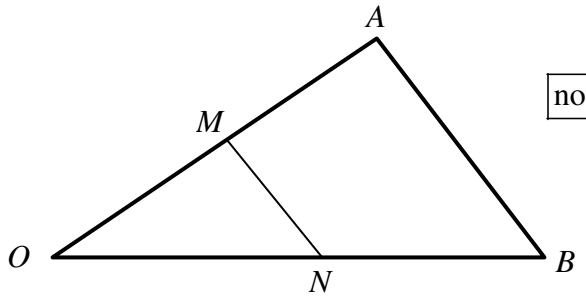
VECTOR PRACTICE

Part A

INTRODUCING VECTOR ALGEBRA AND GEOMETRY

Question 1 (**)

The figure below shows the triangle OAB.



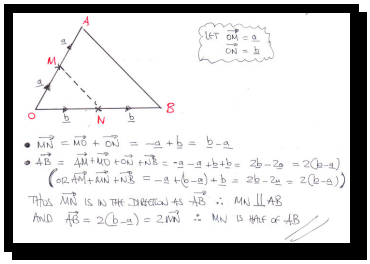
not drawn accurately

The point M is the midpoint of OA and the point N is the midpoint of OB.

Let $\overrightarrow{OM}=\mathbf{a}$ and $\overrightarrow{ON}=\mathbf{b}$.

By finding simplified expressions for $\overrightarrow{MN}$ and $\overrightarrow{AB}$, in terms of $\mathbf{a}$ and $\mathbf{b}$, show that MN is parallel to AB, and half its length.

proof



Question 2 (+)**

$OABC$ is a square.

The point M is the midpoint of AB and the point N is the midpoint of MC.

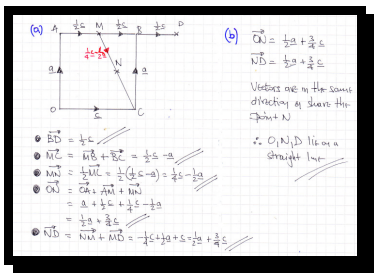
The point D is such so that $\overrightarrow{AD}=\frac{3}{2}\overrightarrow{AB}$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OC}=\mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{BD}$, $\overrightarrow{MC}$, $\overrightarrow{MN}$, $\overrightarrow{ON}$ and $\overrightarrow{ND}$.

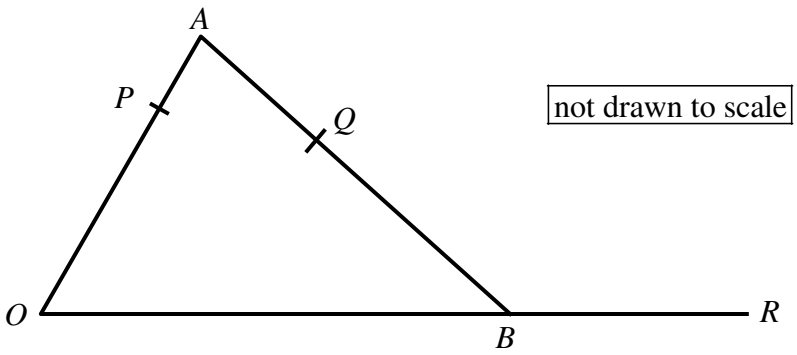
b) Deduce, showing your reasoning, that O, N and D are collinear.

$$\boxed{\overrightarrow{BD}=\frac{1}{2}\mathbf{c}}\ ,\ \boxed{\overrightarrow{MC}=\frac{1}{2}\mathbf{c}-\mathbf{a}}\ ,\ \boxed{\overrightarrow{MN}=\frac{1}{4}\mathbf{c}-\frac{1}{2}\mathbf{a}}\ ,\ \boxed{\overrightarrow{ON}=\frac{1}{2}\mathbf{a}+\frac{3}{4}\mathbf{c}}\ ,\ \boxed{\overrightarrow{ND}=\frac{1}{2}\mathbf{a}+\frac{3}{4}\mathbf{c}}$$



Question 3 (***)

The figure below shows a triangle OAB.



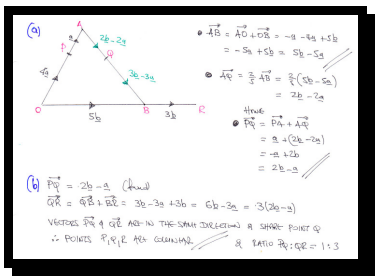
- The point P lies on OA so that $OP:PA=4:1$.
- The point Q lies on AB so that $AQ:QB=2:3$
- The side OB is extended to the point R so that $OB:BR=5:3$.

Let $\overrightarrow{PA}=\mathbf{a}$ and $\overrightarrow{OB}=5\mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AQ}$ and $\overrightarrow{PQ}$.

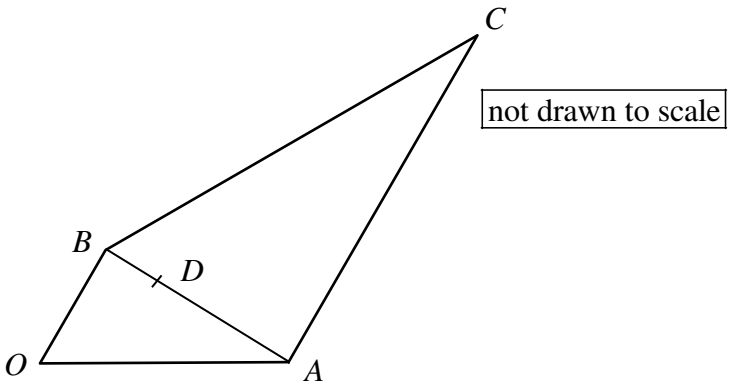
b) Deduce, showing your reasoning, that P, Q and R are collinear and state the ratio of $PQ:QR$.

$\boxed{\overrightarrow{AB}=5\mathbf{b}-5\mathbf{a}}$, $\boxed{\overrightarrow{AQ}=2\mathbf{b}-2\mathbf{a}}$, $\boxed{\overrightarrow{PQ}=2\mathbf{b}-\mathbf{a}}$, $\boxed{PQ:QR=1:3}$



Question 4 (***)

The figure below shows a trapezium $OBCA$ where OB is parallel to AC.



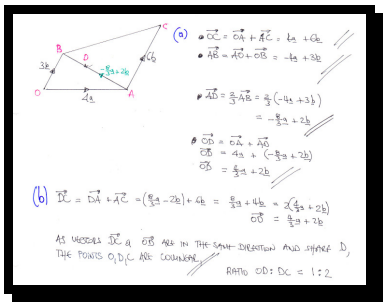
The point D lies on BA so that $BD:DA=1:2$.

Let $\overrightarrow{OA}=4\mathbf{a}$, $\overrightarrow{OB}=3\mathbf{b}$ and $\overrightarrow{AC}=6\mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{OC}$, $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{OD}$.

b) Deduce, showing your reasoning, that O, D and C are collinear and state the ratio of $OD:DC$.

$\boxed{\overrightarrow{OC}=4\mathbf{a}+6\mathbf{b}}$, $\boxed{\overrightarrow{AB}=-4\mathbf{a}+3\mathbf{b}}$, $\boxed{\overrightarrow{AD}=-\frac{8}{3}\mathbf{a}+2\mathbf{b}}$, $\boxed{\overrightarrow{OD}=\frac{4}{3}\mathbf{a}+2\mathbf{b}}$, $\boxed{OD:DC=1:2}$



Question 5 (***)

$OABC$ is a parallelogram and the point M is the midpoint of AB.

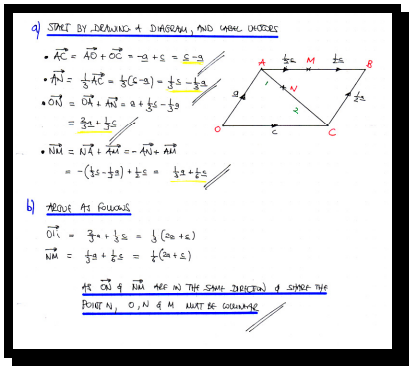
The point N lies on the diagonal AC so that $AN:NC=1:2$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OC}=\mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{AC}$, $\overrightarrow{AN}$, $\overrightarrow{ON}$ and $\overrightarrow{NM}$.

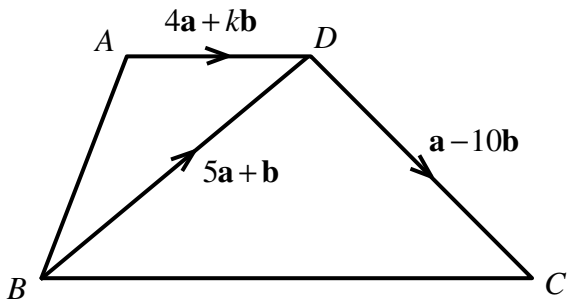
b) Deduce, showing your reasoning, that O, N and M are collinear.

$$\boxed{\overrightarrow{AC}=\mathbf{c}-\mathbf{a}},\ \boxed{\overrightarrow{AN}=\frac{1}{3}\mathbf{c}-\frac{1}{3}\mathbf{a}},\ \boxed{\overrightarrow{ON}=\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{c}},\ \boxed{\overrightarrow{NM}=\frac{1}{3}\mathbf{a}+\frac{1}{6}\mathbf{c}}$$



Question 6 (*+)**

The figure below shows a trapezium $OBCA$ where AD is parallel to BC.



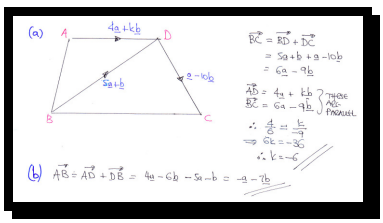
The following information is given for this trapezium.

$\overrightarrow{BD} = 5\mathbf{a} + \mathbf{b}$, $\overrightarrow{DC} = \mathbf{a} - 10\mathbf{b}$ and $\overrightarrow{AD} = 4\mathbf{a} + k\mathbf{b}$, where k is an integer.

a) Find the value of k.

b) Find a simplified expression for $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$\boxed{k = -6}$, $\boxed{\overrightarrow{AB} = -\mathbf{a} - 7\mathbf{b}}$



Question 7 (*+)**

OAB is a triangle with the point P being the midpoint of OB and the point Q being the midpoint of AB.

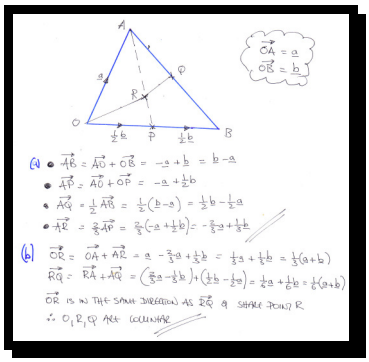
The point R is such so that $\overrightarrow{AR}=\frac{2}{3}\overrightarrow{AP}$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AP}$, $\overrightarrow{AQ}$ and $\overrightarrow{AR}$.

b) By finding simplified expressions, in terms $\mathbf{a}$ and $\mathbf{b}$, for two more suitable vectors, show that the points O, R and Q are collinear.

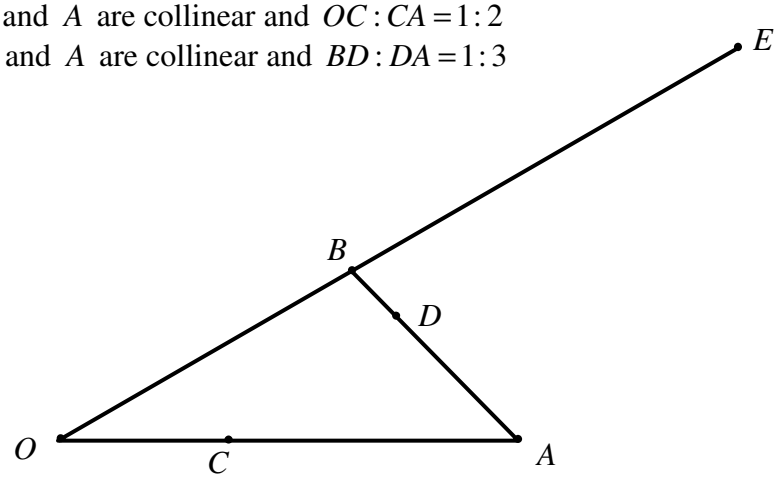
$$\boxed{\overrightarrow{AB}=\mathbf{b}-\mathbf{a}},\ \boxed{\overrightarrow{AP}=\frac{1}{2}\mathbf{b}-\mathbf{a}},\ \boxed{\overrightarrow{AQ}=\frac{1}{2}\mathbf{b}-\frac{1}{2}\mathbf{a}},\ \boxed{\overrightarrow{AR}=\frac{1}{3}\mathbf{b}-\frac{2}{3}\mathbf{a}}$$



Question 8 (*+)**

The figure below shows the points O, C, A, D, B and E, which are related as follows.

- O, B and E are collinear and $OB:BE=1:2$
- O, C and A are collinear and $OC:CA=1:2$
- B, D and A are collinear and $BD:DA=1:3$



Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

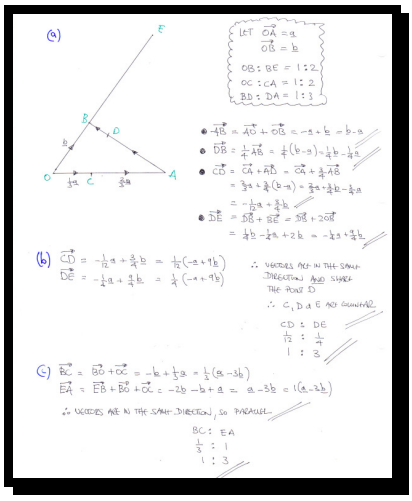
a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{DB}$, $\overrightarrow{CD}$ and $\overrightarrow{DE}$.

b) Show that the points C, D and E are collinear, and find the ratio $CD:DE$.

c) Show further that BC is parallel to EA, and find the ratio $BC:EA$.

$\boxed{\overrightarrow{AB}=\mathbf{b}-\mathbf{a}}$, $\boxed{\overrightarrow{DB}=\frac{1}{4}\mathbf{b}-\frac{1}{4}\mathbf{a}}$, $\boxed{\overrightarrow{CD}=-\frac{1}{12}\mathbf{a}+\frac{3}{4}\mathbf{b}}$, $\boxed{\overrightarrow{DE}=-\frac{1}{4}\mathbf{a}+\frac{9}{4}\mathbf{b}}$,

$\boxed{CD:DE=1:3}$, $\boxed{BC:EA=1:3}$

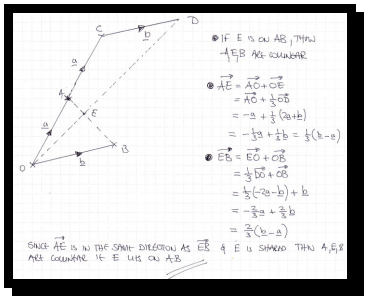


Question 9 (****)

Let $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$, $\overrightarrow{OC}=2\mathbf{a}$ and $\overrightarrow{OD}=2\mathbf{a}+\mathbf{b}$.

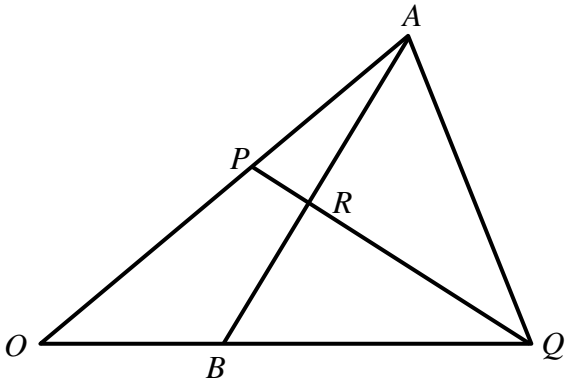
If $\overrightarrow{OE}=\dfrac{1}{3}\overrightarrow{OD}$ prove that the point E lies on the straight line AB.

proof



Question 10 (**+)**

The figure below shows a triangle OAQ.



not drawn to scale

- The point P lies on OA so that $OP:OA=3:5$.
- The point B lies on OQ so that $OB:BQ=1:2$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

a) Given that $\overrightarrow{AR}=h\overrightarrow{AB}$, where h is a scalar parameter with $0<h<1$, show that

$$\overrightarrow{OR}=(1-h)\mathbf{a}+h\mathbf{b}.$$

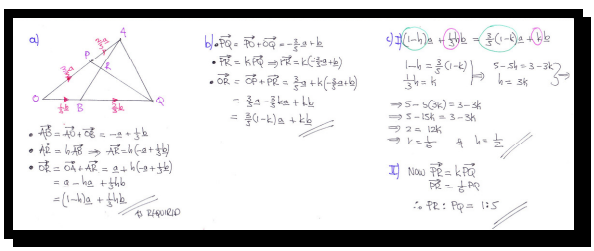
b) Given further that $\overrightarrow{PR}=k\overrightarrow{PQ}$, where k is a scalar parameter with $0<k<1$, find a similar expression for $\overrightarrow{OR}$ in terms of k, $\mathbf{a}$, $\mathbf{b}$.

c) Determine …

i. … the value of k and the value of h.

ii. … the ratio of $\overrightarrow{PR}:\overrightarrow{PQ}$.

$$\boxed{\overrightarrow{OR}=\frac{3}{5}(1-k)\mathbf{a}+k\mathbf{b}},\ \boxed{k=\frac{1}{6}},\ \boxed{h=\frac{1}{2}},\ \boxed{PR:PQ=1:5}$$



Question 11 (*****)

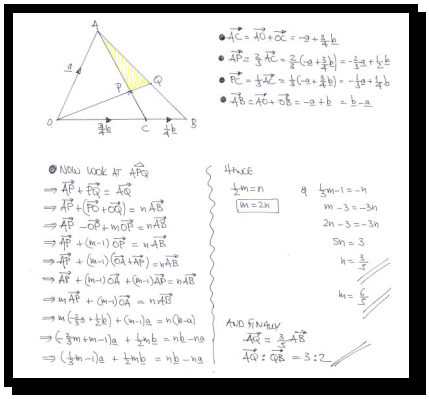
OAB is a triangle and $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

- The point C lies on OB so that $OC:CB=3:1$.
- The point P lies on AC so that $AP:PC=2:1$.
- The point Q lies on AB so that O, P and Q are collinear.

Let $\overrightarrow{OQ}=m\overrightarrow{OP}$ and $\overrightarrow{AQ}=n\overrightarrow{AB}$

Find the value of m and the value of n, and hence write down the ratio $AQ:QB$.

$$\boxed{m=\frac{6}{5},\ n=\frac{3}{5}},\ \boxed{AQ:QB=3:2}$$



Question 12

Find the value of λ and μ, given that the vectors $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

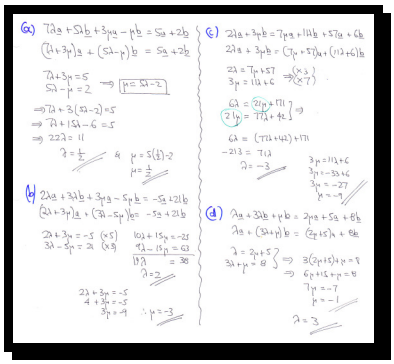
a) $7\lambda\mathbf{a}+5\lambda\mathbf{b}+3\mu\mathbf{a}-\mu\mathbf{b}=5\mathbf{a}+2\mathbf{b}$

b) $2\lambda\mathbf{a}+3\lambda\mathbf{b}+3\mu\mathbf{a}-5\mu\mathbf{b}=-5\mathbf{a}+21\mathbf{b}$

c) $2\lambda\mathbf{a}+3\mu\mathbf{b}=7\mu\mathbf{a}+11\lambda\mathbf{b}+57\mathbf{a}+6\mathbf{b}$

d) $\lambda\mathbf{a}+3\lambda\mathbf{b}+\mu\mathbf{b}=2\mu\mathbf{a}+5\mathbf{a}+8\mathbf{b}$

$\boxed{\lambda=\frac{1}{2},\mu=\frac{1}{2}}$, $\boxed{\lambda=2,\mu=-3}$, $\boxed{\lambda=-3,\mu=-9}$, $\boxed{\lambda=3,\mu=-1}$



VECTOR COMPONENTS AND 3D-COORDINATES

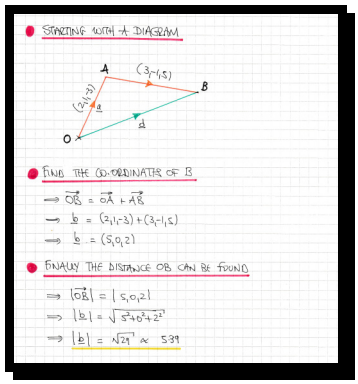
Question 1

Relative to a fixed origin O, the point A has coordinates $(2,1,-3)$.

The point B is such so that $\overrightarrow{AB} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$.

Determine the distance of B from O.

$\boxed{}$, $\boxed{|OB| = \sqrt{29}}$



Question 2

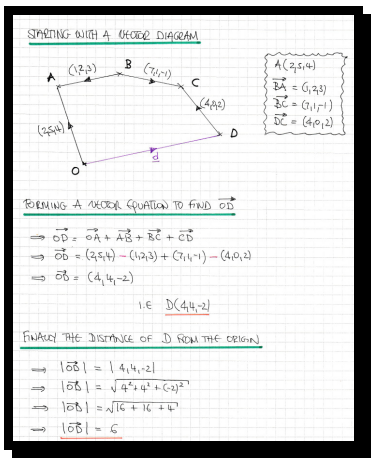
Relative to a fixed origin O, the point A has coordinates $(2,5,4)$.

The points B, C and D are such so that

$$\overrightarrow{BA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{BC} = 7\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{DC} = 4\mathbf{i} + 2\mathbf{k}.$$

Determine the distance of D from the origin.

$\boxed{}$, $\boxed{|OD| = 6}$



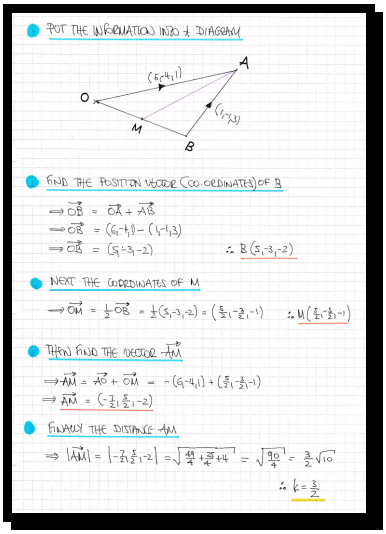
Question 3

Relative to a fixed origin O, the point A has coordinates $(6,-4,1)$.

The point B is such so that $\overrightarrow{BA} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

If the point M is the midpoint of OB, show that $\left|\overrightarrow{AM}\right| = k\sqrt{10}$, where k is a rational constant to be found.

$\square$, $\boxed{k = \frac{3}{2}}$



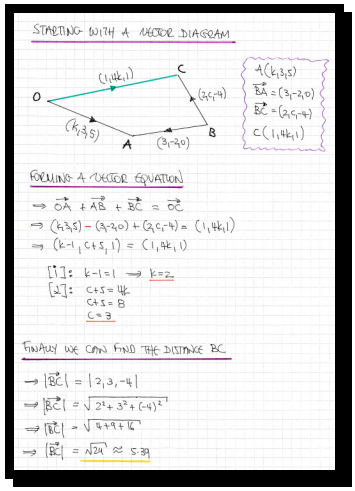
Question 4

Relative to a fixed origin O, the point A has coordinates $(k,3,5)$, where k is a scalar constant.

The points B and C are such so that $\overrightarrow{BA}=3\mathbf{i}-2\mathbf{j}$ and $\overrightarrow{BC}=2\mathbf{i}+c\mathbf{j}-4\mathbf{k}$, where c is a scalar constant.

If the coordinates of C are $(1,4k,1)$, determine the distance BC.

$\boxed{}$, $\boxed{|BC|=\sqrt{29}}$



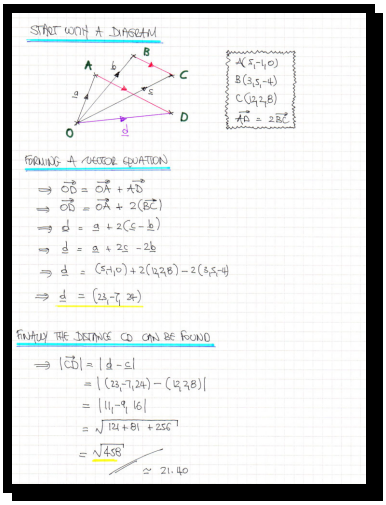
Question 5

The points $A(5,-1,0)$, $B(3,5,-4)$, $C(12,2,8)$ are referred relative to a fixed origin O.

The point D is such so that $\overrightarrow{AD}=2\overrightarrow{BC}$.

Determine the distance CD.

$$\boxed{|CD|=\sqrt{458}\approx 21.40}$$

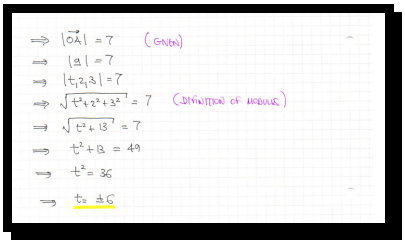


Question 6

The point $A(t,2,3)$, where t is a constant, is referred relative to a fixed origin O.

Given that $\left|\overrightarrow{OA}\right|=7$, find the possible values of t.

$$\boxed{t=\pm 6}$$

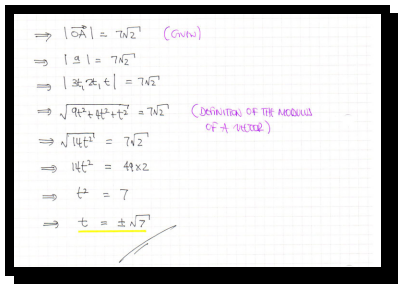


Question 7

The point $A(3t, 2t, t)$, where t is a constant, is referred relative to a fixed origin O.

Given that $\left|\overrightarrow{OA}\right| = 7\sqrt{2}$, find the possible values of t.

$$\boxed{t = \pm\sqrt{7}}$$

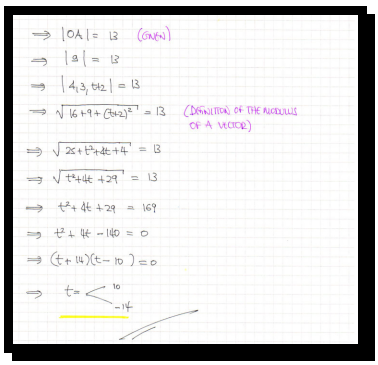


Question 8

The point $A(4, 3, t+2)$, where t is a constant, is referred relative to a fixed origin O.

Given that $\left|\overrightarrow{OA}\right| = 13$, find the possible values of t.

$$\boxed{t = 10, -14}$$

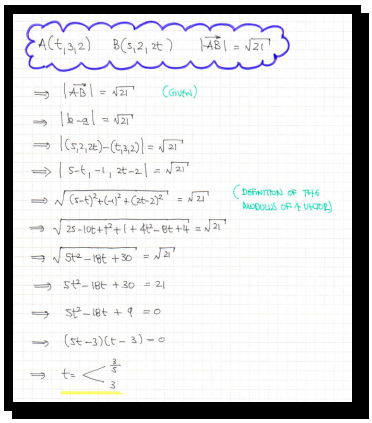


Question 9

The points $A(t,3,2)$ and $B(5,2,2t)$, where t is a scalar constant, are referred relative to a fixed origin O.

Given that $\left|\overrightarrow{AB}\right| = \sqrt{21}$, find the possible values of t.

$$\boxed{t = 3,\ t = \frac{3}{5}}$$



Question 10

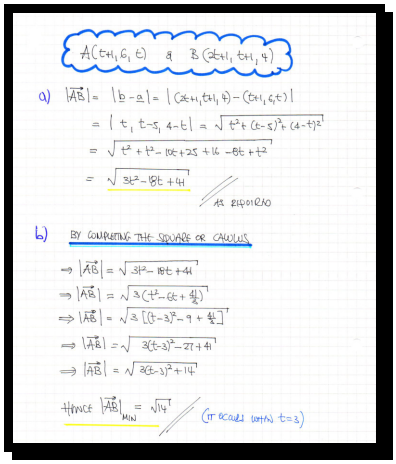
The variable points $A(t+1,6,t)$ and $B(2t+1,t+1,4)$, where t is a scalar variable, are referred relative to a fixed origin O.

a) Show that

$$\left|\overrightarrow{AB}\right| = \sqrt{3t^2-18t+41}\,.$$

b) Hence find the shortest distance between A and B, as t varies.

$$\left|\overrightarrow{AB}\right|_{\min} = \sqrt{14}$$



Question 11

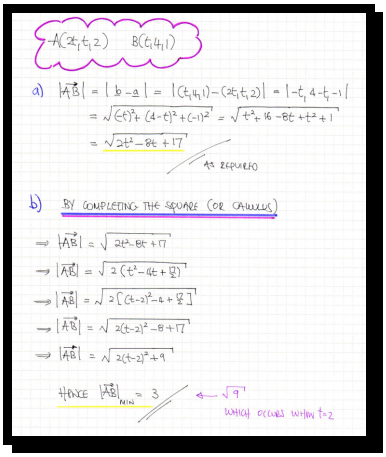
The variable points $A(2t,t,2)$ and $B(t,4,1)$, where t is a scalar variable, are referred relative to a fixed origin O.

a) Show that

$$\left|\overrightarrow{AB}\right| = \sqrt{2t^2 - 8t + 17}\ .$$

b) Hence find the shortest distance between A and B, as t varies.

$$\left|\overrightarrow{AB}\right|_{\min} = 3$$

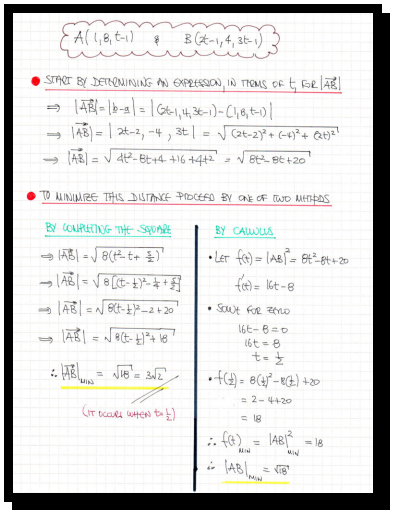


Question 12

The variable points $A(1,8,t-1)$ and $B(2t-1,4,3t-1)$, where t is a scalar variable, are referred relative to a fixed origin O.

Find the shortest distance between A and B, as t varies.

$$\left|\overrightarrow{AB}\right|_{\min}=\sqrt{18}$$

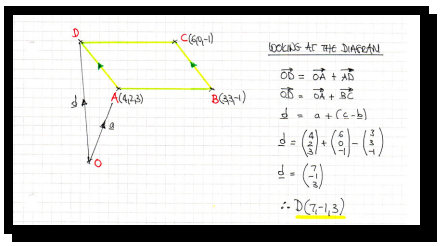


Question 13

The points $A(4,2,3)$, $B(3,3,-1)$ and $C(6,0,-1)$ are referred with respect to a fixed origin O.

If A, B, C and the point D form the parallelogram $ABCD$, use vector algebra to find the coordinates of D.

$$D(7,-1,3)$$

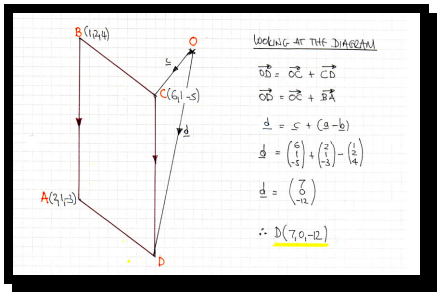


Question 14

The points $A(2,1,-3)$, $B(1,2,4)$ and $C(6,1,-5)$ are referred with respect to a fixed origin O.

If A, B, C and the point D form the parallelogram $ABCD$, use vector algebra to find the coordinates of D.

$$\boxed{D(7,0,-12)}$$

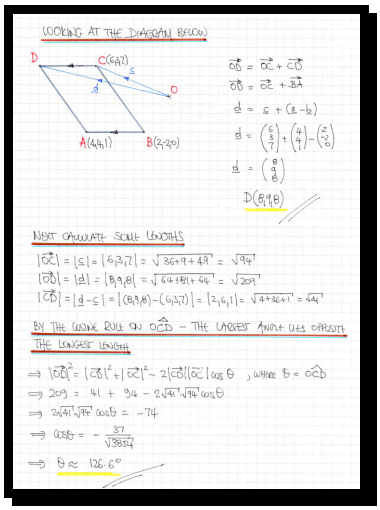


Question 15

The points $A(4,4,1)$, $B(2,-2,0)$ and $C(6,3,7)$ are referred with respect to a fixed origin O.

If A, B, C and the point D form the parallelogram $ABCD$, use vector algebra to find the coordinates of D and hence calculate the angle OCD.

$$\boxed{D(8,9,8)}, \quad \boxed{\measuredangle OCD \approx 126.6^\circ}$$

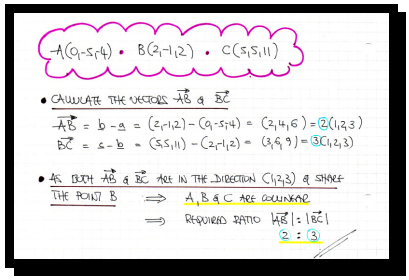


Question 16

The points $A(0,-5,-4)$, $B(2,-1,2)$ and $C(5,5,11)$ are referred with respect to a fixed origin O.

Show that A, B and C are collinear and find the ratio $AB:BC$.

$\boxed{2:3}$

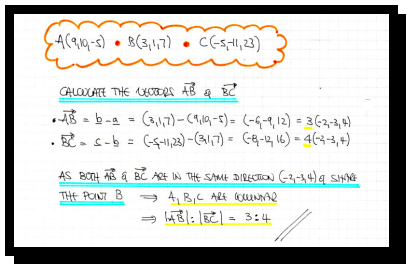


Question 17

The points $A(9,10,-5)$, $B(3,1,7)$ and $C(-5,-11,23)$ are referred with respect to a fixed origin O.

Show that A, B and C are collinear and find the ratio $AB:BC$.

$\boxed{3:4}$



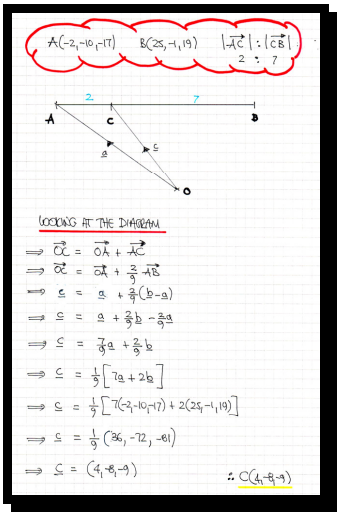
Question 18

The points $A(-2,-10,-17)$ and $B(25,-1,19)$ are referred with respect to a fixed origin O.

The point C is such so that ACB forms a straight line.

Given further that $\dfrac{\left|\overrightarrow{AC}\right|}{\left|\overrightarrow{CB}\right|}=\dfrac{2}{7}$, determine the coordinates of C.

$$\boxed{C(4,-8,-9)}$$



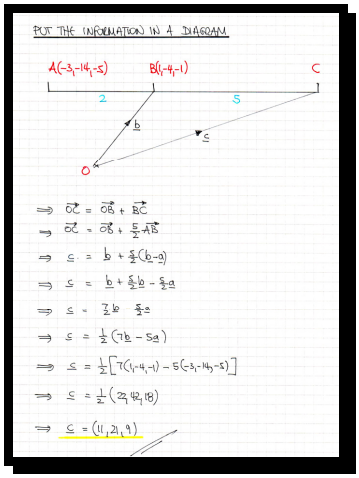
Question 19

The points $A(-3,-14,-5)$ and $B(1,-4,-1)$ are referred relative to a fixed origin O.

The point C is such so that ABC forms a straight line.

Given further that $\dfrac{\left|\overrightarrow{AB}\right|}{\left|\overrightarrow{BC}\right|}=\dfrac{2}{5}$, determine the coordinates of C.

$$\boxed{C(11,21,9)}$$



Question 20

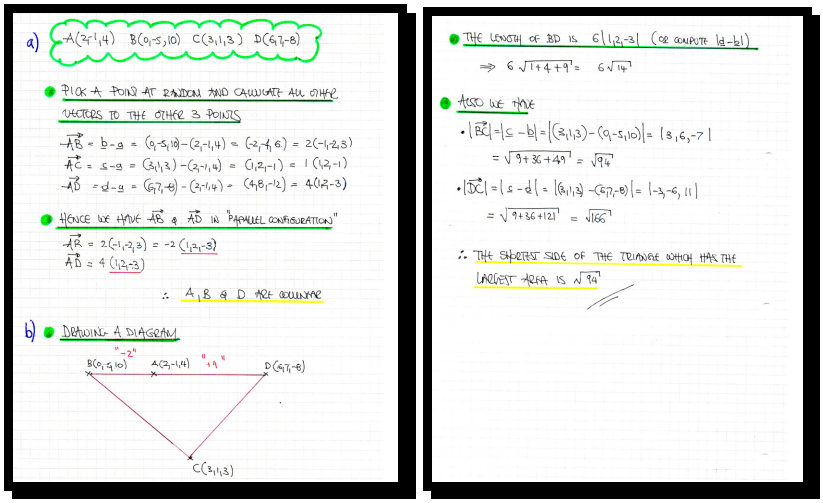
The points $A(2,-1,4)$, $B(0,-5,10)$, $C(3,1,3)$ and $D(6,7,-8)$ are referred relative to a fixed origin O.

a) Use vector algebra to show that three of the above four points are collinear.

A triangle is drawn using three of the above four points as its vertices.

b) Given further that the triangle has the largest possible area, determine, in exact surd form, the length of its shortest side.

$\sqrt{94}$



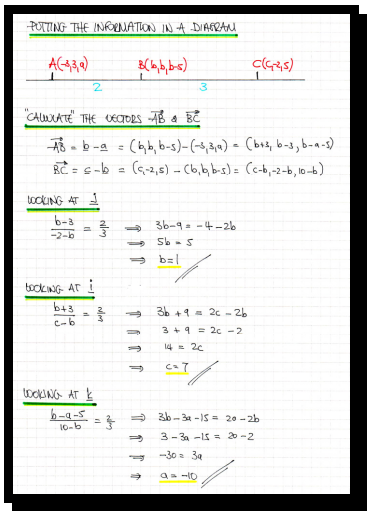
Question 21

The points $A(-3,3,a)$, $B(b,b,b-5)$ and $C(c,-2,5)$, where a, b and c are scalar constants, are referred relative to a fixed origin O.

Ii is further given that A, B and C are collinear and the ratio $\left|\overrightarrow{AB}\right| : \left|\overrightarrow{BC}\right| = 2:3$.

Use vector algebra to find the value of a, the value of b and the value of c.

$$\boxed{[a,b,c]=[-10,1,7]}$$



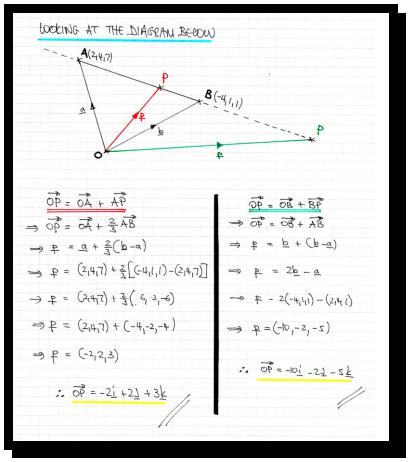
Question 22

With respect to a fixed origin, the points A and B have position vectors $2\mathbf{i}+4\mathbf{j}+7\mathbf{k}$ and $-4\mathbf{i}+\mathbf{j}+\mathbf{k}$, respectively.

The point P lies on the straight line through A and B.

Find the possible position vectors of P if $\left|\overrightarrow{AP}\right|=2\left|\overrightarrow{PB}\right|$.

$$\boxed{\overrightarrow{OP}=\mathbf{p}=-2\mathbf{i}+2\mathbf{j}+3\mathbf{k}}\ ,\quad \boxed{\overrightarrow{OP}=\mathbf{p}=-10\mathbf{i}-2\mathbf{j}-5\mathbf{k}}$$



Question 23

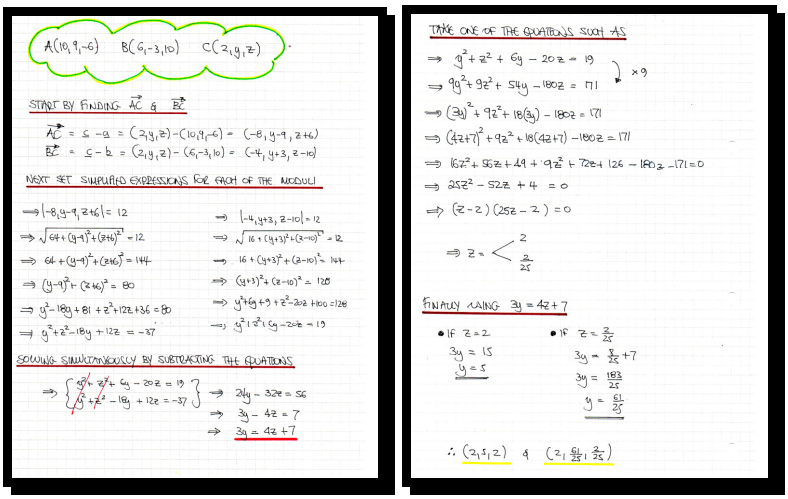
With respect to a fixed origin, the points A and B have position vectors $10\mathbf{i}+9\mathbf{j}-6\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+10\mathbf{k}$, respectively.

The position vector of the point C has $\mathbf{i}$ component equal to 2.

The distance of C from both A and B is 12 units.

Show that one of the two possible position vectors of C is $2\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ and determine the other.

$$\mathbf{c}=2\mathbf{i}+\frac{61}{25}\mathbf{j}+\frac{2}{25}\mathbf{k}$$



ANGLES AND VECTORS

Question 1

Find the angle between each pair of vectors.

a) $4\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $8\mathbf{i}+2\mathbf{j}-3\mathbf{k}$

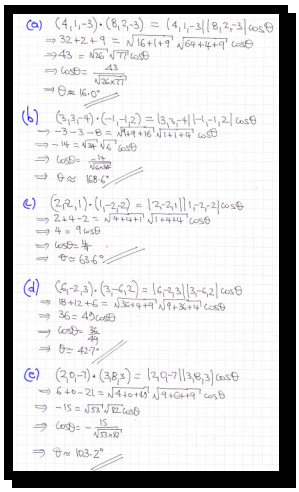
b) $3\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ and $-\mathbf{i}-\mathbf{j}+2\mathbf{k}$

c) $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-2\mathbf{j}-2\mathbf{k}$

d) $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $3\mathbf{i}-6\mathbf{j}+2\mathbf{k}$

e) $2\mathbf{i}-7\mathbf{k}$ and $3\mathbf{i}+8\mathbf{j}+3\mathbf{k}$

$\boxed{16.0°}$, $\boxed{168.6°}$, $\boxed{63.6°}$, $\boxed{42.7°}$, $\boxed{103.2°}$



Question 2

Find the angle between each pair of vectors.

a) $3\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$

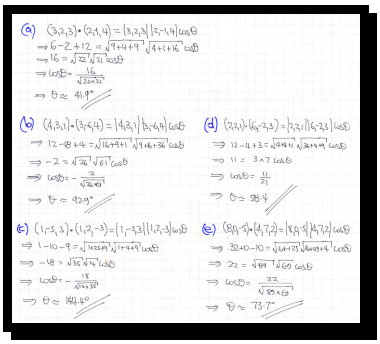
b) $4\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$

c) $\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$

d) $2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

e) $8\mathbf{i}-5\mathbf{k}$ and $4\mathbf{i}+7\mathbf{j}+2\mathbf{k}$

$\boxed{41.9°}$, $\boxed{92.9°}$, $\boxed{144.4°}$, $\boxed{58.4°}$, $\boxed{73.7°}$



Question 3

Find the angle between each pair of vectors.

a) $2\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ and $4\mathbf{i}-\mathbf{j}-\mathbf{k}$

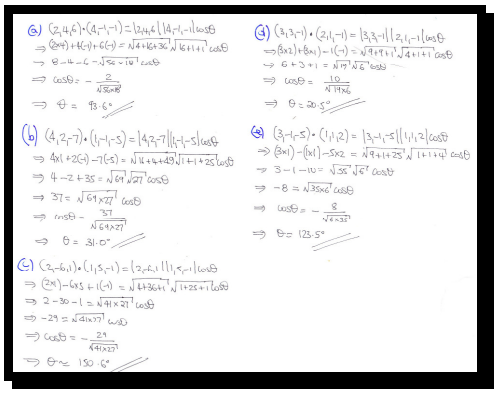
b) $4\mathbf{i}+2\mathbf{j}-7\mathbf{k}$ and $\mathbf{i}-\mathbf{j}-5\mathbf{k}$

c) $2\mathbf{i}-6\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+5\mathbf{j}-\mathbf{k}$

d) $3\mathbf{i}+3\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-\mathbf{k}$

e) $3\mathbf{i}-\mathbf{j}-5\mathbf{k}$ and $\mathbf{i}+\mathbf{j}+2\mathbf{k}$

$\boxed{93.6°}$, $\boxed{31.0°}$, $\boxed{150.6°}$, $\boxed{20.5°}$, $\boxed{123.5°}$



Question 4

Find the angle between each pair of vectors.

a) $\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $3\mathbf{i}-\mathbf{j}+\mathbf{k}$

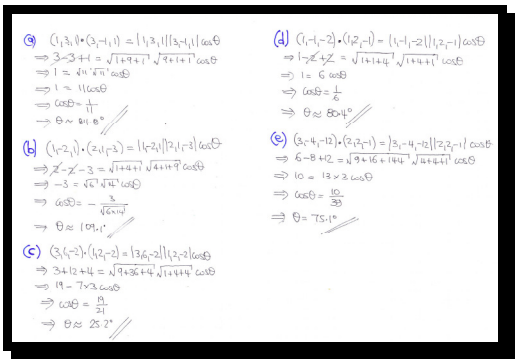
b) $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-3\mathbf{k}$

c) $3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$

d) $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-\mathbf{k}$

e) $3\mathbf{i}-4\mathbf{j}-12\mathbf{k}$ and $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$

$\boxed{84.8°}$, $\boxed{109.1°}$, $\boxed{25.2°}$, $\boxed{80.4°}$, $\boxed{75.1°}$



Question 5

Find the angle $\measuredangle CAB$ for each set of the coordinates given.

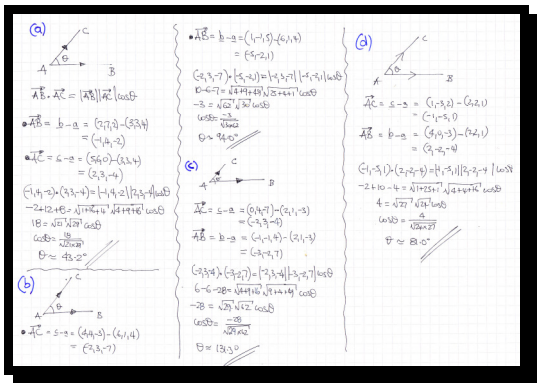
a) $A(3,3,4)$, $B(2,7,2)$, $C(5,6,0)$

b) $A(6,1,4)$, $B(1,-1,5)$, $C(4,4,-3)$

c) $A(2,1,-3)$, $B(-1,-1,4)$, $C(0,4,-7)$

d) $A(2,2,1)$, $B(4,0,-3)$, $C(1,-3,2)$

$\boxed{43.2°}$, $\boxed{94.0°}$, $\boxed{131.3°}$, $\boxed{81.0°}$



Question 6

Find the angle $\measuredangle CAB$ for each set of the coordinates given.

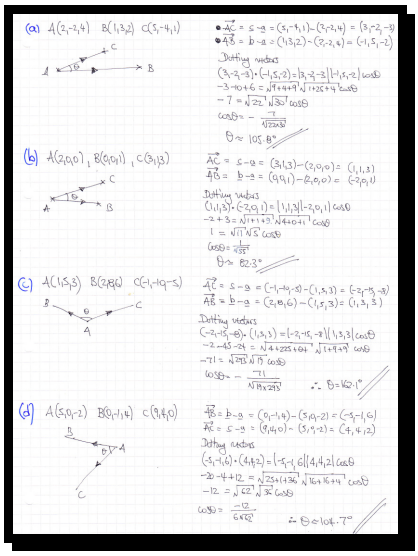
a) $A(2,-2,4)$, $B(1,3,2)$, $C(5,-4,1)$

b) $A(2,0,0)$, $B(0,0,1)$, $C(3,1,3)$

c) $A(1,5,3)$, $B(2,8,6)$, $C(-1,-10,-5)$

d) $A(5,0,-2)$, $B(0,-1,4)$, $C(9,4,0)$

$\boxed{105.8°}$, $\boxed{82.3°}$, $\boxed{162.1°}$, $\boxed{104.7°}$



Question 7

The vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, and λ is a scalar constant.

Find in each case the possible value(s) of λ.

a) $\mathbf{a}=2\mathbf{i}+3\mathbf{j}+\lambda\mathbf{k}$ and $\mathbf{b}=5\mathbf{i}+\lambda\mathbf{j}-5\mathbf{k}$

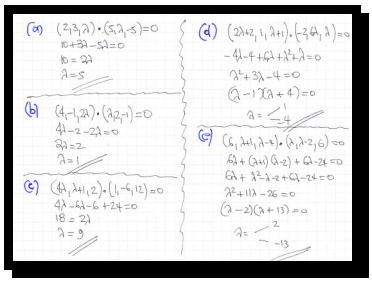
b) $\mathbf{a}=4\mathbf{i}-\mathbf{j}+2\lambda\mathbf{k}$ and $\mathbf{b}=\lambda\mathbf{i}+2\mathbf{j}-\mathbf{k}$

c) $\mathbf{a}=4\lambda\mathbf{i}+(\lambda+1)\mathbf{j}+2\mathbf{k}$ and $\mathbf{b}=\mathbf{i}-6\mathbf{j}+12\mathbf{k}$

d) $\mathbf{a}=(2\lambda+2)\mathbf{i}+\mathbf{j}+(\lambda+1)\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}+6\lambda\mathbf{j}+\lambda\mathbf{k}$

e) $\mathbf{a}=6\mathbf{i}+(\lambda+1)\mathbf{j}+(\lambda-4)\mathbf{k}$ and $\mathbf{b}=\lambda\mathbf{i}+(\lambda-2)\mathbf{j}+6\mathbf{k}$

$\boxed{\lambda_a=5}$, $\boxed{\lambda_b=1}$, $\boxed{\lambda_c=9}$, $\boxed{\lambda_d=1,-4}$, $\boxed{\lambda_e=2,-13}$



Question 8

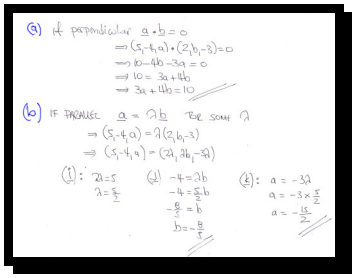
The vectors $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\mathbf{a}=5\mathbf{i}-4\mathbf{j}+a\mathbf{k}\ ,\quad \mathbf{b}=2\mathbf{i}+b\mathbf{j}-3\mathbf{k}\ .$$

a) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular find a relationship between a and b.

b) If instead $\mathbf{a}$ and $\mathbf{b}$ are parallel find the value of a and the value of b.

$\boxed{3a+4b=10}$, $\boxed{a=-\frac{8}{5}}$, $\boxed{b=-\frac{15}{2}}$



Question 9

Find a vector with **integer** components which is perpendicular to **both** the vectors given below.

(Do not use the cross product)

a) $2\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $\mathbf{i}+\mathbf{j}-3\mathbf{k}$

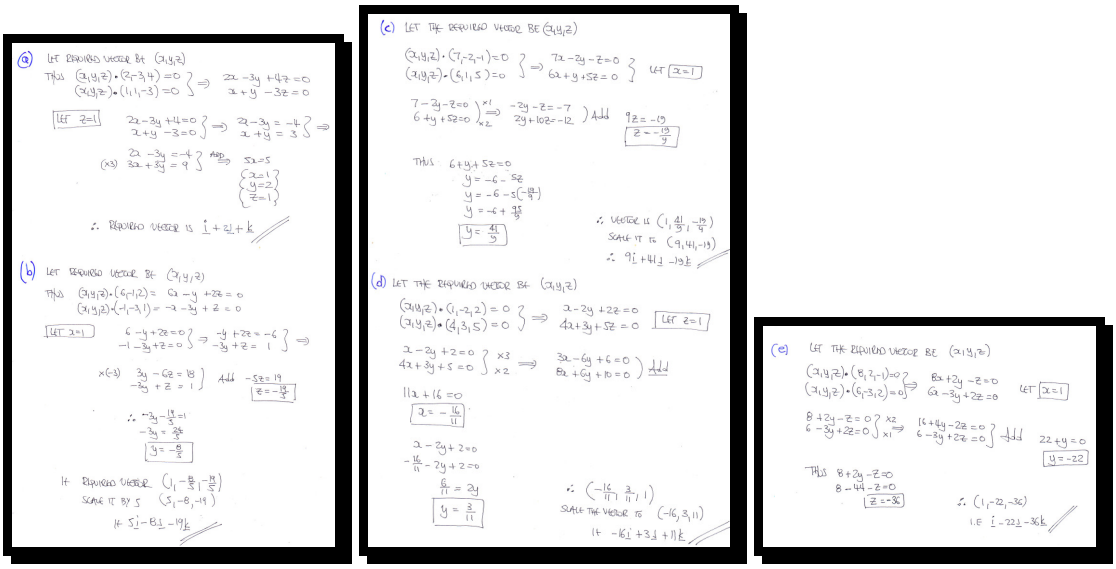
b) $6\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $-\mathbf{i}-3\mathbf{j}+\mathbf{k}$

c) $7\mathbf{i}-2\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}+\mathbf{j}+5\mathbf{k}$

d) $\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ and $4\mathbf{i}+3\mathbf{j}+5\mathbf{k}$

e) $8\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$

$\boxed{\mathbf{i}+2\mathbf{j}+\mathbf{k}}$, $\boxed{5\mathbf{i}-8\mathbf{j}-19\mathbf{k}}$, $\boxed{9\mathbf{i}+41\mathbf{j}-19\mathbf{k}}$, $\boxed{-16\mathbf{i}+3\mathbf{j}+11\mathbf{k}}$, $\boxed{\mathbf{i}-22\mathbf{j}-36\mathbf{k}}$

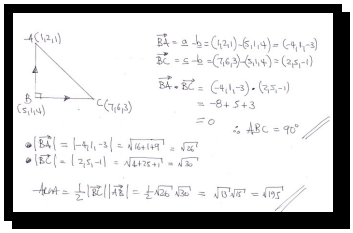


Question 10

The points A, B and C have coordinates $(1,2,1)$, $(5,1,4)$ and $(7,6,3)$, respectively.

Show that $\measuredangle ABC = 90°$ and hence find the exact area of the triangle ABC.

$$\boxed{\text{area} = \sqrt{195}}$$



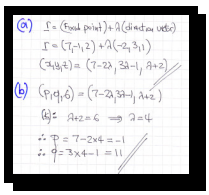
THE EQUATION OF A LINE

Question 1

a) Find a vector equation of the straight line l, that passes through the point $A(7,-1,2)$ and is in the direction $-2\mathbf{i}+3\mathbf{j}+\mathbf{k}$.

b) If $B(p,q,6)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=7\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})},\ \boxed{p=-1},\ \boxed{q=11}$$

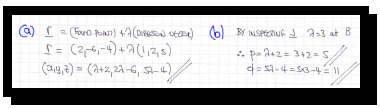


Question 2

a) Find a vector equation of the straight line l, that passes through the point $A(2,-6,-4)$ and is parallel to the vector $\mathbf{i}+2\mathbf{j}+5\mathbf{k}$.

b) If $B(p,0,q)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=2\mathbf{i}-6\mathbf{j}-4\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+5\mathbf{k})},\ \boxed{p=5},\ \boxed{q=11}$$

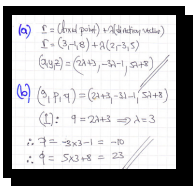


Question 3

a) Find a vector equation of the straight line l, that passes through the point $A(3,-1,8)$ and is in the direction $2\mathbf{i}-3\mathbf{j}+5\mathbf{k}$.

b) If $B(9,p,q)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=3\mathbf{i}-\mathbf{j}+8\mathbf{k}+\lambda(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})},\ \boxed{p=-10},\ \boxed{q=23}$$

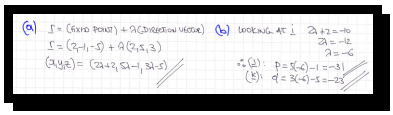


Question 4

a) Find a vector equation of the straight line l that passes through the point $A(2,-1,-5)$ and is in the direction $2\mathbf{i}+5\mathbf{j}+3\mathbf{k}$.

b) If $B(-10,p,q)$ lies on l find the values o p and the value of q.

$$\boxed{\mathbf{r}=2\mathbf{i}-\mathbf{j}-5\mathbf{k}+\lambda(2\mathbf{i}+5\mathbf{j}+3\mathbf{k})},\ \boxed{p=-31},\ \boxed{q=-23}$$

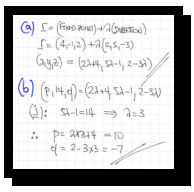


Question 5

a) Determine a vector equation of the straight line l that passes through the point $A(4,-1,2)$ and is in the direction $2\mathbf{i}+5\mathbf{j}-3\mathbf{k}$.

b) If $B(p,14,q)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=4\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+5\mathbf{j}-3\mathbf{k})}\,,\quad \boxed{p=10}\,,\quad \boxed{q=-7}$$

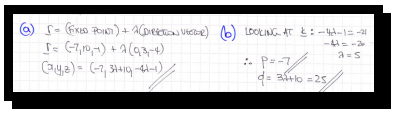


Question 6

a) Determine a vector equation of the straight line l that passes through the point $A(-7,10,-1)$ and is parallel to the vector $3\mathbf{j}-4\mathbf{k}$.

b) If $B(p,q,-21)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=-7\mathbf{i}+10\mathbf{j}-\mathbf{k}+\lambda(3\mathbf{j}-4\mathbf{k})}\,,\quad \boxed{p=-7}\,,\quad \boxed{q=25}$$

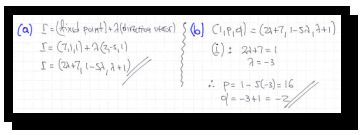


Question 7

a) Determine a vector equation of the straight line l that passes through the point $A(7,1,1)$ and is parallel to the vector $2\mathbf{i}-5\mathbf{j}+\mathbf{k}$.

b) If $B(1,p,q)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=7\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}-5\mathbf{j}+\mathbf{k})}\,,\quad \boxed{p=16}\,,\quad \boxed{q=-2}$$

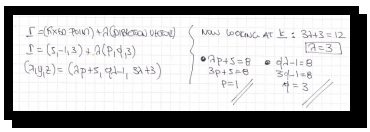


Question 8

The straight line l passes through the point $A(5,-1,3)$ and is parallel to the vector $p\mathbf{i}+q\mathbf{j}+3\mathbf{k}$.

If $B(8,8,12)$ lies on l find the value of p and the value of q.

$$\boxed{p=1}\,,\quad \boxed{q=3}$$

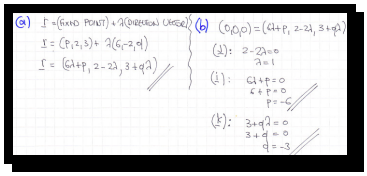


Question 9

a) Determine a vector equation of the straight line l that passes through the point $A(p,2,3)$ and is in the direction $6\mathbf{i}-2\mathbf{j}+q\mathbf{k}$, where p and q are scalar constants.

b) If l passes through the origin, find the value of p and the value of q.

$$\boxed{\mathbf{r}=p\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(6\mathbf{i}-2\mathbf{j}+q\mathbf{k})}, \quad \boxed{p=-6}, \quad \boxed{q=-3}$$

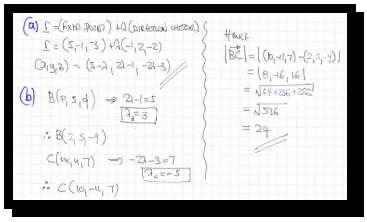


Question 10

a) Determine a vector equation of the straight line l that passes through the point $A(5,-1,-3)$ and is parallel to the vector $-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$.

b) If both the points $B(p,5,q)$ and $C(m,n,7)$ lie on l, find the distance BC.

$$\boxed{\mathbf{r}=5\mathbf{i}-\mathbf{j}-3\mathbf{k}+\lambda(-\mathbf{i}+2\mathbf{j}-2\mathbf{k})}, \quad \boxed{|BC|=24}$$

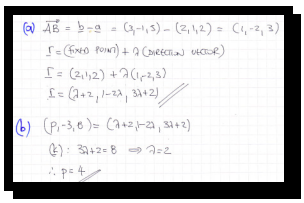


Question 11

a) Determine a vector equation of the straight line l that passes through the points $A(2,1,2)$ and $B(3,-1,5)$.

b) Given that $P(p,-3,8)$ lies on l find the value of p.

$$\boxed{\mathbf{r}=2\mathbf{i}+\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+3\mathbf{k})}, \quad \boxed{p=4}$$

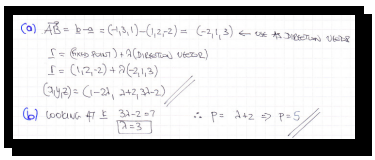


Question 12

a) Find a vector equation of the straight line l that passes through the points $A(1,2,-2)$ and $B(-1,3,1)$.

b) Given that $P(-5,p,7)$ lies on l find the value of p.

$$\boxed{\mathbf{r}=\mathbf{i}+2\mathbf{j}-2\mathbf{k}+\lambda(-2\mathbf{i}+\mathbf{j}+3\mathbf{k})}, \quad \boxed{p=5}$$

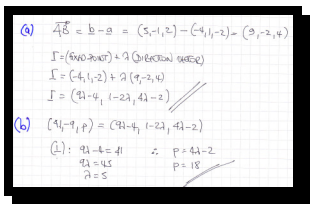


Question 13

a) Determine a vector equation of the straight line l that passes through the points $A(-4,1,-2)$ and $B(5,-1,2)$.

b) Given that $P(41,-9,p)$ lies on l find the value of p.

$$\boxed{\mathbf{r}=-4\mathbf{i}+\mathbf{j}-2\mathbf{k}+\lambda(9\mathbf{i}-2\mathbf{j}+4\mathbf{k})}\,,\quad \boxed{p=18}$$

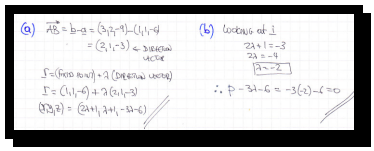


Question 14

a) Find a vector equation of the straight line l that passes through the points $A(1,1,-6)$ and $B(3,2,-9)$.

b) Given that $P(-3,-1,p)$ lies on l find the value of p.

$$\boxed{\mathbf{r}=\mathbf{i}+\mathbf{j}-6\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}-3\mathbf{k})}\,,\quad \boxed{p=0}$$

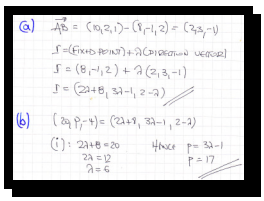


Question 15

a) Determine a vector equation of the straight line l that passes through the points $A(8,-1,2)$ and $B(10,2,1)$.

b) Given that $P(20,p,-4)$ lies on l find the value of p.

$$\boxed{\mathbf{r}=8\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-\mathbf{k})}, \quad \boxed{p=17}$$

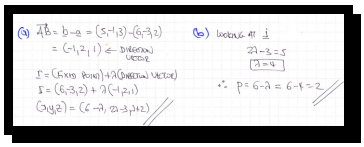


Question 16

a) Find a vector equation of the straight line l that passes through the points $A(6,-3,2)$ and $B(5,-1,3)$.

b) Given that $P(p,5,6)$ lies on l find the value of p.

$$\boxed{\mathbf{r}=6\mathbf{i}-3\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}+2\mathbf{j}+\mathbf{k})}, \quad \boxed{p=2}$$

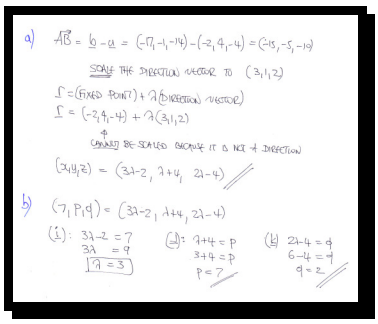


Question 17

a) Determine a vector equation of the straight line l that passes through the points $A(-2,4,-4)$ and $B(-17,-1,-14)$.

b) Given that $P(7,p,q)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=-2\mathbf{i}+4\mathbf{j}-4\mathbf{k}+\lambda(3\mathbf{i}+2\mathbf{j}+\mathbf{k})}\,,\quad \boxed{p=7}\,,\quad \boxed{q=2}$$

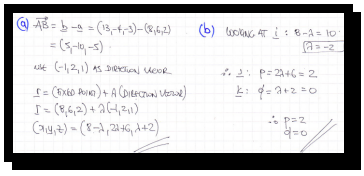


Question 18

a) Find a vector equation of the straight line l that passes through the points $A(8,6,2)$ and $B(13,-4,-3)$.

b) Given that $C(10,p,q)$ lies on l find the value of p and the value of q.

$$\boxed{\mathbf{r}=8\mathbf{i}+6\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}+2\mathbf{j}+\mathbf{k})}\,,\quad \boxed{p=2}\,,\quad \boxed{q=0}$$

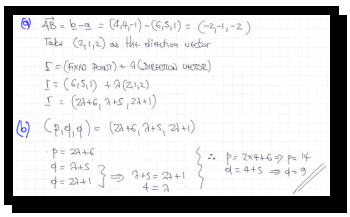


Question 19

a) Determine a vector equation of the straight line l that passes through the points $A(6,5,1)$ and $B(4,4,-1)$.

b) Given the point $C(p,q,q)$ lies on l find the value of p and the value of q.

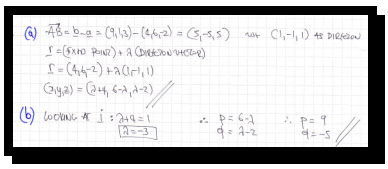
$$\boxed{\mathbf{r}=6\mathbf{i}+5\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}+2\mathbf{k})}\,,\quad \boxed{p=14}\,,\quad \boxed{q=9}$$



Question 20

Given that the points $A(4,6,-2)$, $B(9,1,3)$ and $C(1,p,q)$ lie on a straight line find a vector equation for the straight line and hence find the value of p and the value of q.

$$\boxed{\mathbf{r}=4\mathbf{i}+6\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+\mathbf{k})}\,,\quad \boxed{p=9}\,,\quad \boxed{q=-5}$$



Question 21

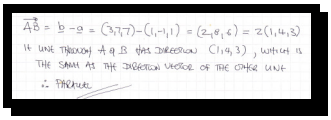
Show that the straight line with vector equation

$$\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}),$$

where λ is a scalar parameter

and the straight line through the points $A(1,-1,1)$ and $B(3,7,7)$ are parallel.

proof

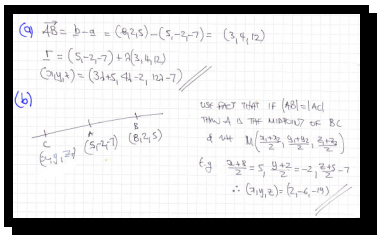


Question 22

a) Find a vector equation of the straight line l that passes through the points $A(5,-2,-7)$ and $B(8,2,5)$.

b) Find the coordinates of the point C which also lies on l with $|AB| = |AC|$,

$\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} - 7\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k})$, $C(2,-6,-19)$

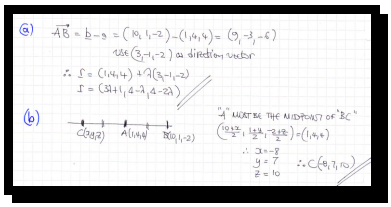


Question 23

a) Find a vector equation of the straight line l through the points $A(1,4,4)$ and $B(10,1,-2)$.

b) Find the coordinates of the point C, given that it lies on l so that $|AB|=|AC|$.

$$\boxed{\mathbf{r}=\mathbf{i}+4\mathbf{j}+4\mathbf{k}+\lambda(3\mathbf{i}-\mathbf{j}-2\mathbf{k})}, \quad \boxed{B(-8,7,10)}$$



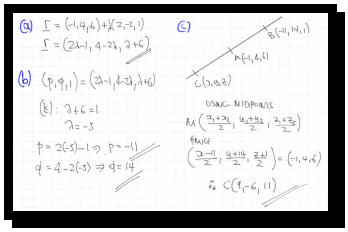
Question 24

a) Find a vector equation of the straight line l that passes through the point $A(-1,4,6)$ and is parallel to the vector $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$.

b) Given that $B(p,q,1)$ lies on l, find the value of p and the value of q.

c) Find the coordinates of the point C, given that it lies on l with $|AB|=|AC|$.

$$\boxed{\mathbf{r}=-\mathbf{i}+4\mathbf{j}+6\mathbf{k}+\lambda(2\mathbf{i}-2\mathbf{j}+\mathbf{k})}, \quad \boxed{p=-11}, \quad \boxed{q=14}, \quad \boxed{C(9,-6,11)}$$

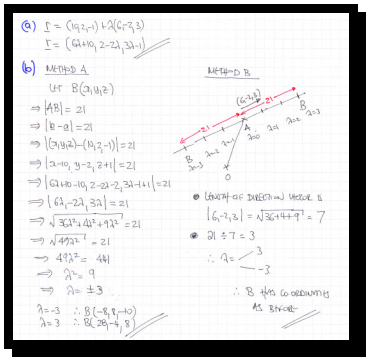


Question 25

a) Find a vector equation of the straight line l that passes through the point $A(10,2,-1)$ and is parallel to the vector $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$.

b) Find the two possible sets of coordinates of the point B given that it lies on l and that $|AB|=21$ units.

$$\boxed{\mathbf{r}=10\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(6\mathbf{i}-2\mathbf{j}+3\mathbf{k})}\ ,\quad \boxed{B(28,-4,8)\ \text{ or }\ B(-8,8,-10)}$$

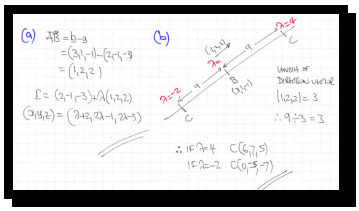


Question 26

a) Find a vector equation of the straight line l which passes through the points $A(2,-1,-3)$ and $B(3,1,-1)$.

b) Find the two possible sets of coordinates of the point C given that it also lies on l and that $|BC|=9$ units.

$$\boxed{\mathbf{r}=2\mathbf{i}-\mathbf{j}-3\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})}\ ,\quad \boxed{C(6,7,5)\ \text{ or }\ C(0,-5,-7)}$$



Question 27

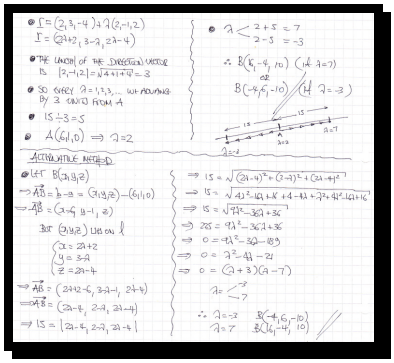
The point $A(6,1,0)$ lies on the straight line l with equation

$$\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),$$

where λ is a scalar parameter.

If the point B also lies on l so that the distance AB is 15 units find the possible coordinates of the point B.

$$\boxed{B(16,-4,10) \text{ or } B(-4,6,-10)}$$



Question 28

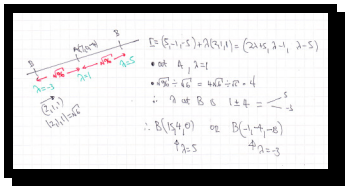
The point $A(7,0,-4)$ lies on the straight line l with equation

$$\mathbf{r} = 5\mathbf{i} - \mathbf{j} - 5\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k}),$$

where λ is a scalar parameter.

If the point B also lies on l so that $|AB| = \sqrt{96}$, find the possible coordinates of B.

$$\boxed{B(15,4,0) \text{ or } B(-1,-4,-8)}$$



Question 29

For each of the pairs of the straight lines shown below,

i. … prove that they intersect.

ii. … find the coordinates of their point of intersection.

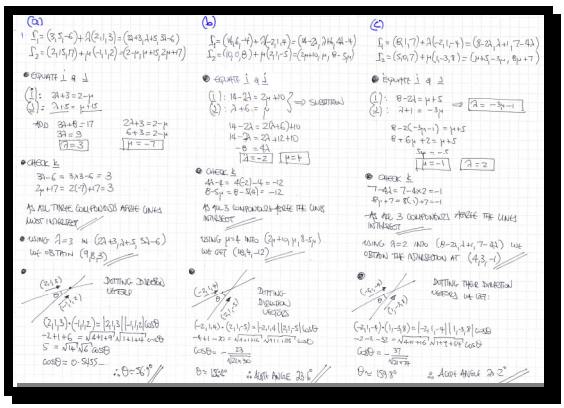
iii. … calculate the <u>acute</u> angle between them.

a)
$$\mathbf{r}_1 = 3\mathbf{i}+5\mathbf{j}-6\mathbf{k}+\lambda\left(2\mathbf{i}+\mathbf{j}+3\mathbf{k}\right)$$
$$\mathbf{r}_2 = 2\mathbf{i}+15\mathbf{j}+17\mathbf{k}+\mu\left(-\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$$

b)
$$\mathbf{r}_1 = 14\mathbf{i}+6\mathbf{j}-4\mathbf{k}+\lambda\left(-2\mathbf{i}+\mathbf{j}+4\mathbf{k}\right)$$
$$\mathbf{r}_2 = 10\mathbf{i}+8\mathbf{k}+\mu\left(2\mathbf{i}+\mathbf{j}-5\mathbf{k}\right)$$

c)
$$\mathbf{r}_1 = 8\mathbf{i}+\mathbf{j}+7\mathbf{k}+\lambda\left(-2\mathbf{i}+\mathbf{j}-4\mathbf{k}\right)$$
$$\mathbf{r}_2 = 5\mathbf{i}+7\mathbf{k}+\mu\left(\mathbf{i}-3\mathbf{j}+8\mathbf{k}\right)$$

$\boxed{(9,8,3)\ \&\ 56.9°}$, $\boxed{(18,4,-12)\ \&\ 23.6°}$, $\boxed{(4,3,-1)\ \&\ 20.2°}$



Question 30

For each of the pairs of the straight lines shown below,

i. … prove that they intersect.

ii. … find the coordinates of their point of intersection.

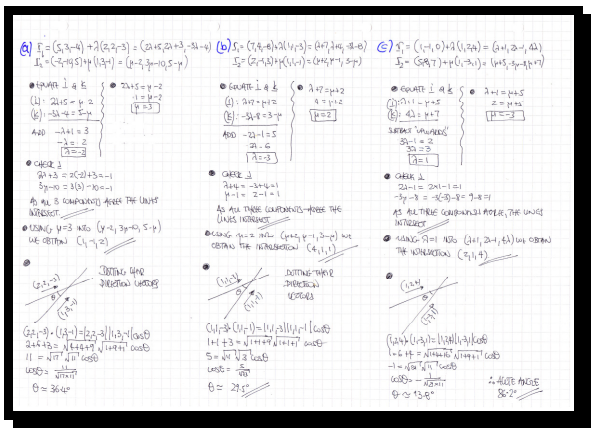
iii. … calculate the <u>acute</u> angle between them.

a)
$$\mathbf{r}_1 = 5\mathbf{i}+3\mathbf{j}-4\mathbf{k}+\lambda\left(2\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)$$
$$\mathbf{r}_2 = -2\mathbf{i}-10\mathbf{j}+5\mathbf{k}+\mu\left(\mathbf{i}+3\mathbf{j}-\mathbf{k}\right)$$

b)
$$\mathbf{r}_1 = 7\mathbf{i}+4\mathbf{j}-8\mathbf{k}+\lambda\left(\mathbf{i}+\mathbf{j}-3\mathbf{k}\right)$$
$$\mathbf{r}_2 = 2\mathbf{i}-\mathbf{j}+3\mathbf{k}+\mu\left(\mathbf{i}+\mathbf{j}-\mathbf{k}\right)$$

c)
$$\mathbf{r}_1 = \mathbf{i}-\mathbf{j}+\lambda\left(\mathbf{i}+2\mathbf{j}+4\mathbf{k}\right)$$
$$\mathbf{r}_2 = 5\mathbf{i}-8\mathbf{j}+7\mathbf{k}+\mu\left(\mathbf{i}-3\mathbf{j}+\mathbf{k}\right)$$

$\boxed{(1,-1,2)\ \&\ 36.4°}$, $\boxed{(4,1,1)\ \&\ 29.5°}$, $\boxed{(2,1,4)\ \&\ 86.2°}$



Question 31

For each of the pairs of the straight lines shown below,

i. … prove that they intersect.

ii. … find the coordinates of their point of intersection.

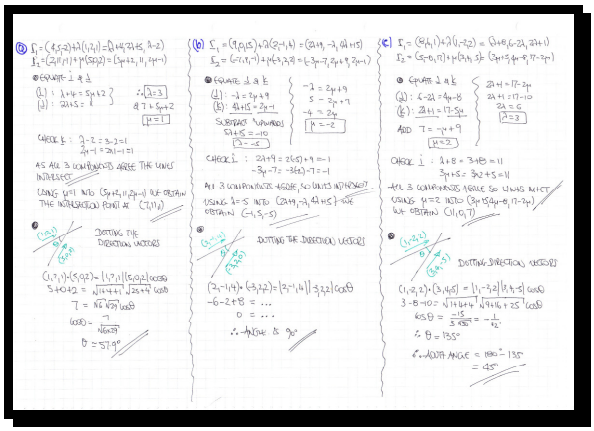
iii. … calculate the acute angle between them.

a)
$$\mathbf{r}_1 = 4\mathbf{i}+5\mathbf{j}-2\mathbf{k}+\lambda\left(\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)$$
$$\mathbf{r}_2 = 2\mathbf{i}+11\mathbf{j}-\mathbf{k}+\mu\left(5\mathbf{i}+2\mathbf{k}\right)$$

b)
$$\mathbf{r}_1 = 9\mathbf{i}+15\mathbf{k}+\lambda\left(2\mathbf{i}-\mathbf{j}+4\mathbf{k}\right)$$
$$\mathbf{r}_2 = -7\mathbf{i}+9\mathbf{j}-\mathbf{k}+\mu\left(-3\mathbf{i}+2\mathbf{j}+2\mathbf{k}\right)$$

c)
$$\mathbf{r}_1 = 8\mathbf{i}+6\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}-2\mathbf{j}+2\mathbf{k}\right)$$
$$\mathbf{r}_2 = 5\mathbf{i}-8\mathbf{j}+17\mathbf{k}+\mu\left(3\mathbf{i}+4\mathbf{j}-5\mathbf{k}\right)$$

$\boxed{(7,11,1)\ \&\ 57.9°}$, $\boxed{(-1,5,-5)\ \&\ 90°}$, $\boxed{(11,0,7)\ \&\ 45°}$



Question 32

For each of the pairs of the straight lines shown below,

i. … prove that they intersect.

ii. … find the coordinates of their point of intersection.

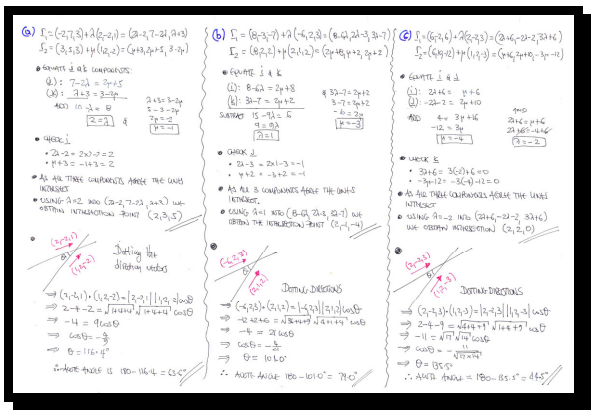
iii. … calculate the acute angle between them.

a)
$$\mathbf{r}_1 = -2\mathbf{i}+7\mathbf{j}+3\mathbf{k}+\lambda\left(2\mathbf{i}-2\mathbf{j}+\mathbf{k}\right)$$
$$\mathbf{r}_2 = 3\mathbf{i}+5\mathbf{j}+3\mathbf{k}+\mu\left(\mathbf{i}+2\mathbf{j}-2\mathbf{k}\right)$$

b)
$$\mathbf{r}_1 = 8\mathbf{i}-3\mathbf{j}-7\mathbf{k}+\lambda\left(-6\mathbf{i}+2\mathbf{j}+3\mathbf{k}\right)$$
$$\mathbf{r}_2 = 8\mathbf{i}+2\mathbf{j}+2\mathbf{k}+\mu\left(2\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$$

c)
$$\mathbf{r}_1 = 6\mathbf{i}-2\mathbf{j}+6\mathbf{k}+\lambda\left(2\mathbf{i}-2\mathbf{j}+3\mathbf{k}\right)$$
$$\mathbf{r}_2 = 6\mathbf{i}+10\mathbf{j}-12\mathbf{k}+\mu\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)$$

$\boxed{(2,3,5)\ \&\ 63.6°}$, $\boxed{(2,-1,-4)\ \&\ 79.0°}$, $\boxed{(2,2,0)\ \&\ 44.5°}$

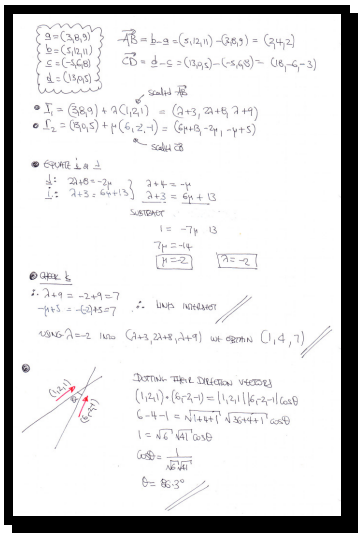


Question 33

Prove that the straight line through $A(3,8,9)$ and $B(5,12,11)$ intersects with the straight line through $C(-5,6,8)$ and $D(13,0,5)$.

Find the point of intersection and the acute angle between the two straight lines.

$\boxed{(1,4,7)}$, $\boxed{86.3°}$



SHORTEST DISTANCES INVOLVING LINES

Question 1

The straight line l has vector equation

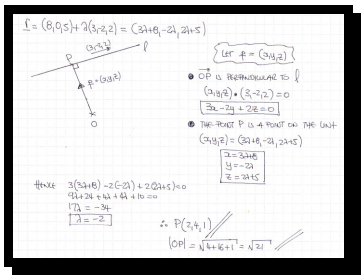
$$\mathbf{r} = 8\mathbf{i} + 5\mathbf{k} + \lambda\left(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}\right),$$

where λ is a scalar parameter.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

Find the coordinates of P and the distance OP.

$$\boxed{P\left(2,4,1\right)}, \quad \boxed{\left|OP\right| = \sqrt{21}}$$



Question 2

The straight line l has vector equation

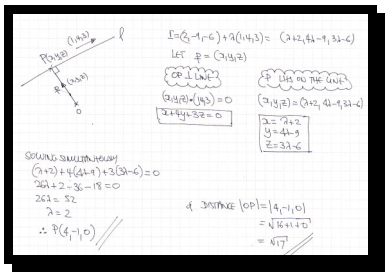
$$\mathbf{r} = 2\mathbf{i} - 9\mathbf{j} - 6\mathbf{k} + \lambda\left(\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}\right)$$

where λ is a scalar parameter.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

Find the coordinates of P and the distance OP.

$$\boxed{P\left(4,-1,0\right)}, \quad \boxed{\left|OP\right| = \sqrt{17}}$$



Question 3

The straight line l has vector equation

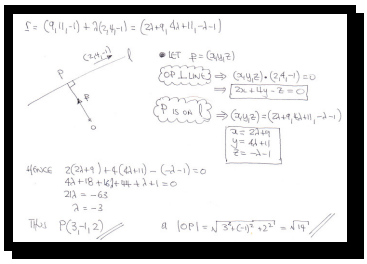
$$\mathbf{r} = 9\mathbf{i} + 11\mathbf{j} - \mathbf{k} + \lambda\left(2\mathbf{i} + 4\mathbf{j} - \mathbf{k}\right)$$

where λ is a scalar parameter.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

Find the coordinates of P and the distance OP.

$$\boxed{P\left(3,-1,2\right)}\,,\quad \boxed{\left|OP\right| = \sqrt{14}}$$



Question 4

The straight line l has vector equation

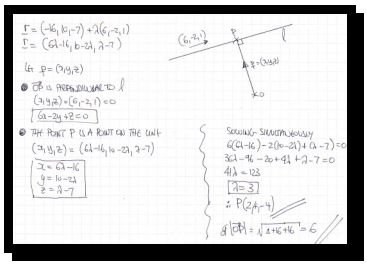
$$\mathbf{r} = -16\mathbf{i} + 10\mathbf{j} - 7\mathbf{k} + \lambda\left(6\mathbf{i} - 2\mathbf{j} + \mathbf{k}\right)$$

where λ is a scalar parameter.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

Find the coordinates of P and the distance OP.

$$\boxed{P\left(2,4,-4\right)}, \quad \boxed{\left|OP\right| = 6}$$



Question 5

The straight line l has vector equation

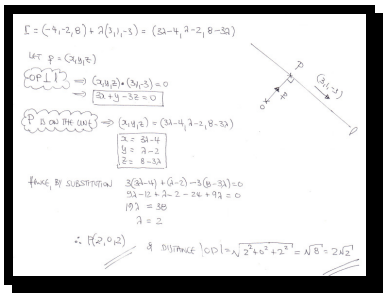
$$\mathbf{r} = -4\mathbf{i} - 2\mathbf{j} + 8\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - 3\mathbf{k})$$

where λ is a scalar parameter.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

Find the coordinates of P and the distance OP.

$$\boxed{P(2,0,2)}, \quad \boxed{|OP| = 2\sqrt{2}}$$



Question 6

The straight line l has vector equation

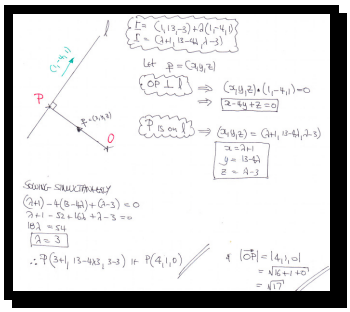
$$\mathbf{r}=\mathbf{i}+13\mathbf{j}-3\mathbf{k}+\lambda\left(\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)$$

where λ is a scalar parameter.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

Find the coordinates of P and the distance OP.

$$\boxed{P\left(4,1,0\right)},\ \boxed{\left|OP\right|=\sqrt{17}}$$



Question 7

The straight line l has vector equation

$$\mathbf{r} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$$

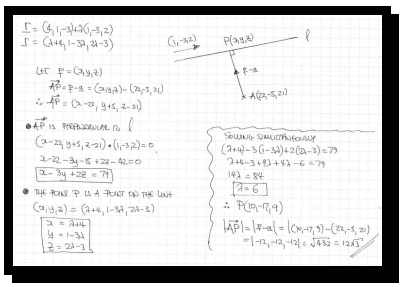
where λ is a scalar parameter.

The point A has coordinates $(22, -5, 21)$.

The point P lies on l so that AP is perpendicular to l.

Find the coordinates of P and the distance AP.

$$\boxed{P(10, -17, 9)}, \quad \boxed{|AP| = 12\sqrt{3}}$$



Question 8

The straight line l has vector equation

$$\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{k}),$$

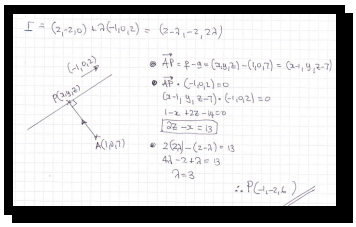
where λ is a scalar parameter.

The point A has coordinates $(1,0,7)$.

The point P lies on l so that AP is perpendicular to l.

Find the coordinates of P.

$$\boxed{P(-1,-2,6)}$$



Question 9

The straight line l has vector equation

$$\mathbf{r} = 17\mathbf{i} + 6\mathbf{j} + 47\mathbf{k} + \lambda(\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}),$$

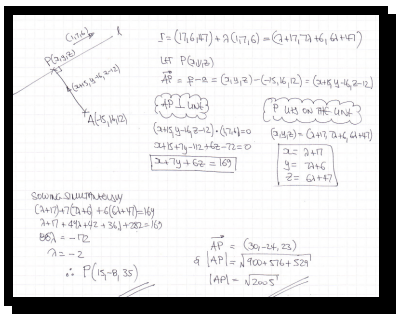
where λ is a scalar parameter.

The point A has coordinates $(-15,16,12)$.

The point P lies on l so that AP is perpendicular to l.

Find the coordinates of P and the distance AP.

$$\boxed{P(15,-8,35)}, \quad \boxed{|AP| = \sqrt{2005}}$$



Question 10

The parallel lines l_1 and l_2 have respective vector equations

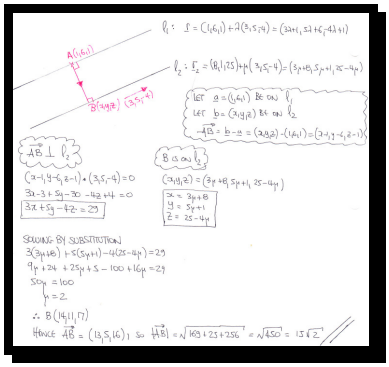
$$\mathbf{r}_1 = \mathbf{i} + 6\mathbf{j} + \mathbf{k} + \lambda\left(3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}\right)$$

$$\mathbf{r}_2 = 8\mathbf{i} + \mathbf{j} + 25\mathbf{k} + \mu\left(3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}\right)$$

where λ and μ are scalar parameters.

Find the distance between l_1 and l_2.

$$\boxed{15\sqrt{2}}$$



Question 11

The parallel lines l_1 and l_2 have respective vector equations

$$\mathbf{r}_1 = 8\mathbf{i} + 3\mathbf{j} + \lambda(2\mathbf{i} - \mathbf{k})$$

$$\mathbf{r}_2 = -\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{k})$$

where λ and μ are scalar parameters.

Find the distance between l_1 and l_2.

[3]

