WORK and VECTORS

Question 1 (**)

The vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors perpendicular to each other.*

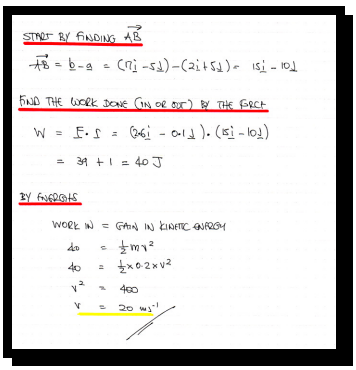
A bead of mass 0.2 kg is threaded on a smooth straight horizontal wire.

The bead is at rest at the point A with position vector $(2\mathbf{i}+5\mathbf{j})$ m.

A single force $(2.6\mathbf{i}-0.1\mathbf{j})$ N acts on the bead and moves it to the point B with position vector $(17\mathbf{i}-5\mathbf{j})$ m.

Find the speed of the bead at B.

$\boxed{}$, $\boxed{20\text{ ms}^{-1}}$



Question 2 (**)

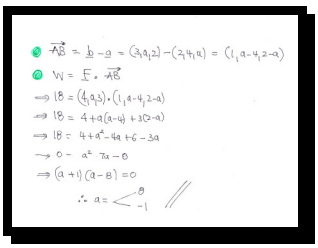
The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle P moves from the point A, with position vector $(2\mathbf{i}+4\mathbf{j}+a\mathbf{k})$ m, where a is a constant, to the point B, with position vector $(3\mathbf{i}+a\mathbf{j}+2\mathbf{k})$ m, under the action of a constant force $\mathbf{F}=(4\mathbf{i}+a\mathbf{j}+3\mathbf{k})$ N.

The work done by $\mathbf{F}$, as it moves P from A to B, is 18 J.

Find the possible values of a.

$$\boxed{a=-1,\ \ a=8}$$



Question 3 (***)

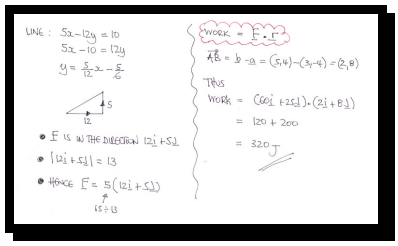
A small bead is threaded on a smooth, straight horizontal wire which passes through the point $A(3,-4)$ and the point $B(5,4)$.

The bead moves under the action of a single horizontal force $\mathbf{F}$ of magnitude 65 N, whose line of action is parallel to the straight line with equation

$$5x-12y=10\,.$$

Given that all distances are measured in m, find the work done by $\mathbf{F}$ as it moves the bead from A to B.

$\boxed{320\text{ J}}$



Question 4 (***)

The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle of mass 5 kg is initially at rest at the point P with position vector $(4\mathbf{i}-\mathbf{j}+7\mathbf{k})$ m when is acted by a force $\mathbf{F}$ which causes it to move to the point Q with position vector $(9\mathbf{i}+9\mathbf{j}-3\mathbf{k})$ m.

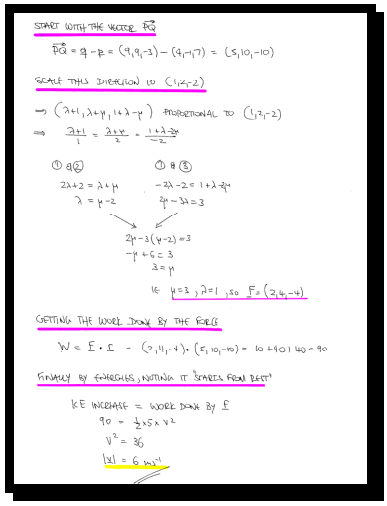
It is further given that

$$\mathbf{F}=\left[(\lambda+1)\mathbf{i}+(\lambda+\mu)\mathbf{j}+(1+\lambda-2\mu)\mathbf{k}\right] \text{ N},$$

where λ and μ are scalar constants.

If $\mathbf{F}$ is acting in the direction PQ, determine the speed of the particle as it passes Q.

$\boxed{}$, $\boxed{6 \text{ ms}^{-1}}$



Question 5 (****)

The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

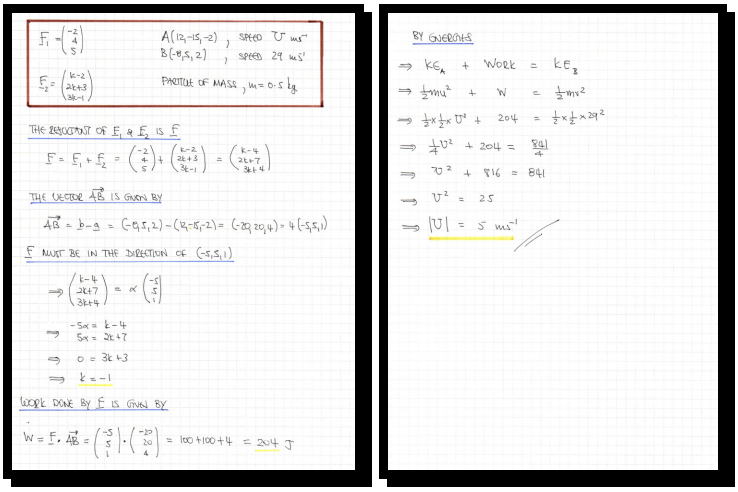
A particle, of mass 0.5 kg, passes through the point A whose position vector is $(12\mathbf{i}-15\mathbf{j}-2\mathbf{k})\text{ m}$, with speed $U\text{ ms}^{-1}$. The particle is moving due to the action of the following two constant forces, $\mathbf{F}_1$, and $\mathbf{F}_2$.

$$\mathbf{F}_1=\begin{pmatrix}-2\\4\\5\end{pmatrix}\text{N}\quad\text{and}\quad\mathbf{F}_2=\begin{pmatrix}k-2\\2k+3\\3k-1\end{pmatrix}\text{N},$$

where k is a scalar constant.

Determine the value of U, given further that it passes through the point B, whose position vector is $(-8\mathbf{i}+5\mathbf{j}+2\mathbf{k})\text{ m}$, with speed 29 ms^{-1}.

$\boxed{}$, $\boxed{U=5\text{ ms}^{-1}}$



Question6 (****)

The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle of mass 2 kg, which is free to move in any direction, is acted by two forces

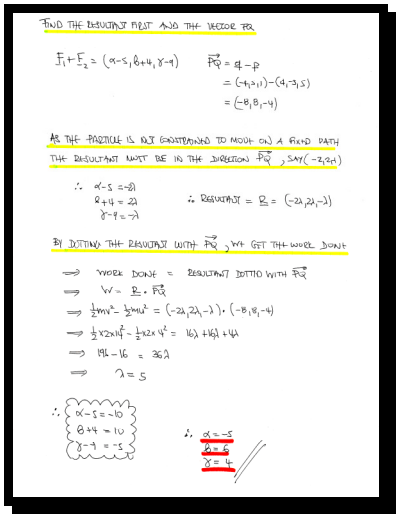
$$\mathbf{F}_1=(-5\mathbf{i}+4\mathbf{j}-9\mathbf{k})\text{ N} \quad \text{and} \quad \mathbf{F}_2=(\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k})\text{ N},$$

where α, β and γ are scalar constants.

These two forces cause the particle to move **directly** from the point P with position vector $(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$ m to the point Q with position vector $(-4\mathbf{i}+5\mathbf{j}+\mathbf{k})$ m.

If the respective speeds of the particle at P and Q are 4 ms^{-1} and 14 ms^{-1}, determine the values of α, β and γ.

$\boxed{}$, $\boxed{[\alpha,\beta,\gamma]=[-5,6,4]}$



Question 7 (****)

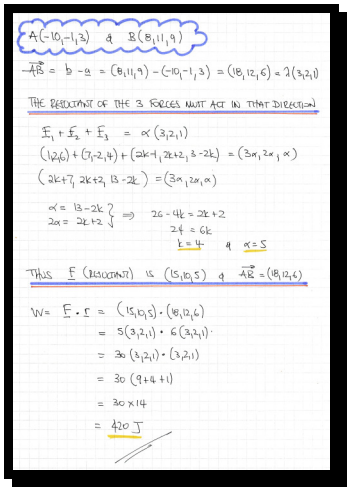
The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle P moves from the point A, with position vector $(-10\mathbf{i}-\mathbf{j}+3\mathbf{k})$ m to the point B, with position vector $(8\mathbf{i}+11\mathbf{j}+9\mathbf{k})$ m, under the action of the following three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

- $\mathbf{F}_1=(\mathbf{i}+2\mathbf{j}+6\mathbf{k})$ N.
- $\mathbf{F}_2=(7\mathbf{i}-2\mathbf{j}+4\mathbf{k})$ N
- $\mathbf{F}_3=\left[(2k-1)\mathbf{i}+(2k+2)\mathbf{j}+(3-2k)\mathbf{k}\right]$ N, where k is a scalar constant.

Determine the work done by the three forces in moving the particle from A to B.

$\boxed{\phantom{W=420\text{ J}}}$, $\boxed{W=420\text{ J}}$



Question 8 (*****)

The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to each other.*

A particle, of mass 0.5 kg, passes through the point A whose position vector is $(14\mathbf{i}-10\mathbf{j})\text{ m}$, with speed 5 ms^{-1}. The particle is moving due to the action of the following two constant forces, $\mathbf{F}_1$, and $\mathbf{F}_2$.

$$\mathbf{F}_1=\begin{pmatrix}-\lambda\\2\lambda\\\nu\end{pmatrix}\text{N}\quad\text{and}\quad\mathbf{F}_2=\begin{pmatrix}\mu-2\\2\mu+3\\3\mu-1\end{pmatrix}\text{N},$$

where λ, μ and ν are scalar constants.

It further given that the particle passes through the point B, whose position vector is $(-6\mathbf{i}+10\mathbf{j}+4\mathbf{k})\text{ m}$, with speed 29 ms^{-1}.

Determine the value of each of the constants λ, μ and ν.

$\boxed{}$, $\boxed{\lambda=2}$, $\boxed{\mu=-1}$, $\boxed{\nu=5}$

