VECTOR DIFFERENTIAL EQUATIONS

First Order

Question 1 (+)**

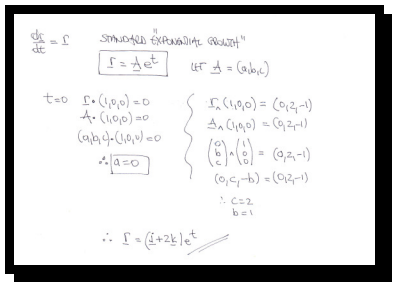
A particle P is moving on the Cartesian plane so that its position vector $\mathbf{r}$ m at time t s satisfies the differential equation

$$\frac{d\mathbf{r}}{dt}=\mathbf{r}\,.$$

When $t=0$, $\mathbf{r}\cdot\mathbf{i}=0$ and $\mathbf{r}\wedge\mathbf{i}=2\mathbf{j}-\mathbf{k}$.

Express $\mathbf{r}$ in terms of t.

$$\boxed{\mathbf{r}=(\mathbf{j}+2\mathbf{k})\mathrm{e}^{t}}$$



Question 2 (+)**

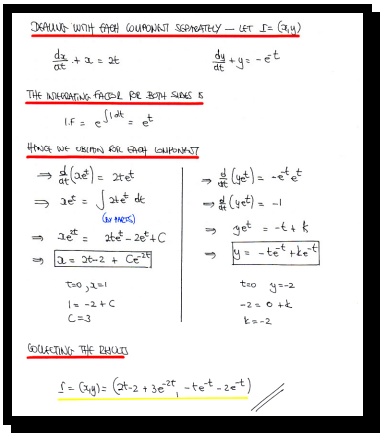
A particle moves in a plane so that its position vector, $\mathbf{r}$ m at time t s, satisfies the differential equation

$$\frac{d\mathbf{r}}{dt}+\mathbf{r}=2t\mathbf{i}-\mathrm{e}^{-t}\,\mathbf{j}.$$

When $t=0$ the particle is at the point with position vector $(\mathbf{i}-2\mathbf{j})$ m.

Express $\mathbf{r}$ in terms of t.

$\square$, $\boxed{\mathbf{r}=\left(2t-2+3\mathrm{e}^{-t}\right)\mathbf{i}-\left(t\mathrm{e}^{-t}+2\mathrm{e}^{-t}\right)\mathbf{j}}$



Question 3 (***)

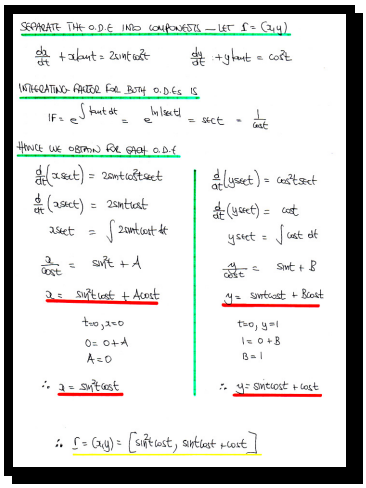
A particle moves in a plane so that its position vector, $\mathbf{r}$ m at time t s, satisfies the differential equation

$$\frac{d\mathbf{r}}{dt}+(\tan t)\mathbf{r}=\left(2\sin t\cos^2 t\right)\mathbf{i}+\left(\cos^2 t\right)\mathbf{j}, \qquad 0\le t<\frac{\pi}{2}.$$

When $t=0$ the particle is at the point with position vector $\mathbf{j}$ m.

Express $\mathbf{r}$ in terms of t.

$\boxed{}$, $\boxed{\mathbf{r}=\left(\sin^2 t\cos t\right)\mathbf{i}+\left(\sin t\cos t+\cos t\right)\mathbf{j}}$



Second Order

Question 1 (**)

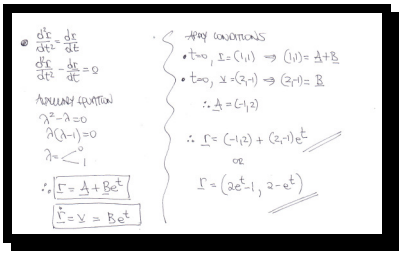
A particle P is moving on the Cartesian plane so that its position vector $\mathbf{r}$ m at time t s satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2}=\frac{d\mathbf{r}}{dt}.$$

When $t=0$, P has position vector $(\mathbf{i}+\mathbf{j})$ m and moving with velocity $(2\mathbf{i}-\mathbf{j})\,\text{ms}^{-1}$.

Express $\mathbf{r}$ in terms of t.

$$\boxed{\mathbf{r}=(-\mathbf{i}+2\mathbf{j})+(2\mathbf{i}-\mathbf{j})\mathrm{e}^{t}\quad\text{or}\quad\mathbf{r}=\left(2\mathrm{e}^{t}-1\right)\mathbf{i}+\left(2-\mathrm{e}^{t}\right)\mathbf{j}}$$



Question 2 **(**+)**

A particle P is moving on the Cartesian plane so that its position vector $\mathbf{r}$ m at time t s satisfies the differential equation

$$\frac{d^2\mathbf{r}}{dt^2}-\frac{d\mathbf{r}}{dt}=6\left(\mathbf{r}+t\,\mathbf{i}-2\mathbf{j}\right).$$

When $t=0$, P has position vector $\left(\mathbf{i}+2\mathbf{j}\right)$m and moving with velocity $\left(3\mathbf{i}-\mathbf{j}\right)\text{ms}^{-1}$.

Express $\mathbf{r}$ in terms of t.

$\square$,

$$\mathbf{r}=\frac{1}{15}\text{e}^{3t}\left(17\mathbf{i}-3\mathbf{j}\right)-\frac{1}{10}\text{e}^{-2t}\left(3\mathbf{i}-2\mathbf{j}\right)\text{e}^{t}+\frac{1}{6}\left(\mathbf{i}+12\mathbf{j}\right)-t\,\mathbf{i}$$

or

$$\mathbf{r}=\left(\frac{17}{15}\text{e}^{3t}-\frac{3}{10}\text{e}^{-2t}-t+\frac{1}{6}\right)\mathbf{i}+\left(-\frac{1}{5}\text{e}^{3t}+\frac{1}{5}\text{e}^{-2t}+2\right)\mathbf{j}$$

