DIFFERENTIATION

Question 1 **(**)**

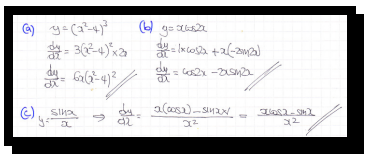
Differentiate each of the following expressions with respect to x, simplifying the final answers as far as possible

a) $y=\left(x^2-4\right)^3$

b) $y=x\cos 2x$

c) $y=\dfrac{\sin x}{x}$

$\square$, $\boxed{\dfrac{dy}{dx}=6x\left(x^2-4\right)^2}$, $\boxed{\dfrac{dy}{dx}=\cos 2x-2x\sin 2x}$, $\boxed{\dfrac{dy}{dx}=\dfrac{x\cos x-\sin x}{x^2}}$



Question 2 **(**)**

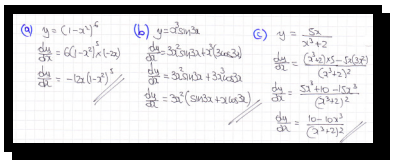
Differentiate each of the following expressions with respect to x, simplifying the final answers as far as possible.

a) $y=\left(1-x^2\right)^6$

b) $y=x^3\sin 3x$

c) $y=\dfrac{5x}{x^3+2}$

$\square$, $\boxed{\dfrac{dy}{dx}=-12x\left(1-x^2\right)^5}$, $\boxed{\dfrac{dy}{dx}=3x^2\left(\sin 3x+x\cos 3x\right)}$, $\boxed{\dfrac{dy}{dx}=\dfrac{10\left(1-x^3\right)}{\left(x^3+2\right)^2}}$



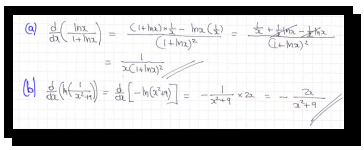
Question 3 **(***)**

Differentiate each of the following expressions with respect to x, writing the final answers as simplified fractions.

a) $y=\dfrac{\ln x}{1+\ln x}$.

b) $y=\ln\left(\dfrac{1}{x^2+9}\right)$.

$\square$, $\boxed{\dfrac{dy}{dx}=\dfrac{1}{x(1+\ln x)^2}}$, $\boxed{\dfrac{dy}{dx}=-\dfrac{2x}{x^2+9}}$



Question 4 **(***)**

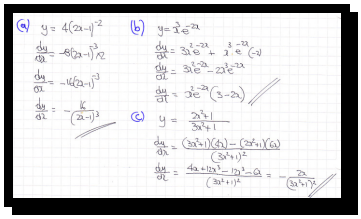
Differentiate each of the following expressions with respect to x, simplifying the final answers as far as possible

a) $y=\dfrac{4}{(2x-1)^2}$.

b) $y=x^3\mathrm{e}^{-2x}$.

c) $y=\dfrac{2x^2+1}{3x^2+1}$.

$\square$, $\boxed{\dfrac{dy}{dx}=-\dfrac{16}{(2x-1)^3}}$, $\boxed{\dfrac{dy}{dx}=x^2(3-2x)\mathrm{e}^{-2x}}$, $\boxed{\dfrac{dy}{dx}=-\dfrac{2x}{\left(3x^2+1\right)^2}}$



Question 5 **(***)**

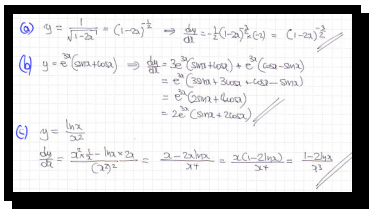
Differentiate each of the following expressions with respect to x, simplifying the final answers where possible.

a) $y=\dfrac{1}{\sqrt{1-2x}}$.

b) $y=\mathrm{e}^{3x}\left(\sin x+\cos x\right)$.

c) $y=\dfrac{\ln x}{x^2}$.

$\square$, $\boxed{\dfrac{dy}{dx}=\left(1-2x\right)^{-\frac{3}{2}}}$, $\boxed{\dfrac{dy}{dx}=2\mathrm{e}^{3x}\left(\sin x+2\cos x\right)}$, $\boxed{\dfrac{dy}{dx}=\dfrac{1-2\ln x}{x^3}}$



Question 6 (***)

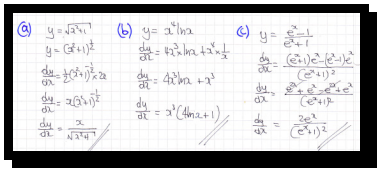
Differentiate each of the following expressions with respect to x, simplifying the final answers where possible.

a) $y=\sqrt{x^2-1}$

b) $y=x^4\ln x$

c) $y=\dfrac{\mathrm{e}^x-1}{\mathrm{e}^x+1}$

$\square$, $\boxed{\dfrac{dy}{dx}=x\left(x^2-1\right)^{-\frac{1}{2}}}$, $\boxed{\dfrac{dy}{dx}=4x^3\ln x+x^3}$, $\boxed{\dfrac{dy}{dx}=\dfrac{2\mathrm{e}^x}{\left(\mathrm{e}^x+1\right)^2}}$



Question 7 **(***+)**

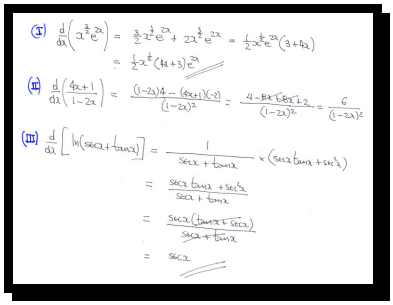
Show clearly that …

i. … $\dfrac{d}{dx}\left(x^{\frac{3}{2}}\,\mathrm{e}^{2x}\right)=\dfrac{1}{2}\left(4x+3\right)x^{\frac{1}{2}}\,\mathrm{e}^{2x}$.

ii. … $\dfrac{d}{dx}\left(\dfrac{4x+1}{1-2x}\right)=\dfrac{6}{\left(1-2x\right)^2}$.

iii. … $\dfrac{d}{dx}\left(\ln\left(\sec x+\tan x\right)\right)=\sec x$.

proof



Question 8 **(***+)**

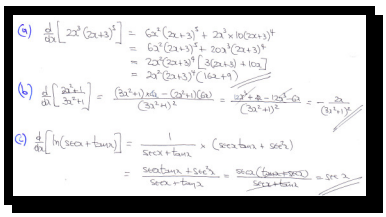
Show clearly that …

i. … $\dfrac{d}{dx}\left[2x^3\left(2x+3\right)^5\right]=2x^2\left(16x+9\right)\left(2x+3\right)^4$.

ii. … $\dfrac{d}{dx}\left[\dfrac{2x^2+1}{3x^2+1}\right]=-\dfrac{2x}{\left(3x^2+1\right)^2}$.

iii. … $\dfrac{d}{dx}\left[\ln\left(\sec x+\tan x\right)\right]=\sec x$.

proof



Question 9 **(***+)**

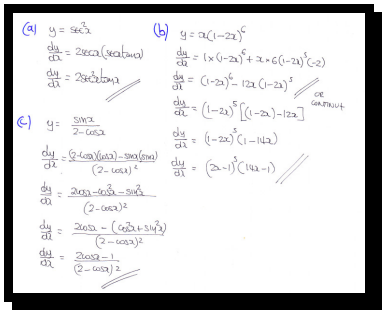
Differentiate each of the following expressions with respect to x, simplifying the final answer as far as possible.

a) $y=\sec^2 x$.

b) $y=x(1-2x)^6$.

c) $y=\dfrac{\sin x}{2-\cos x}$.

$$\boxed{\frac{dy}{dx}=2\sec^2 x\tan x},\quad \boxed{\frac{dy}{dx}=(14x-1)(2x-1)^5=(1-14x)(1-2x)^5},\quad \boxed{\frac{dy}{dx}=\frac{2\cos x-1}{(2-\cos x)^2}}$$



Question 10 (****)

Differentiate each the following expressions with respect to x, simplifying the final answers as far as possible.

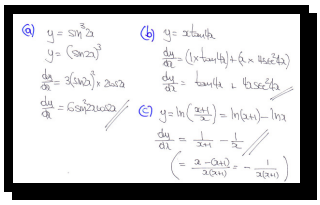
(Fractional answers must not involve double fractions)

a) $y=\sin^3 2x$.

b) $y=x\tan 4x$.

c) $y=\ln\left(\dfrac{x+1}{x}\right)$.

$$\boxed{\frac{dy}{dx}=6\sin^2 2x\cos 2x}\,,\quad \boxed{\frac{dy}{dx}=\tan 4x+4x\sec^2 4x}\,,\quad \boxed{\frac{dy}{dx}=-\frac{1}{x^2+x}}$$



Question 11 (****)

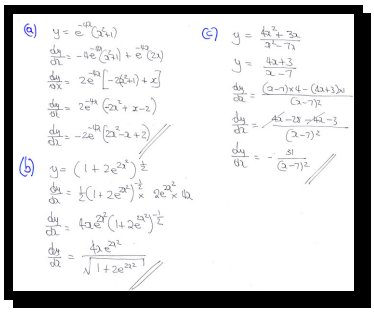
Differentiate each of the following expressions with respect to x, simplifying the answers as far as possible.

a) $y=\mathrm{e}^{-4x}\left(x^2+1\right)$.

b) $y=\sqrt{1+2\mathrm{e}^{2x^2}}$.

c) $y=\dfrac{4x^2+3x}{x^2-7x}$.

$$\boxed{\frac{dy}{dx}=-2\mathrm{e}^{-4x}\left(2x^2-x+2\right)},\quad \boxed{\frac{dy}{dx}=\frac{4x\mathrm{e}^{2x^2}}{\sqrt{1+2\mathrm{e}^{2x^2}}}},\quad \boxed{\frac{dy}{dx}=-\frac{31}{(x-7)^2}}$$



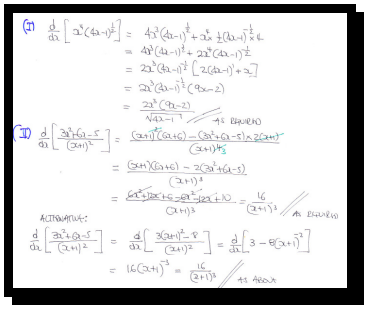
Question 12 **(****)**

Prove that …

i. … $\dfrac{d}{dx}\left(x^4\sqrt{4x-1}\right)=\dfrac{2x^3(9x-2)}{\sqrt{4x-1}}$.

ii. … $\dfrac{d}{dx}\left(\dfrac{3x^2+6x-5}{(x+1)^2}\right)=\dfrac{16}{(x+1)^3}$.

proof



Question 13 (****)

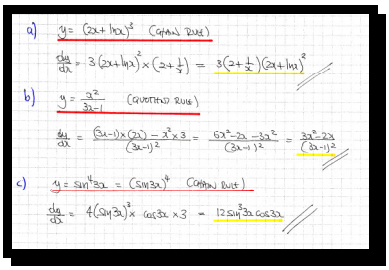
Differentiate each of the following expressions with respect to x.

a) $y=(2x+\ln x)^3$.

b) $y=\dfrac{x^2}{3x-1}$.

c) $y=\sin^4 3x$.

$\square$, $\boxed{\dfrac{dy}{dx}=3(2x+\ln x)^2\left(2+\dfrac{1}{x}\right)}$, $\boxed{\dfrac{dy}{dx}=\dfrac{3x^2-2x}{(3x-1)^2}}$, $\boxed{\dfrac{dy}{dx}=12\sin^3 3x\cos 3x}$



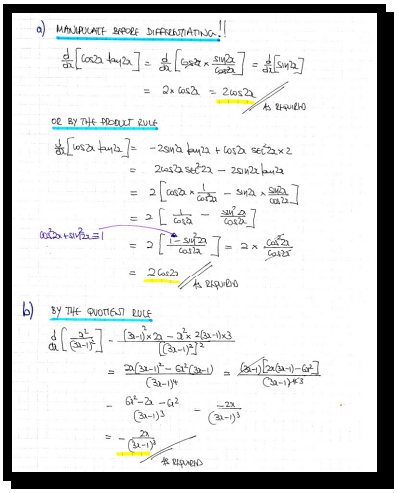
Question 14 **(****)**

Show, with detailed workings, that

a) $\dfrac{d}{dx}\left(\cos 2x \tan 2x\right) = 2\cos 2x$.

b) $\dfrac{d}{dx}\left(\dfrac{x^2}{(3x-1)^2}\right) = -\dfrac{2x}{(3x-1)^3}$.

[ ], [proof]



Question 15 **(*****)**

Show clearly that …

i. … $\dfrac{d}{dx}\left(\dfrac{x-4}{\sqrt{x}+2}\right)=\dfrac{1}{2\sqrt{x}}$

ii. … $\dfrac{d}{dx}\left(\dfrac{4x-8\sqrt{x}+3}{\left(\sqrt{x}-1\right)^2}\right)=\dfrac{1}{\sqrt{x}\left(\sqrt{x}-1\right)^3}$.

[ ], [proof]

