VECTOR PRACTICE

Part B

THE CROSS PRODUCT

Question 1

Find in each of the following cases $\mathbf{a} \wedge \mathbf{b}$, where the vectors $\mathbf{a}$ and $\mathbf{b}$ are

a) $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j}$

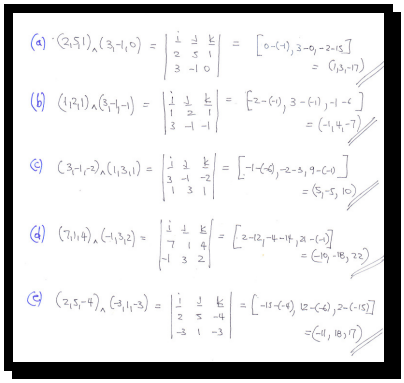
b) $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$

c) $\mathbf{a} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$

d) $\mathbf{a} = 7\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

e) $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

$\boxed{\mathbf{i} + 3\mathbf{j} - 17\mathbf{k}}$, $\boxed{-\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}}$, $\boxed{5\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}}$, $\boxed{-10\mathbf{i} - 18\mathbf{j} + 22\mathbf{k}}$, $\boxed{-11\mathbf{i} + 18\mathbf{j} + 17\mathbf{k}}$

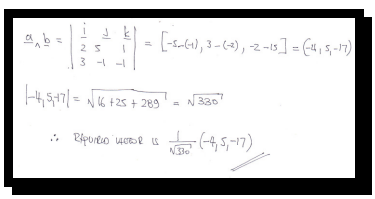


Question 2

Find a **unit** vector perpendicular to both

$$\mathbf{a} = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}\,.$$

$$\boxed{\frac{1}{\sqrt{330}}\left(-4\mathbf{i} + 5\mathbf{j} + 17\mathbf{k}\right)}$$



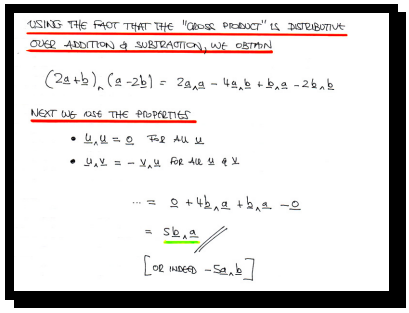
Question 3

The vectors $\mathbf{a}$ and $\mathbf{b}$, are not parallel.

Simplify fully

$$(2\mathbf{a}+\mathbf{b})\wedge(\mathbf{a}-2\mathbf{b}).$$

$$\boxed{5\mathbf{b}\wedge\mathbf{a}=-5\mathbf{a}\wedge\mathbf{b}}$$



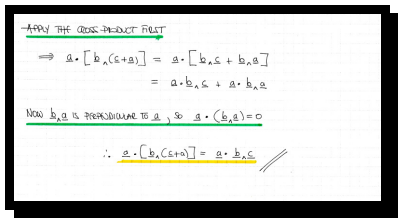
Question 4

The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are not parallel.

Simplify fully

$$\mathbf{a}\cdot\left[\mathbf{b}\wedge(\mathbf{c}+\mathbf{a})\right].$$

$$\boxed{\mathbf{a}\cdot(\mathbf{b}\wedge\mathbf{c})}$$



Question 5

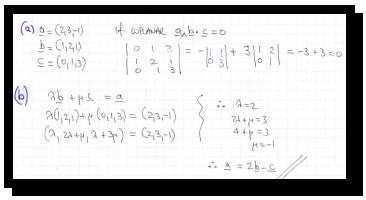
The following vectors are given

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$
$$\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$$
$$\mathbf{c} = \mathbf{j} + 3\mathbf{k}$$

a) Show that the three vectors are coplanar.

b) Express $\mathbf{a}$ in terms of $\mathbf{b}$ and $\mathbf{c}$.

$$\boxed{\mathbf{a} = 2\mathbf{b} - \mathbf{c}}$$



Question 6

The following three vectors are given

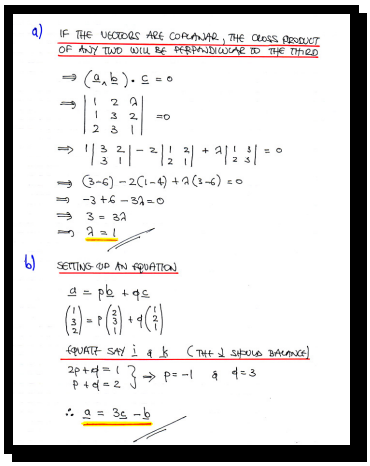
$$\mathbf{a}=\mathbf{i}+3\mathbf{j}+2\mathbf{k}$$
$$\mathbf{b}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}$$
$$\mathbf{c}=\mathbf{i}+2\mathbf{j}+\lambda\mathbf{k}$$

where λ is a scalar constant.

a) If the three vectors given above are coplanar, find the value of λ.

b) Express $\mathbf{a}$ in terms of $\mathbf{b}$ and $\mathbf{c}$.

$\boxed{\lambda=1}$, $\boxed{\mathbf{a}=3\mathbf{c}-\mathbf{b}}$


a) IF THE VECTORS ARE COPLANAR, THE CROSS PRODUCT OF ANY TWO WILL BE PERPENDICULAR TO THE THIRD

$\Rightarrow (\underline{a}\times\underline{b})\cdot\underline{c}=0$

$\Rightarrow \begin{vmatrix} 1 & 2 & \lambda \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 1\begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + \lambda\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 0$

$\Rightarrow (3-6)-2(1-4)+\lambda(3-6)=0$

$\Rightarrow -3+6-3\lambda=0$

$\Rightarrow 3=3\lambda$

$\Rightarrow \lambda=1$

b) SETTING UP AN EQUATION

$\underline{a}=p\underline{b}+q\underline{c}$

$\begin{pmatrix}1\\3\\2\end{pmatrix}=p\begin{pmatrix}2\\3\\1\end{pmatrix}+q\begin{pmatrix}1\\2\\1\end{pmatrix}$

EQUATE SAY $\underline{i}$ & $\underline{k}$ (THE $\underline{j}$ SHOULD BALANCE)

$\left.\begin{matrix}2p+q=1\\ p+q=2\end{matrix}\right\}\Rightarrow p=-1 \ \& \ q=3$

$\therefore \underline{a}=3\underline{c}-\underline{b}$

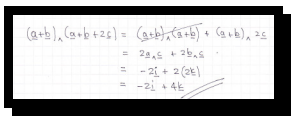
Question 7

The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such so that

$$\mathbf{c} \wedge \mathbf{a} = \mathbf{i} \quad \text{and} \quad \mathbf{b} \wedge \mathbf{c} = 2\mathbf{k} .$$

Express $(\mathbf{a}+\mathbf{b}) \wedge (\mathbf{a}+\mathbf{b}+2\mathbf{c})$ in terms of $\mathbf{i}$ and $\mathbf{k}$.

$$\boxed{-2\mathbf{i}+4\mathbf{k}}$$

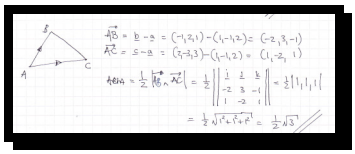


CROSS PRODUCT GEOMETRIC APPLICATIONS

Question 1

Find the area of the triangle with vertices at $A(1,-1,2)$, $B(-1,2,1)$ and $C(2,-3,3)$.

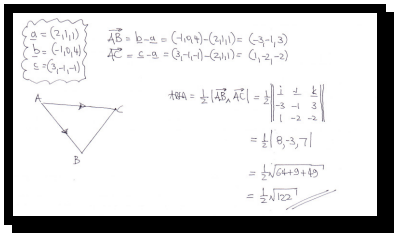
$$\boxed{\frac{1}{2}\sqrt{3}}$$



Question 2

Find the area of the triangle with vertices at $A(2,1,1)$, $B(-1,0,4)$ and $C(3,-1,-1)$.

$$\boxed{\frac{1}{2}\sqrt{122}}$$

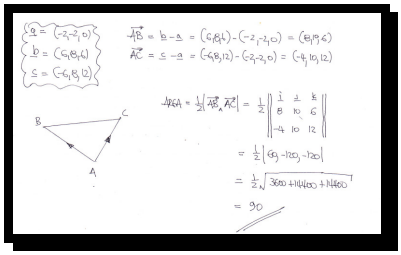


Question 3

A triangle has vertices at $A(-2,-2,0)$, $B(6,8,6)$ and $C(-6,8,12)$.

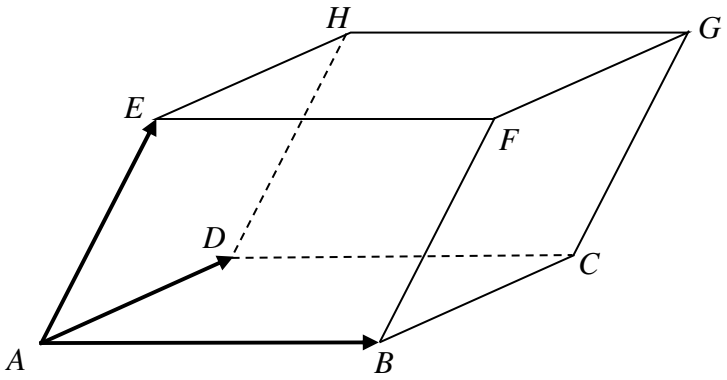
Find the area of the triangle ABC.

90



Question 4

A parallelepiped has vertices at the points $A(2,1,t)$, $B(3,3,2)$, $D(4,0,5)$ and $E(1,-2,7)$, where t is a scalar constant.



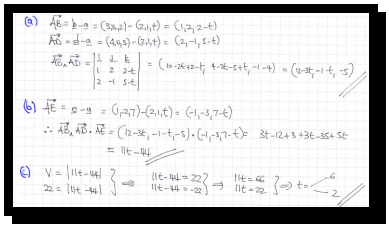
a) Calculate $\overrightarrow{AB} \wedge \overrightarrow{AD}$, in terms of t.

b) Find the value of $\overrightarrow{AB} \wedge \overrightarrow{AD} \cdot \overrightarrow{AE}$

The volume of the parallelepiped is 22 cubic units.

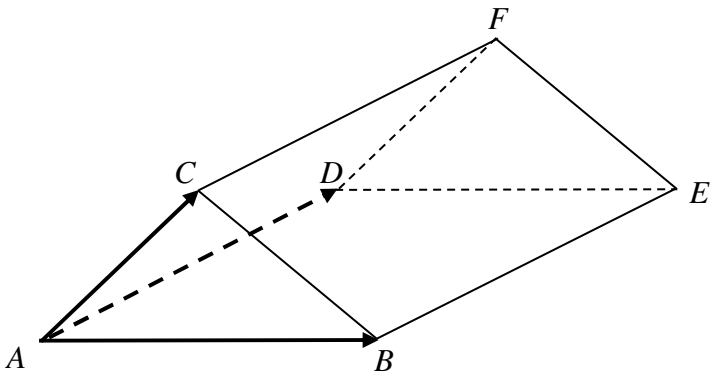
c) Determine the possible values of t.

$\boxed{(12-3t)\mathbf{i}+(-t-1)\mathbf{j}-5\mathbf{k}}$, $\boxed{11t-44}$, $\boxed{t=2,6}$



Question 5

A triangular prism has vertices at the points $A(3,3,3)$, $B(1,3,t)$, $C(5,1,5)$ and $F(8,0,10)$, where t is a scalar constant.



The face ABC is parallel to the face DEF and the lines AD, BE and CF are parallel to each other.

a) Calculate $\overrightarrow{AB} \wedge \overrightarrow{AC}$, in terms of t.

b) Find the value of $\overrightarrow{AB} \wedge \overrightarrow{AC} \cdot \overrightarrow{AD}$, in terms of t.

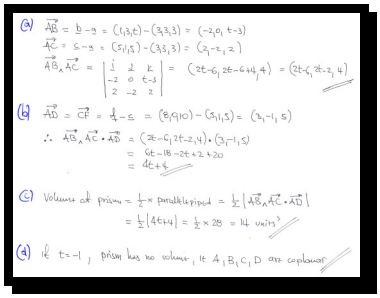
The value of t is taken to be 6.

c) Determine the volume of the prism for this value of t.

d) Explain the geometrical significance if $t=-1$.

$\boxed{(2t-6)\mathbf{i}+(2t-2)\mathbf{j}+4\mathbf{k}}$, $\boxed{4t+4}$, $\boxed{V=14 \text{ cubic units}}$,

$\boxed{A,B,C,D \text{ are coplanar, so no volume}}$



Question 6

A tetrahedron has vertices at the points $A(-3,6,4)$, $B(0,11,0)$, $C(4,1,28)$ and $D(7,k,24)$, where k is a scalar constant.

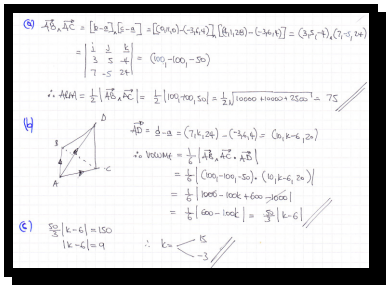
a) Calculate the area of the triangle ABC.

b) Find the volume of the tetrahedron $ABCD$, in terms of k.

The volume of the tetrahedron is 150 cubic units.

c) Determine the possible values of k.

$$\boxed{\text{area} = 75}, \quad \boxed{\text{volume} = \frac{50}{3}|k-6|}, \quad \boxed{k = -3, 15}$$



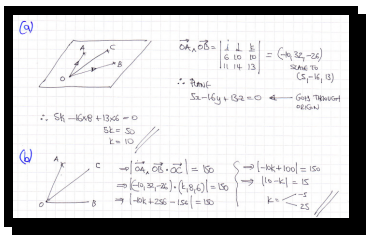
Question 7

With respect to a fixed origin O the points A, B and C, have respective coordinates $(6,10,10)$, $(11,14,13)$ and $(k,8,6)$, where k is a constant.

a) Given that all the three points lie on a plane which contains the origin, find the value of k.

b) Given instead that OA, OB, OC are edges of a parallelepiped of volume 150 cubic units determine the possible values of k.

$\boxed{k=10}$, $\boxed{k=-5,25}$



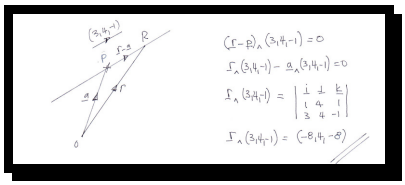
LINES

Question 1

Find an equation of the straight line that passes through the point $P(1,4,1)$ and is parallel to the vector $3\mathbf{i}+4\mathbf{j}-\mathbf{k}$.

Give the answer in the form $\mathbf{r}\wedge\mathbf{a}=\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

$$\boxed{\mathbf{r}\wedge(3\mathbf{i}+4\mathbf{j}-\mathbf{k})=-8\mathbf{i}+4\mathbf{j}-8\mathbf{k}}$$

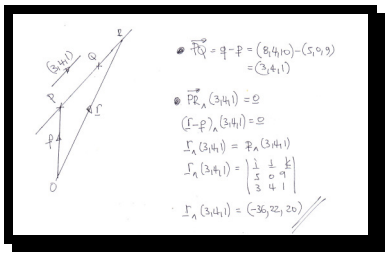


Question 2

Find an equation of the straight line that passes through the points $P(5,0,9)$ and $Q(8,4,10)$.

Give the answer in the form $\mathbf{r}\wedge\mathbf{a}=\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

$$\boxed{\mathbf{r}\wedge(3\mathbf{i}+4\mathbf{j}+\mathbf{k})=-36\mathbf{i}+22\mathbf{j}+20\mathbf{k}}$$



Question 3

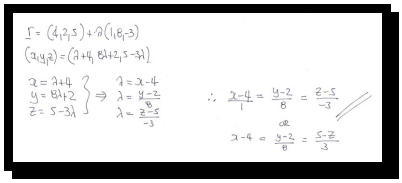
A straight line has equation

$$\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}),$$

where λ is a scalar constant.

Convert the above equation into Cartesian form.

$$\boxed{\frac{x-4}{1} = \frac{y-2}{8} = \frac{z-5}{-3}}$$



Question 4

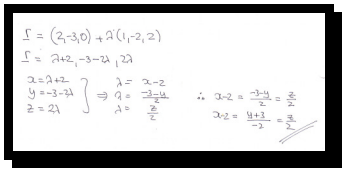
A straight line has equation

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

where λ is a scalar parameter.

Convert the above equation into Cartesian form.

$$\boxed{\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z}{2}}$$



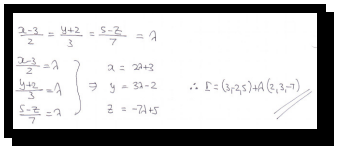
Question 5

Convert the equation of the straight line

$$\frac{x-3}{2}=\frac{y+2}{3}=\frac{5-z}{7}$$

into a vector equation of the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

$$\boxed{\mathbf{r}=3\mathbf{i}-2\mathbf{j}+5\mathbf{k}+\lambda\left(2\mathbf{i}+3\mathbf{j}-7\mathbf{k}\right)}$$



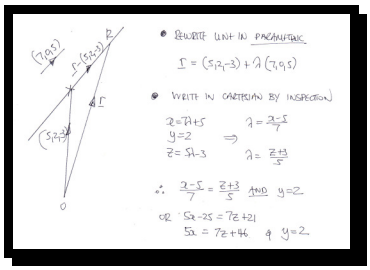
Question 6

A straight line has equation

$$\left[\mathbf{r}-\left(5\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)\right]\wedge\left(7\mathbf{i}+5\mathbf{k}\right)=\mathbf{0}$$

Convert the above equation into the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

$$\boxed{\frac{x-5}{7}=\frac{z+3}{5},\ y=2}$$



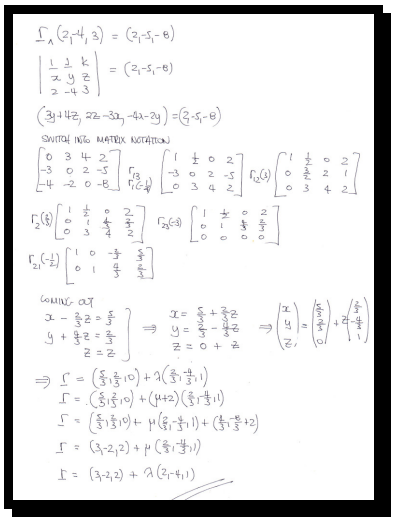
Question 7

A straight line has equation

$$\mathbf{r}\wedge\left(2\mathbf{i}-4\mathbf{j}+3\mathbf{k}\right)=\left(2\mathbf{i}-5\mathbf{j}-8\mathbf{k}\right)$$

Convert the above equation into the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

$$\boxed{\mathbf{r}=3\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\lambda\left(2\mathbf{i}-4\mathbf{j}+3\mathbf{k}\right)\quad\text{or}\quad\mathbf{r}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda\left(2\mathbf{i}-4\mathbf{j}+3\mathbf{k}\right)}$$



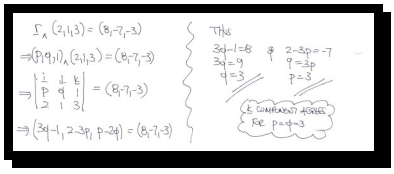
Question 8

If the point $A\left(p,q,1\right)$ lies on the straight line with vector equation

$$\mathbf{r}\wedge\left(2\mathbf{i}+\mathbf{j}+3\mathbf{k}\right)=\left(8\mathbf{i}-7\mathbf{j}-3\mathbf{k}\right),$$

find the value of each of the scalar constants p and q.

$$\boxed{p=q=3}$$



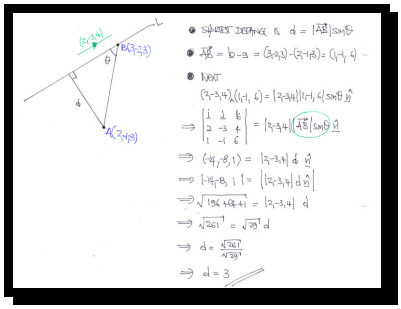
Question 9

The straight line L has equation

$$\left[\mathbf{r}-\left(3\mathbf{i}-2\mathbf{j}+3\mathbf{k}\right)\right]\wedge\left(2\mathbf{i}-3\mathbf{k}+4\mathbf{k}\right)=0\,.$$

Use a method involving the cross product to show that the shortest distance of the point $(2,-1,-3)$ from L is 3 units.

proof

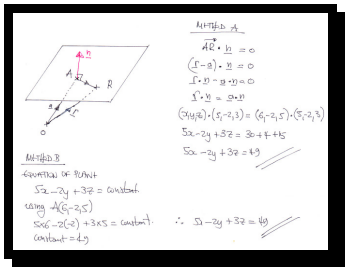


PLANES

Question 1

Find a Cartesian equation of the plane that passes through the point $A(6,-2,5)$, and its normal is in the direction $5\mathbf{i}-2\mathbf{j}+3\mathbf{k}$.

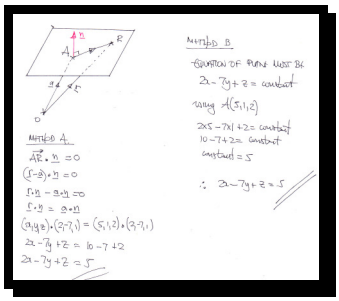
$$\boxed{5x-2y+3z=49}$$



Question 2

Find a Cartesian equation of the plane that passes through the point $A(5,1,2)$, and its normal is in the direction $2\mathbf{i}-7\mathbf{j}+\mathbf{k}$.

$$\boxed{2x-7y+z=5}$$

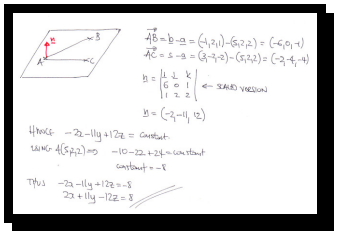


Question 3

Find a Cartesian equation of the plane that passes through the points

$$A(5,2,2)\,,\quad B(-1,2,1)\quad \text{and}\quad C(3,-2,-2)\,.$$

$$\boxed{2x+11y-12z=8}$$



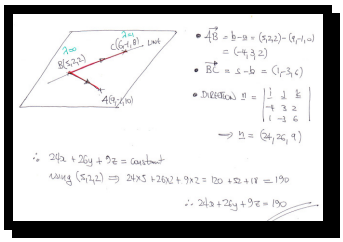
Question 4

Determine a Cartesian equation of the plane that contains the point $A(9,-1,0)$ and the straight line with vector equation

$$\mathbf{r}=5\mathbf{i}+2\mathbf{j}+2\mathbf{k}+\lambda\left(\mathbf{i}-3\mathbf{j}+6\mathbf{k}\right),$$

where λ is a scalar parameter.

$$\boxed{24x+26y+9z=190}$$



Question 5

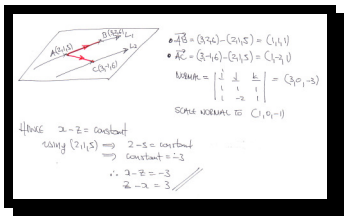
Find a Cartesian equation of the plane that contains the parallel straight lines with vector equations

$$\mathbf{r}_1 = 2\mathbf{i}+\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}+\mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = 3\mathbf{i}-\mathbf{j}+6\mathbf{k}+\mu(\mathbf{i}+\mathbf{j}+\mathbf{k}),$$

where λ and μ are scalar parameters.

.

$$\boxed{z-x=3}$$



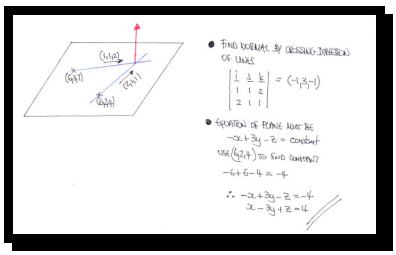
Question 6

Find a Cartesian equation of the plane that contains the intersecting straight lines with vector equations

$$\mathbf{r}_1 = 6\mathbf{i}+3\mathbf{j}+7\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}+2\mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = 6\mathbf{i}+2\mathbf{j}+4\mathbf{k}+\mu(2\mathbf{i}+\mathbf{j}+\mathbf{k}),$$

where λ and μ are scalar parameters.

$$\boxed{x-3y+z=4}$$

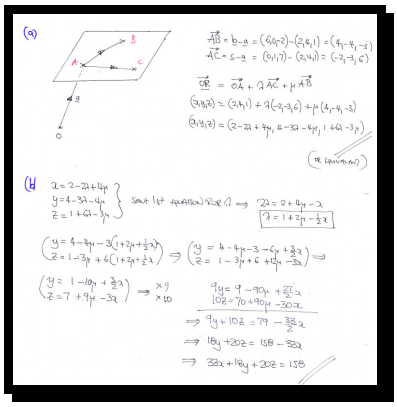


Question 7

a) Find a set of parametric equations for the plane that passes through the points $A(2,4,1)$, $B(6,0,-2)$ and $C(0,1,7)$.

b) Eliminate the parameters to obtain a Cartesian equation of the plane.

$$\boxed{(x,y,z)=(2-2\lambda+4\mu,4-3\lambda-4\mu,1+6\lambda-3\mu)}\,,\quad \boxed{33x+18y+20z=158}$$



GEOMETRIC PROBLEMS

(WITH PLANES AND LINES)

Question 1

Find the coordinates of the point of intersection of the plane with equation

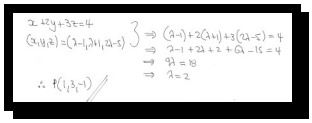
$$x+2y+3z=4$$

and the straight line with equation

$$\mathbf{r}=-\mathbf{i}+\mathbf{j}-5\mathbf{k}+\lambda\left(\mathbf{i}+\mathbf{j}+2\mathbf{k}\right),$$

where λ is a scalar parameter.

$$\boxed{(1,3,-1)}$$



Question 2

Find the coordinates of the point of intersection of the plane with equation

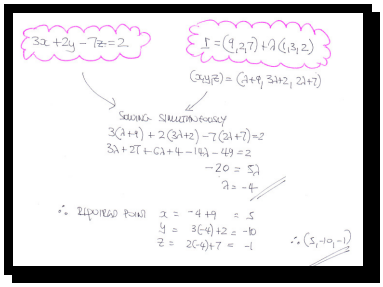
$$3x+2y-7z=2$$

and the straight line with equation

$$\mathbf{r}=9\mathbf{i}+2\mathbf{j}+7\mathbf{k}+\lambda\left(\mathbf{i}+3\mathbf{j}+2\mathbf{k}\right),$$

where λ is a scalar parameter.

$$\boxed{(5,-10,-1)}$$

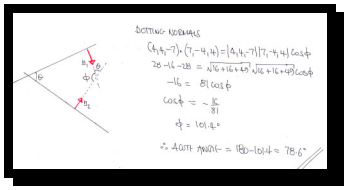


Question 3

Find the size of the acute angle formed by the planes with Cartesian equations

$$4x+4y-7z=13 \quad \text{and} \quad 7x-4y+4z=6\,.$$

$\boxed{78.6°}$

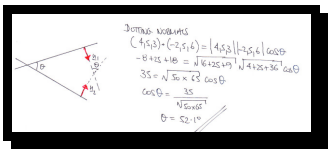


Question 4

Find the size of the acute angle between the planes with Cartesian equations

$$4x+5y+3z=82 \quad \text{and} \quad -2x+5y+6z=124\,.$$

$\boxed{52.1°}$



Question 5

Find the size of the acute angle between the plane with equation

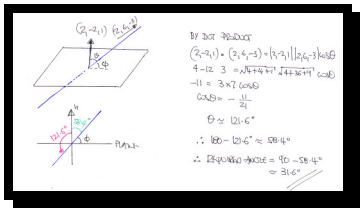
$$2x-2y+z=12$$

and the straight line with equation

$$\mathbf{r}=7\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda\left(2\mathbf{i}+6\mathbf{j}-3\mathbf{k}\right),$$

where λ is a scalar parameter.

$\boxed{31.6°}$



Question 6

Find the size of the acute angle formed between the plane with Cartesian equation

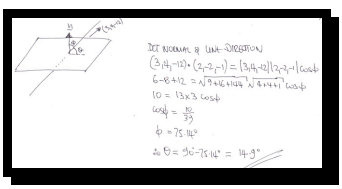
$$2x-2y-z=2$$

and the straight line with vector equation

$$\mathbf{r}=2\mathbf{i}+\mathbf{j}-5\mathbf{k}+\lambda\left(3\mathbf{i}+4\mathbf{j}-12\mathbf{k}\right),$$

where λ is a scalar parameter.

$\boxed{14.9°}$



Question 7

Find the size of the acute angle between the plane with equation

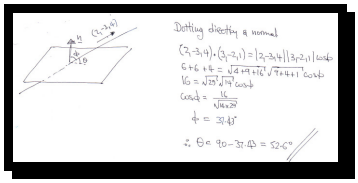
$$3x-2y+z=5$$

and the straight line with equation

$$\mathbf{r}=-3\mathbf{i}-2\mathbf{j}+3\mathbf{k}+\lambda\left(2\mathbf{i}-3\mathbf{j}+4\mathbf{k}\right),$$

where λ is a scalar parameter.

$\boxed{52.6°}$

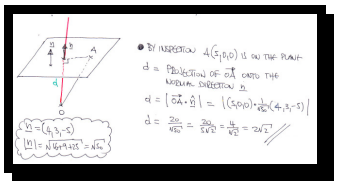


Question 8

Find shortest distance of the origin O from the plane with equation

$$4x+3y-5z=20.$$

$\boxed{2\sqrt{2}}$

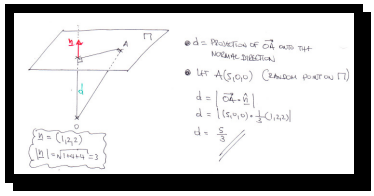


Question 9

Find shortest distance of the origin O from the plane with equation

$$x+2y+2z=5\,.$$

$$\boxed{\frac{5}{3}}$$

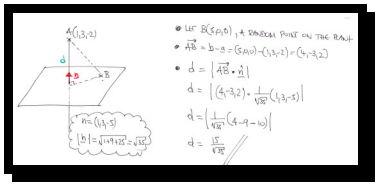


Question 10

Find shortest distance from the point $A(1,3,-2)$ to the plane with Cartesian equation

$$x+3y-5z=5\,.$$

$$\boxed{\frac{15}{\sqrt{35}}}$$

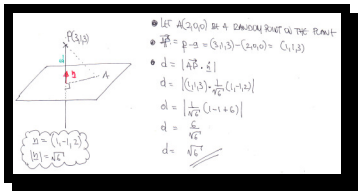


Question 11

Find shortest distance from the point $P(3,1,3)$ to the plane with Cartesian equation

$$x - y + 2z = 2 .$$

$\boxed{\sqrt{6}}$

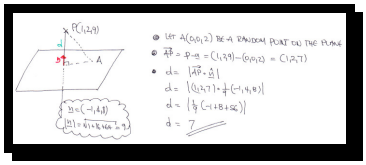


Question 12

Find shortest distance of the point $P(1,2,9)$ from the plane with Cartesian equation

$$-x + 4y + 8z = 16 .$$

$\boxed{7}$

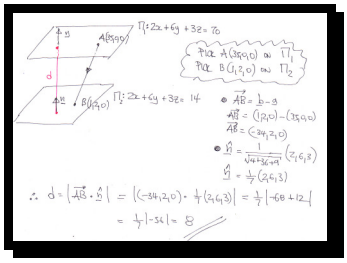


Question 13

Find the distance between the parallel planes with Cartesian equations

$$2x+6y+3z=70 \qquad \text{and} \qquad 2x+6y+3z=14\,.$$

$\boxed{8}$



Question 14

The straight line with vector equation

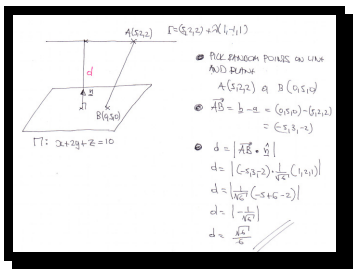
$$\mathbf{r}=(\lambda+5)\mathbf{i}+(2-\lambda)\mathbf{j}+(\lambda+2)\mathbf{k}\,,$$

where λ is a scalar parameter, is parallel to the plane with Cartesian equation

$$x+2y+z=10\,.$$

Find the distance between the plane and the straight line.

$\boxed{\dfrac{\sqrt{6}}{6}}$

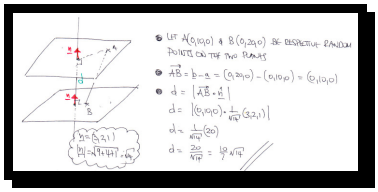


Question 15

Find the distance between the parallel planes with Cartesian equations

$$3x+2y+z=20 \qquad \text{and} \qquad 3x+2y+z=40\,.$$

$$\boxed{\frac{10}{7}\sqrt{14}}$$



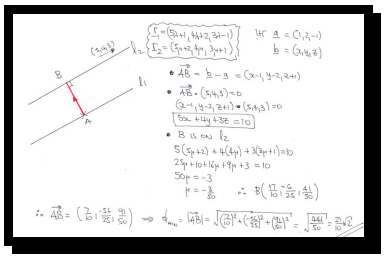
Question 16

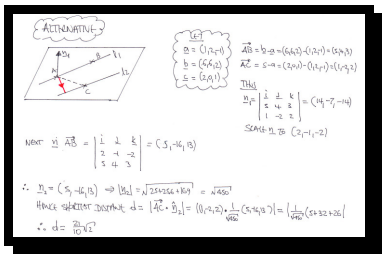
Find the distance between the parallel straight lines with vector equations

$$\mathbf{r}_1=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda\left(5\mathbf{i}+4\mathbf{j}+3\mathbf{k}\right) \quad \text{and} \quad \mathbf{r}_2=2\mathbf{i}+\mathbf{k}+\mu\left(5\mathbf{i}+4\mathbf{j}+3\mathbf{k}\right),$$

where λ and μ are a scalar parameters.

$$\boxed{\frac{21}{10}\sqrt{2}}$$



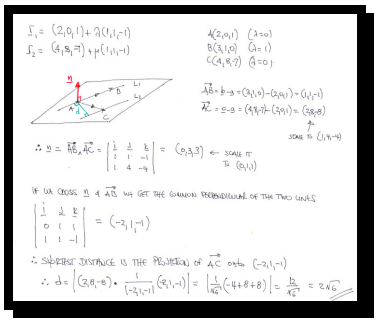


Question 17

Find the distance between the parallel straight lines with vector equations

$$\left[\mathbf{r}-\left(2\mathbf{i}+\mathbf{k}\right)\right]\wedge\left(\mathbf{i}+\mathbf{j}-\mathbf{k}\right)=\mathbf{0} \quad \text{and} \quad x-4=y-8=-z-7\,.$$

$$\boxed{2\sqrt{6}}$$



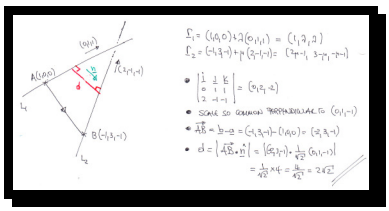
Question 18

Find the shortest distance between the skew straight lines with vector equations

$$\mathbf{r}_1=\mathbf{i}+\lambda\left(\mathbf{j}+\mathbf{k}\right) \quad \text{and} \quad \mathbf{r}_2=-\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu\left(2\mathbf{i}-\mathbf{j}-\mathbf{k}\right),$$

where λ and μ are a scalar parameters.

$$\boxed{2\sqrt{2}}$$



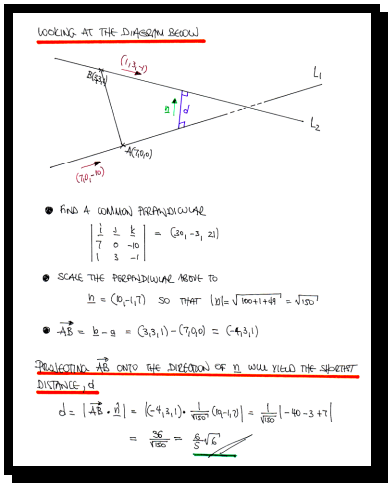
Question 19

Find the shortest distance between the skew straight lines with vector equations

$$\mathbf{r}_1 = 7\mathbf{i} + \lambda(7\mathbf{i} - 10\mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = 3\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 3\mathbf{j} - \mathbf{k}),$$

where λ and μ are a scalar parameters.

$$\frac{6}{5}\sqrt{6}$$



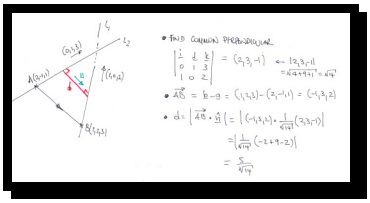
Question 20

Find the shortest distance between the skew straight lines with vector equations

$$\mathbf{r}_1 = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{k}),$$

where λ and μ are a scalar parameters.

$$\frac{5}{\sqrt{14}}$$



Question 21

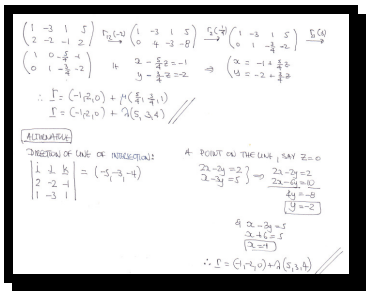
Find the intersection of the planes with Cartesian equations

$$2x-2y-z=2 \quad \text{and} \quad x-3y+z=5,$$

giving the answer in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$,

where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

$$\boxed{\mathbf{r}=-\mathbf{i}-2\mathbf{j}+\lambda\left(5\mathbf{i}+3\mathbf{j}+4\mathbf{k}\right)}$$



Question 22

Show that the planes with Cartesian equations

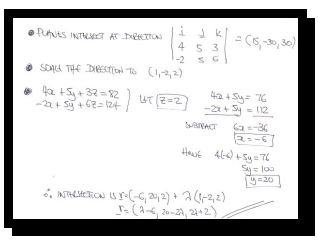
$$4x+5y+3z=82 \quad \text{and} \quad -2x+5y+6z=124$$

intersect along the straight line with equation

$$\mathbf{r}=\left(\lambda-6\right)\mathbf{i}+\left(20-2\lambda\right)\mathbf{j}+\left(2\lambda+2\right)\mathbf{k},$$

where λ is a scalar parameter.

proof



Question 23

The planes Π_1 and Π_2 have Cartesian equations:

$$\Pi_1 : x-2y+2z=0$$

$$\Pi_2 : 3x-2y-z=5$$

Show that the two planes intersect along the straight line with Cartesian equation

$$\frac{x-4}{6}=\frac{y-3}{7}=\frac{z-1}{4}.$$

proof

