V, $\boxed{(x, y) = \left(-\frac{1}{2}, 2\right)}$

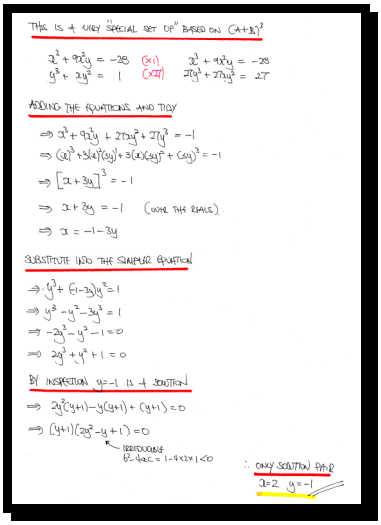


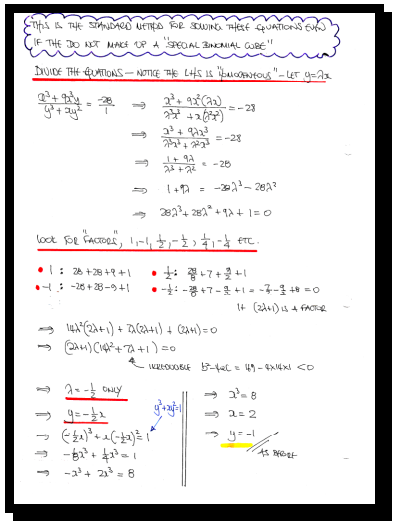
Question 93 (*****)

Given that $x \in \mathbb{R}$ and $y \in \mathbb{R}$, find the solution pair for the following system of simultaneous equations

$$x^3 + 9x^2 y = -28$$
$$y^3 + xy^2 = 1.$$

$\square$, $\boxed{(x, y) = (2, -1)}$



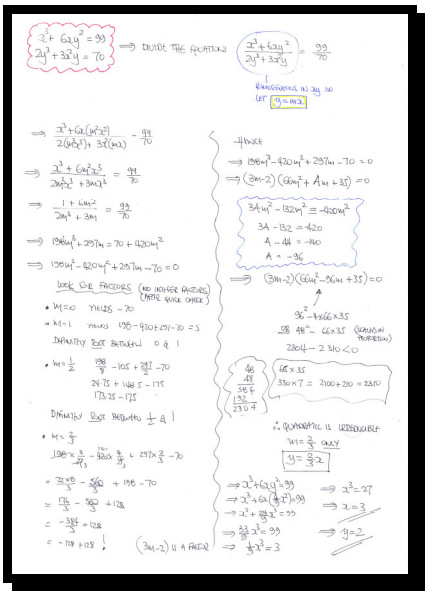


Question 94 (*****)

Use an algebraic method to find the real solutions for the following system of simultaneous equations

$$x^3+6xy^2=99$$
$$2y^3+3x^2y=70\,.$$

$$\boxed{(x,y)=(3,2)}$$



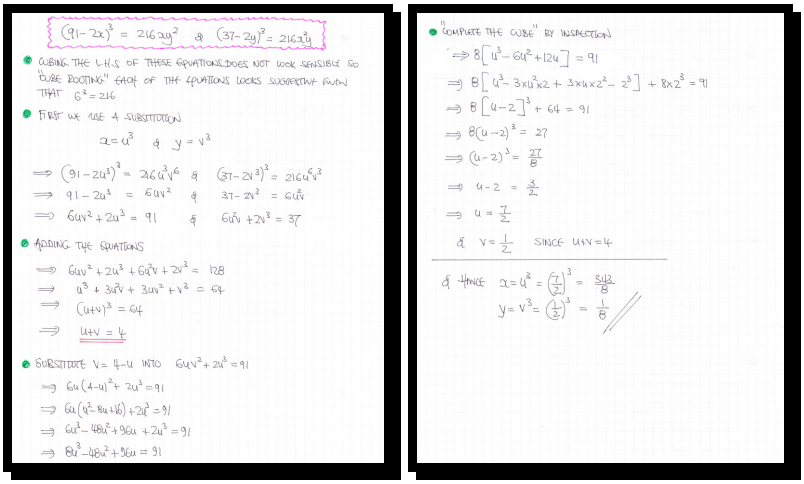
Question 95 **(*****)**

Solve the following simultaneous equations

$$(91-2x)^3=216xy^2 \qquad \text{and} \qquad (37-2y)^3=216x^2y\,,$$

where $x\in\mathbb{R}$, $y\in\mathbb{R}$.

$\square$, $\boxed{x=\frac{343}{8},\ y=\frac{1}{8}}$



Question 96 (*****)

A system of simultaneous equations is given below

$$\begin{aligned} x+y+z&=1\\ x^2+y^2+z^2&=21\\ x^3+y^3+z^3&=55\,. \end{aligned}$$

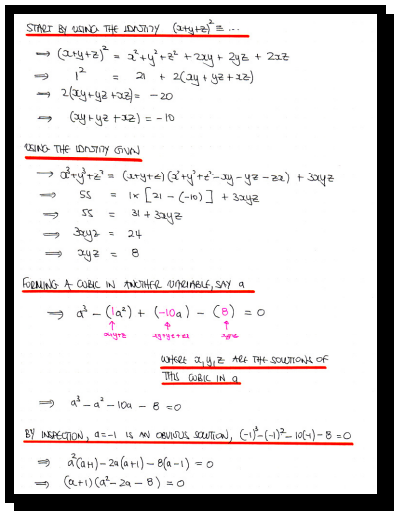
By forming an auxiliary cubic equation find the solution to the above system.

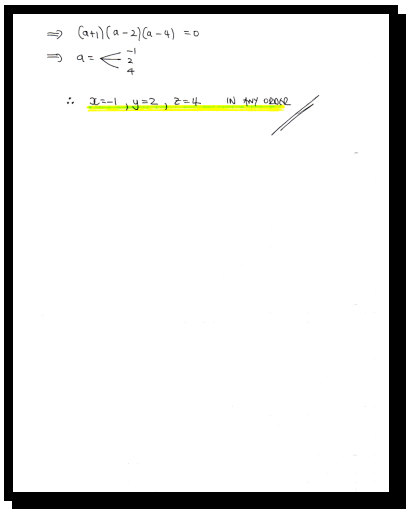
You may find the identity

$$x^3+y^3+z^3\equiv(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)+3xyz\,,$$

useful in this question.

$\boxed{}$, $\boxed{x,y,z=-2,-1,4 \text{ in any order}}$





Question 97 **(*****)**

The real numbers a, b, c and d satisfy

$$\frac{a}{b}=\frac{c}{d},\ a\neq b\neq c\neq d\neq 0.$$

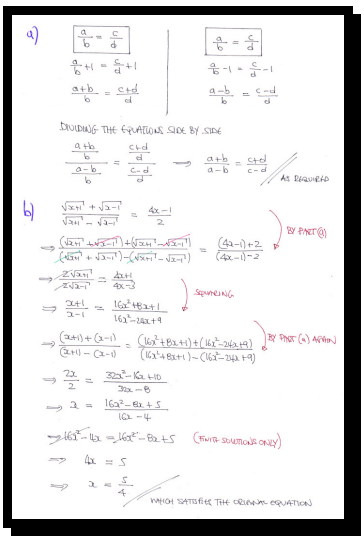
a) Show that

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}.$$

b) By using the result of part **(a)** or otherwise solve the equation

$$\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}=\frac{4x-1}{2},\ x>1.$$

$\square$, $\boxed{x=\frac{5}{4}}$

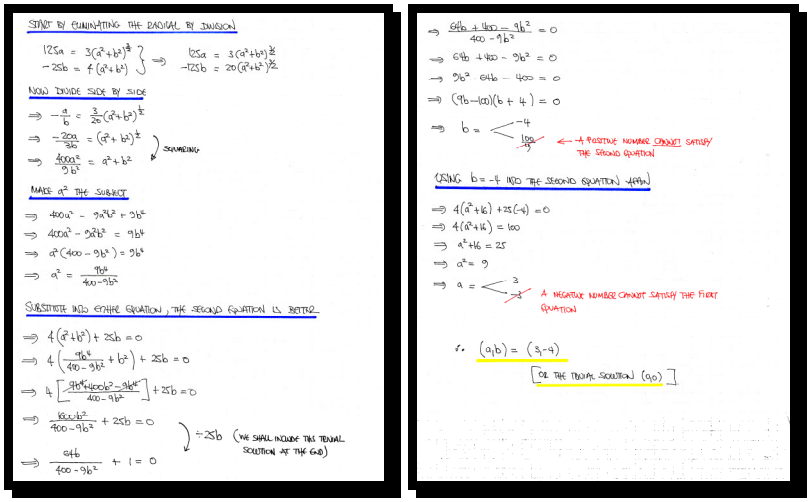


Question 98 (*****)

If $a \in \mathbb{R}$, $b \in \mathbb{R}$, solve the following simultaneous equations.

$$3\left(a^2+b^2\right)^{\frac{3}{2}}-125a=0 \qquad \text{and} \qquad 4\left(a^2+b^2\right)+25b=0 .$$

$\boxed{}$, $\boxed{(a,b)=(3,-4)}$



Question 99 (*****)

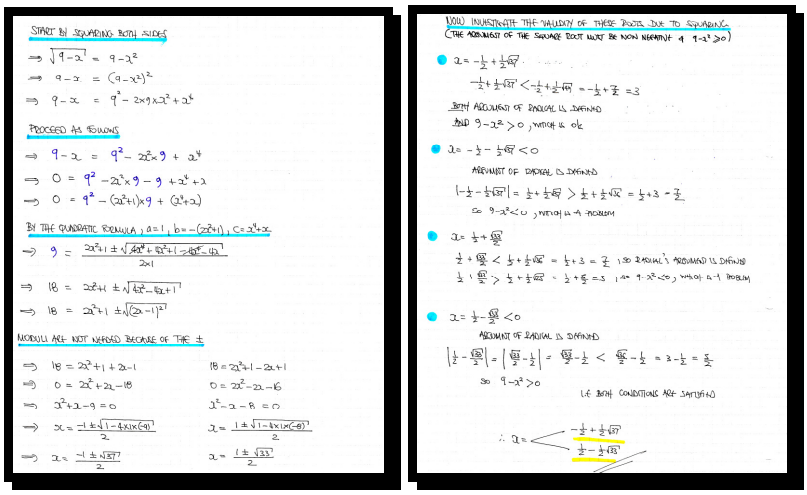
It is required to find the real solutions of the equation

$$\sqrt{9-x} = 9-x^2 .$$

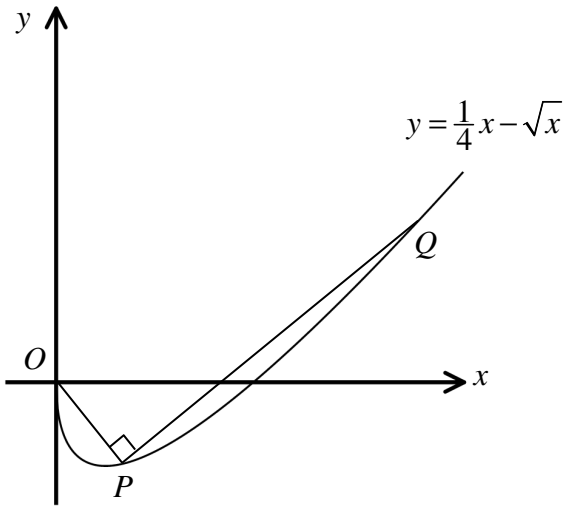
Solve the equation by considering a quadratic equation with variable coefficients in x.

You must fully justify the validity of your answers.

V, ☐, proof



Question 100 **(*****)**



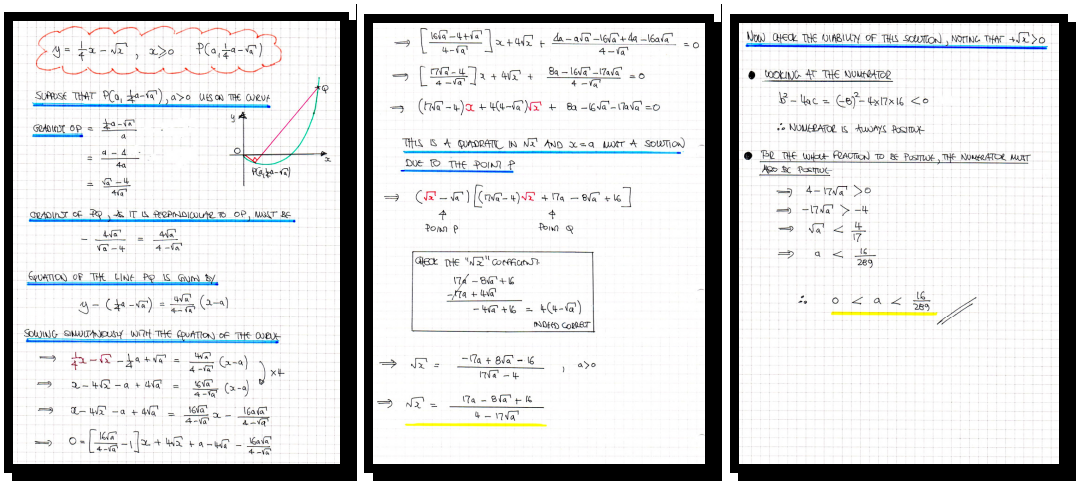
The figure above show the curve with equation

$$y=\frac{1}{4}x-\sqrt{x}\ ,\ x\in\mathbb{R}\ ,\ x\geq 0\ .$$

The points P and Q lie on the curve, so that $\angle OPQ=90°$, where O is the origin.

Determine the range of possible values of the x coordinate of P.

$$\boxed{}\ ,\ \boxed{0<x<\frac{16}{289}}$$



Question 101 **(*****)**

Find, in exact trigonometric form where appropriate, the real solutions of the following polynomial equation

$$x^7-7x^6-21x^5+35x^4+35x^3-21x^2-7x+1=0\,.$$

$\square$, $\boxed{x=\tan\left(\frac{\pi}{28}\right)}$, $\boxed{x=\tan\left(\frac{5\pi}{28}\right)}$, $\boxed{x=\tan\left(\frac{9\pi}{28}\right)}$, $\boxed{x=\tan\left(\frac{13\pi}{28}\right)}$, $\boxed{x=\tan\left(\frac{17\pi}{28}\right)}$, $\boxed{x=\tan\left(\frac{3\pi}{4}\right)=-1}$, $\boxed{x=\tan\left(\frac{25\pi}{28}\right)}$

