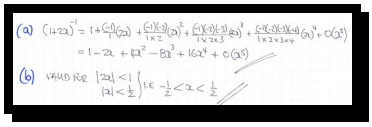
BINOMIAL SERIES EXPANSIONS

Question 1

a) Expand $(1+2x)^{-1}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-2x+4x^2-8x^3+16x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

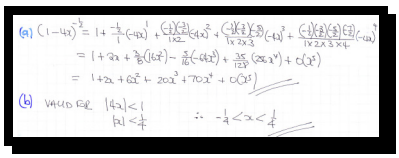


Question 2

a) Expand $(1-4x)^{-\frac{1}{2}}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+2x+6x^2+20x^3+70x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

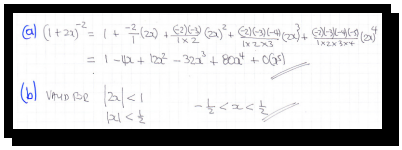


Question 3

a) Expand $(1+2x)^{-2}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-4x+12x^2-32x^3+80x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

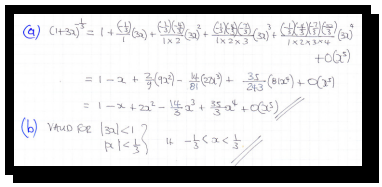


Question 4

a) Expand $(1+3x)^{-\frac{1}{3}}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-x+2x^2-\frac{14}{3}x^3+\frac{35}{3}x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{3}<x<\frac{1}{3}}$$

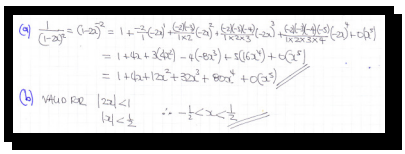


Question 5

a) Expand $\dfrac{1}{(1-2x)^2}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+4x+12x^2+32x^3+80x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

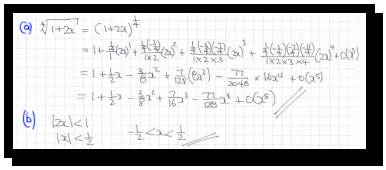


Question 6

a) Expand $\sqrt[4]{1+2x}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{7}{16}x^3-\frac{77}{128}x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

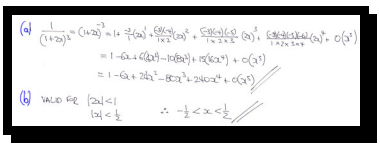


Question 7

a) Expand $\dfrac{1}{(1+2x)^3}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-6x+24x^2-80x^3+240x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

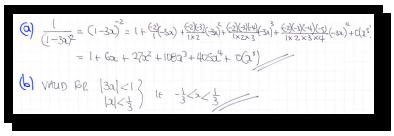


Question 8

a) Expand $\dfrac{1}{(1-3x)^2}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+6x+27x^2+108x^3+405x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{3}<x<\frac{1}{3}}$$

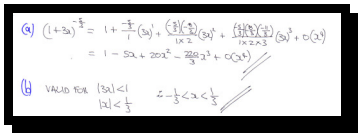


Question 9

a) Expand $(1+3x)^{-\frac{5}{3}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-5x+20x^2-\frac{220}{3}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{3}<x<\frac{1}{3}}$$

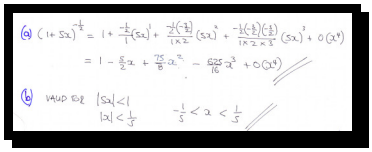


Question 10

a) Expand $(1+5x)^{-\frac{1}{2}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-\frac{5}{2}x+\frac{75}{8}x^2-\frac{625}{16}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{5}<x<\frac{1}{5}}$$

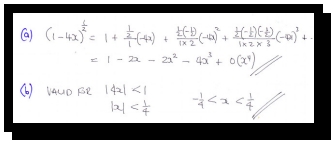


Question 11

a) Expand $(1-4x)^{\frac{1}{2}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-2x-2x^2-4x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

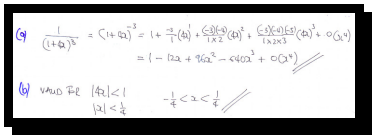


Question 12

a) Expand $\dfrac{1}{(1+4x)^3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-12x+96x^2-640x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

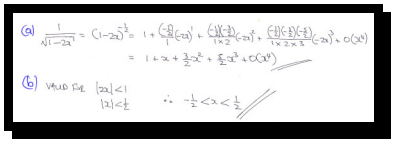


Question 13

a) Expand $\dfrac{1}{\sqrt{1-2x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$



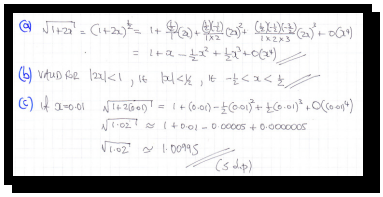
Question 14

a) Expand $\sqrt{1+2x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

c) By using $x=0.01$ in the above expansion find an approximation to $\sqrt{1.02}$, giving the answer correct to 5 decimal places.

$$\boxed{1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}, \quad \boxed{1.00995}$$



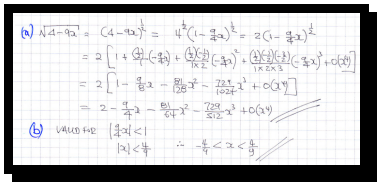
MORE BINOMIAL EXPANSIONS

Question 1

a) Expand $\sqrt{4-9x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{9}{4}x-\frac{81}{64}x^2-\frac{729}{512}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{4}{9}<x<\frac{4}{9}}$$

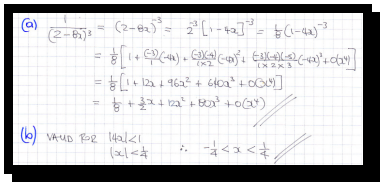


Question 2

a) Expand $\dfrac{1}{(2-5x)^2}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+\frac{125}{8}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{5}<x<\frac{2}{5}}$$

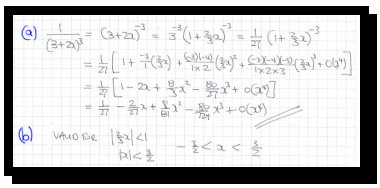


Question 3

a) Expand $\dfrac{1}{(3+2x)^3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{27}-\frac{2}{27}x+\frac{8}{81}x^2-\frac{80}{729}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{3}{2}<x<\frac{3}{2}}$$

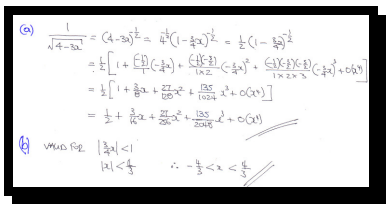


Question 4

a) Expand $\dfrac{1}{\sqrt{4-3x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\frac{135}{2048}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{4}{3}<x<\frac{4}{3}}$$

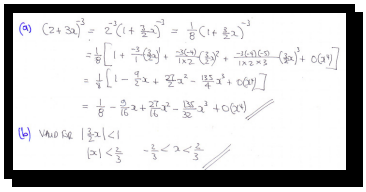


Question 5

a) Expand $(2+3x)^{-3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2-\frac{135}{32}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}$$

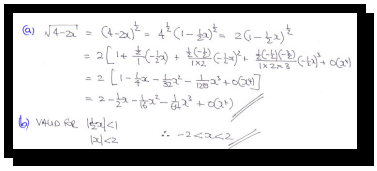


Question 6

a) Expand $\sqrt{4-2x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{1}{2}x-\frac{1}{16}x^2-\frac{1}{64}x^3+O\left(x^4\right)}, \quad \boxed{-2<x<2}$$

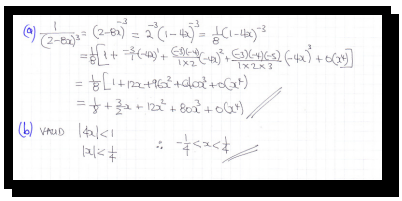


Question 7

a) Expand $\dfrac{1}{(2-8x)^3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{8}+\frac{3}{2}x+12x^2+80x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

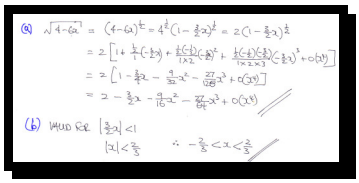


Question 8

a) Expand $\sqrt{4-6x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{3}{2}x-\frac{9}{16}x^2-\frac{27}{64}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}$$

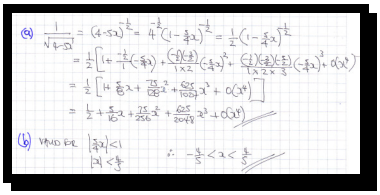


Question 9

a) Expand $\dfrac{1}{\sqrt{4-5x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\frac{625}{2048}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{4}{5}<x<\frac{4}{5}}$$

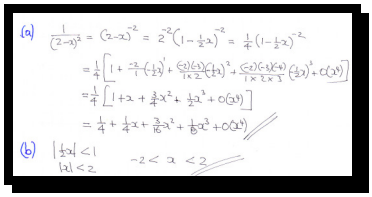


Question 10

a) Expand $\dfrac{1}{(2-x)^2}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{4}+\frac{1}{4}x+\frac{3}{16}x^2+\frac{1}{8}x^3+O\left(x^4\right)}, \quad \boxed{-2<x<2}$$

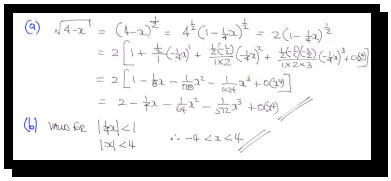


Question 11

a) Expand $\sqrt{4-x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{1}{4}x-\frac{1}{64}x^2-\frac{1}{512}x^3+O\left(x^4\right)}, \quad \boxed{-4<x<4}$$

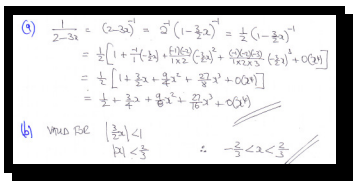


Question 12

a) Expand $\dfrac{1}{2-3x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{2}+\frac{3}{4}x+\frac{9}{8}x^2+\frac{27}{16}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}$$

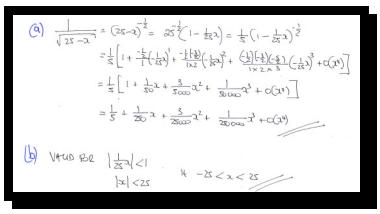


Question 13

a) Expand $\dfrac{1}{\sqrt{25-x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{5}+\frac{1}{250}x+\frac{3}{25000}x^2+\frac{1}{250000}x^3+O\left(x^4\right)}, \quad \boxed{-25<x<25}$$

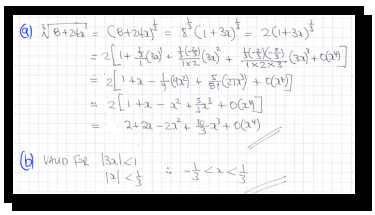


Question 14

a) Expand $\sqrt[3]{8+24x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2+2x-2x^2+\frac{10}{3}x^3+O\left(x^4\right)}, \quad \boxed{-2<x<2}$$



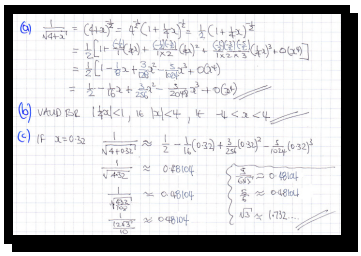
Question 15

a) Expand $\dfrac{1}{\sqrt{4+x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

c) By substituting $x=0.32$ into the expansion show that $\sqrt{3}\approx 1.732$.

$$\boxed{\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2-\frac{5}{2048}x^3+O\left(x^4\right)}, \quad \boxed{-4<x<4}$$

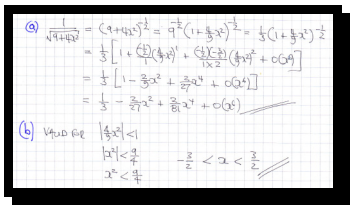


Question 16

a) Expand $\dfrac{1}{\sqrt{9+4x^2}}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{3}-\frac{2}{27}x^2+\frac{2}{81}x^4+O\left(x^6\right)}, \quad \boxed{-\frac{3}{2}<x<\frac{3}{2}}$$



Question 17

a) Expand $(2+3x)^{-2}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

c) Find the coefficient of x^6 in the above expansion.

$$\boxed{\frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2-\frac{27}{8}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}, \quad \boxed{\frac{5103}{256}}$$

