KINEMATICS
&
DYNAMICS
(SIMPLE DIFFERENTIAL EQUATIONS)

Question 1 (**)

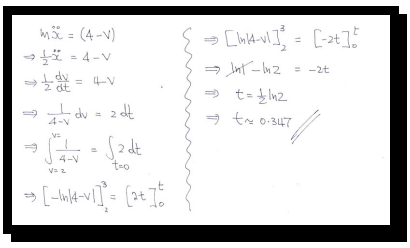
A toy of mass 0.5 kg, is placed on a slope.

The toy is set in motion down the slope with an initial speed 2 ms^{-1}.

The resultant force acting on the particle has magnitude $(4-v)\text{ N}$, where $v\text{ ms}^{-1}$ is the speed of the toy at time $t\text{ s}$ after it was set in motion.

By modelling the toy as a particle, determine the value of t when the speed of the toy reaches 3 ms^{-1}.

$$\boxed{t=\frac{1}{2}\ln 2\approx 0.347\text{ s}}$$



Question 2 (**)

A particle P is moving on the positive x axis with acceleration of magnitude

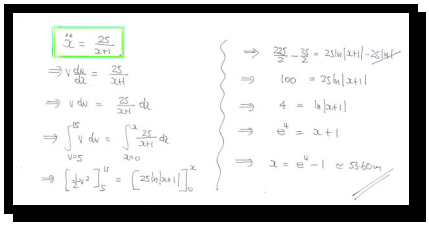
$$\frac{25}{x+1}\ \text{ms}^{-2},$$

acting away from the origin O.

When the displacement of P from O is x m, its speed is v ms^{-1}.

Given that P passes through O with speed 5 ms^{-1}, calculate the distance of P from O when its speed reaches 15 ms^{-1}.

$$\boxed{d \approx 53.598...\ \text{m}}$$



Question 3 (**)

A particle P, of mass 2 kg, is moving on the positive x axis.

At time t s, its displacement from the origin O is x m and its velocity is v ms^{-1}.

When $x=1$, $v=1$.

The resultant force acting on P has magnitude

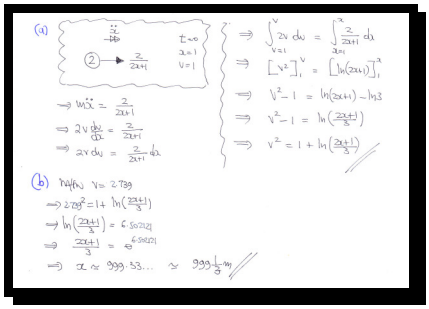
$$\frac{2}{2x+1}\text{ N},$$

acting away from O.

a) Find an expression for v^2 in terms of x.

b) Find the distance of the particle from O when its speed is 2.739 ms^{-1}.

$$\boxed{v^2=1+\ln\left(\frac{2x+1}{3}\right)},\quad \boxed{d\approx 999\tfrac{1}{3}\text{ m}}$$



Question 4 (**)

A particle P is moving on the positive x axis with acceleration of magnitude

$$\frac{4}{5}\mathrm{e}^{-0.1x}\ \mathrm{ms}^{-2},$$

acting away from the origin O.

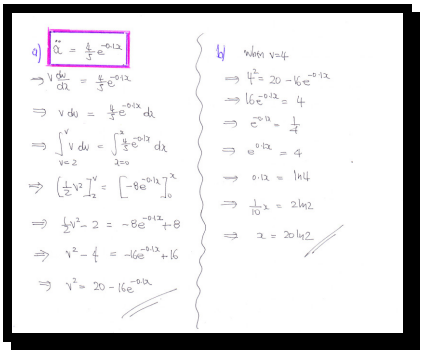
When the displacement of P from O is x m, its velocity is $v\ \mathrm{ms}^{-1}$.

When $x=0$, $v=2$.

a) Find an expression for v^2 in terms of x.

b) Determine the exact value of x when $v=4$.

$\boxed{v^2=20-16\mathrm{e}^{-0.1x}}$, $\boxed{x=20\ln 2}$



Question 5 (**)

A particle P is moving on the x axis with acceleration of magnitude

$$2x-\frac{3}{2}x^2 \text{ ms}^{-2},$$

acting away from the origin O.

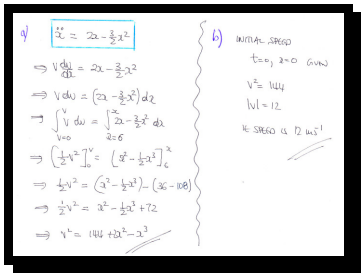
When the displacement of P from O is x m, its velocity is v ms^{-1}.

P passes through O when $t=0$ and comes to instantaneous rest at $x=6$.

a) Find an expression for v^2 in terms of x.

b) Determine the initial speed of P.

$$\boxed{v^2=144+2x^2-x^3}, \quad \boxed{\text{speed}=12 \text{ ms}^{-1}}$$



Question 6 (+)**

A particle P of mass 2 kg, is moving on a horizontal x axis, in the positive x direction. The only force acting on P is F, which acts in the positive x direction.

The magnitude of F is

$$1+4\sqrt[3]{t}\,,$$

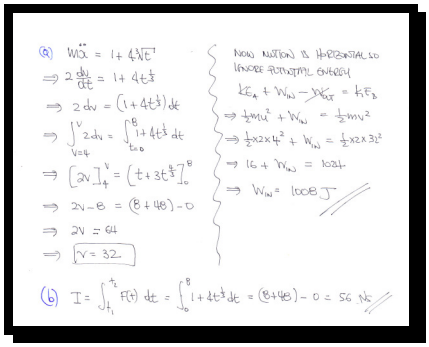
where t is the time in seconds since the particle had velocity 4 ms^{-1}.

For the interval $0\le t\le 8$, determine …

a) … the work done by F.

b) … the impulse of F.

$\boxed{W_{\text{in}}=1008\text{ J}}$, $\boxed{I=56\text{ Ns}}$



Question 7 (+)**

A particle P starts from rest and moves on the x axis with acceleration of magnitude

$$\frac{60}{(t+3)^2}\ \text{N},$$

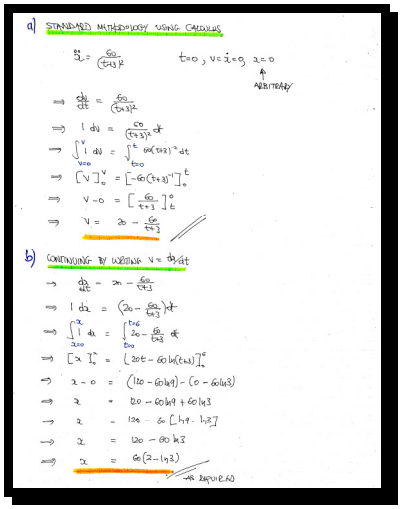
acting in the direction of x increasing.

a) Find an expression for the velocity of P, in terms of t.

b) Show that the distance covered by P in the first 6 s of its motion is

$$60(2-\ln 3)\ \text{m}.$$

$\square$, $\boxed{v=20-\dfrac{60}{t+3}}$



Question 8 (+)**

A particle P of mass 2 kg starts from rest and moves on the x axis.

At time t s, the resultant force acting on P has magnitude

$$\frac{90}{(t+3)^2}\ \text{N},$$

acting in the direction of x increasing.

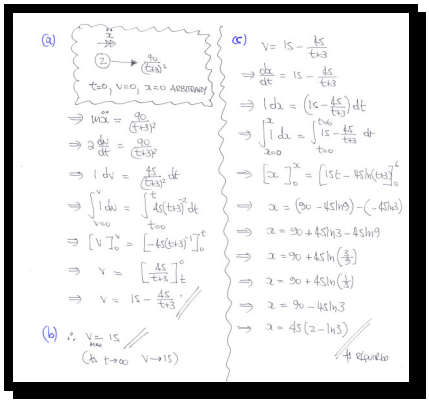
a) Find an expression for the velocity of P, in terms of t.

b) State the limiting value for the velocity of P.

c) Show that the distance covered by P in the first 6 s of its motion is

$$45(2-\ln 3)\ \text{m}.$$

$$\boxed{v=15-\frac{45}{t+3}},\quad \boxed{v_{\max}=15\ \text{ms}^{-1}}$$



Question 9 (***)

A particle P is moving on the x axis, starting from the origin O, and moving in the direction of x increasing with speed 8 ms^{-1}.

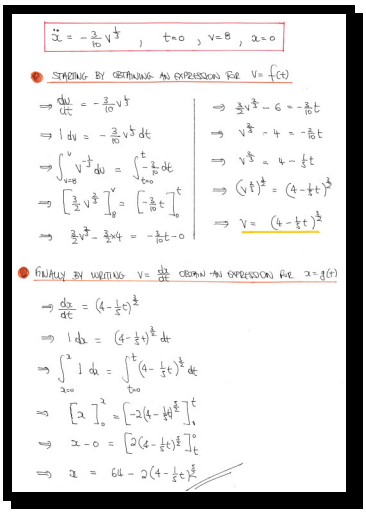
The acceleration of P is in the direction of x **decreasing** and has magnitude

$$\frac{3}{10}v^{\frac{1}{3}}\text{ ms}^{-2},$$

where v is the velocity of the particle at time t.

Find an expression for the displacement of P in terms of t.

$\square$, $\boxed{x=64-2\left(4-\frac{1}{5}t\right)^{\frac{5}{2}}}$



Question 10 (***)

A particle P, of mass $0.5\,\text{kg}$, is projected horizontally with speed $u\ \text{ms}^{-1}$ from a fixed origin O on a smooth horizontal plane. When P has been moving for $t\,\text{s}$, its speed is $v\ \text{ms}^{-1}$ and its displacement from O is $x\,\text{m}$.

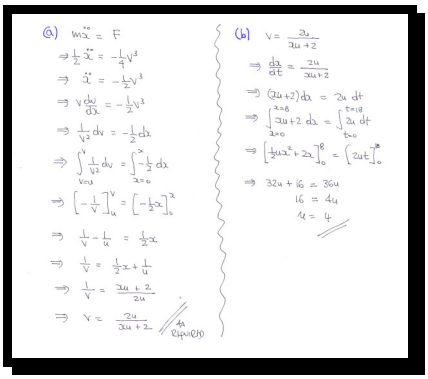
The only force acting on P is a resistive force of magnitude $\frac{1}{4}v^3$.

a) Show clearly that

$$v=\frac{2u}{ux+2}.$$

b) Given further that when $t=18$, $x=8$, determine the value of u.

$$\boxed{u=4}$$



Question 11 (***)

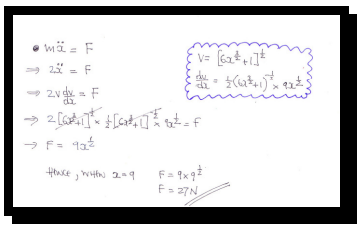
A particle of mass 2 kg is moving along the positive x axis under the action of a single force of magnitude F N, which acts along the x axis in the direction of x increasing.

When the particle is x m from the origin O, it is moving away from O with speed

$$\sqrt{8x^{\frac{3}{2}}+1} \text{ ms}^{-1}.$$

Find the value of F when the particle is 9 m away from O.

$$\boxed{F=27}$$



Question 12 (***)

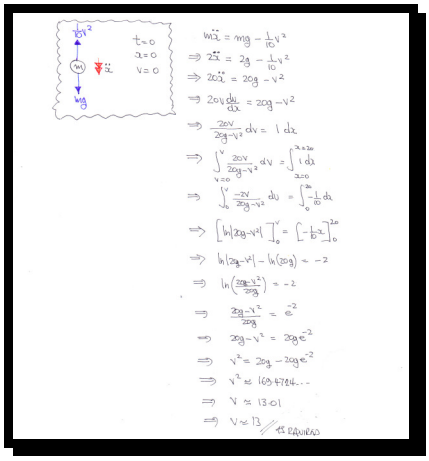
A particle P, of mass 2 kg, is released from rest and falls vertically.

When P has fallen a distance x m it has speed v ms^{-1}.

P is falling under the action of its weight and air resistance of magnitude $\frac{1}{10}v^2$ N.

Show that at the instant when P has fallen through a distance of 20 m its speed is approximately 13 ms^{-1}.

proof



Question 13 (***)

A particle of mass 0.25 kg, falls vertically from rest, and when the particle has been falling for t s, its speed is $v\text{ ms}^{-1}$.

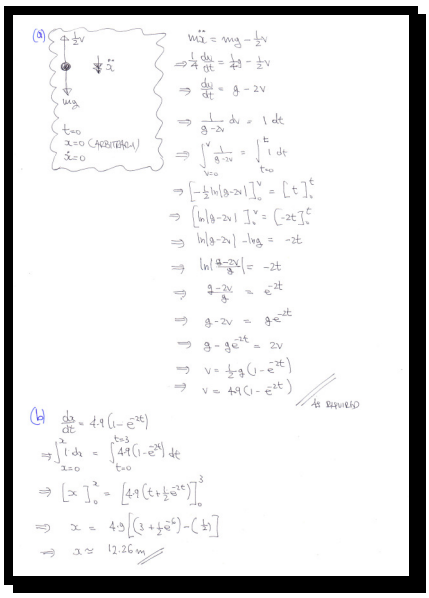
A resistive force of magnitude $\frac{1}{2}v$ is acting on the particle as it falls.

a) Show clearly that

$$v = 4.9\left(1-\mathrm{e}^{-2t}\right).$$

b) Calculate, correct to two decimal places, the distance the particle falls in the first 3 s of its motion.

$$\boxed{d \approx 12.26\text{ m}}$$



Question 14 (***)

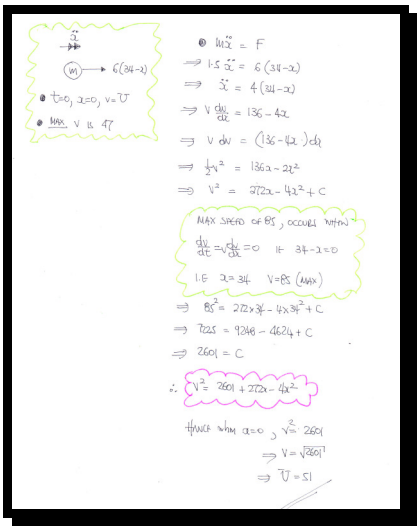
A particle P of mass 1.5 kg is moving along a horizontal x axis.

At time $t = 0$ s, P is passes through the origin O with speed U ms^{-1}, moving in the direction of x increasing.

At time t s, $|OP| = x$ and the resultant force F acting on P has magnitude $6(34 - x)$ N.

Given that for $t > 0$ the greatest speed of P is 85 ms^{-1}, determine the value of U.

$$\boxed{U = 51}$$



Question 15 (*+)**

A particle P, of mass $\frac{1}{6}$ kg, is moving on a straight horizontal line under the action of the force

$$F=\left(\frac{1}{t}-1\right)\left(\frac{1}{t}+1\right) \text{ N},$$

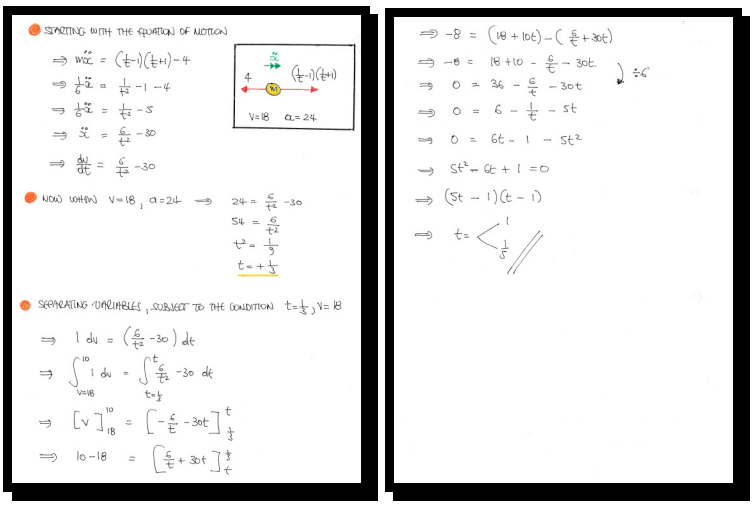
where t s is the time the particle is in motion, $t>0$.

The motion of P is resisted by a constant force of magnitude 4 N.

It is further given that when the velocity of P is 18 ms^{-1} its acceleration is 24 ms^{-2}.

Determine the values of t when the velocity of P is 10 ms^{-1}.

$\square$, $\boxed{t=1,\ \frac{1}{5}}$



Question 16 (*+)**

A particle P, of mass 0.2 kg, is moving on the positive x axis under the action of a force F which is directed towards the origin O.

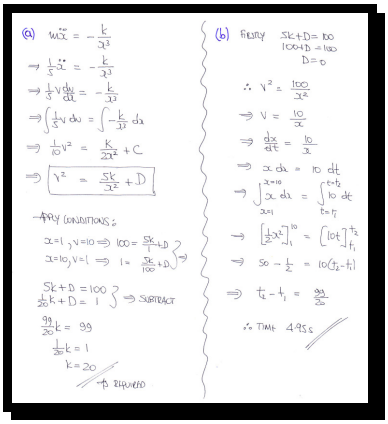
The magnitude of F is given by $\dfrac{k}{x^3}$, where k is the positive constant and x is the distance OP.

P has a speed of 10 ms^{-1}, away from the origin, when $OP=1$ m and sometime later it has a speed of 1 ms^{-1}, away from the origin, when $OP=10$ m.

a) Show that $k=20$.

b) Calculate the time it took P to reduce its speed from 10 ms^{-1} to 1 ms^{-1}.

$$\boxed{t=4.95\text{ s}}$$



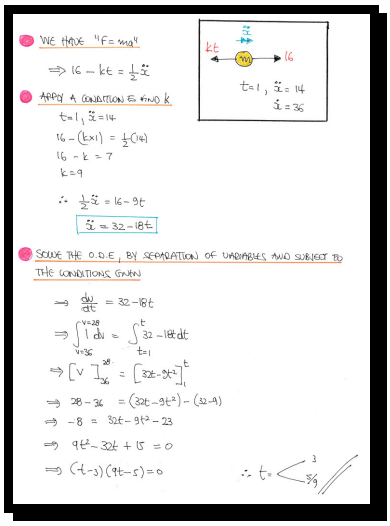
Question 17 **(***+)**

A particle P, of mass 0.5 kg, is moving on a straight horizontal line under the action of a constant force of magnitude 16 N. The motion of P is resisted by a force whose magnitude is proportional to the time $t\text{ s}$, where t is measured from a given instant.

When $t=1$, the velocity of P is 36 ms^{-1} and its acceleration is 14 ms^{-2}.

Determine the values of t when the velocity of P is 28 ms^{-1}.

$\square$, $\boxed{t=3,\ \frac{5}{9}}$



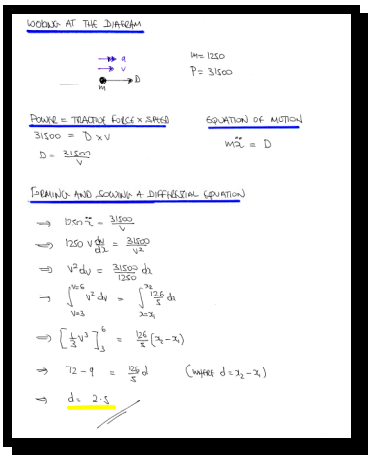
Question 18 (*+)**

A car of mass 1250 kg is accelerating along a straight, horizontal road with the engine of the car producing a constant power of magnitude 31.5 kW .

The car is modelled as a particle with any other resistances to its motion ignored.

Find the distance covered by the car as its speed increases from 3 ms^{-1} to 6 ms^{-1}.

$\boxed{}$, $\boxed{d = 2.5\text{ m}}$



Question 19 (*+)**

A car of mass 1600 kg moves along a straight horizontal road.

At time t s the resultant force acting on the car has magnitude

$$\frac{96000}{kt^2}\text{ N},$$

where k is a positive constant.

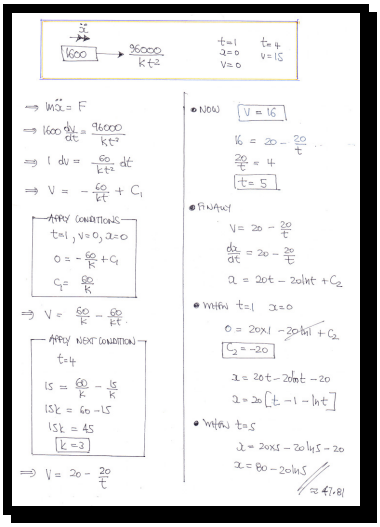
The resultant force acts in the direction of motion of the car.

At time t s , $t \geqslant 1$, the speed of the car is v ms^{-1} and the car is at a distance x m from a fixed point O, on the road.

When $t = 1$ the car is at rest at O and when $t = 4$ the speed of the car is 15 ms^{-1}.

Find the value of x when $v = 16$.

$$\boxed{x = 80 - 20\ln 5 \approx 47.81\text{ m}}$$



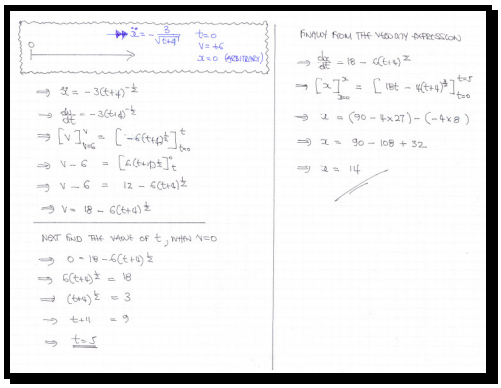
Question 20 (*+)**

At time $t=0$ s, a particle is at the origin O, and moving with speed 6 ms^{-1}, in the positive x direction.

For $t \geq 0$, the particle moves with acceleration of magnitude $-\dfrac{3}{\sqrt{t+4}}$, which is directed towards O.

Find the distance of the particle from O, when it comes to instantaneous rest.

$\boxed{x=14 \text{ m}}$



Question 21 (*+)**

A charged particle is accelerated in an electromagnetic field.

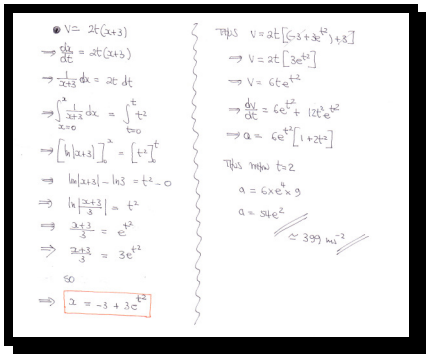
Its velocity, v ms^{-1}, is given by

$$v = 2t\left(x+3\right),$$

where x m is the distance of the particle from a fixed origin O, at time t s.

Given that $x = 0$ when $t = 0$, determine the acceleration of the particle when $t = 2$.

$$\boxed{a = 54\mathrm{e}^4 \approx 399 \text{ ms}^{-2}}$$



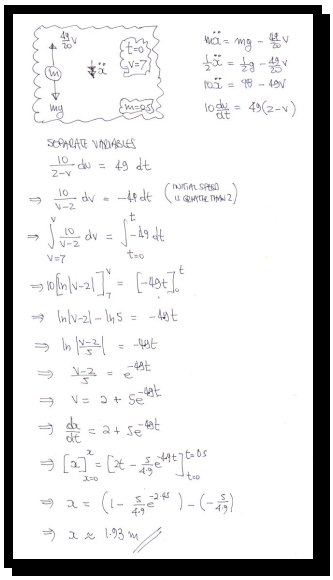
Question 22 (*+)**

A particle P, of mass 0.5 kg, is projected vertically downwards, in a viscous fluid with an initial speed 7 ms^{-1}. When P has been moving for $t\text{ s}$, its speed is $v\text{ ms}^{-1}$.

A resistive force of magnitude $\frac{49}{20}v$ is acting on P.

Calculate the approximate distance P covers in the first 0.5 s of its motion.

$$\boxed{d \approx 1.93\text{ m}}$$



Question 23 (*+)**

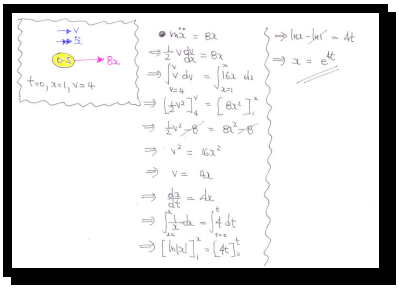
A particle P of mass 0.5 kg is moving in a straight line on a smooth horizontal plane.

At time $t = 0$ s, P is at a distance of 1 m from the origin O and moving away from O with speed 4 ms^{-1}.

The resultant force acting on P has magnitude $8x$ N away from O, where x is the distance OP.

Find a simplified expression for the speed of P at time t.

$$\boxed{x = \mathrm{e}^{4t}}$$



Question 24 (*+)**

A particle of mass 0.5 kg is moving along the positive x axis.

At time t s, where $t \geq 0$, the particle has displacement x m from the origin O, its velocity is v ms^{-1} and its acceleration is a ms^{-2}.

When $t = 0$, $x = 0$.

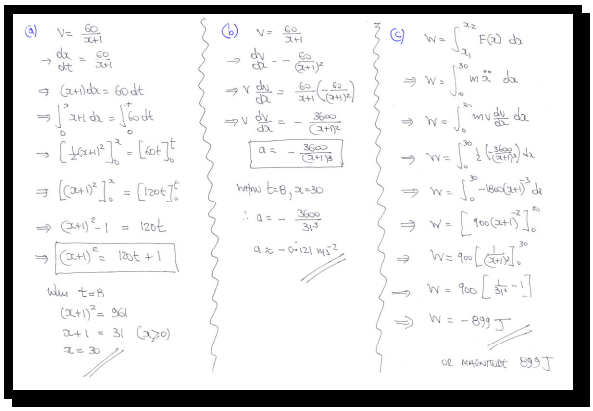
Given that $v = \dfrac{60}{x+1}$, $x \geq 0$, determine for the time interval $0 \leq t \leq 8$ …

a) … the displacement of the particle at the end of this time interval.

b) … the acceleration of the particle at the end of this time interval.

c) … the magnitude of the work done by the resultant force acting on this particle during this time interval.

$\boxed{x = 30 \text{ m}}$, $\boxed{a \approx -0.121 \text{ ms}^{-1}}$, $\boxed{|W| \approx 899 \text{ J}}$



Question 25 (*+)**

A particle, of mass 0.5 kg, is moving on a straight line, under the action of a single force of magnitude

$$\left[\frac{25}{x^2}-\frac{50}{x^3}\right]\text{ N},\ x>0,$$

where x is the distance of the particle from a fixed origin O.

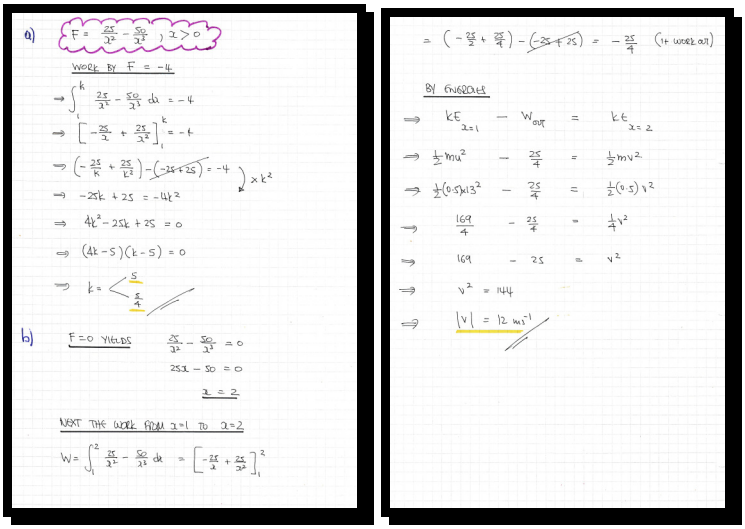
The particle is released from the point where $x=1$, with speed 13 ms^{-1}, in the direction of x increasing.

It is further given that in moving the particle from $x=1$ to a point where $x=k$, $k>1$, the force does work of -4 J.

a) Determine the possible values of k.

b) Find the least speed of the particle in its consequent motion.

$\boxed{}$, $\boxed{k=5\cup k=\frac{5}{4}}$, $\boxed{V_{\min}=12\text{ ms}^{-1}}$



Question 26 (****)

A particle P is moving on the x axis, starting from rest at the origin O.

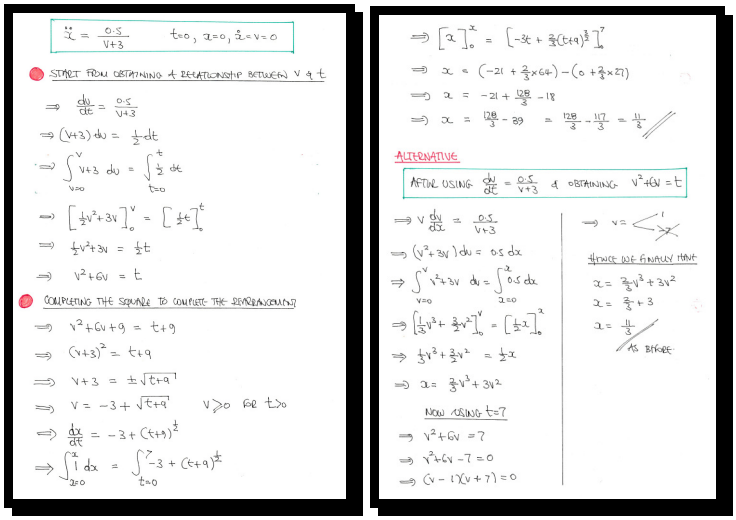
The acceleration of P is in the direction of x increasing and has magnitude

$$\frac{0.5}{v+3}\ \text{ms}^{-2},$$

where v is the subsequent velocity of the particle.

Find the distance P covers in the first 7 seconds of its motion.

$\boxed{}$, $\boxed{d=\frac{11}{3}}$



Question 27 (****)

A particle is moving along the positive x axis.

At time t s, the particle is at a distance of x m from the origin O and is moving away from O with speed v ms^{-1}.

The particle is moving in such a way so that the rate of change of change of its speed with respect to the distance covered is 0.5 s^{-1}.

It is further given that $x=4$ and $v=3$ when $t=0$.

a) Find the value of t when $x=16$.

b) Determine the acceleration of the particle when $x=16$.

$$\boxed{t=2\ln 3\approx 2.20 \text{ s}}, \quad \boxed{a=4.5 \text{ ms}^{-2}}$$

