MOTION ON AXISYMETRIC SURFACES

Question 1 (***)

A particle of mass m is moving on a smooth axisymmetric surface with equation

$$z = f(r),$$

where r measures the distance from the **vertical** axis of symmetry, and z is the vertical distance along that axis, measured from an arbitrary origin O.

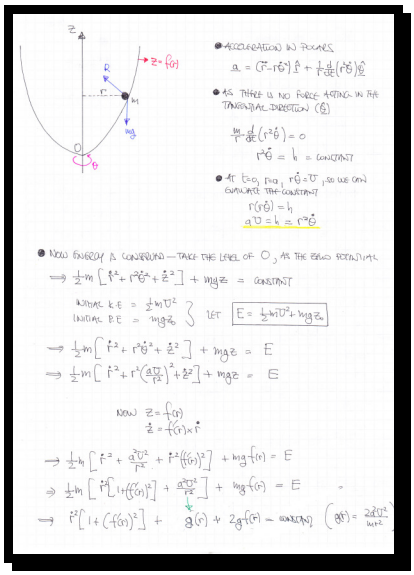
The particle is set in motion at a point on the surface where $r = a$, with horizontal speed U, tangential to the surface.

If air resistance can be ignored, show that

$$\dot{r}^2\left[1+\left[f'(r)\right]^2\right]+2g\,f(r)+g(r)=\text{constant},$$

where g is a function to be found.

$$\boxed{g(r)=\frac{2a^2U^2}{mr^2}}$$



Question 2 (*+)**

A particle of mass m is moving on the smooth outer surface of a right circular cone, of semi-vertical angle α, $\alpha < \frac{1}{2}\pi$, whose vertex is uppermost and its axis vertical.

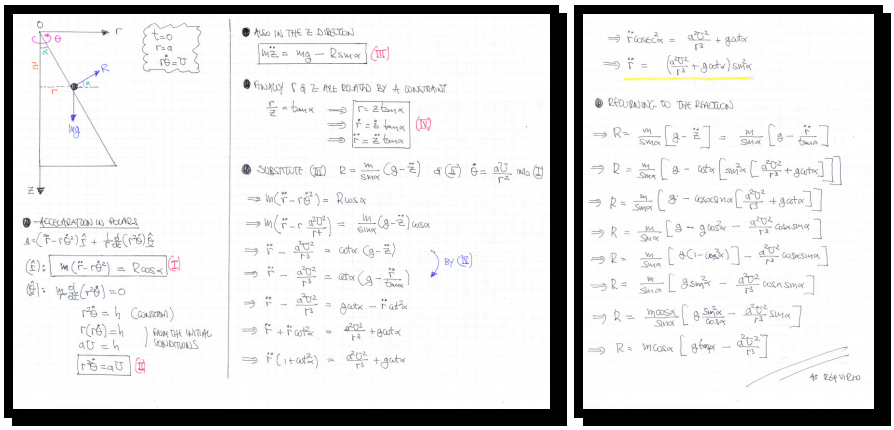
The particle is set in motion at a point on the surface of the cone where the radius is a, with horizontal speed U, tangential to the surface of the cone.

At a general point on the cone with radius is r, the reaction force on the particle is R.

Ignoring air resistance, show that

$$R = m\left[g\tan\alpha - \frac{a^2U^2}{r^3} \right]\cos\alpha .$$

proof



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