DIFFERENTIATION PRACTICE

THE CHAIN RULE WITH ALGEBRAIC FUNCTIONS

Question 1

1. $y=(2x+1)^4$

2. $y=(3x-2)^6$

3. $y=(8x-1)^{-1}$

4. $y=(6x+1)^{\frac{1}{2}}$

5. $y=(4x+3)^{\frac{3}{2}}$

6. $y=(1-x)^4$

7. $y=2(5x-2)^3$

8. $y=2(3x+1)^{\frac{1}{2}}$

9. $y=4(1-5x)^{-\frac{1}{2}}$

10. $y=6(1-2x)^{\frac{1}{3}}$

11. $y=(3x^2+1)^5$

12. $y=(2x^2-3x)^4$

13. $y=(5-2x+x^2)^{-2}$

14. $y=\dfrac{4}{(5x+9)^3}$

15. $y=\dfrac{3}{\sqrt{3-2x}}$

16. $y=\dfrac{1}{4x+1}$

17. $y=\dfrac{2}{3(2x+7)^2}$

18. $y=\dfrac{3}{x^2+1}$

19. $y=\dfrac{4}{\sqrt{4-3x^2}}$

20. $y=\dfrac{1}{1-x^3}$

Question 2

1. $y=(4x+1)^3$

2. $y=(2x-1)^7$

3. $y=(6x-5)^{-1}$

4. $y=(4x+1)^{\frac{1}{2}}$

5. $y=(6x-3)^{\frac{5}{2}}$

6. $y=(1-2x)^8$

7. $y=4(2x-3)^5$

8. $y=3(6x-1)^{\frac{1}{2}}$

9. $y=6(1-3x)^{-\frac{1}{2}}$

10. $y=9(1-5x)^{\frac{1}{3}}$

11. $y=(2x^2-1)^5$

12. $y=(3x^2-4x)^3$

13. $y=(1-4x+x^2)^{-2}$

14. $y=\dfrac{2}{(4x-1)^3}$

15. $y=\dfrac{2}{\sqrt{5-2x}}$

16. $y=\dfrac{3}{2x+1}$

17. $y=\dfrac{3}{2\left(4x+1\right)^2}$

18. $y=\dfrac{4}{x^2-3}$

19. $y=\dfrac{5}{6\sqrt{1-3x^2}}$

20. $y=\dfrac{4}{4-x^3}$

Question 3

1. $y=(6x-5)^3$

2. $y=(1-3x)^6$

3. $y=(2x-5)^{-2}$

4. $y=(8x+3)^{\frac{1}{2}}$

5. $y=3(4x-1)^{\frac{5}{2}}$

6. $y=4\left(1-\frac{1}{2}x\right)^6$

7. $y=8\left(\frac{15}{16}x-3\right)^{1.2}$

8. $y=5\left(\frac{3}{5}x-1\right)^{\frac{1}{3}}$

9. $y=2\left(1-\frac{5}{2}x\right)^{1.4}$

10. $y=\dfrac{2}{4x-1}$

11. $y=\left(4x^2-3\right)^3$

12. $y=\left(4x^2+2x\right)^5$

13. $y=\left(5-4x-2x^2\right)^{-3}$

14. $y=\dfrac{5}{\left(4x^2-3\right)^2}$

15. $y=\dfrac{2}{x^4+1}$

16. $y=\dfrac{5}{2\sqrt{2x+1}}$

17. $y=\dfrac{5}{6\sqrt{1-x^3}}$

18. $y=\dfrac{4}{1+2\sqrt{x}}$

19. $y=\sqrt{2+\sqrt{x}}$

20. $y=\sqrt{3x+(2x+1)^4}$

THE CHAIN RULE WITH EXPONENTIALS & LOGS

Question 4

1. $y = \mathrm{e}^{2x}$
2. $y = \ln 2x$
3. $y = \mathrm{e}^{4x-1}$
4. $y = \ln(3x-4)$
5. $y = \mathrm{e}^{-3x}$
6. $y = \ln(x^2-4)$
7. $y = 3\mathrm{e}^{x^2}$
8. $y = (\ln x)^4$
9. $y = (1+\mathrm{e}^{x})^4$
10. $y = \ln x^4$
11. $y = 3(1+\mathrm{e}^{2x})^3$
12. $y = (2x+\ln x)^4$
13. $y = (4+\mathrm{e}^{-5x})^4$
14. $y = (x^3-3\ln x)^5$
15. $y = (x^2+\mathrm{e}^{2x})^4$

16. $y=\sqrt{5x-3\ln x}$

17. $y=\sqrt{1+\mathrm{e}^{2x}}$

18. $y=\ln\left(\mathrm{e}^{2x}+3\right)$

19. $y=\mathrm{e}^{(2x+1)^5}$

20. $y=\ln\left(\mathrm{e}^{4}+1\right)$

Question 5

1. $y = \mathrm{e}^{4x}$

2. $y = \ln 4x$

3. $y = \mathrm{e}^{2x-5}$

4. $y = \ln(5x-4)$

5. $y = 3\mathrm{e}^{-2x}$

6. $y = 3\ln\left(x^2+1\right)$

7. $y = 2\mathrm{e}^{-x^2}$

8. $y = 2(\ln x)^3$

9. $y = 2\left(1+\mathrm{e}^{x}\right)^3$

10. $y = 2\ln x^{\frac{1}{2}}$

11. $y = 4\sqrt{2+\mathrm{e}^{-2x}}$

12. $y = \dfrac{3}{2x-\ln x}$

13. $y = 10\left(x^2+\mathrm{e}^{-\frac{1}{2}x}\right)^3$

14. $y = \dfrac{1}{2}\left(\mathrm{e}^{3x}-3\ln x\right)^{\frac{4}{3}}$

15. $y = \left(\mathrm{e}^{2x}+\mathrm{e}^{-2x}\right)^4$

16. $y = \dfrac{2}{\sqrt{4x-\ln x}}$

17. $y=\sqrt{1+2\mathrm{e}^{2x^2}}$

18. $y=\ln\left(\ln x\right)$

19. $y=\mathrm{e}^{\mathrm{e}^x}$

20. $y=\ln\left(\mathrm{e}^{4+\ln 2}+1\right)$

Question 6

1. $y = \mathrm{e}^{3x}$

2. $y = \ln 5x$

3. $y = \mathrm{e}^{3x+2}$

4. $y = \ln\left(2x+7\right)$

5. $y = 4\mathrm{e}^{-5x}$

6. $y = 2\ln\left(4x^2-3\right)$

7. $y = \mathrm{e}^{3x} - 3\mathrm{e}^{-x^2}$

8. $y = \ln x^5 + \left(\ln x\right)^5$

9. $y = 3\left(2-\mathrm{e}^{x}\right)^4$

10. $y = \ln\sqrt{x} + \sqrt{\ln x}$

11. $y = 3\sqrt{1+\mathrm{e}^{-3x}}$

12. $y = \dfrac{1}{x^3+\ln x}$

13. $y = 10\left(x^4 - 2\mathrm{e}^{-\frac{1}{2}x}\right)^4$

14. $y = \dfrac{1}{3}\left(\mathrm{e}^{6x}+3\ln x\right)^{\frac{1}{3}}$

15. $y = \left(2\mathrm{e}^{3x}-3\mathrm{e}^{-2x}\right)^3$

16. $y = \dfrac{4}{\sqrt{2x-3\ln x}}$

17. $y=\sqrt{1-2\mathrm{e}^{-3x^2}}$

18. $y=\ln\left(\ln\left(\ln x\right)\right)$

19. $y=2\mathrm{e}^{2\mathrm{e}^{2x}}$

20. $y=3\ln\left(\mathrm{e}^{\sqrt{5}+\ln 2}-\ln 3\right)$

THE CHAIN RULE WITH SINES & COSINES

Question 7

1. $y = \sin 4x$
2. $y = 2\sin 3x$
3. $y = \cos 3x$
4. $y = 6\cos\left(\frac{2}{3}x\right)$
5. $y = 4\sin\left(\frac{x}{2}\right)$
6. $y = 3\sin\left(5x - 1\right)$
7. $y = 3\cos\left(2x - \frac{\pi}{3}\right)$
8. $y = 2\cos\left(\frac{\pi}{4} - 2x\right)$
9. $y = 6\cos\left(1 - \frac{3x}{2}\right)$
10. $y = 2\sin x^4$
11. $y = 2\sin^4 x$
12. $y = 4\cos x^3$
13. $y = 4\cos^3 x$
14. $y = 3\sin^5 x$
15. $y = 2\cos\sqrt{x}$
16. $y = 2\sin^3 2x$

17. $y = 2\left(3\cos 2x + 1\right)^4$

18. $y = \sqrt{1 - 2\cos x}$

19. $y = \left(2\sin 3x - 3\cos 2x\right)^3$

20. $y = \sin^3\left(\frac{\pi}{2}\right)$

Question 8

1. $y = \sin 5x$
2. $y = 4\sin 2x$
3. $y = \cos 4x$
4. $y = 8\cos\left(\frac{1}{2}x\right)$
5. $y = 4\sin\left(\frac{x}{4}\right)$
6. $y = 2\sin\left(3x - 2\right)$
7. $y = 5\cos\left(3x - \frac{\pi}{4}\right)$
8. $y = 4\cos\left(\frac{\pi}{2} - 3x\right)$
9. $y = 8\cos\left(3 - \frac{5x}{2}\right)$
10. $y = \frac{1}{2}\sin x^6$
11. $y = \frac{1}{2}\sin^6 x$
12. $y = 10\cos x^5$
13. $y = 10\cos^5 x$
14. $y = 3\sin^7 x$
15. $y = 2\cos\left(x^{\frac{3}{2}}\right)$
16. $y = \frac{1}{6}\sin^4 3x$
17. $y = \frac{1}{3}\left(2\sin 3x + 3\right)^5$

18. $y=\sqrt{1-\cos 6x}$

19. $y=\left(4\sin 3x-3\cos 4x\right)^3$

20. $y=\sin\left(\frac{\pi}{4}\right)$

THE CHAIN RULE WITH TAN, COT, SEC, COSEC

Question 9

1. $y = 4\tan 3x$

2. $y = 2\tan\left(2x + \frac{\pi}{4}\right)$

3. $y = 3\tan^4 x$

4. $y = 3\tan 2x$

5. $y = 12\tan\left(\frac{\pi x}{4}\right)$

6. $y = \cot 2x$

7. $y = 3\tan 2x - \cot 3x$

8. $y = 4\sec 2x$

9. $y = 2\operatorname{cosec} 3x$

10. $y = 4\sec\frac{x}{2} - 6\operatorname{cosec}\frac{2x}{3}$

11. $y = 2\cot 4x - 2\sec 3x$

12. $y = 3\tan 5x - 6\operatorname{cosec} 2x$

13. $y = \tan^6 x$

14. $y = 3\cot^4 x$

15. $y = 3\sec^2 x$

16. $y = 4\operatorname{cosec}^4 x$

17. $y = 2\tan^4 3x$

18. $y = 2\cot^2 4x$

19. $y = 4\sec^4 2x$

20. $y = 6\operatorname{cosec}^3\left(\frac{x}{2}\right)$

Question 10

1. $y = 3\tan 5x$

2. $y = 4\tan\left(3x + \frac{\pi}{3}\right)$

3. $y = \frac{1}{2}\tan^6 x$

4. $y = 10\tan\frac{1}{2}x$

5. $y = 12\tan\left(\frac{5\pi x}{6}\right)$

6. $y = \cot 7x$

7. $y = 4\tan 3x - 2\cot 2x$

8. $y = 3\sec 4x$

9. $y = 6\operatorname{cosec} 2x$

10. $y = 6\sec\frac{x}{3} - 4\operatorname{cosec}\frac{3x}{4}$

11. $y = 7\cot 2x - 3\sec 3x$

12. $y = 2\tan 7x - 7\operatorname{cosec} 2x$

13. $y = \tan^3 x$

14. $y = 8\cot^5 x$

15. $y = \frac{1}{2}\sec^4 x$

16. $y = \frac{3}{4}\operatorname{cosec}^6 x$

17. $y = 2\tan^6 2x$

18. $y = 2\cot^3 3x$

19. $y = 3\sec^3 3x$

20. $y = 12\operatorname{cosec}^3\left(\frac{x}{4}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS

Question 11

1. $y = \sin 2x$
2. $y = 3\cos 2x$
3. $y = 4\tan 3x$
4. $y = 6\sin\left(\frac{1}{2}x\right)$
5. $y = 3\cos\left(\frac{x}{3}\right)$
6. $y = 2\sin(3x-1)$
7. $y = 2\cos\left(4x-\frac{\pi}{3}\right)$
8. $y = 2\tan\left(2x+\frac{\pi}{4}\right)$
9. $y = 9\cos\left(\frac{\pi}{6}-3x\right)$
10. $y = 2\sin x^3$
11. $y = 2\sin^3 x$
12. $y = 4\cos x^2$
13. $y = 5\cos^2 x$
14. $y = 3\tan^4 x$

15. $y = 4\sin\sqrt{x}$

16. $y = \sin^5 2x$

17. $y = (3\sin x + 2)^4$

18. $y = \sqrt{1+4\sin x}$

19. $y = (\sin x - \cos x)^3$

20. $y = \sin^3\left(\frac{\pi}{6}\right)$

Question 12

1. $y = 3\sin 3x$

2. $y = 2\cos 4x$

3. $y = 3\tan 2x$

4. $y = 4\sin\left(\frac{3}{2}x\right)$

5. $y = 2\cos\left(\frac{x}{4}\right)$

6. $y = \frac{2}{3}\sin\left(6x - 5\right)$

7. $y = 3\cos\left(\frac{\pi}{4} - 4x\right)$

8. $y = 12\tan\left(\frac{\pi x}{4}\right)$

9. $y = \frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6} - \sqrt{3}x\right)$

10. $y = \sin x^4$

11. $y = \sin^4 x$

12. $y = \cos^3 x$

13. $y = 2\sin^5 x$

14. $y = \tan^6 x$

15. $y = 4\cos^2 x$

16. $y = 3\cot^4 x$

17. $y = 3\sec^2 x$

18. $y = 4\operatorname{cosec}^4 x$

19. $y = 2\sin^5 2x$

20. $y = 4\cos^3 2x$

21. $y = 2\tan^4 3x$

22. $y = 2\cot^2 4x$

23. $y = 4\sec^4 2x$

24. $y = 6\operatorname{cosec}^3\left(\frac{x}{2}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS, EXPONENTIALS AND LOGARITHMS

Question 13

1. $y = 4\mathrm{e}^{\sin x}$
2. $y = \sin\left(\mathrm{e}^{2x}\right)$
3. $y = \ln\left(\sin x\right)$
4. $y = \sin\left(\ln x\right)$
5. $y = \mathrm{e}^{2\tan x}$
6. $y = \tan\left(\mathrm{e}^{-x}\right)$
7. $y = \cos\left(3\ln x\right)$
8. $y = 2\mathrm{e}^{\cos 2x}$
9. $y = \sin^4\left(\mathrm{e}^{x}\right)$
10. $y = \mathrm{e}^{\sin^2 x}$

MIXED CHAIN RULE

Question 14

1. $y=(3x+1)^8$
2. $y=\ln 2x$
3. $y=3\sin 2x+2\cos 3x$
4. $y=\mathrm{e}^{3-2x}$
5. $y=\ln\left(x^2+1\right)$
6. $y=\dfrac{4}{(2x-1)^2}$
7. $y=4\cot 3x$
8. $y=\ln\left(4-x^3\right)$
9. $y=\tan\left(2x^2+3\right)$
10. $y=2\sin 4x-3\cos 2x$
11. $y=2\mathrm{e}^{3x}$
12. $y=\ln(\sin x)$
13. $y=\cos(\ln x)$
14. $y=\mathrm{e}^{\sin x}$
15. $y=4\cos 3x-2\sin 4x$

16. $y=\dfrac{3}{(4x-2)^3}$

17. $y=(3-6x)^{\frac{5}{2}}$

18. $y=\ln\left(2x^2+3x-1\right)$

19. $y=\mathrm{e}^{x^2}$

20. $y=\sin\left(x^2\right)$

21. $y=\sin^3 x$

22. $y=\cos\left(x^2-1\right)$

23. $y=3\cot 4x$

24. $y=\mathrm{e}^{\tan x}$

25. $y=\left(\mathrm{e}^{2x}+2\right)^3$

26. $y=3(2x+1)^6$

27. $y=3\ln 4x$

28. $y=4\sin 3x-3\cos 2x$

29. $y=4\mathrm{e}^{1-4x}$

30. $y=3\ln\left(2x^2+1\right)$

31. $y=\dfrac{4}{\sqrt{2x-1}}$

32. $y=4\tan 2x$

33. $y=\ln(\sin 2x)$

34. $y = \tan^4 x$

35. $y = 2\sin\left(\frac{1}{2}x\right) - 3\cos\left(\frac{2}{3}x\right)$

36. $y = 2\mathrm{e}^{x^2}$

37. $y = \ln\left(3\sin 2x\right)$

38. $y = 2\cos\left(2\ln x\right)$

39. $y = 2\mathrm{e}^{\sin 3x}$

40. $y = 2\sin^4 3x$

41. $y = \frac{2}{\left(2x-1\right)^4}$

42. $y = \left(3-6\mathrm{e}^x\right)^{\frac{3}{2}}$

43. $y = \ln\left(\sec x + \tan x\right)$

44. $y = 2\mathrm{e}^{-x^4}$

45. $y = 4\sin\left(\sqrt{x}\right)$

46. $y = 4\sin^3 2x$

47. $y = \cos\left(\mathrm{e}^{2x}-1\right)$

48. $y = 6\tan 2x - 2\cot 3x$

49. $y = \mathrm{e}^{4\tan^2 x}$

50. $y = \ln\left(\cos 3x\right)$

THE PRODUCT RULE

Question 15

1. $y = x \sin x$

2. $y = 4x^2 \cos x$

3. $y = x^4 \mathrm{e}^x$

4. $y = x^3 \mathrm{e}^{-2x}$

5. $y = x^2 (2x-1)^5$

6. $y = 2\mathrm{e}^{3x} (3x-1)^4$

7. $y = 3\mathrm{e}^{-4x} \sin 2x$

8. $y = \cos 2x \tan 2x$

9. $y = (2x-1)^4 \sqrt{x}$

10. $y = (2x+1)^{\frac{3}{2}} (6x-1)^{\frac{1}{2}}$

Question 16

1. $y = x\cos x$

2. $y = 2x^4 \sin x$

3. $y = x^3 \mathrm{e}^{-x}$

4. $y = 4x\mathrm{e}^{2x}$

5. $y = x^2(3x-1)^4$

6. $y = 3\mathrm{e}^{-2x}(2x-1)^3$

7. $y = \mathrm{e}^{-2x}\tan 2x$

8. $y = \sin 2x \cot 2x$

9. $y = 2(1-4x)^3\sqrt{x}$

10. $y = (6x+1)^{\frac{1}{2}}(2x-1)^{-\frac{1}{2}}$

MIXED PRODUCT RULE

Question 17

1. $y = x^4 (4x-1)^3$

2. $y = 2x^3 (2x+3)^5$

3. $y = 6x^{\frac{1}{2}} (2x-1)^4$

4. $y = \mathrm{e}^{3x} \cos x$

5. $y = x^2 \mathrm{e}^{4x}$

6. $y = (4x+1)\mathrm{e}^{2x}$

7. $y = x^2 \tan x$

8. $y = 3x^2 \sin 2x$

9. $y = x^3 \tan 2x$

10. $y = x^4 \ln x$

11. $y = \mathrm{e}^{4x} \cos x$

12. $y = 4x^{\frac{1}{2}} \ln x$

13. $y = 4\mathrm{e}^{-x} \tan 2x$

14. $y = \mathrm{e}^{2x} (4\sin 2x + 3\cos 2x)$

15. $y = (2x+1)\cot 4x$

16. $y = (3x^2 - 4x)\tan 2x$

17. $y = x^4 \sin^2 x$

18. $y = 3x^2 \sec 2x$

19. $y = (4x+5)^{\frac{3}{2}} \mathrm{e}^{-2x}$

20. $y = x^3 \left(\sin 2x - 3\cos 2x\right)$

21. $y = \mathrm{e}^{6x} \cos^3 x$

22. $y = \sin x \tan^2 x$

23. $y = 4x^3 \operatorname{cosec} 3x$

24. $y = 3\mathrm{e}^{-4x} \cot 6x$

25. $y = 4x^{\frac{5}{2}} \sin^5 x$

26. $y = 5x \ln\left(x^2 - 2\right)$

27. $y = \sin x \tan x$

28. $y = x^2 \sin 4x$

29. $y = \mathrm{e}^{2x} \cos 3x$

30. $y = \left(4x-1\right)\mathrm{e}^{-x}$

31. $y = x^3 \tan 2x$

32. $y = x^4 \left(4x-1\right)^3$

33. $y = \left(3-2x^2\right)\cos 2x$

34. $y = \left(3x-1\right)^{\frac{1}{2}} \mathrm{e}^{x}$

35. $y = 2x^4 \ln x$

36. $y = \mathrm{e}^{x}\left(\sin x - \cos x\right)$

37. $y = \mathrm{e}^{-2x}\left(4x-1\right)^{3}$

38. $y = \left(x^{2}-2x+1\right)\mathrm{e}^{2x}$

39. $y = \left(4x+1\right)^{3}\left(1-3x\right)^{2}$

40. $y = x^{4}\sqrt{4x-1}$

41. $y = x\sin^{2} x$

42. $y = x^{-2}\ln x$

43. $y = \operatorname{cosec} x \cot x$

44. $y = \sqrt{x}\sin 2x$

45. $y = \left(4x+3\right)\tan 2x$

46. $y = \left(3x-1\right)^{5}\left(2x+1\right)^{\frac{3}{2}}$

47. $y = x^{2}\sin^{4} x$

48. $y = 6\mathrm{e}^{2x}\cos^{3} x$

49. $y = x^{2}\left(\mathrm{e}^{x}+\mathrm{e}^{-x}\right)$

50. $y = \left(1-2x\right)^{\frac{3}{2}}\left(3x+1\right)^{-\frac{1}{2}}$

51. $y = x^{5}\sqrt{x^{2}-1}$

THE QUOTIENT RULE

Question 18

1. $y=\dfrac{2x-5}{3x-1}$

2. $y=\dfrac{4x-1}{1-5x}$

3. $y=\dfrac{2x^2+1}{3x^2-1}$

4. $y=\dfrac{\ln x}{x}$

5. $y=\dfrac{\sin x}{x}$

6. $y=\dfrac{\mathrm{e}^{2x}}{x}$

7. $y=\dfrac{x^2}{(3x-1)^2}$

8. $y=\dfrac{\sin x}{\cos x}$

9. $y=\dfrac{x^2-1}{2x+3}$

10. $y=\dfrac{8x^2+8x+3}{(2x+1)^2}$

Question 19

1. $y=\dfrac{4x-3}{2x-5}$

2. $y=\dfrac{3x-1}{1-2x}$

3. $y=\dfrac{5x^2+1}{2x^2-3}$

4. $y=\dfrac{\ln x}{x^3}$

5. $y=\dfrac{\sin 2x}{x^2}$

6. $y=\dfrac{\mathrm{e}^{3x}}{2x}$

7. $y=\dfrac{4x^2}{(2x+3)^2}$

8. $y=\dfrac{\cos x}{\sin x}$

9. $y=\dfrac{3x^2+2}{x-1}$

10. $y=\dfrac{3x^2-6x+4}{(x-1)^2}$

MIXED QUOTIENT RULE

Question 20

1. $y = \dfrac{4x+3}{2x-3}$

2. $y = \dfrac{3-4x}{2x+1}$

3. $y = \dfrac{2x^2+1}{3x^2+1}$

4. $y = \dfrac{1+\cos x}{1+\sin x}$

5. $y = \dfrac{\ln x}{x^2}$

6. $y = \dfrac{x^2+1}{x^2-2}$

7. $y = \dfrac{3x^2+2}{x^2+5}$

8. $y = \dfrac{1-\cos x}{1+\cos x}$

9. $y = \dfrac{\sec x}{\tan x}$

10. $y = \dfrac{\mathrm{e}^x+2}{\mathrm{e}^x-2}$

11. $y = \dfrac{2x-1}{\sqrt{x+1}}$

12. $y = \dfrac{\sin^2 x}{\tan x}$

13. $y=\dfrac{\ln x}{x^4}$

14. $y=\dfrac{\sin 2x}{x}$

15. $y=\dfrac{x^2-1}{\sqrt{x+1}}$

16. $y=\dfrac{3x}{(4x-2)^3}$

17. $y=\dfrac{3\mathrm{e}^x}{2\mathrm{e}^x-1}$

18. $y=\dfrac{4x-1}{2x+1}$

19. $y=\dfrac{1-2x}{3x+2}$

20. $y=\dfrac{4x^3+1}{2x^3+1}$

21. $y=\dfrac{1-\sin x}{1+\sin x}$

22. $y=\dfrac{\ln 2x}{x^3}$

23. $y=\dfrac{2x^2+3}{x^2-1}$

24. $y=\dfrac{2x^2+3}{x+1}$

25. $y=\dfrac{1+\cos x}{1-\cos x}$

26. $y=\dfrac{\sec x}{\sin x}$

27. $y=\dfrac{\mathrm{e}^{2x}+2}{\mathrm{e}^{2x}-1}$

28. $y=\dfrac{2x-5}{\sqrt{4x+1}}$

29. $y=\dfrac{\cos^2 x}{\tan x}$

30. $y=\dfrac{\ln x}{\sqrt{x}}$

31. $y=\dfrac{\sin 2x}{x^2}$

32. $y=\dfrac{4x}{\sqrt{2x+1}}$

33. $y=\dfrac{x^3}{(2x-1)^4}$

34. $y=\dfrac{4\mathrm{e}^{x}}{\mathrm{e}^{x}+2}$

Question 21

Prove that:

1. $\frac{d}{dx}\left(\mathrm{e}^{2x}\left(4\sin 2x+3\cos 2x\right)\right)=2\mathrm{e}^{2x}\left(\sin 2x+7\cos 2x\right)$ **(**)**

2. $\frac{d}{dx}\left(\mathrm{e}^{2x}\left(x^2-4x-2\right)\right)=2\mathrm{e}^{2x}\left(x^2-3x-4\right)$ **(**)**

3. $\frac{d}{dx}\left(\frac{4x}{4x-3}\right)=-\frac{12}{\left(4x-3\right)^2}$ **(**)**

4. $\frac{d}{dx}\left(\frac{4x+3}{2x-3}\right)=-\frac{18}{\left(2x-3\right)^2}$ **(**)**

5. $\frac{d}{dx}\left(\frac{3-4x}{2x+1}\right)=-\frac{10}{\left(2x+1\right)^2}$ **(**)**

6. $\frac{d}{dx}\left(\frac{2x^2+1}{3x^2+1}\right)=-\frac{2x}{\left(3x^2+1\right)^2}$ **(**)**

7. $\frac{d}{dx}\left(\ln\left(\sec x+\tan x\right)\right)=\sec x$ **(***)**

8. $\frac{d}{dx}\left(x^4\left(4x-1\right)^3\right)=4x^3\left(7x-1\right)\left(4x-1\right)^2$ **(***)**

9. $\frac{d}{dx}\left(2x^3\left(2x+3\right)^5\right)=2x^2\left(16x+9\right)\left(2x+3\right)^4$ **(***)**

10. $\frac{d}{dx}\left(x^4\left(4x-1\right)^3\right)=4x^3\left(7x-1\right)\left(4x-1\right)^2$ **(***)**

11. $\frac{d}{dx}\left(2\mathrm{e}^{-3x}\left(2x+1\right)^{\frac{3}{2}}\right)=-12x\mathrm{e}^{-3x}\left(2x+1\right)^{\frac{1}{2}}$ **(****)**

12. $\frac{d}{dx}\left(\frac{x-4}{\sqrt{x}+2}\right)=\frac{1}{2\sqrt{x}}$ **(****+)**

13. $\dfrac{d}{dx}\left(\mathrm{e}^{-2x}\left(4x-1\right)^{3}\right)=2\left(7-4x\right)\left(4x-1\right)^{2}\mathrm{e}^{-2x}$ **(***)**

14. $\dfrac{d}{dx}\left(\left(x^{2}-2x+1\right)\mathrm{e}^{2x}\right)=2x\left(x-1\right)\mathrm{e}^{2x}$ **(***)**

15. $\dfrac{d}{dx}\left(x^{4}\sqrt{4x-1}\right)=\dfrac{2x^{3}\left(9x-2\right)}{\sqrt{4x-1}}$ **(****)**

16. $\dfrac{d}{dx}\left(4\sqrt{x}\ln x\right)=\dfrac{2\left(2+\ln x\right)}{\sqrt{x}}$ **(****)**

17. $\dfrac{d}{dx}\left(\left(4x+5\right)^{\frac{3}{2}}\mathrm{e}^{-2x}\right)=-4\left(2x+1\right)\sqrt{4x+5}\,\mathrm{e}^{-2x}$ **(****)**

18. $\dfrac{d}{dx}\left(6\sqrt{x}\left(2x-1\right)^{4}\right)=\dfrac{3\left(18x-1\right)\left(2x-1\right)^{3}}{\sqrt{x}}$ **(****)**

19. $\dfrac{d}{dx}\left(\dfrac{1-\cos x}{1+\cos x}\right)=\dfrac{2\sin x}{\left(1+\cos x\right)^{2}}$ **(***)**

20. $\dfrac{d}{dx}\left(\dfrac{3+\sin 2x}{2+\cos 2x}\right)=\dfrac{6\sin 2x+4\cos 2x+2}{\left(2+\cos 2x\right)^{2}}$ **(***)**

21. $\dfrac{d}{dx}\left(\dfrac{5x^{2}-10x+8}{\left(x-1\right)^{2}}\right)=-\dfrac{6}{\left(x-1\right)^{3}}$ **(***)**

22. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\tan x}\right)=-\operatorname{cosec}x\cot x$ **(****)**

23. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{x}+2}{\mathrm{e}^{x}-2}\right)=-\dfrac{4}{\left(\mathrm{e}^{x}-2\right)^{2}}$ **(***)**

24. $\dfrac{d}{dx}\left(\dfrac{2x+1}{\sqrt{x+1}}\right)=\dfrac{2x+3}{2\left(x+1\right)^{\frac{3}{2}}}$ **(****)**

25. $\dfrac{d}{dx}\left(\dfrac{x-1}{\sqrt{x}+1}\right)=\dfrac{1}{2\sqrt{x}}$ **(****)**

26. $\dfrac{d}{dx}\left(\dfrac{\sin^2 x}{\tan x}\right)=\cos 2x$ **(****)**

27. $\dfrac{d}{dx}\left(\dfrac{3\mathrm{e}^{x}}{2\mathrm{e}^{x}-1}\right)=-\dfrac{3\mathrm{e}^{x}}{\left(2\mathrm{e}^{x}-1\right)^2}$ **(***)**

28. $\dfrac{d}{dx}\left(\dfrac{4x-1}{2x+1}\right)=\dfrac{6}{\left(2x+1\right)^2}$ **(**)**

29. $\dfrac{d}{dx}\left(\dfrac{1-2x}{3x+2}\right)=-\dfrac{7}{\left(3x+2\right)^2}$ **(**)**

30. $\dfrac{d}{dx}\left(\dfrac{4x^3+1}{2x^3+1}\right)=\dfrac{6x^2}{\left(2x^3+1\right)^2}$ **(**)**

31. $\dfrac{d}{dx}\left(\dfrac{1-\sin x}{1+\sin x}\right)=-\dfrac{2\cos x}{\left(1+\sin x\right)^2}$ **(**)**

32. $\dfrac{d}{dx}\left(\ln\left(\dfrac{1-\sin x}{1+\sin x}\right)\right)=-\sec x$ **(****)**

33. $\dfrac{d}{dx}\left(\ln\left(\dfrac{x+1}{x-1}\right)\right)=-\dfrac{2}{x^2-1}$ **(***)**

34. $\dfrac{d}{dx}\left(\dfrac{2\sin 3x}{1+\cos 3x}\right)=\dfrac{6}{1+\cos 3x}$ **(***)**

35. $\dfrac{d}{dx}\left(\dfrac{2x^2+3}{x^2-1}\right)=-\dfrac{10x}{\left(x^2-1\right)^2}$ **(**)**

36. $\dfrac{d}{dx}\left(\dfrac{2x^2+3}{x+1}\right)=\dfrac{2x^2+4x-3}{\left(x+1\right)^2}$ **(***)**

37. $\dfrac{d}{dx}\left(\dfrac{3x^2+6x-5}{\left(x+1\right)^2}\right)=\dfrac{16}{\left(x+1\right)^3}$ **(***)**

38. $\dfrac{d}{dx}\left(\dfrac{1+\cos x}{1-\cos x}\right)=-\cot\dfrac{x}{2}\operatorname{cosec}^2\dfrac{x}{2}$ **(*****)**

39. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\sin x}\right)=\sec^2 x-\operatorname{cosec}^2 x$ **(****)**

40. $\dfrac{d}{dx}\left(\ln\left(\dfrac{1}{1-\sin x}\right)\right)=\dfrac{1+\sin x}{\cos x}$ **(****)**

41. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{2x}+2}{\mathrm{e}^{2x}-1}\right)=-\dfrac{6\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^2}$ **(***)**

42. $\dfrac{d}{dx}\left(\dfrac{2x-5}{\sqrt{4x+1}}\right)=\dfrac{4\left(x+3\right)}{\left(4x+1\right)^{\frac{3}{2}}}$ **(****)**

43. $\dfrac{d}{dx}\left(\dfrac{\ln x}{\sqrt{x}}\right)=\dfrac{2-\ln x}{2x^{\frac{3}{2}}}$ **(****)**

44. $\dfrac{d}{dx}\left(\dfrac{4x}{\sqrt{2x+1}}\right)=\dfrac{4\left(x+1\right)}{\left(2x+1\right)^{\frac{3}{2}}}$ **(****)**

45. $\dfrac{d}{dx}\left(\dfrac{x^3}{\left(2x-1\right)^4}\right)=-\dfrac{x^2\left(2x+3\right)}{\left(2x-1\right)^5}$ **(****)**

46. $\dfrac{d}{dx}\left(\ln\sqrt{4x+1}\right)=\dfrac{2}{4x+1}$ **(**)**

47. $\dfrac{d}{dx}\left(\dfrac{4\mathrm{e}^x}{\mathrm{e}^x+2}\right)=\dfrac{8\mathrm{e}^x}{\left(\mathrm{e}^x+2\right)^2}$ **(***)**

48. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\sin x}\right)=-4\operatorname{cosec}2x\cot 2x$ **(****)**

49. $\dfrac{d}{dx}\left(\ln\left(\tan x\right)\right)=2\operatorname{cosec}2x$ **(***)**

50. $\dfrac{d}{dx}\left(\dfrac{2x\left(x^2+6x+12\right)}{\left(x+2\right)^3}\right)=\dfrac{48}{\left(x+2\right)^4}$ (****)

51. $\dfrac{d}{dx}\left(\sqrt{\dfrac{x+1}{x-1}}\right)=-\dfrac{1}{\left(x-1\right)\sqrt{x^2-1}}$ (****)

52. $\dfrac{d}{dx}\left(\ln\left(\dfrac{1+\cos x}{1-\cos x}\right)\right)=-2\operatorname{cosec} x$ (****)

53. $\dfrac{d}{dx}\left(\cos 2x+\tan x\sin 2x\right)=0$ (*****)

54. $\dfrac{d}{dx}\left(\ln\left(\dfrac{\sqrt{\mathrm{e}^x+1}-1}{\sqrt{\mathrm{e}^x+1}+1}\right)\right)=\dfrac{1}{\sqrt{\mathrm{e}^x+1}}$ (*****)

55. $\dfrac{d}{dx}\left(\ln\left(\dfrac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right)\right)=\dfrac{2}{x\sqrt{1-x^2}}$ (*****)

56. $\dfrac{d}{dx}\left(\dfrac{\cos 2x}{\sqrt{1+\sin 2x}}\right)=-\sin x-\cos x$ (*****)

57. $\dfrac{d}{dx}\left(\ln\left(x+\sqrt{x^2+8}\right)-\dfrac{x}{x^2+8}\right)=\dfrac{x^2}{\left(x^2+8\right)^{\frac{3}{2}}}$ (*****)

58. $\dfrac{d}{dx}\left(\dfrac{\sqrt{\mathrm{e}^{2x}-9}}{\mathrm{e}^x}\right)=\dfrac{9}{\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-9}}$ (****)

59. $\dfrac{d}{dx}\left(\ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2x}-9}\right)\right)=\dfrac{\mathrm{e}^x}{\sqrt{\mathrm{e}^{2x}-9}}$ (*****)

60. $\dfrac{d}{dx}\left(\dfrac{\sqrt{\mathrm{e}^{2x}+4}}{\mathrm{e}^x}\right)=-\dfrac{4}{\mathrm{e}^x\sqrt{\mathrm{e}^{2x}+4}}$ (****)

61. $\dfrac{d}{dx}\left(\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-1}-\ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2x}-1}\right)\right)=2\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-1}$ (*****)

THE CHAIN RULE WITH ALGEBRAIC FUNCTIONS

Question 1

1. $\frac{d}{dx}\left((2x+1)^4\right)=8(2x+1)^3$

2. $\frac{d}{dx}\left((3x-2)^6\right)=18(3x-2)^5$

3. $\frac{d}{dx}\left((8x-1)^{-1}\right)=-8(8x-1)^{-2}$

4. $\frac{d}{dx}\left((6x+1)^{\frac{1}{2}}\right)=3(6x+1)^{-\frac{1}{2}}$

5. $\frac{d}{dx}\left((4x+3)^{\frac{3}{2}}\right)=6(4x+3)^{\frac{1}{2}}$

6. $\frac{d}{dx}\left((1-x)^4\right)=-4(1-x)^3$

7. $\frac{d}{dx}\left(2(5x-2)^3\right)=30(5x-2)^2$

8. $\frac{d}{dx}\left(2(3x+1)^{\frac{1}{2}}\right)=3(3x+1)^{-\frac{1}{2}}$

9. $\frac{d}{dx}\left(4(1-5x)^{-\frac{1}{2}}\right)=10(1-5x)^{-\frac{3}{2}}$

10. $\frac{d}{dx}\left(6(1-2x)^{\frac{1}{3}}\right)=-4(1-2x)^{-\frac{2}{3}}$

11. $\frac{d}{dx}\left((3x^2+1)^5\right)=30x(3x^2+1)^4$

12. $\frac{d}{dx}\left((2x^2-3x)^4\right)=4(4x-3)(2x^2-3x)^3$

13. $y=\left(5-2x+x^2\right)^{-2}=4(1-x)\left(5-2x+x^2\right)^{-3}$

14. $\dfrac{d}{dx}\left(\dfrac{4}{(5x+9)^3}\right)=-\dfrac{60}{(5x+9)^4}$

15. $\dfrac{d}{dx}\left(\dfrac{3}{\sqrt{3-2x}}\right)=\dfrac{3}{(3-2x)^{\frac{3}{2}}}$

16. $\dfrac{d}{dx}\left(\dfrac{1}{4x+1}\right)=-\dfrac{4}{(4x+1)^2}$

17. $\dfrac{d}{dx}\left(\dfrac{2}{3(2x+7)^2}\right)=-\dfrac{8}{3(2x+7)^3}$

18. $\dfrac{d}{dx}\left(\dfrac{3}{x^2+1}\right)=-\dfrac{6x}{\left(x^2+1\right)^2}$

19. $\dfrac{d}{dx}\left(\dfrac{4}{\sqrt{4-3x^2}}\right)=\dfrac{12x}{\left(4-3x^2\right)^{\frac{3}{2}}}$

20. $\dfrac{d}{dx}\left(\dfrac{1}{1-x^3}\right)=\dfrac{3x^2}{\left(1-x^3\right)^2}$

Question 2

1. $\dfrac{d}{dx}\left((4x+1)^3\right)=12(4x+1)^2$

2. $\dfrac{d}{dx}\left((2x-1)^7\right)=14(2x-1)^6$

3. $\dfrac{d}{dx}\left((6x-5)^{-1}\right)=-6(6x-5)^{-2}$

4. $\dfrac{d}{dx}\left((4x+1)^{\frac{1}{2}}\right)=2(4x+1)^{-\frac{1}{2}}$

5. $\dfrac{d}{dx}\left((6x-3)^{\frac{5}{2}}\right)=15(6x-3)^{\frac{3}{2}}$

6. $\dfrac{d}{dx}\left((1-2x)^8\right)=-16(1-2x)^7$

7. $\dfrac{d}{dx}\left(4(2x-3)^5\right)=40(2x-3)^4$

8. $\dfrac{d}{dx}\left(3(6x-1)^{\frac{1}{2}}\right)=9(6x-1)^{-\frac{1}{2}}$

9. $\dfrac{d}{dx}\left(6(1-3x)^{-\frac{1}{2}}\right)=9(1-3x)^{-\frac{3}{2}}$

10. $\dfrac{d}{dx}\left(9(1-5x)^{\frac{1}{3}}\right)=-15(1-5x)^{-\frac{2}{3}}$

11. $\dfrac{d}{dx}\left((2x^2-1)^5\right)=20x(2x^2-1)^4$

12. $\dfrac{d}{dx}\left((3x^2-4x)^3\right)=6(3x-2)(3x^2-4x)^2$

13. $\dfrac{d}{dx}\left((1-4x+x^2)^{-2}\right)=-4(x-2)(1-4x+x^2)^{-3}$

14. $\dfrac{d}{dx}\left(\dfrac{2}{(4x-1)^3}\right)=\dfrac{-24}{(4x-1)^4}$

15. $\dfrac{d}{dx}\left(\dfrac{2}{\sqrt{5-2x}}\right)=\dfrac{2}{(5-2x)^{\frac{3}{2}}}$

16. $\dfrac{d}{dx}\left(\dfrac{3}{2x+1}\right)=-\dfrac{6}{(2x+1)^2}$

17. $\dfrac{d}{dx}\left(\dfrac{3}{2(4x+1)^2}\right)=-\dfrac{12}{(4x+1)^3}$

18. $\dfrac{d}{dx}\left(\dfrac{4}{x^2-3}\right)=-\dfrac{8x}{\left(x^2-3\right)^2}$

19. $\dfrac{d}{dx}\left(\dfrac{5}{6\sqrt{1-3x^2}}\right)=\dfrac{5x}{2\left(1-3x^2\right)^{\frac{3}{2}}}$

20. $\dfrac{d}{dx}\left(\dfrac{4}{4-x^3}\right)=\dfrac{12x^2}{\left(4-x^3\right)^2}$

Question 3

1. $\frac{d}{dx}\left((6x-5)^3\right)=18(6x-5)^2$

2. $\frac{d}{dx}\left((1-3x)^6\right)=-18(1-3x)^5$

3. $\frac{d}{dx}\left((2x-5)^{-2}\right)=-4(2x-5)^{-3}$

4. $\frac{d}{dx}\left((8x+3)^{\frac{1}{2}}\right)=4(8x+3)^{-\frac{1}{2}}$

5. $\frac{d}{dx}\left(3(4x-1)^{\frac{5}{2}}\right)=30(4x-1)^{\frac{3}{2}}$

6. $\frac{d}{dx}\left(4\left(1-\frac{1}{2}x\right)^6\right)=-12\left(1-\frac{1}{2}x\right)^5$

7. $\frac{d}{dx}\left(8\left(\frac{15}{16}x-3\right)^{1.2}\right)=9\left(\frac{15}{16}x-3\right)^{0.2}$

8. $\frac{d}{dx}\left(5\left(\frac{3}{5}x-1\right)^{\frac{1}{3}}\right)=\left(\frac{3}{5}x-1\right)^{-\frac{2}{3}}$

9. $\frac{d}{dx}\left(2\left(1-\frac{5}{2}x\right)^{1.4}\right)=-7\left(1-\frac{5}{2}x\right)^{0.4}$

10. $\frac{d}{dx}\left(\frac{2}{4x-1}\right)=-\frac{8}{(4x-1)^2}$

11. $\frac{d}{dx}\left((4x^2-3)^3\right)=24x(4x^2-3)^2$

12. $\frac{d}{dx}\left((4x^2+2x)^5\right)=10(4x+1)(4x^2+2x)^4$

13. $\frac{d}{dx}\left((5-4x-2x^2)^{-3}\right)=12(x+1)(5-4x-2x^2)^{-4}$

14. $\dfrac{d}{dx}\left(\dfrac{5}{\left(4x^2-3\right)^2}\right)=-\dfrac{80x}{\left(4x^2-3\right)^3}$

15. $\dfrac{d}{dx}\left(\dfrac{2}{x^4+1}\right)=-\dfrac{8x^3}{\left(x^4+1\right)^{-2}}$

16. $\dfrac{d}{dx}\left(\dfrac{5}{2\sqrt{2x+1}}\right)=-\dfrac{5}{2\left(2x+1\right)^{\frac{3}{2}}}$

17. $\dfrac{d}{dx}\left(\dfrac{5}{6\sqrt{1-x^3}}\right)=\dfrac{5x^2}{4\left(1-x^3\right)^{\frac{3}{2}}}$

18. $\dfrac{d}{dx}\left(\dfrac{4}{1+2\sqrt{x}}\right)=-\dfrac{4}{\sqrt{x}\left(1+2\sqrt{x}\right)^2}$

19. $\dfrac{d}{dx}\left(\sqrt{2+\sqrt{x}}\right)=\dfrac{1}{4\sqrt{x}\sqrt{2+\sqrt{x}}}=\dfrac{1}{4\sqrt{2x+x\sqrt{x}}}$

20. $\dfrac{d}{dx}\left(\sqrt{3x+\left(2x+1\right)^4}\right)=\dfrac{3+8\left(2x+1\right)^3}{2\sqrt{3x+\left(2x+1\right)^4}}$

THE CHAIN RULE WITH EXPONENTIALS & LOGS

Question 4

1. $\frac{d}{dx}\left(\mathrm{e}^{2x}\right)=2\mathrm{e}^{2x}$

2. $\frac{d}{dx}\left(\ln 2x\right)=\frac{1}{x}$

3. $\frac{d}{dx}\left(\mathrm{e}^{4x-1}\right)=4\mathrm{e}^{4x-1}$

4. $\frac{d}{dx}\left(\ln\left(3x-4\right)\right)=\frac{3}{3x-4}$

5. $\frac{d}{dx}\left(\mathrm{e}^{-3x}\right)=-3\mathrm{e}^{-3x}$

6. $\frac{d}{dx}\left(\ln\left(x^2-4\right)\right)=\frac{2x}{x^2-4}$

7. $\frac{d}{dx}\left(3\mathrm{e}^{x^2}\right)=6x\mathrm{e}^{x^2}$

8. $\frac{d}{dx}\left(\left(\ln x\right)^4\right)=\frac{4\left(\ln x\right)^3}{x}$

9. $\frac{d}{dx}\left(\left(1+\mathrm{e}^{x}\right)^4\right)=4\mathrm{e}^{x}\left(1+\mathrm{e}^{x}\right)^3$

10. $\frac{d}{dx}\left(\ln x^4\right)=\frac{4}{x}$

11. $\frac{d}{dx}\left(3\left(1+\mathrm{e}^{2x}\right)^3\right)=18\mathrm{e}^{2x}\left(1+\mathrm{e}^{2x}\right)^2$

12. $\frac{d}{dx}\left(\left(2x+\ln x\right)^4\right)=4\left(2x+\ln x\right)^3\left(2+\frac{1}{x}\right)$

13. $\dfrac{d}{dx}\left(\left(4+\mathrm{e}^{-5x}\right)^4\right)=-20\mathrm{e}^{-5x}\left(4+\mathrm{e}^{-5x}\right)^3$

14. $\dfrac{d}{dx}\left(\left(x^3-3\ln x\right)^5\right)=15\left(x^3-3\ln x\right)^4\left(x^2-\dfrac{1}{x}\right)$

15. $\dfrac{d}{dx}\left(\left(x^2+\mathrm{e}^{2x}\right)^4\right)=8\left(x+\mathrm{e}^{2x}\right)\left(x^2+\mathrm{e}^{2x}\right)^3$

16. $\dfrac{d}{dx}\left(\sqrt{5x-3\ln x}\right)=\dfrac{1}{2\sqrt{5x-3\ln x}}\left(5-\dfrac{3}{x}\right)=\dfrac{5x-3}{2x\sqrt{5x-3\ln x}}$

17. $\dfrac{d}{dx}\left(\sqrt{1+\mathrm{e}^{2x}}\right)=\dfrac{\mathrm{e}^{2x}}{\sqrt{1+\mathrm{e}^{2x}}}$

18. $\dfrac{d}{dx}\left(\ln\left(\mathrm{e}^{2x}+3\right)\right)=\dfrac{2\mathrm{e}^{2x}}{\mathrm{e}^{2x}+3}$

19. $\dfrac{d}{dx}\left(\mathrm{e}^{(2x+1)^5}\right)=10\left(2x+1\right)^4\mathrm{e}^{(2x+1)^5}$

20. $\dfrac{d}{dx}\left(\ln\left(\mathrm{e}^4+1\right)\right)=0$

Question 5

1. $\dfrac{d}{dx}\left(\mathrm{e}^{4x}\right)=4\mathrm{e}^{4x}$

2. $\dfrac{d}{dx}\left(\ln 4x\right)=\dfrac{1}{x}$

3. $\dfrac{d}{dx}\left(\mathrm{e}^{2x-5}\right)=2\mathrm{e}^{2x-5}$

4. $\dfrac{d}{dx}\left(\ln\left(5x-4\right)\right)=\dfrac{5}{5x-4}$

5. $\dfrac{d}{dx}\left(3\mathrm{e}^{-2x}\right)=-6\mathrm{e}^{-2x}$

6. $\dfrac{d}{dx}\left(3\ln\left(x^2+1\right)\right)=\dfrac{6x}{x^2+1}$

7. $\dfrac{d}{dx}\left(2\mathrm{e}^{-x^2}\right)=-4x\mathrm{e}^{-x^2}$

8. $\dfrac{d}{dx}\left(2\left(\ln x\right)^3\right)=\dfrac{6}{x}\left(\ln x\right)^2$

9. $\dfrac{d}{dx}\left(2\left(1+\mathrm{e}^{x}\right)^3\right)=6\mathrm{e}^{x}\left(1+\mathrm{e}^{x}\right)^2$

10. $\dfrac{d}{dx}\left(2\ln x^{\frac{1}{2}}\right)=\dfrac{1}{x}$

11. $\dfrac{d}{dx}\left(4\sqrt{2+\mathrm{e}^{-2x}}\right)=-\dfrac{4\mathrm{e}^{-2x}}{\sqrt{2+\mathrm{e}^{-2x}}}$

12. $\dfrac{d}{dx}\left(\dfrac{3}{2x-\ln x}\right)=\dfrac{3\left(\frac{1}{x}-2\right)}{\left(2x-\ln x\right)^2}=\dfrac{3\left(1-2x\right)}{x\left(2x-\ln x\right)^2}$

13. $\dfrac{d}{dx}\left(10\left(x^2+\mathrm{e}^{-\frac{1}{2}x}\right)^3\right)=15\left(4x-\mathrm{e}^{-\frac{1}{2}x}\right)\left(x^2+\mathrm{e}^{-\frac{1}{2}x}\right)^2$

14. $\dfrac{d}{dx}\left(\dfrac{1}{2}\left(\mathrm{e}^{3x}-3\ln x\right)^{\frac{4}{3}}\right)=2\left(\mathrm{e}^{3x}-\dfrac{1}{x}\right)\left(\mathrm{e}^{3x}-3\ln x\right)^{\frac{1}{3}}$

15. $\dfrac{d}{dx}\left(\left(\mathrm{e}^{2x}+\mathrm{e}^{-2x}\right)^4\right)=8\left(\mathrm{e}^{2x}-\mathrm{e}^{-2x}\right)\left(\mathrm{e}^{2x}+\mathrm{e}^{-2x}\right)^3$

16. $\dfrac{d}{dx}\left(\dfrac{2}{\sqrt{4x-\ln x}}\right)=\left(\dfrac{1}{x}-4\right)\left(4x-\ln x\right)^{-\frac{3}{2}}=\dfrac{1-4x}{x\left(4x-\ln x\right)^{\frac{3}{2}}}$

17. $\dfrac{d}{dx}\left(\sqrt{1+2\mathrm{e}^{2x^2}}\right)=\dfrac{4x\mathrm{e}^{2x^2}}{\sqrt{1+2\mathrm{e}^{2x^2}}}$

18. $\dfrac{d}{dx}\left(\ln\left(\ln x\right)\right)=\dfrac{1}{x\ln x}$

19. $\dfrac{d}{dx}\left(\mathrm{e}^{\mathrm{e}^x}\right)=\mathrm{e}^x\,\mathrm{e}^{\mathrm{e}^x}$

20. $\dfrac{d}{dx}\left(\ln\left(\mathrm{e}^{4+\ln 2}+1\right)\right)=0$

Question 6

1. $\dfrac{d}{dx}\left(\mathrm{e}^{3x}\right)=3\mathrm{e}^{3x}$

2. $\dfrac{d}{dx}\left(\ln 5x\right)=\dfrac{1}{x}$

3. $\dfrac{d}{dx}\left(\mathrm{e}^{3x+2}\right)=3\mathrm{e}^{3x+2}$

4. $\dfrac{d}{dx}\left(\ln\left(2x+7\right)\right)=\dfrac{2}{2x+7}$

5. $\dfrac{d}{dx}\left(4\mathrm{e}^{-5x}\right)=-20\mathrm{e}^{-5x}$

6. $\dfrac{d}{dx}\left(2\ln\left(4x^2-3\right)\right)=\dfrac{16x}{4x^2-3}$

7. $\dfrac{d}{dx}\left(\mathrm{e}^{3x}-3\mathrm{e}^{-x^2}\right)=3\mathrm{e}^{3x}+6x\mathrm{e}^{-x^2}$

8. $\dfrac{d}{dx}\left(\ln x^5+\left(\ln x\right)^5\right)=\dfrac{5}{x}+\dfrac{5}{x}\left(\ln x\right)^4=\dfrac{5}{x}\left[1+\left(\ln x\right)^4\right]$

9. $\dfrac{d}{dx}\left(3\left(2-\mathrm{e}^{x}\right)^4\right)=-12\mathrm{e}^{x}\left(2-\mathrm{e}^{x}\right)^4$

10. $\dfrac{d}{dx}\left(\ln\sqrt{x}+\sqrt{\ln x}\right)=\dfrac{1}{2x}+\dfrac{1}{2x\sqrt{\ln x}}=\dfrac{1}{2x}\left(1+\dfrac{1}{\sqrt{\ln x}}\right)$

11. $\dfrac{d}{dx}\left(3\sqrt{1+\mathrm{e}^{-3x}}\right)=-\dfrac{9\mathrm{e}^{-3x}}{2\sqrt{1+\mathrm{e}^{-3x}}}$

12. $\dfrac{d}{dx}\left(\dfrac{1}{x^3+\ln x}\right)=-\left(x^3+\ln x\right)^{-2}\times\left(3x^2+\dfrac{1}{x}\right)=-\dfrac{3x^3+1}{x\left(x^3+\ln x\right)^2}$

13. $\dfrac{d}{dx}\left(10\left(x^4-2\mathrm{e}^{-\frac{1}{2}x}\right)^4\right)=40\left(4x^3+\mathrm{e}^{-\frac{1}{2}x}\right)\left(x^4-2\mathrm{e}^{-\frac{1}{2}x}\right)^3$

14. $\dfrac{d}{dx}\left(\dfrac{1}{3}\left(\mathrm{e}^{6x}+3\ln x\right)^{\frac{1}{3}}\right)=\dfrac{1}{9}\left(6\mathrm{e}^{6x}+\dfrac{3}{x}\right)\left(\mathrm{e}^{6x}+3\ln x\right)^{-\frac{2}{3}}=\dfrac{1}{3}\left(2\mathrm{e}^{6x}+\dfrac{1}{x}\right)\left(\mathrm{e}^{6x}+3\ln x\right)^{-\frac{2}{3}}$

15. $\dfrac{d}{dx}\left(\left(2\mathrm{e}^{3x}-3\mathrm{e}^{-2x}\right)^3\right)=3\left(2\mathrm{e}^{3x}-3\mathrm{e}^{-2x}\right)^2\left(6\mathrm{e}^{3x}+6\mathrm{e}^{-2x}\right)=18\left(2\mathrm{e}^{3x}-3\mathrm{e}^{-2x}\right)^2\left(\mathrm{e}^{3x}+\mathrm{e}^{-2x}\right)$

16. $\dfrac{d}{dx}\left(\dfrac{4}{\sqrt{2x-3\ln x}}\right)=-4\left(2-\dfrac{3}{x}\right)\left(2x-3\ln x\right)^{-\frac{3}{2}}=\dfrac{4\left(3-2x\right)}{x\left(2x-3\ln x\right)^{\frac{3}{2}}}$

17. $\dfrac{d}{dx}\left(\sqrt{1-2\mathrm{e}^{-3x^2}}\right)=6x\mathrm{e}^{-3x^2}\left(1-2\mathrm{e}^{-3x^2}\right)^{-\frac{1}{2}}=\dfrac{6x\mathrm{e}^{-3x^2}}{\sqrt{1-2\mathrm{e}^{-3x^2}}}$

18. $\dfrac{d}{dx}\Big(\ln\big(\ln\left(\ln x\right)\big)\Big)=\dfrac{1}{x\left(\ln x\right)\big[\ln\left(\ln x\right)\big]}$

19. $\dfrac{d}{dx}\left(2\mathrm{e}^{2\mathrm{e}^{2x}}\right)=8\mathrm{e}^{2x}\,\mathrm{e}^{2\mathrm{e}^{2x}}$

20. $\dfrac{d}{dx}\left(3\ln\left(\mathrm{e}^{\sqrt{5}+\ln 2}-\ln 3\right)\right)=0$

THE CHAIN RULE WITH SINES & COSINES

Question 7

1. $\frac{d}{dx}(\sin 4x) = 4\cos 4x$

2. $\frac{d}{dx}(2\sin 3x) = 6\cos 3x$

3. $\frac{d}{dx}(\cos 3x) = -3\sin 3x$

4. $\frac{d}{dx}\left(6\cos\left(\frac{2}{3}x\right)\right) = -4\sin\left(\frac{2}{3}x\right)$

5. $\frac{d}{dx}\left(4\sin\left(\frac{x}{2}\right)\right) = 2\cos\left(\frac{x}{2}\right)$

6. $\frac{d}{dx}(3\sin(5x-1)) = 15\cos(5x-1)$

7. $\frac{d}{dx}\left(3\cos\left(2x-\frac{\pi}{3}\right)\right) = -6\sin\left(2x-\frac{\pi}{3}\right)$

8. $\frac{d}{dx}\left(2\cos\left(\frac{\pi}{4}-2x\right)\right) = 4\sin\left(\frac{\pi}{4}-2x\right)$

9. $\frac{d}{dx}\left(6\cos\left(1-\frac{3x}{2}\right)\right) = 9\sin\left(1-\frac{3x}{2}\right)$

10. $\frac{d}{dx}\left(2\sin x^4\right) = 8x^3\cos x^4$

11. $\frac{d}{dx}\left(2\sin^4 x\right) = 8\sin^3 x\cos x$

12. $\frac{d}{dx}\left(4\cos x^3\right) = -12x^2\sin x^3$

13. $\dfrac{d}{dx}\left(4\cos^3 x\right)=-12\cos^2 x\sin x$

14. $\dfrac{d}{dx}\left(3\sin^5 x\right)=15\sin^4 x\cos x$

15. $\dfrac{d}{dx}\left(2\cos\sqrt{x}\right)=-\dfrac{\sin\sqrt{x}}{\sqrt{x}}$

16. $\dfrac{d}{dx}\left(2\sin^3 2x\right)=12\sin^2 2x\cos 2x$

17. $\dfrac{d}{dx}\left(2\left(3\cos 2x+1\right)^4\right)=-48\sin 2x\left(3\cos 2x+1\right)^3$

18. $\dfrac{d}{dx}\left(\sqrt{1-2\cos x}\right)=\dfrac{\sin x}{\sqrt{1-2\cos x}}$

19. $\dfrac{d}{dx}\left(\left(2\sin 3x-3\cos 2x\right)^3\right)=18\left(\cos 3x+\sin 2x\right)\left(2\sin 3x-3\cos 2x\right)^2$

20. $\dfrac{d}{dx}\left(\sin^3\left(\frac{\pi}{2}\right)\right)=0$

Question 8

1. $\frac{d}{dx}(\sin 5x) = 5\cos 5x$

2. $\frac{d}{dx}(4\sin 2x) = 8\cos 2x$

3. $\frac{d}{dx}(\cos 4x) = -4\sin 4x$

4. $\frac{d}{dx}\left(8\cos\left(\frac{1}{2}x\right)\right) = -4\sin\left(\frac{1}{2}x\right)$

5. $\frac{d}{dx}\left(4\sin\left(\frac{x}{4}\right)\right) = \cos\left(\frac{x}{4}\right)$

6. $\frac{d}{dx}(2\sin(3x-2)) = 6\cos(3x-2)$

7. $\frac{d}{dx}\left(5\cos\left(3x-\frac{\pi}{4}\right)\right) = -15\sin\left(3x-\frac{\pi}{4}\right)$

8. $\frac{d}{dx}\left(4\cos\left(\frac{\pi}{2}-3x\right)\right) = 12\sin\left(\frac{\pi}{2}-3x\right)$

9. $\frac{d}{dx}\left(8\cos\left(3-\frac{5x}{2}\right)\right) = 20\sin\left(3-\frac{5x}{2}\right)$

10. $\frac{d}{dx}\left(\frac{1}{2}\sin x^6\right) = 3x^5\cos x^6$

11. $\frac{d}{dx}\left(\frac{1}{2}\sin^6 x\right) = 3\sin^5 x\cos x$

12. $\frac{d}{dx}\left(10\cos x^5\right) = -50x^4\sin x^5$

13. $\frac{d}{dx}\left(10\cos^5 x\right) = -50\cos^4 x\sin x$

14. $\frac{d}{dx}\left(3\sin^7 x\right)=21\sin^6 x\cos x$

15. $\frac{d}{dx}\left(2\cos\left(x^{\frac{3}{2}}\right)\right)=-3x^{\frac{1}{2}}\sin\left(x^{\frac{3}{2}}\right)$

16. $\frac{d}{dx}\left(\frac{1}{6}\sin^4 3x\right)=2\cos 3x\sin^3 3x$

17. $\frac{d}{dx}\left(\frac{1}{3}\left(2\sin 3x+3\right)^5\right)=10\cos 3x\left(2\sin 3x+3\right)^4$

18. $\frac{d}{dx}\left(\sqrt{1-\cos 6x}\right)=\frac{3\sin 6x}{\sqrt{1-\cos 6x}}$

19. $\frac{d}{dx}\left(\left(4\sin 3x-3\cos 4x\right)^3\right)=36\left(\cos 3x+\sin 4x\right)\left(4\sin 3x-3\cos 4x\right)^2$

20. $\frac{d}{dx}\left(\sin\left(\frac{\pi}{4}\right)\right)=0$

THE CHAIN RULE WITH TAN, COT, SEC, COSEC

Question 9

1. $\frac{d}{dx}(4\tan 3x)=12\sec^2 3x$

2. $\frac{d}{dx}\left(2\tan\left(2x+\frac{\pi}{4}\right)\right)=4\sec^2\left(2x+\frac{\pi}{4}\right)$

3. $\frac{d}{dx}(3\tan^4 x)=12\tan^3 x\sec^2 x$

4. $\frac{d}{dx}(3\tan 2x)=6\sec^2 2x$

5. $\frac{d}{dx}\left(12\tan\left(\frac{\pi x}{4}\right)\right)=3\pi\sec^2\left(\frac{\pi x}{4}\right)$

6. $\frac{d}{dx}(\cot 2x)=-2\operatorname{cosec}^2 2x$

7. $\frac{d}{dx}(3\tan 2x-\cot 3x)=6\sec^2 2x+3\operatorname{cosec}^2 3x$

8. $\frac{d}{dx}(4\sec 2x)=8\sec 2x\tan 2x$

9. $\frac{d}{dx}(2\operatorname{cosec} 3x)=-6\operatorname{cosec} 3x\cot 3x$

10. $\frac{d}{dx}\left(4\sec\frac{x}{2}-6\operatorname{cosec}\frac{2x}{3}\right)=2\sec\frac{x}{2}\tan\frac{x}{2}+4\operatorname{cosec}\frac{2x}{3}\cot\frac{2x}{3}$

11. $\frac{d}{dx}(2\cot 4x-2\sec 3x)=-8\operatorname{cosec}^2 4x-6\sec 3x\tan 3x$

12. $\frac{d}{dx}(3\tan 5x-6\operatorname{cosec} 2x)=15\sec^2 5x+12\operatorname{cosec} 2x\cot 2x$

13. $\dfrac{d}{dx}\left(\tan^6 x\right)=6\tan^5 x\sec^2 x$

14. $\dfrac{d}{dx}\left(3\cot^4 x\right)=-12\cot^3 x\operatorname{cosec}^2 x$

15. $\dfrac{d}{dx}\left(3\sec^2 x\right)=6\sec^2 x\tan x$

16. $\dfrac{d}{dx}\left(4\operatorname{cosec}^4 x\right)=-16\operatorname{cosec}^4 x\cot x$

17. $\dfrac{d}{dx}\left(2\tan^4 3x\right)=24\tan^3 3x\sec^2 3x$

18. $\dfrac{d}{dx}\left(2\cot^2 4x\right)=-16\cot 4x\operatorname{cosec}^2 4x$

19. $\dfrac{d}{dx}\left(4\sec^4 2x\right)=32\sec^4 2x\tan 2x$

20. $\dfrac{d}{dx}\left(6\operatorname{cosec}^3\left(\frac{x}{2}\right)\right)=-9\operatorname{cosec}^3\left(\frac{x}{2}\right)\cot\left(\frac{x}{2}\right)$

Question 10

1. $\frac{d}{dx}\left(3\tan 5x\right)=15\sec^2 5x$

2. $\frac{d}{dx}\left(4\tan\left(3x+\frac{\pi}{3}\right)\right)=12\sec^2\left(3x+\frac{\pi}{3}\right)$

3. $\frac{d}{dx}\left(\frac{1}{2}\tan^6 x\right)=3\tan^5 x\sec^2 x$

4. $\frac{d}{dx}\left(10\tan\frac{1}{2}x\right)=5\sec^2\frac{1}{2}x$

5. $\frac{d}{dx}\left(12\tan\left(\frac{5\pi x}{6}\right)\right)=10\pi\sec^2\left(\frac{5\pi x}{6}\right)$

6. $\frac{d}{dx}\left(\cot 7x\right)=-7\operatorname{cosec}^2 7x$

7. $\frac{d}{dx}\left(4\tan 3x-2\cot 2x\right)=12\sec^2 3x+4\operatorname{cosec}^2 2x$

8. $\frac{d}{dx}\left(3\sec 4x\right)=12\sec 4x\tan 4x$

9. $\frac{d}{dx}\left(6\operatorname{cosec} 2x\right)=-12\operatorname{cosec} 2x\cot 2x$

10. $\frac{d}{dx}\left(6\sec\frac{x}{3}-4\operatorname{cosec}\frac{3x}{4}\right)=2\sec\frac{x}{3}\tan\frac{x}{3}+3\operatorname{cosec}\frac{3x}{4}\cot\frac{3x}{4}$

11. $\frac{d}{dx}\left(7\cot 2x-3\sec 3x\right)=-14\operatorname{cosec}^2 2x-9\sec 3x\tan 3x$

12. $\frac{d}{dx}\left(2\tan 7x-7\operatorname{cosec} 2x\right)=14\sec^2 7x+14\operatorname{cosec} 2x\cot 2x$

13. $\frac{d}{dx}\left(\tan^3 x\right)=3\tan^2 x\sec^2 x$

14. $\dfrac{d}{dx}\left(8\cot^5 x\right) = -40\cot^4 x \operatorname{cosec}^2 x$

15. $\dfrac{d}{dx}\left(\frac{1}{2}\sec^4 x\right) = 2\sec^4 x \tan x$

16. $\dfrac{d}{dx}\left(\frac{3}{4}\operatorname{cosec}^6 x\right) = -\frac{9}{2}\operatorname{cosec}^6 x \cot x$

17. $\dfrac{d}{dx}\left(2\tan^6 2x\right) = 24\tan^5 2x \sec^2 2x$

18. $\dfrac{d}{dx}\left(2\cot^3 3x\right) = -18\cot^2 3x \operatorname{cosec}^2 3x$

19. $\dfrac{d}{dx}\left(3\sec^3 3x\right) = 27\sec^3 3x \tan 3x$

20. $\dfrac{d}{dx}\left(12\operatorname{cosec}^3\left(\frac{x}{4}\right)\right) = -9\operatorname{cosec}^3\left(\frac{x}{4}\right)\cot\left(\frac{x}{4}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS

Question 11

1. $\frac{d}{dx}(\sin 2x) = 2\cos 2x$

2. $\frac{d}{dx}(3\cos 2x) = -6\sin 2x$

3. $\frac{d}{dx}(4\tan 3x) = 12\sec^2 3x$

4. $\frac{d}{dx}\left(6\sin\left(\frac{1}{2}x\right)\right) = 3\cos\left(\frac{1}{2}x\right)$

5. $\frac{d}{dx}\left(3\cos\left(\frac{x}{3}\right)\right) = -\sin\left(\frac{x}{3}\right)$

6. $\frac{d}{dx}(2\sin(3x-1)) = 6\cos(3x-1)$

7. $\frac{d}{dx}\left(2\cos\left(4x-\frac{\pi}{3}\right)\right) = -8\sin\left(4x-\frac{\pi}{3}\right)$

8. $\frac{d}{dx}\left(2\tan\left(2x+\frac{\pi}{4}\right)\right) = 4\sec^2\left(2x+\frac{\pi}{4}\right)$

9. $\frac{d}{dx}\left(9\cos\left(\frac{\pi}{6}-3x\right)\right) = 27\sin\left(\frac{\pi}{6}-3x\right)$

10. $\frac{d}{dx}(2\sin x^3) = 6x^2\cos x^3$

11. $\frac{d}{dx}(2\sin^3 x) = 6\sin^2 x\cos x$

12. $\frac{d}{dx}(4\cos x^2) = -8x\sin x^2$

13. $\dfrac{d}{dx}\left(5\cos^2 x\right)=-10\cos x\sin x$

14. $\dfrac{d}{dx}\left(3\tan^4 x\right)=12\tan^3 x\sec^2 x$

15. $\dfrac{d}{dx}\left(4\sin\sqrt{x}\right)=\dfrac{2}{\sqrt{x}}\cos\sqrt{x}$

16. $\dfrac{d}{dx}\left(\sin^5 2x\right)=10\sin^4 2x\cos 2x$

17. $\dfrac{d}{dx}\left(\left(3\sin x+2\right)^4\right)=12\cos x\left(3\sin x+2\right)^3$

18. $\dfrac{d}{dx}\left(\sqrt{1+4\sin x}\right)=\dfrac{2\cos x}{\sqrt{1+4\sin x}}$

19. $\dfrac{d}{dx}\left(\left(\sin x-\cos x\right)^3\right)=3\left(\cos x+\sin x\right)\left(\sin x-\cos x\right)^2$

20. $\dfrac{d}{dx}\left(\sin^3\left(\dfrac{\pi}{6}\right)\right)=0$

Question 12

1. $\frac{d}{dx}(3\sin 3x) = 9\cos 3x$

2. $\frac{d}{dx}(2\cos 4x) = -8\sin 4x$

3. $\frac{d}{dx}(3\tan 2x) = 6\sec^2 2x$

4. $\frac{d}{dx}\left(4\sin\left(\frac{3}{2}x\right)\right) = 6\cos\left(\frac{3}{2}x\right)$

5. $\frac{d}{dx}\left(2\cos\left(\frac{x}{4}\right)\right) = -\frac{1}{2}\sin\left(\frac{x}{4}\right)$

6. $\frac{d}{dx}\left(\frac{2}{3}\sin(6x-5)\right) = 4\cos(6x-5)$

7. $\frac{d}{dx}\left(3\cos\left(\frac{\pi}{4}-4x\right)\right) = 12\sin\left(\frac{\pi}{4}-4x\right)$

8. $\frac{d}{dx}\left(12\tan\left(\frac{\pi x}{4}\right)\right) = 3\pi\sec^2\left(\frac{\pi x}{4}\right)$

9. $\frac{d}{dx}\left(\frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6}-\sqrt{3}x\right)\right) = \sin\left(\frac{\pi}{6}-\sqrt{3}x\right)$

10. $\frac{d}{dx}(\sin x^4) = 4x^3\cos x^4$

11. $\frac{d}{dx}(\sin^4 x) = 4\sin^3 x\cos x$

12. $\frac{d}{dx}(\cos^3 x) = -3\cos^2 x\sin x$

13. $\frac{d}{dx}(2\sin^5 x) = 10\sin^4 x\cos x$

14. $\dfrac{d}{dx}\left(\tan^6 x\right)=6\tan^5 x\sec^2 x$

15. $\dfrac{d}{dx}\left(4\cos^2 x\right)=-8\cos x\sin x$

16. $\dfrac{d}{dx}\left(3\cot^4 x\right)=-12\cot^3 x\operatorname{cosec}^2 x$

17. $\dfrac{d}{dx}\left(3\sec^2 x\right)=6\sec^2 x\tan x$

18. $\dfrac{d}{dx}\left(4\operatorname{cosec}^4 x\right)=-16\cot x\operatorname{cosec}^4 x$

19. $\dfrac{d}{dx}\left(2\sin^5 2x\right)=20\sin^4 2x\cos 2x$

20. $\dfrac{d}{dx}\left(4\cos^3 2x\right)=-24\cos^2 2x\sin 2x$

21. $\dfrac{d}{dx}\left(2\tan^4 3x\right)=24\tan^3 3x\sec^2 3x$

22. $\dfrac{d}{dx}\left(2\cot^2 4x\right)=-16\cot 4x\operatorname{cosec}^2 4x$

23. $\dfrac{d}{dx}\left(4\sec^4 2x\right)=32\sec^4 2x\tan 2x$

24. $\dfrac{d}{dx}\left(6\operatorname{cosec}^3\left(\dfrac{x}{2}\right)\right)=-9\operatorname{cosec}^3\left(\dfrac{x}{2}\right)\cot\left(\dfrac{x}{2}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS, EXPONENTIALS AND LOGARITHMS

Question 13

1. $\frac{d}{dx}\left(4\mathrm{e}^{\sin x}\right)=4\mathrm{e}^{\sin x}\cos x$

2. $\frac{d}{dx}\left(\sin\left(\mathrm{e}^{2x}\right)\right)=2\mathrm{e}^{2x}\cos\left(\mathrm{e}^{2x}\right)$

3. $\frac{d}{dx}\left(\ln\left(\sin x\right)\right)=\cot x$

4. $\frac{d}{dx}\left(\sin\left(\ln x\right)\right)=\frac{\cos\left(\ln x\right)}{x}$

5. $\frac{d}{dx}\left(\mathrm{e}^{2\tan x}\right)=2\mathrm{e}^{2\tan x}\sec^2 x$

6. $\frac{d}{dx}\left(\tan\left(\mathrm{e}^{-x}\right)\right)=-\mathrm{e}^{-x}\sec^2\left(\mathrm{e}^{-x}\right)$

7. $\frac{d}{dx}\left(\cos\left(3\ln x\right)\right)=-\frac{3\sin\left(3\ln x\right)}{x}$

8. $\frac{d}{dx}\left(2\mathrm{e}^{\cos 2x}\right)=-4\mathrm{e}^{\cos 2x}\sin 2x$

9. $\frac{d}{dx}\left(\sin^4\left(\mathrm{e}^{x}\right)\right)=4\mathrm{e}^{x}\sin^3\left(\mathrm{e}^{x}\right)\cos\left(\mathrm{e}^{x}\right)$

10. $\frac{d}{dx}\left(\mathrm{e}^{\sin^2 x}\right)=2\mathrm{e}^{\sin^2 x}\sin x\cos x$

MIXED CHAIN RULE

Question 14

1. $\frac{d}{dx}\left((3x+1)^8\right)=24(3x+1)^7$

2. $\frac{d}{dx}(\ln 2x)=\frac{1}{x}$

3. $\frac{d}{dx}(3\sin 2x+2\cos 3x)=6\cos 2x-6\sin 3x$

4. $\frac{d}{dx}\left(\mathrm{e}^{3-2x}\right)=-2\mathrm{e}^{3-2x}$

5. $\frac{d}{dx}\left(\ln\left(x^2+1\right)\right)=\frac{2x}{x^2+1}$

6. $\frac{d}{dx}\left(\frac{4}{(2x-1)^2}\right)=-\frac{16}{(2x-1)^3}$

7. $\frac{d}{dx}(4\cot 3x)=-12\operatorname{cosec}^2 3x$

8. $\frac{d}{dx}\left(\ln\left(4-x^3\right)\right)=-\frac{3x^2}{4-x^3}$

9. $\frac{d}{dx}\left(\tan\left(2x^2+3\right)\right)=4x\sec^2\left(2x^2+3\right)$

10. $\frac{d}{dx}(2\sin 4x-3\cos 2x)=8\cos 4x+6\sin 2x$

11. $\frac{d}{dx}\left(2\mathrm{e}^{3x}\right)=6\mathrm{e}^{3x}$

12. $\frac{d}{dx}(\ln(\sin x))=\cot x$

13. $\dfrac{d}{dx}\left(\cos\left(\ln x\right)\right)=-\dfrac{1}{x}\sin\left(\ln x\right)$

14. $\dfrac{d}{dx}\left(\mathrm{e}^{\sin x}\right)=\mathrm{e}^{\sin x}\cos x$

15. $\dfrac{d}{dx}\left(4\cos 3x-2\sin 4x\right)=-12\sin 3x-8\cos 4x$

16. $\dfrac{d}{dx}\left(\dfrac{3}{\left(4x-2\right)^{3}}\right)=-\dfrac{36}{\left(4x-2\right)^{4}}$

17. $\dfrac{d}{dx}\left(\left(3-6x\right)^{\frac{5}{2}}\right)=-15\left(3-6x\right)^{\frac{3}{2}}$

18. $\dfrac{d}{dx}\left(\ln\left(2x^{2}+3x-1\right)\right)=\dfrac{4x+3}{2x^{2}+3x-1}$

19. $\dfrac{d}{dx}\left(\mathrm{e}^{x^{2}}\right)=2x\mathrm{e}^{x^{2}}$

20. $\dfrac{d}{dx}\left(\sin\left(x^{2}\right)\right)=2x\cos\left(x^{2}\right)$

21. $\dfrac{d}{dx}\left(\sin^{3}x\right)=3\sin^{2}x\cos x$

22. $\dfrac{d}{dx}\left(\cos\left(x^{2}-1\right)\right)=-2x\sin\left(x^{2}-1\right)$

23. $\dfrac{d}{dx}\left(3\cot 4x\right)=-12\operatorname{cosec}^{2}4x$

24. $\dfrac{d}{dx}\left(\mathrm{e}^{\tan x}\right)=\mathrm{e}^{\tan x}\sec^{2}x$

25. $\dfrac{d}{dx}\left(\left(\mathrm{e}^{2x}+2\right)^{3}\right)=6\mathrm{e}^{2x}\left(\mathrm{e}^{2x}+2\right)^{2}$

26. $\dfrac{d}{dx}\left(3\left(2x+1\right)^{6}\right)=36\left(2x+1\right)^{5}$

27. $\dfrac{d}{dx}\left(3\ln 4x\right)=\dfrac{3}{x}$

28. $\dfrac{d}{dx}\left(4\sin 3x-3\cos 2x\right)=12\cos 3x+6\sin 2x$

29. $\dfrac{d}{dx}\left(4\mathrm{e}^{1-4x}\right)=-16\mathrm{e}^{1-4x}$

30. $\dfrac{d}{dx}\left(3\ln\left(2x^2+1\right)\right)=\dfrac{12x}{2x^2+1}$

31. $\dfrac{d}{dx}\left(\dfrac{4}{\sqrt{2x-1}}\right)=-4\left(2x-1\right)^{-\frac{3}{2}}$

32. $\dfrac{d}{dx}\left(4\tan 2x\right)=8\sec^2 2x$

33. $\dfrac{d}{dx}\left(\ln\left(\sin 2x\right)\right)=2\cot 2x$

34. $\dfrac{d}{dx}\left(\tan^4 x\right)=4\tan^3 x\sec^2 x$

35. $\dfrac{d}{dx}\left(2\sin\left(\dfrac{1}{2}x\right)-3\cos\left(\dfrac{2}{3}x\right)\right)=\cos\left(\dfrac{1}{2}x\right)+2\sin\left(\dfrac{2}{3}x\right)$

36. $\dfrac{d}{dx}\left(2\mathrm{e}^{x^2}\right)=4x\mathrm{e}^{x^2}$

37. $\dfrac{d}{dx}\left(\ln\left(3\sin 2x\right)\right)=2\cot 2x$

38. $\dfrac{d}{dx}\left(2\cos\left(2\ln x\right)\right)=-\dfrac{4}{x}\sin\left(2\ln x\right)$

39. $\dfrac{d}{dx}\left(2\mathrm{e}^{\sin 3x}\right)=6\mathrm{e}^{\sin 3x}\cos 3x$

40. $\dfrac{d}{dx}\left(2\sin^4 3x\right)=24\sin^3 3x\cos 3x$

41. $\dfrac{d}{dx}\left(\dfrac{2}{(2x-1)^4}\right)=-\dfrac{16}{(2x-1)^5}$

42. $\dfrac{d}{dx}\left(\left(3-6\mathrm{e}^{x}\right)^{\frac{3}{2}}\right)=-9\mathrm{e}^{x}\left(3-6\mathrm{e}^{x}\right)^{\frac{1}{2}}$

43. $\dfrac{d}{dx}\left(\ln\left(\sec x+\tan x\right)\right)=\sec x$

44. $\dfrac{d}{dx}\left(2\mathrm{e}^{-x^4}\right)=-8x^3\,\mathrm{e}^{-x^4}$

45. $\dfrac{d}{dx}\left(4\sin\left(\sqrt{x}\right)\right)=\dfrac{2\cos\sqrt{x}}{\sqrt{x}}$

46. $\dfrac{d}{dx}\left(4\sin^3 2x\right)=24\sin^2 2x\cos 2x$

47. $\dfrac{d}{dx}\left(\cos\left(\mathrm{e}^{2x}-1\right)\right)=-2\mathrm{e}^{2x}\sin\left(\mathrm{e}^{2x}-1\right)$

48. $\dfrac{d}{dx}\left(6\tan 2x-2\cot 3x\right)=12\sec^2 2x+6\operatorname{cosec}^2 3x$

49. $\dfrac{d}{dx}\left(\mathrm{e}^{4\tan^2 x}\right)=8\mathrm{e}^{4\tan^2 x}\tan x\sec^2 x$

50. $\dfrac{d}{dx}\left(\ln\left(\cos 3x\right)\right)=-3\tan 3x$

THE PRODUCT RULE

Question 15

1. $\dfrac{d}{dx}\left(x\sin x\right)=\sin x+x\cos x$

2. $\dfrac{d}{dx}\left(4x^2\cos x\right)=8x\cos x-4x^2\sin x$

3. $\dfrac{d}{dx}\left(x^4\,\mathrm{e}^x\right)=4x^3\,\mathrm{e}^x+x^4\,\mathrm{e}^x=x^3\left(x+4\right)\mathrm{e}^x$

4. $\dfrac{d}{dx}\left(x^3\,\mathrm{e}^{-2x}\right)=3x^2\,\mathrm{e}^{-2x}-2x^3\,\mathrm{e}^{-2x}=x^2\left(3-2x\right)\mathrm{e}^{-2x}$

5. $\dfrac{d}{dx}\left(x^2\left(2x-1\right)^5\right)=2x\left(2x-1\right)^5+10x^2\left(2x-1\right)^4=2x\left(2x-1\right)^4\left(7x-1\right)$

6. $\dfrac{d}{dx}\left(2\mathrm{e}^{3x}\left(3x-1\right)^4\right)=6\mathrm{e}^{3x}\left(3x-1\right)^4+24\mathrm{e}^{3x}\left(3x-1\right)^3=18\mathrm{e}^{3x}\left(3x-1\right)^3\left(x+1\right)$

7. $\dfrac{d}{dx}\left(3\mathrm{e}^{-4x}\sin 2x\right)=-12\mathrm{e}^{-4x}\sin 2x+6\mathrm{e}^{-4x}\cos 2x=6\mathrm{e}^{-4x}\left(\cos 2x-2\sin 2x\right)$

8. $\dfrac{d}{dx}\left(\cos 2x\tan 2x\right)=-2\sin 2x\tan 2x+2\cos 2x\sec^2 2x=2\cos 2x$

9. $\dfrac{d}{dx}\left(\left(2x-1\right)^4\sqrt{x}\right)=8\left(2x-1\right)^3x^{\frac{1}{2}}+\dfrac{1}{2}\left(2x-1\right)^4x^{-\frac{1}{2}}=\dfrac{1}{2}x^{-\frac{1}{2}}\left(2x-1\right)^3\left(18x-1\right)$

10. $\dfrac{d}{dx}\left(\left(2x+1\right)^{\frac{3}{2}}\left(6x-1\right)^{\frac{1}{2}}\right)=3\left(2x+1\right)^{\frac{1}{2}}\left(6x-1\right)^{\frac{1}{2}}+3\left(2x+1\right)^{\frac{3}{2}}\left(6x-1\right)^{-\frac{1}{2}}=24x\left(2x+1\right)^{\frac{1}{2}}\left(6x-1\right)^{-\frac{1}{2}}$

Question 16

1. $\dfrac{d}{dx}\left(x\cos x\right)=\cos x-x\sin x$

2. $\dfrac{d}{dx}\left(2x^4\sin x\right)=8x^3\sin x+2x^4\cos x=2x^3\left(4\sin x+x\cos x\right)$

3. $\dfrac{d}{dx}\left(x^3\mathrm{e}^{-x}\right)=3x^2\mathrm{e}^{-x}-x^3\mathrm{e}^{-x}=x^2\left(3-x\right)\mathrm{e}^{-x}$

4. $\dfrac{d}{dx}\left(4x\mathrm{e}^{2x}\right)=4\mathrm{e}^{2x}+8x\mathrm{e}^{2x}=4\left(2x+1\right)\mathrm{e}^{2x}$

5. $\dfrac{d}{dx}\left(x^2\left(3x-1\right)^4\right)=2x\left(3x-1\right)^4+12x^2\left(3x-1\right)^3=2x\left(9x-1\right)\left(3x-1\right)^3$

6. $\dfrac{d}{dx}\left(3\mathrm{e}^{-2x}\left(2x-1\right)^3\right)=-6\mathrm{e}^{-2x}\left(2x-1\right)^3+18\mathrm{e}^{-2x}\left(2x-1\right)^2=12\mathrm{e}^{-2x}\left(2-x\right)\left(2x-1\right)^2$

7. $\dfrac{d}{dx}\left(\mathrm{e}^{-2x}\tan 2x\right)=-2\mathrm{e}^{-2x}\tan 2x+2\mathrm{e}^{-2x}\sec^2 2x=2\mathrm{e}^{-2x}\left(\sec^2 2x-\tan 2x\right)$

8. $\dfrac{d}{dx}\left(\sin 2x\cot 2x\right)=2\cos 2x\cot 2x-2\sin 2x\operatorname{cosec}^2 2x=-2\sin 2x$

9. $\dfrac{d}{dx}\left(2\left(1-4x\right)^3\sqrt{x}\right)=-24x^{\frac{1}{2}}\left(1-4x\right)^2+x^{-\frac{1}{2}}\left(1-4x\right)^3=x^{-\frac{1}{2}}\left(1-28x\right)\left(1-4x\right)^2$

10. $\dfrac{d}{dx}\left(\left(6x+1\right)^{\frac{1}{2}}\left(2x-1\right)^{-\frac{1}{2}}\right)=3\left(6x+1\right)^{-\frac{1}{2}}\left(2x-1\right)^{-\frac{1}{2}}-\left(6x+1\right)^{\frac{1}{2}}\left(2x-1\right)^{-\frac{3}{2}}=-4\left(6x+1\right)^{-\frac{1}{2}}\left(2x-1\right)^{-\frac{3}{2}}$

MIXED PRODUCT RULE

Question 17

1. $\frac{d}{dx}\left(x^4(4x-1)^3\right)=4x^3(4x-1)^3+12x^4(4x-1)^2=4x^3(7x-1)(4x-1)^2$

2. $\frac{d}{dx}\left(2x^3(2x+3)^5\right)=6x^2(2x+3)^5+20x^3(2x+3)^4=2x^2(16x+9)(2x+3)^4$

3. $\frac{d}{dx}\left(6x^{\frac{1}{2}}(2x-1)^4\right)=3x^{-\frac{1}{2}}(2x-1)^4+48x^{\frac{1}{2}}(2x-1)^3=3x^{-\frac{1}{2}}(2x-1)^3(18x-1)$

4. $\frac{d}{dx}\left(\mathrm{e}^{3x}\cos x\right)=3\mathrm{e}^{3x}\cos x-\mathrm{e}^{3x}\sin x=\mathrm{e}^{3x}(3\cos x-\sin x)$

5. $\frac{d}{dx}\left(x^2\,\mathrm{e}^{4x}\right)=2x\mathrm{e}^{4x}+4x^2\,\mathrm{e}^{4x}=2x(2x+1)\mathrm{e}^{4x}$

6. $\frac{d}{dx}\left((4x+1)\mathrm{e}^{2x}\right)=4\mathrm{e}^{2x}+2(4x+1)\mathrm{e}^{2x}=2(4x+3)\mathrm{e}^{2x}$

7. $\frac{d}{dx}\left(x^2\tan x\right)=2x\tan x+x^2\sec^2 x=x\left(2\tan x+x\sec^2 x\right)$

8. $\frac{d}{dx}\left(3x^2\sin 2x\right)=6x\sin 2x+6x^2\cos 2x=6x(\sin 2x+x\cos 2x)$

9. $\frac{d}{dx}\left(x^3\tan 2x\right)=3x^2\tan 2x+2x^3\sec^2 2x=x^2\left(3\tan 2x+2x\sec^2 2x\right)$

10. $\frac{d}{dx}\left(x^4\ln x\right)=4x^3\ln x+x^3=x^3(4\ln x+1)$

11. $\frac{d}{dx}\left(\mathrm{e}^{4x}\cos x\right)=4\mathrm{e}^{4x}\cos x-\mathrm{e}^{4x}\sin x=\mathrm{e}^{4x}(4\cos x-\sin x)$

12. $\frac{d}{dx}\left(4x^{\frac{1}{2}}\ln x\right)=2x^{-\frac{1}{2}}\ln x+4x^{-\frac{1}{2}}=2x^{-\frac{1}{2}}(\ln x+2)$

13. $\dfrac{d}{dx}\left(4\mathrm{e}^{-x}\tan 2x\right)=-4\mathrm{e}^{-x}\tan 2x+8\mathrm{e}^{-x}\sec^2 2x=4\mathrm{e}^{-x}\left(2\sec^2 2x-\tan 2x\right)$

14. $\dfrac{d}{dx}\left(\mathrm{e}^{2x}\left(4\sin 2x+3\cos 2x\right)\right)=2\mathrm{e}^{2x}\left(4\sin 2x+3\cos 2x\right)+\mathrm{e}^{2x}\left(8\cos 2x-6\sin 2x\right)$

$$=2\mathrm{e}^{2x}\left(\sin 2x+7\cos 2x\right)$$

15. $\dfrac{d}{dx}\left((2x+1)\cot 4x\right)=2\cot 4x-4(2x+1)\operatorname{cosec}^2 4x$

16. $\dfrac{d}{dx}\left(\left(3x^2-4x\right)\tan 2x\right)=(6x-4)\tan 2x+2\left(3x^2-4x\right)\sec^2 2x$

17. $\dfrac{d}{dx}\left(x^4\sin^2 x\right)=2x^3\sin^2 x+2x^4\sin x\cos x=2x^3\sin x\left(2\sin x+x\cos x\right)$

18. $\dfrac{d}{dx}\left(3x^2\sec 2x\right)=6x\sec 2x+6x^2\sec 2x\tan 2x=6x\sec 2x\left(1+x\tan 2x\right)$

19. $\dfrac{d}{dx}\left((4x+5)^{\frac{3}{2}}\mathrm{e}^{-2x}\right)=6(4x+5)^{\frac{1}{2}}\mathrm{e}^{-2x}-2(4x+5)^{\frac{3}{2}}\mathrm{e}^{-2x}=-4(2x+1)(4x+5)^{\frac{1}{2}}\mathrm{e}^{-2x}$

20. $\dfrac{d}{dx}\left(x^3\left(\sin 2x-3\cos 2x\right)\right)=3x^2\left(\sin 2x-3\cos 2x\right)+2x^3\left(\cos 2x+3\sin 2x\right)$

21. $\dfrac{d}{dx}\left(\mathrm{e}^{6x}\cos^3 x\right)=6\mathrm{e}^{6x}\cos^3 x-3\mathrm{e}^{6x}\cos^2 x\sin x=3\mathrm{e}^{6x}\cos^2 x\left(2\cos x-\sin x\right)$

22. $\dfrac{d}{dx}\left(\sin x\tan^2 x\right)=\cos x\tan^2 x+2\sin x\tan x\sec^2 x$

23. $\dfrac{d}{dx}\left(4x^3\operatorname{cosec}3x\right)=12x^2\operatorname{cosec}3x-12x^3\operatorname{cosec}3x\cot 3x=12x^2\operatorname{cosec}3x\left(1-x\cot 3x\right)$

24. $\dfrac{d}{dx}\left(3\mathrm{e}^{-4x}\cot 6x\right)=-12\mathrm{e}^{-4x}\cot 6x-18\mathrm{e}^{-4x}\operatorname{cosec}^2 6x=-6\mathrm{e}^{-4x}\left(2\cot 6x+3\operatorname{cosec}^2 6x\right)$

25. $\dfrac{d}{dx}\left(4x^{\frac{5}{2}}\sin^5 x\right)=10x^{\frac{3}{2}}\sin^5 x+20x^{\frac{5}{2}}\sin^4 x\cos x=10x^{\frac{3}{2}}\sin^4 x\left(\sin x+2x\cos x\right)$

26. $\dfrac{d}{dx}\left(5x\ln\left(x^2-2\right)\right)=5\ln\left(x^2-2\right)+\dfrac{10x^2}{x^2-2}$

27. $\dfrac{d}{dx}\left(\sin x\tan x\right)=\sin x\sec^2 x+\cos x\tan x$

28. $\dfrac{d}{dx}\left(x^2\sin 4x\right)=2x\left(\sin 4x+2x\cos 4x\right)$

29. $\dfrac{d}{dx}\left(\mathrm{e}^{2x}\cos 3x\right)=2\mathrm{e}^{2x}\cos 3x-3\mathrm{e}^{2x}\sin 3x=\mathrm{e}^{2x}\left(2\cos 3x-3\sin 3x\right)$

30. $\dfrac{d}{dx}\left(\left(4x-1\right)\mathrm{e}^{-x}\right)=4\mathrm{e}^{-x}-\left(4x-1\right)\mathrm{e}^{-x}=\mathrm{e}^{-x}\left(5-4x\right)$

31. $\dfrac{d}{dx}\left(x^3\tan 2x\right)=3x^2\tan 2x+2x^3\sec^2 2x=x^2\left(3\tan 2x+2x\sec^2 2x\right)$

32. $\dfrac{d}{dx}\left(x^4\left(4x-1\right)^3\right)=4x^3\left(4x-1\right)^3+12x^4\left(4x-1\right)^2=4x^3\left(7x-1\right)\left(4x-1\right)^2$

33. $\dfrac{d}{dx}\left(\left(3-2x^2\right)\cos 2x\right)=-4x\cos 2x-2\left(3-2x^2\right)\sin 2x$

34. $\dfrac{d}{dx}\left(\left(3x-1\right)^{\frac{1}{2}}\mathrm{e}^x\right)=\dfrac{3}{2}\left(3x-1\right)^{-\frac{1}{2}}\mathrm{e}^x+\left(3x-1\right)^{\frac{1}{2}}\mathrm{e}^x=\dfrac{\left(6x+1\right)\mathrm{e}^x}{2\sqrt{3x-1}}$

35. $\dfrac{d}{dx}\left(2x^4\ln x\right)=2x^3\left(1+4\ln x\right)$

36. $\dfrac{d}{dx}\left(\mathrm{e}^x\left(\sin x-\cos x\right)\right)=\mathrm{e}^x\left(\sin x-\cos x\right)+\mathrm{e}^x\left(\cos x+\sin x\right)=2\mathrm{e}^x\sin x$

37. $\dfrac{d}{dx}\left(\mathrm{e}^{-2x}\left(4x-1\right)^3\right)=-2\mathrm{e}^{-2x}\left(4x-1\right)^3+12\mathrm{e}^{-2x}\left(4x-1\right)^2=2\left(7-4x\right)\left(4x-1\right)^2\mathrm{e}^{-2x}$

38. $\dfrac{d}{dx}\left(\left(x^2-2x+1\right)\mathrm{e}^{2x}\right)=2\left(x^2-2x+1\right)\mathrm{e}^{2x}+\left(2x-2\right)\mathrm{e}^{2x}=2x\left(x-1\right)\mathrm{e}^{2x}$

39. $\dfrac{d}{dx}\left(\left(4x+1\right)^3\left(1-3x\right)^2\right)=12\left(4x+1\right)^2\left(1-3x\right)^2-6\left(4x+1\right)^3\left(1-3x\right)=6\left(1-3x\right)\left(1-10x\right)\left(4x+1\right)^2$

40. $\dfrac{d}{dx}\left(x^4\sqrt{4x-1}\right)=4x^3\left(4x-1\right)^{\frac{1}{2}}+2x^4\left(4x-1\right)^{-\frac{1}{2}}=\dfrac{2x^3\left(9x-2\right)}{\sqrt{4x-1}}$

41. $\dfrac{d}{dx}\left(x\sin^2 x\right)=\sin x\left(\sin x+2x\cos x\right)$

42. $\dfrac{d}{dx}\left(x^{-2}\ln x\right)=-2x^{-3}+x^{-3}=\dfrac{1-2\ln x}{x^3}$

43. $\dfrac{d}{dx}\left(\operatorname{cosec} x\cot x\right)=-\operatorname{cosec} x\left(\cot^2 x+\operatorname{cosec}^2 x\right)$

44. $\dfrac{d}{dx}\left(\sqrt{x}\sin 2x\right)=\dfrac{1}{2}x^{-\frac{1}{2}}\sin 2x+2x^{\frac{1}{2}}\cos 2x=\dfrac{\sin 2x+4x\cos 2x}{2\sqrt{x}}$

45. $\dfrac{d}{dx}\left(\left(4x+3\right)\tan 2x\right)=4\tan 2x+2\left(4x+3\right)\sec^2 2x$

46. $\dfrac{d}{dx}\left(\left(3x-1\right)^5\left(2x+1\right)^{\frac{3}{2}}\right)=15\left(3x-1\right)^4\left(2x+1\right)^{\frac{3}{2}}+3\left(3x-1\right)^5\left(2x+1\right)^{\frac{1}{2}}$

$$=3\left(13x+4\right)\left(3x-1\right)^4\left(2x+1\right)^{\frac{1}{2}}$$

47. $\dfrac{d}{dx}\left(x^2\sin^4 x\right)=2x\sin^4 x+4x^2\sin^3 x\cos x=2x\sin^3 x\left(\sin x+2x\cos x\right)$

48. $\dfrac{d}{dx}\left(6\mathrm{e}^{2x}\cos^3 x\right)=12\mathrm{e}^{2x}\cos^3 x-18\mathrm{e}^{2x}\cos^2 x\sin x=6\mathrm{e}^{2x}\cos^2 x\left(2\cos x-3\sin x\right)$

49. $\dfrac{d}{dx}\left(x^2\left(\mathrm{e}^x+\mathrm{e}^{-x}\right)\right)=2x\left(\mathrm{e}^x+\mathrm{e}^{-x}\right)+x^2\left(\mathrm{e}^x-\mathrm{e}^{-x}\right)=x\left(x+2\right)\mathrm{e}^x+2x\left(1-x\right)\mathrm{e}^{-x}$

50. $\dfrac{d}{dx}\left(\left(1-2x\right)^{\frac{3}{2}}\left(3x+1\right)^{-\frac{1}{2}}\right)=-3\left(1-2x\right)^{\frac{1}{2}}\left(3x+1\right)^{-\frac{1}{2}}-\dfrac{3}{2}\left(1-2x\right)^{\frac{3}{2}}\left(3x+1\right)^{-\frac{3}{2}}$

$$=-\frac{3}{2}\left(4x+3\right)\left(1-2x\right)^{\frac{1}{2}}\left(3x+1\right)^{-\frac{3}{2}}$$

51. $\dfrac{d}{dx}\left(x^5\sqrt{x^2-1}\right)=5x^4\left(x^2-1\right)^{\frac{1}{2}}+x^6\left(x^2-1\right)^{-\frac{1}{2}}=\dfrac{x^4\left(6x^2-5\right)}{\sqrt{x^2-1}}$

THE QUOTIENT RULE

Question 18

1. $\dfrac{d}{dx}\left(\dfrac{2x-5}{3x-1}\right)=\dfrac{13}{(3x-1)^2}$

2. $\dfrac{d}{dx}\left(\dfrac{4x-1}{1-5x}\right)=-\dfrac{1}{(1-5x)^2}$

3. $\dfrac{d}{dx}\left(\dfrac{2x^2+1}{3x^2-1}\right)=-\dfrac{10x}{\left(3x^2-1\right)^2}$

4. $\dfrac{d}{dx}\left(\dfrac{\ln x}{x}\right)=\dfrac{1-\ln x}{x^2}$

5. $\dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right)=\dfrac{x\cos x-\sin x}{x^2}$

6. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{2x}}{x}\right)=\dfrac{2x\mathrm{e}^{2x}-\mathrm{e}^{2x}}{x^2}=\dfrac{\mathrm{e}^{2x}(2x-1)}{x^2}$

7. $\dfrac{d}{dx}\left(\dfrac{x^2}{(3x-1)^2}\right)=\dfrac{2x(3x-1)^2-6x^2(3x-1)}{(3x-1)^4}=-\dfrac{2x}{(3x-1)^3}$

8. $\dfrac{d}{dx}\left(\dfrac{\sin x}{\cos x}\right)=\dfrac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}=\dfrac{1}{\cos^2 x}=\sec^2 x$

9. $\dfrac{d}{dx}\left(\dfrac{x^2-1}{2x+3}\right)=\dfrac{2x(2x+3)-2\left(x^2-1\right)}{(2x+3)^2}=\dfrac{2x^2+6x+2}{(2x+3)^2}$

10. $\dfrac{d}{dx}\left(\dfrac{8x^2+8x+3}{(2x+1)^2}\right)=\dfrac{(16x+8)(2x+1)^2-4\left(8x^2+8x+3\right)(2x+1)}{(2x+1)^4}=-\dfrac{4}{(2x+1)^3}$

Question 19

1. $\dfrac{d}{dx}\left(\dfrac{4x-3}{2x-5}\right)=-\dfrac{14}{(2x-5)^2}$

2. $\dfrac{d}{dx}\left(\dfrac{3x-1}{1-2x}\right)=\dfrac{1}{(1-2x)^2}$

3. $\dfrac{d}{dx}\left(\dfrac{5x^2+1}{2x^2-3}\right)=-\dfrac{34x}{(2x^2-3)^2}$

4. $\dfrac{d}{dx}\left(\dfrac{\ln x}{x^3}\right)=\dfrac{1-3\ln x}{x^4}$

5. $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{x^2}\right)=\dfrac{2x^2\cos 2x-2x\sin 2x}{x^4}=\dfrac{2(x\cos 2x-\sin 2x)}{x^3}$

6. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{3x}}{2x}\right)=\dfrac{6x\mathrm{e}^{3x}-2\mathrm{e}^{3x}}{4x^2}=\dfrac{\mathrm{e}^{3x}(3x-1)}{2x^2}$

7. $\dfrac{d}{dx}\left(\dfrac{4x^2}{(2x+3)^2}\right)=\dfrac{8x(2x+3)^2-16x^2(2x+3)}{(2x+3)^4}=\dfrac{24x}{(2x+3)^3}$

8. $\dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right)=\dfrac{\sin x(-\sin x)-\cos x\cos x}{\sin^2 x}=-\dfrac{1}{\sin^2 x}=-\operatorname{cosec}^2 x$

9. $\dfrac{d}{dx}\left(\dfrac{3x^2+2}{x-1}\right)=\dfrac{6x(x-1)-(3x^2+2)}{(x-1)^2}=\dfrac{3x^2-6x-2}{(x-1)^2}$

10. $\dfrac{d}{dx}\left(\dfrac{3x^2-6x+4}{(x-1)^2}\right)=\dfrac{(6x-6)(x-1)^2-2(3x^2-6x+4)(x-1)}{(x-1)^4}=-\dfrac{2}{(x-1)^3}$

MIXED QUOTIENT RULE

Question 20

1. $\dfrac{d}{dx}\left(\dfrac{4x+3}{2x-3}\right)=\dfrac{4(2x-3)-2(4x+3)}{(2x-3)^2}=-\dfrac{18}{(2x-3)^2}$

2. $\dfrac{d}{dx}\left(\dfrac{3-4x}{2x+1}\right)=\dfrac{-4(2x+1)-2(3-4x)}{(2x+1)^2}=-\dfrac{10}{(2x+1)^2}$

3. $\dfrac{d}{dx}\left(\dfrac{2x^2+1}{3x^2+1}\right)=\dfrac{4x\left(3x^2+1\right)-6x\left(2x^2+1\right)}{\left(3x^2+1\right)^2}=-\dfrac{2x}{\left(3x^2+1\right)^2}$

4. $\dfrac{d}{dx}\left(\dfrac{1+\cos x}{1+\sin x}\right)=\dfrac{-\sin x(1+\sin x)-(1+\cos x)\cos x}{(1+\sin x)^2}=-\dfrac{1+\sin x+\cos x}{(1+\sin x)^2}$

5. $\dfrac{d}{dx}\left(\dfrac{\ln x}{x^2}\right)=\dfrac{1-2\ln x}{x^3}$

6. $\dfrac{d}{dx}\left(\dfrac{x^2+1}{x^2-2}\right)=\dfrac{2x\left(x^2-2\right)-2x\left(x^2+1\right)}{\left(x^2-2\right)^2}=-\dfrac{6x}{\left(x^2-2\right)^2}$

7. $\dfrac{d}{dx}\left(\dfrac{3x^2+2}{x^2+5}\right)=\dfrac{6x\left(x^2+5\right)-2x\left(3x^2+2\right)}{\left(x^2+5\right)^2}=\dfrac{26x}{\left(x^2+5\right)^2}$

8. $\dfrac{d}{dx}\left(\dfrac{1-\cos x}{1+\cos x}\right)=\dfrac{(1+\cos x)\sin x-(-\sin x)(1-\cos x)}{(1+\cos x)^2}=\dfrac{2\sin x}{(1+\cos x)^2}$

9. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\tan x}\right)=\dfrac{\tan x\left(\sec x\tan x\right)-\sec x\left(\sec^2 x\right)}{\tan^2 x}=-\operatorname{cosec} x\cot x$

10. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^x+2}{\mathrm{e}^x-2}\right)=\dfrac{\mathrm{e}^x\left(\mathrm{e}^x-2\right)-\mathrm{e}^x\left(\mathrm{e}^x+2\right)}{\left(\mathrm{e}^x-2\right)^2}=-\dfrac{4\mathrm{e}^x}{\left(\mathrm{e}^x-2\right)^2}$

11. $\dfrac{d}{dx}\left(\dfrac{2x+1}{\sqrt{x+1}}\right)=\dfrac{2(x+1)^{\frac{1}{2}}-\frac{1}{2}(x+1)^{-\frac{1}{2}}(2x+1)}{x+1}=\dfrac{2x+3}{2(x+1)^{\frac{3}{2}}}$

12. $\dfrac{d}{dx}\left(\dfrac{\sin^2 x}{\tan x}\right)=\dfrac{\tan x(2\sin x\cos x)-\sin^2 x\sec^2 x}{\tan^2 x}=\cos^2 x-\sin^2 x=\cos 2x$

13. $\dfrac{d}{dx}\left(\dfrac{\ln x}{x^4}\right)=\dfrac{x^4\times\frac{1}{x}-4x^3\ln x}{x^8}=\dfrac{x^3(1-4\ln x)}{x^8}=\dfrac{1-4\ln x}{x^5}$

14. $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{x}\right)=\dfrac{2x\cos 2x-\sin 2x}{x^2}$

15. $\dfrac{d}{dx}\left(\dfrac{x^2-1}{\sqrt{x+1}}\right)=\dfrac{2(x+1)^{\frac{1}{2}}-\frac{1}{2}(x+1)^{-\frac{1}{2}}(x^2-1)}{x+1}=\dfrac{5+4x-x^2}{2(x+1)^{\frac{3}{2}}}$

16. $\dfrac{d}{dx}\left(\dfrac{3x}{(4x-2)^3}\right)=\dfrac{3(4x-2)^3-36x(4x-2)^2}{(4x-2)^6}=-\dfrac{24x+6}{(4x-2)^4}=-\dfrac{3(4x+1)}{8(2x-1)^4}$

17. $\dfrac{d}{dx}\left(\dfrac{3\mathrm{e}^x}{2\mathrm{e}^x-1}\right)=\dfrac{3(2\mathrm{e}^x-1)\mathrm{e}^x-2\mathrm{e}^x(3\mathrm{e}^x)}{(2\mathrm{e}^x-1)^2}=-\dfrac{3\mathrm{e}^x}{(2\mathrm{e}^x-1)^2}$

18. $\dfrac{d}{dx}\left(\dfrac{4x-1}{2x+1}\right)=\dfrac{6}{(2x+1)^2}$

19. $\dfrac{d}{dx}\left(\dfrac{1-2x}{3x+2}\right)=-\dfrac{7}{(3x+2)^2}$

20. $\dfrac{d}{dx}\left(\dfrac{4x^3+1}{2x^3+1}\right)=\dfrac{6x^2}{(2x^3+1)^2}$

21. $\dfrac{d}{dx}\left(\dfrac{1-\sin x}{1+\sin x}\right)=\dfrac{-\cos x(1+\sin x)-\cos x(1-\sin x)}{(1+\sin x)^2}=-\dfrac{2\cos x}{(1+\sin x)^2}$

22. $\dfrac{d}{dx}\left(\dfrac{\ln 2x}{x^3}\right)=\dfrac{x^2-3x^2\ln 2x}{x^6}=\dfrac{1-3\ln 2x}{x^4}$

23. $\dfrac{d}{dx}\left(\dfrac{2x^2+3}{x^2-1}\right)=\dfrac{4x\left(x^2-1\right)-2x\left(2x^2+3\right)}{\left(x^2-1\right)^2}=-\dfrac{10x}{\left(x^2-1\right)^2}$

24. $\dfrac{d}{dx}\left(\dfrac{2x^2+3}{x+1}\right)=\dfrac{4x\left(x+1\right)-\left(2x^2+3\right)}{\left(x+1\right)^2}=\dfrac{2x^2+4x-3}{\left(x+1\right)^2}$

25. $\dfrac{d}{dx}\left(\dfrac{1+\cos x}{1-\cos x}\right)=\dfrac{-\sin x\left(1-\cos x\right)-\sin x\left(1+\cos x\right)}{\left(1-\cos x\right)^2}=\dfrac{-2\sin x}{\left(1-\cos x\right)^2}=-\cot\dfrac{x}{2}\operatorname{cosec}^2\dfrac{x}{2}$

26. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\sin x}\right)=\dfrac{\sin x\sec x\tan x-\sec x\cos x}{\sin^2 x}=\sec^2 x-\operatorname{cosec}^2 x=-4\operatorname{cosec}2x\cot 2x$

27. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{2x}+2}{\mathrm{e}^{2x}-1}\right)=\dfrac{2\mathrm{e}^{2x}\left(\mathrm{e}^{2x}-1\right)-2\mathrm{e}^{2x}\left(\mathrm{e}^{2x}+2\right)}{\left(\mathrm{e}^{2x}-1\right)^2}=-\dfrac{6\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^2}$

28. $\dfrac{d}{dx}\left(\dfrac{2x-5}{\sqrt{4x+1}}\right)=\dfrac{2\left(4x+1\right)^{\frac{1}{2}}-2\left(4x+1\right)^{-\frac{1}{2}}\left(2x-5\right)}{4x+1}=\dfrac{4\left(x+3\right)}{\left(4x+1\right)^{\frac{3}{2}}}$

29. $\dfrac{d}{dx}\left(\dfrac{\cos^2 x}{\tan x}\right)=\dfrac{-2\sin x\cos x\tan x-\cos^2 x\sec^2 x}{\tan^2 x}=-\dfrac{2\sin^2 x+1}{\tan^2 x}$

30. $\dfrac{d}{dx}\left(\dfrac{\ln x}{\sqrt{x}}\right)=\dfrac{x^{\frac{1}{2}}\times\dfrac{1}{x}-\dfrac{1}{2}x^{-\frac{1}{2}}\ln x}{x}=\dfrac{2-\ln x}{2x^{\frac{3}{2}}}$

31. $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{x^2}\right)=\dfrac{2x^2\cos 2x-2x\sin 2x}{x^4}=\dfrac{2x\cos 2x-2\sin 2x}{x^3}$

32. $\dfrac{d}{dx}\left(\dfrac{4x}{\sqrt{2x+1}}\right)=\dfrac{4\left(2x+1\right)^{\frac{1}{2}}-4x\left(2x+1\right)^{-\frac{1}{2}}}{2x+1}=\dfrac{4\left(x+1\right)}{\left(2x+1\right)^{\frac{3}{2}}}$

33. $\dfrac{d}{dx}\left(\dfrac{x^3}{\left(2x-1\right)^4}\right)=\dfrac{3x^2\left(2x-1\right)^4-8x^3\left(2x-1\right)^3}{\left(2x-1\right)^8}=-\dfrac{x^2\left(2x+3\right)}{\left(2x-1\right)^5}$

34. $\dfrac{d}{dx}\left(\dfrac{4\mathrm{e}^{x}}{\mathrm{e}^{x}+2}\right)=\dfrac{8\mathrm{e}^{x}}{\left(\mathrm{e}^{x}+2\right)^{2}}$