CURVE SKETCHING
EXAM QUESTIONS

Question 1 (**)

$$f(x)=x^2+6x+10,\ x\in\mathbb{R}.$$

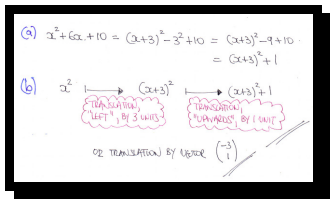
a) Express $f(x)$ in the form

$$f(x)=(x+a)^2+b,$$

where a and b are integers.

b) Describe geometrically the transformations which map the graph of x^2 onto the graph of $f(x)$.

$\boxed{a=3}$, $\boxed{b=1}$, $\boxed{\text{translation by } \begin{pmatrix}-3\\1\end{pmatrix}}$



Question 2 (+)**

The curve C has equation

$$y=(x-a)^2+b,$$

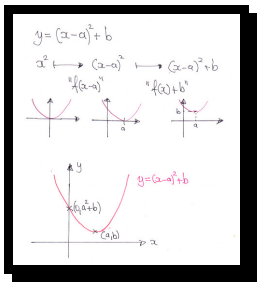
where a, b are positive constants.

By considering the two transformations that map the graph of $y=x^2$ onto the graph of C, or otherwise, sketch the graph of C.

The sketch must include the coordinates, in terms of a, b, of …

- … all the points where the curve meets the coordinate axes.
- … the maximum point of the curve.

graph



Question 3 (***)

$$f(x)=\sqrt{x}\,,\ x\in\mathbb{R}\,,\ x\geq 0\,.$$

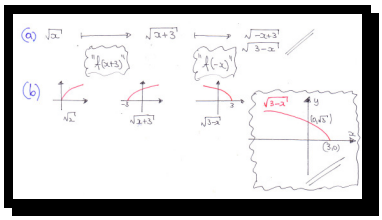
The graph of $f(x)$ is translated by 3 units in the negative x direction, followed by a reflection in the y axis, forming the graph of $g(x)$.

a) Find the equation of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include the coordinates of all the points where the curve meets the coordinate axes.

$$\boxed{g(x)=\sqrt{3-x}}$$



Question 4 (***)

The curve C has equation

$$y = x^3 - 9x .$$

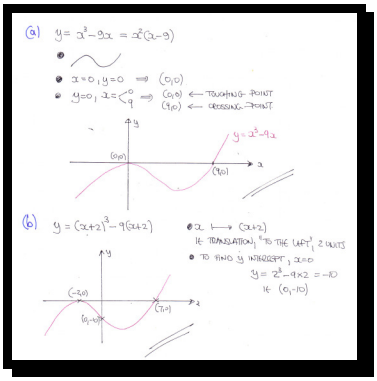
a) Sketch the graph of C.

b) Hence sketch on a separate diagram the graph of

$$y = (x+2)^3 - 9(x+2).$$

Both sketches must include the coordinates of all the points where each of the curves meets the coordinate axes.

graph



Question 5 (***)

A curve is defined by the equation

$$f(x)=(x-a)(x-b)^2(x-c)^3,\ x\in\mathbb{R},$$

where a, b and c are constants.

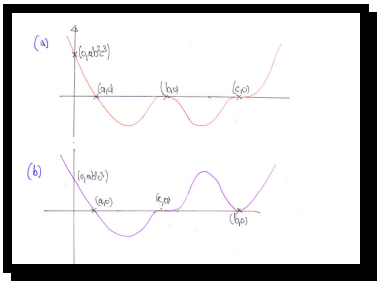
Sketch the graph of $f(x)$ in each of the following cases.

a) $0<a<b<c$

b) $0<a<c<b$.

Each sketch must clearly show any intercepts with the coordinate axes, in terms of a, b and c, where appropriate.

graph



Question 6 (*+)**

$$f(x)=x^2-2x-8,\ x\in\mathbb{R}.$$

a) Express $f(x)$ in the form $f(x)=(x+a)^2+b$, where a and b are integers.

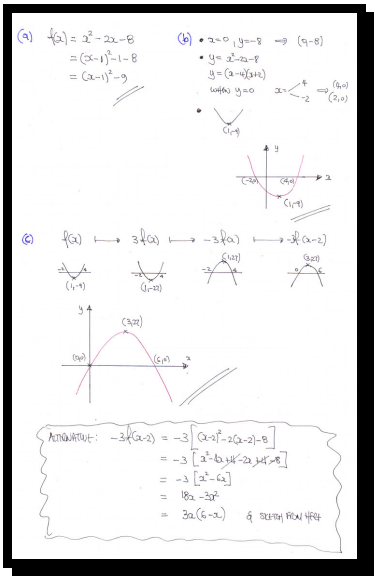
b) Sketch the graph of $f(x)$.

a) By considering a series of three geometrical transformations, sketch the graph of $y=-3f(x-2)$.

Both sketches must include the coordinates of …

- … all the points where the curves meets the coordinate axes.
- … the minimum or maximum points of the curves.

$\boxed{a=-1}$, $\boxed{b=-9}$



Question 7 (*+)**

The curve C has equation

$$y = 9 - (x-2)^2 .$$

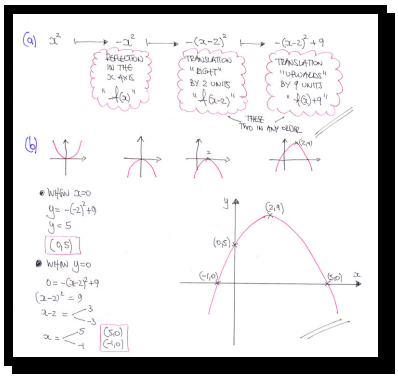
a) Describe geometrically the three transformations that map the graph of $y = x^2$ onto the graph of C.

b) Hence, sketch the graph of C.

The sketch must include the coordinates of

- … all the points where the curve meets the coordinate axes.
- … the maximum point of the curve.

reflection in the x axis, translation "right" by 2 units,
translation "upwards" by 9 units



Question 8 (*+)**

The curve C has equation

$$y=\frac{2x+3}{x-2},\ x\in\mathbb{R},\ x\neq 2.$$

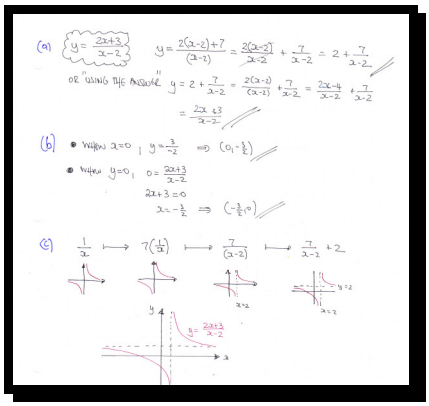
a) Show clearly that

$$\frac{2x+3}{x-2}\equiv 2+\frac{7}{x-2}.$$

b) Find the coordinates of the points where C meets the coordinate axes.

c) Sketch the graph of C showing clearly the equations of any asymptotes.

$$\boxed{\left(0,-\frac{3}{2}\right),\left(-\frac{3}{2},0\right)}$$



Question 9 **(***+)**

$$f(x)=\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

$$g(x)=\frac{1}{x+2}+2,\ x\in\mathbb{R},\ x\neq -2.$$

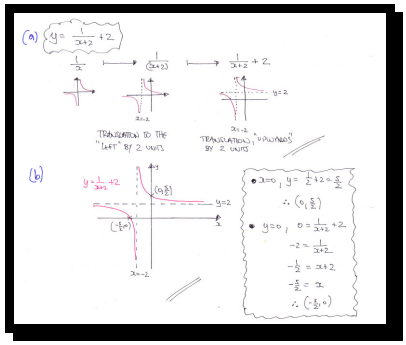
a) Describe mathematically the two transformations that map the graph of $f(x)$ onto the graph of $g(x)$.

b) Sketch the graph of $g(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of any asymptotes of the curve.

translation "left" by 2 units, followed by translation "upwards" by 2 unit



Question 10 **(***+)**

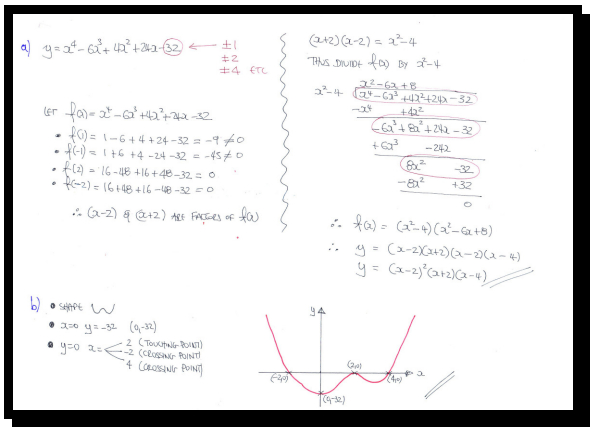
The curve C has equation

$$y = x^4 - 6x^3 + 4x^2 + 24x - 32 .$$

a) Express y as the product of four linear factors.

b) Hence the graph of C, showing clearly the coordinates of any points where the graph of C meets the coordinate axes.

$$\boxed{y = (x+2)(x-4)(x-2)^2}$$



Question 11 (****)

$$f(x)=\frac{x-2}{x-3},\ x\in\mathbb{R},\ x\neq 3.$$

b) Express $f(x)$ in the form

$$f(x)=a+\frac{1}{x+b},$$

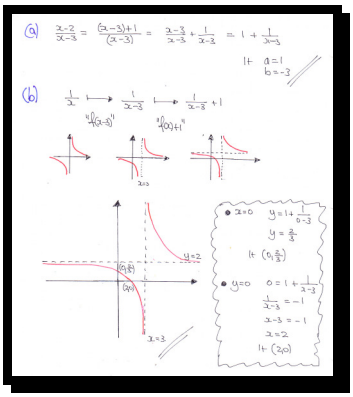
where a and b are integers.

c) By considering a series of transformations which map the graph of $\frac{1}{x}$ onto the graph of $f(x)$, sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

$\boxed{a=1}$, $\boxed{b=-3}$



Question 12 (****)

A cubic curve C has equation

$$y=(3-x)(4+x)^2.$$

a) Sketch the graph of C.

The sketch must include any points where the graph meets the coordinate axes.

b) Sketch in separate diagrams the graph of …

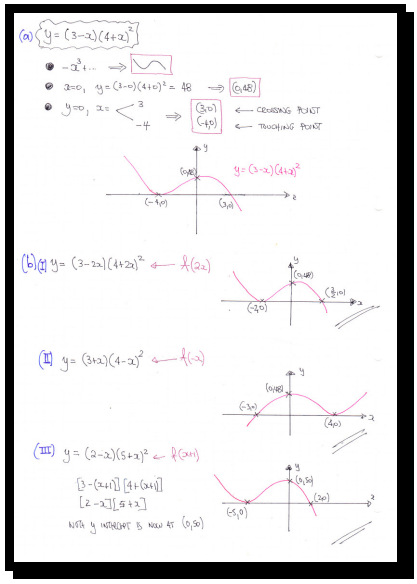
i. … $y=(3-2x)(4+2x)^2$.

ii. … $y=(3+x)(4-x)^2$.

iii. … $y=(2-x)(5+x)^2$.

The sketches must include any points where each of the graphs meets the coordinate axes.

graph



Question 13 (****)

The curve C has equation $y=f(x)$ given by

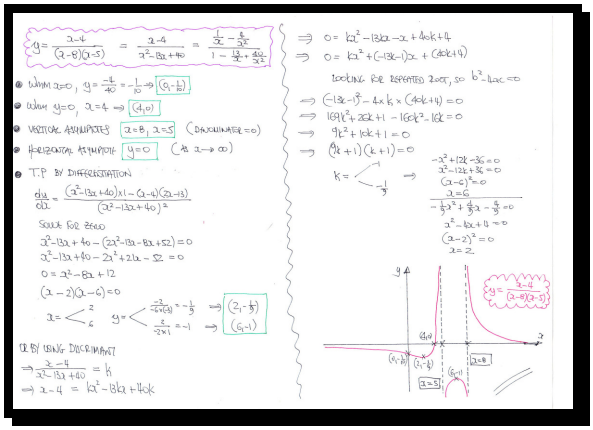
$$f(x)=\frac{x-4}{(x-5)(x-8)},\ x\in\mathbb{R},\ x\neq 5,\ x\neq 8.$$

Sketch the graph of C.

Indicate clearly in the sketch …

- … the equations of the asymptotes
- … the coordinates of any intersections of C with the coordinate axes.
- … the coordinates of any turning points of C.

graph



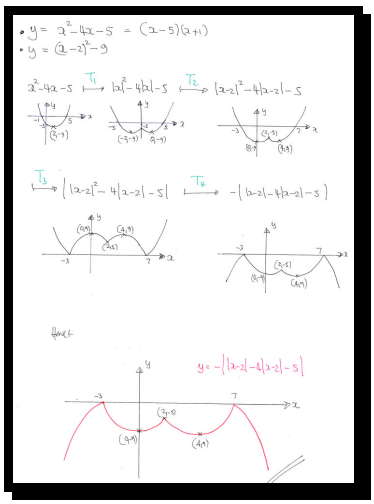
Question 14 (*****)

By considering a sequence of four transformations, or otherwise, sketch the graph of

$$y=-\left||x-2|^2-4|x-2|-5\right|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$$\boxed{(-3,0),(7,0),(0,-9),(2,-5)}$$



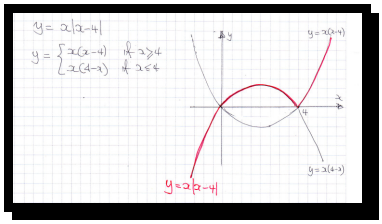
Question 15 (*****)

By considering the graphs of two separate curves, or otherwise, sketch the graph of

$$y=x|x-4|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$$\boxed{(0,0),(4,0)}$$



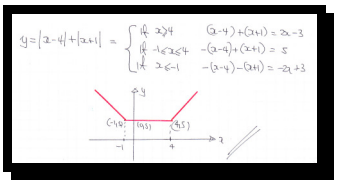
Question 16 (*****)

By considering the graphs of three separate lines, or otherwise, sketch the graph of

$$y=|x-4|+|x+1|$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$$\boxed{(-1,5),(0,5),(4,5)}$$



Question 17 (*****)

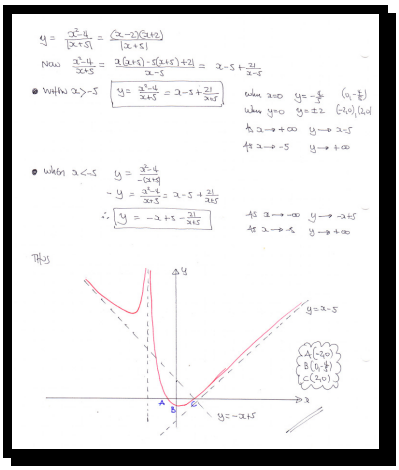
The curve C has equation

$$y=\frac{x^2-4}{|x+5|},\ x\in\mathbb{R},\ x\neq -5.$$

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the asymptotes of the curve.

$$\boxed{\text{graph}}$$



Created by T. Madas