VECTOR EXAM QUESTIONS

Part A

Question 1 **(**)**

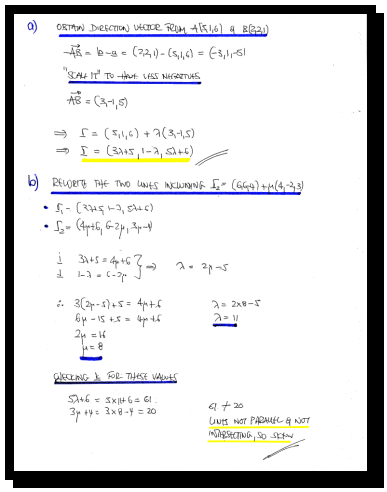
The straight line l_1 passes through the points with coordinates $(5,1,6)$ and $(2,2,1)$.

a) Find a vector equation of l_1.

A different straight line l_2 passes though the point $C(6,6,-4)$ and is parallel to the vector $4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$.

b) Show clearly that l_1 and l_2 are skew.

$\boxed{}$, $\boxed{\mathbf{r}=5\mathbf{i}+\mathbf{j}+6\mathbf{k}+\lambda(3\mathbf{i}-\mathbf{j}+5\mathbf{k})}$



Question 2 (**)

Relative to a fixed origin O, the respective position vectors of three points A, B and C are

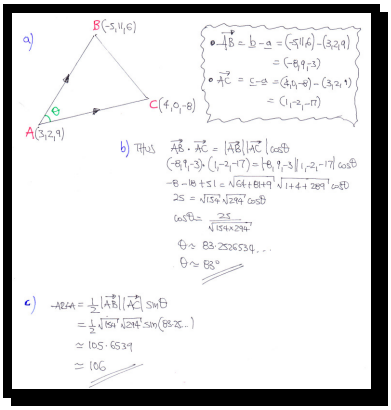
$$\begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix}, \quad \begin{pmatrix} -5 \\ 11 \\ 6 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 4 \\ 0 \\ -8 \end{pmatrix}.$$

a) Determine, in component form, the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

b) Hence find, to the nearest degree, the angle BAC.

c) Calculate the area of the triangle BAC.

$$\boxed{\overrightarrow{AB} = -8\mathbf{i} + 9\mathbf{j} - 3\mathbf{k}}, \quad \boxed{\overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - 17\mathbf{k}}, \quad \boxed{\theta \approx 83^\circ}, \quad \boxed{\text{area} \approx 106}$$



Question 3 (**)

The straight line l_1 passes through the points $A(2,5,9)$ and $B(6,0,10)$.

a) Find a vector equation for l_1.

The straight line l_2 has vector equation

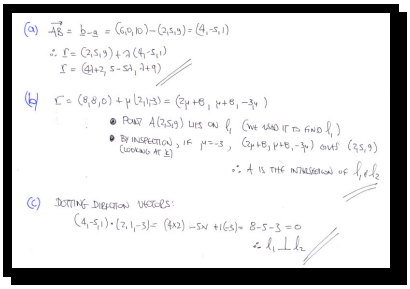
$$\mathbf{r}=\begin{pmatrix}8\\8\\0\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-3\end{pmatrix},$$

where μ is a scalar parameter.

b) Show that the point A is the intersection of l_1 and l_2.

c) Show further that l_1 and l_2 are perpendicular to each other.

$$\boxed{\mathbf{r}=2\mathbf{i}+5\mathbf{j}+9\mathbf{k}+\lambda\left(4\mathbf{i}-5\mathbf{j}+\mathbf{k}\right)}$$



Question 4 (**)

Relative to a fixed origin O, the points A and B have respective position vectors

$$\mathbf{i}+7\mathbf{j}+5\mathbf{k} \text{ and } 5\mathbf{i}+\mathbf{j}-5\mathbf{k}.$$

a) Find a vector equation of the straight line l_1 which passes through A and B.

The straight line l_2 has vector equation

$$\mathbf{r}_2 = 5\mathbf{i}-4\mathbf{j}+4\mathbf{k}+\mu\left(\mathbf{i}-4\mathbf{j}+2\mathbf{k}\right),$$

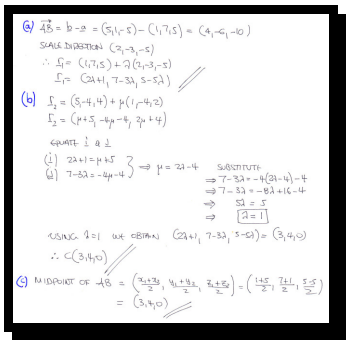
where μ is a scalar parameter.

The point C is the point of intersection between l_1 and l_2.

b) Find the position vector of C.

c) Show that C is the midpoint of AB.

$$\boxed{\mathbf{r}=\mathbf{i}+7\mathbf{j}+5\mathbf{k}+\lambda\left(2\mathbf{i}-3\mathbf{j}-5\mathbf{k}\right)}, \quad \boxed{\overrightarrow{OC}=3\mathbf{i}+4\mathbf{j}}$$


(a) $\overrightarrow{AB} = \mathbf{b}-\mathbf{a} = (5,1,-5)-(1,7,5) = (4,-6,-10)$

SCALE DIVISION $(2,-3,-5)$

$\therefore l_1 = (1,7,5)+\lambda(2,-3,-5)$

$l_1 = (2\lambda+1,\ 7-3\lambda,\ 5-5\lambda)$

(b) $l_2 = (5,-4,4)+\mu(1,-4,2)$

$l_2 = (\mu+5,\ -4\mu-4,\ 2\mu+4)$

EQUATE $\mathbf{i}$ & $\mathbf{j}$

(i) $2\lambda+1 = \mu+5$ ②

(ii) $7-3\lambda = -4\mu-4$ ① $\Rightarrow \mu = 2\lambda-4$

SUBSTITUTE

$\Rightarrow 7-3\lambda = -4(2\lambda-4)-4$

$\Rightarrow 7-3\lambda = -8\lambda+16-4$

$\Rightarrow 5\lambda = 5$

$\Rightarrow \boxed{\lambda = 1}$

USING $\lambda=1$ IN EITHER $(2\lambda+1,\ 7-3\lambda,\ 5-5\lambda) = (3,4,0)$

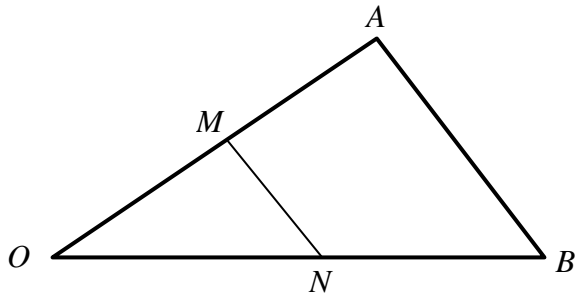
$\therefore C(3,4,0)$

(c) MIDPOINT OF $AB = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = \left(\frac{1+5}{2}, \frac{7+1}{2}, \frac{5-5}{2}\right)$

$= (3,4,0)$

Question 5 (**)

The figure below shows the triangle OAB.

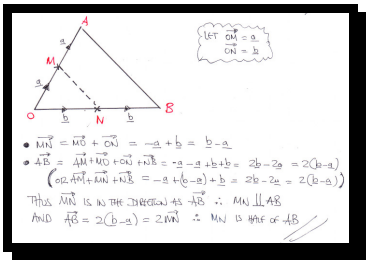


The point M is the midpoint of OA and the point N is the midpoint of OB.

Let $\overrightarrow{OM}=\mathbf{a}$ and $\overrightarrow{ON}=\mathbf{b}$.

By finding simplified expressions for $\overrightarrow{MN}$ and $\overrightarrow{AB}$, in terms of $\mathbf{a}$ and $\mathbf{b}$, show that MN is parallel to AB, and half its length.

proof



Question 6 (+)**

The points $A(2,4,4)$, $B(6,8,4)$, $C(6,4,0)$, $D(2,0,0)$ and $M(4,4,2)$ are given.

The straight line l_1 has equation

$$\mathbf{r}_1 = 6\mathbf{i}+4\mathbf{j}+\lambda(\mathbf{i}+\mathbf{j}),$$

where λ is a scalar parameter.

The straight line l_2 passes through the points C and M.

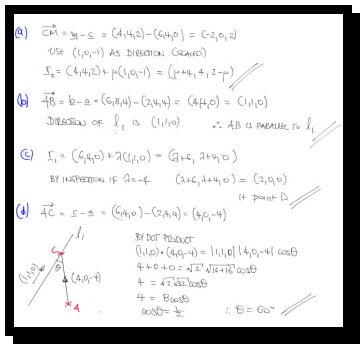
a) Find a vector equation of l_2.

b) Show that $\overrightarrow{AB}$ is parallel to l_1.

c) Verify that D lies on l_1.

d) Find the acute angle between $\overrightarrow{AC}$ and l_1.

$\boxed{\mathbf{r}_2 = 4\mathbf{i}+4\mathbf{j}+2\mathbf{k}+\mu(\mathbf{i}-\mathbf{k})}$, $\boxed{60°}$



Question 7 (+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 4\mathbf{i}+3\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}+4\mathbf{j}+3\mathbf{k}\right)$$

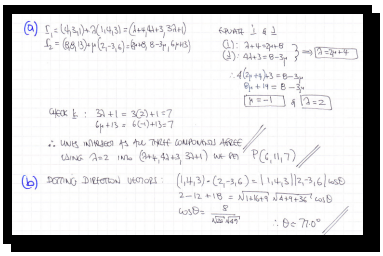
$$\mathbf{r}_2 = 8\mathbf{i}+8\mathbf{j}+13\mathbf{k}+\mu\left(2\mathbf{i}-3\mathbf{j}+6\mathbf{k}\right),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point P and find its coordinates.

b) Calculate the acute angle between l_1 and l_2.

$$\boxed{P(6,11,7)}, \quad \boxed{77.0°}$$



Question 8 (+)**

Relative to a fixed origin O, the points P and Q have respective position vectors

$$(-7\mathbf{j}+4\mathbf{k}) \text{ and } (3\mathbf{i}-8\mathbf{j}+2\mathbf{k}).$$

The straight line l_1 passes through the points P and Q.

a) Determine a vector equation for l_1.

The straight line l_2 has vector equation

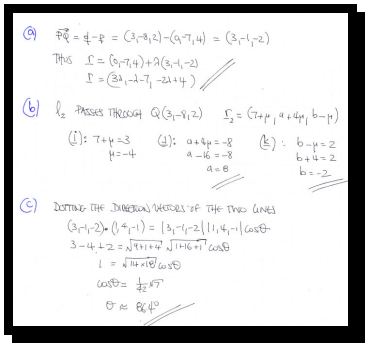
$$\mathbf{r}=(7\mathbf{i}+a\mathbf{j}+b\mathbf{k})+\mu(\mathbf{i}+4\mathbf{j}-\mathbf{k}),$$

where a and b are scalar constants, and μ is a scalar parameter.

b) Given that l_1 and l_2 intersect at Q, find the value of a and the value of b.

c) Calculate the acute angle between l_1 and l_2.

$\square$, $\boxed{\mathbf{r}=-7\mathbf{j}+4\mathbf{k}+\lambda(3\mathbf{i}-\mathbf{j}-2\mathbf{k})}$, $\boxed{a=8}$, $\boxed{b=-2}$, $\boxed{86.4°}$



Question 9 (+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i}+2\mathbf{j}+\lambda(\mathbf{i}+\mathbf{j})$$

$$\mathbf{r}_2 = 2\mathbf{i}+5\mathbf{j}+7\mathbf{k}+\mu(2\mathbf{i}+\mathbf{j}-\mathbf{k}),$$

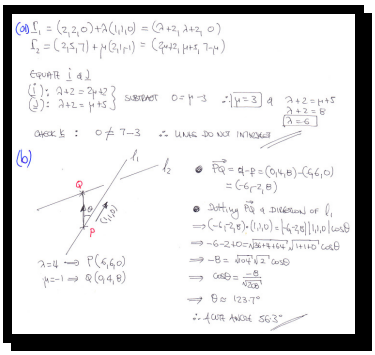
where λ and μ are scalar parameters.

a) Show that l_1 and l_2 **do not** intersect.

The point P lies on l_1 where $\lambda = 4$ and the point Q lies on l_2 where $\mu = -1$.

b) Find the acute angle between PQ and l_1.

[ ], [56.3°]



Question 10 (+)**

Relative to a fixed origin O, the points A, B and C have respective position vectors

$$\mathbf{a}=8\mathbf{i}+\mathbf{j},\quad \mathbf{b}=5\mathbf{j}+8\mathbf{k}\quad \text{and}\quad \mathbf{c}=14\mathbf{i}+\mathbf{j}+15\mathbf{k}.$$

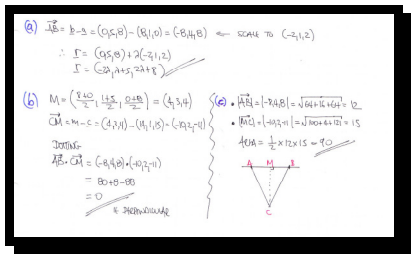
a) Find a vector equation of the straight line which passes through A and B.

The point M is the midpoint of AB.

b) Show that CM is perpendicular to AB.

c) Determine the area of the triangle ABC.

$$\boxed{\mathbf{r}=5\mathbf{j}+8\mathbf{k}+\lambda\left(-2\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)},\quad \boxed{\text{area}=90}$$



Question 11 (+)**

With respect to a fixed origin O, the respective position vectors of the points A, B and C are

$$\begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 12 \\ 4 \\ 7 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 10 \\ -3 \\ 7 \end{pmatrix}.$$

a) Find the position vector of the midpoint of AC.

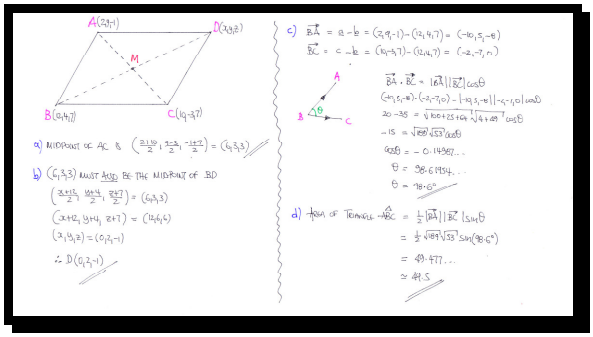
The point D is such so that $ABCD$ is a parallelogram.

b) Determine the position vector of D.

c) Calculate, correct to one decimal place, the angle ABC.

d) Hence, calculate the area of the triangle ABC.

$\boxed{6\mathbf{i}+3\mathbf{j}+3\mathbf{k}}$, $\boxed{2\mathbf{j}-\mathbf{k}}$, $\boxed{\theta \approx 98.6°}$, $\boxed{\text{area} \approx 49.5}$



Question 12 (+)**

$OABC$ is a square.

The point M is the midpoint of AB and the point N is the midpoint of MC.

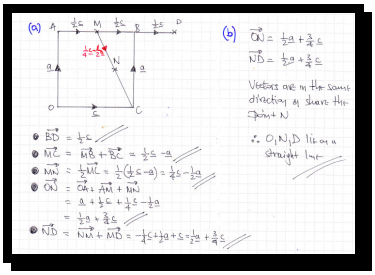
The point D is such so that $\overrightarrow{AD}=\frac{3}{2}\overrightarrow{AB}$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OC}=\mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{BD}$, $\overrightarrow{MC}$, $\overrightarrow{MN}$, $\overrightarrow{ON}$ and $\overrightarrow{ND}$.

b) Deduce, showing your reasoning, that O, N and D are collinear.

$$\boxed{\overrightarrow{BD}=\frac{1}{2}\mathbf{c}},\ \boxed{\overrightarrow{MC}=\frac{1}{2}\mathbf{c}-\mathbf{a}},\ \boxed{\overrightarrow{MN}=\frac{1}{4}\mathbf{c}-\frac{1}{2}\mathbf{a}},\ \boxed{\overrightarrow{ON}=\frac{1}{2}\mathbf{a}+\frac{3}{4}\mathbf{c}},\ \boxed{\overrightarrow{ND}=\frac{1}{2}\mathbf{a}+\frac{3}{4}\mathbf{c}}$$



Question 13 (***)

Relative to a fixed origin O, the points P and Q have respective position vectors

$$5\mathbf{i}+2\mathbf{k} \text{ and } 3\mathbf{i}+3\mathbf{j}.$$

a) Determine a vector equation of the straight line l which passes through the points P and Q.

The straight line m has a vector equation

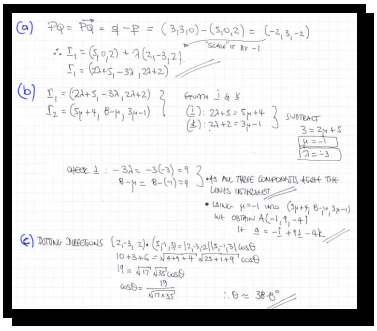
$$\mathbf{r}=4\mathbf{i}+8\mathbf{j}-\mathbf{k}+\mu\left(5\mathbf{i}-\mathbf{j}+3\mathbf{k}\right),$$

where μ is a scalar parameter.

b) Show that l and m intersect at some point A and find its position vector.

c) Find the size of the acute angle θ, formed by l and m.

$$\boxed{\mathbf{r}=5\mathbf{i}+2\mathbf{k}+\lambda\left(2\mathbf{i}-3\mathbf{j}+2\mathbf{k}\right)},\ \boxed{\overrightarrow{OA}=-\mathbf{i}+9\mathbf{j}-4\mathbf{k}},\ \boxed{\theta\approx 38.8^\circ}$$



Question 14 (***)

The points $A(2,10,7)$ and $B(0,15,12)$ are given.

a) Determine a vector equation of the straight line l_1 that passes through the points A and B.

The vector equation of the straight line l_2 is

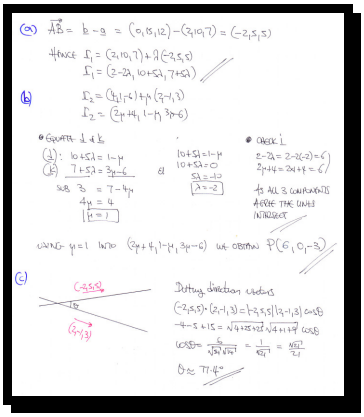
$$\mathbf{r}_2 = 4\mathbf{i}+\mathbf{j}-6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}+3\mathbf{k}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 intersect at some point P and find its coordinates.

c) Calculate the acute angle between l_1 and l_2.

$\boxed{}$, $\boxed{\mathbf{r}_1 = 2\mathbf{i}+10\mathbf{j}+7\mathbf{k}+\lambda(-2\mathbf{i}+5\mathbf{j}+5\mathbf{k})}$, $\boxed{P(6,0,-3)}$, $\boxed{77.4°}$



Question 15 (***)

Relative to a fixed origin O the following position vectors are given.

$$\overrightarrow{OA}=\begin{pmatrix}1\\6\\11\end{pmatrix},\ \overrightarrow{OB}=\begin{pmatrix}4\\3\\5\end{pmatrix},\ \overrightarrow{OC}=\begin{pmatrix}6\\1\\1\end{pmatrix}\ \text{and}\ \overrightarrow{OD}=\begin{pmatrix}2\\8\\9\end{pmatrix}.$$

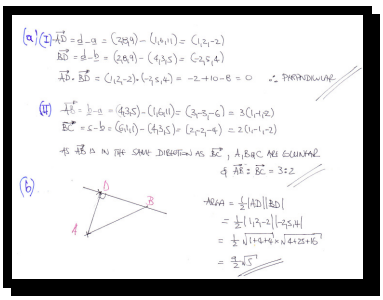
a) Show clearly that …

i. … $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BD}$.

ii. … the points A, B and C are collinear and state the ratio $AB:BC$.

b) Determine the **exact** area of the triangle ABD.

$\boxed{AB:BC=3:2}$, $\boxed{\text{area}=\frac{9}{2}\sqrt{5}}$



Question 16 (***)

Relative to a fixed origin O, the points A and B have respective position vectors

$$4\mathbf{i}+3\mathbf{j}-3\mathbf{k} \quad \text{and} \quad 2\mathbf{i}+3\mathbf{k}\,.$$

a) Determine a vector equation of the straight line l_1 which passes through the points A and B.

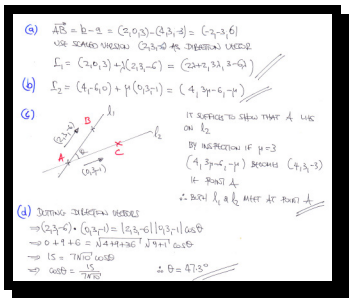
The straight line l_2 passes through the point C with position vector $4\mathbf{i}-6\mathbf{j}$ and is parallel to the vector $3\mathbf{j}-\mathbf{k}$.

b) Write down a vector equation of l_2.

c) Show that l_1 and l_2 intersect at the point A.

d) Find the acute angle between l_1 and l_2.

$$\boxed{\mathbf{r}_1=2\mathbf{i}+3\mathbf{k}+\lambda\left(2\mathbf{i}+3\mathbf{j}-6\mathbf{k}\right)}\,,\quad \boxed{\mathbf{r}_2=4\mathbf{i}-6\mathbf{k}+\mu\left(3\mathbf{j}-\mathbf{k}\right)}\,,\quad \boxed{47.3^\circ}$$



Question 17 (***)

Relative to a fixed origin O, the straight lines L and M have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r}_2 = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} + s \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix},$$

where t and s are scalar parameters.

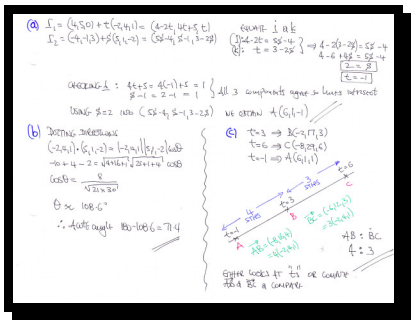
a) Show that L and M intersect at some point A and find its coordinates.

b) Find the size of the acute angle θ, formed by L and M.

The points B and C lie on the L where $t = 3$ and $t = 6$, respectively.

c) Find the ratio $AB : BC$.

$\square$, $\boxed{A(6,1,-1)}$, $\boxed{\theta \approx 71.4}$, $\boxed{AB : BC = 4 : 3}$



Question 18 (***)

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i}+\mathbf{j}+\lambda\left(-\mathbf{i}+2\mathbf{j}+3\mathbf{k}\right)$$

$$\mathbf{r}_2 = \mathbf{i}+5\mathbf{j}+4\mathbf{k}+\mu\left(\mathbf{i}-2\mathbf{k}\right)$$

where λ and μ are scalar parameters.

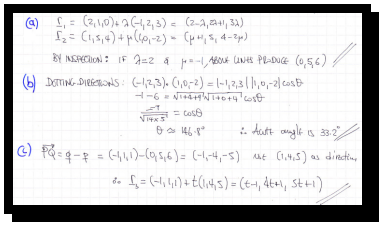
a) Verify that both l_1 and l_2 pass through the point P, whose position vector is $5\mathbf{j}+6\mathbf{k}$.

b) Find the acute angle between l_1 and l_2 .

The point Q has position vector $-\mathbf{i}+\mathbf{j}+\mathbf{k}$.

c) Find a vector equation of the straight line l_3 that passes through the point Q, so that all three straight lines intersect.

$\boxed{33.2°}$, $\boxed{\mathbf{r}_3 = -\mathbf{i}+\mathbf{j}+\mathbf{k}+t\left(\mathbf{i}+4\mathbf{j}+5\mathbf{k}\right)}$



Question 19 (***)

With respect to a fixed origin O, the point A has position vector $8\mathbf{i}-6\mathbf{j}+5\mathbf{k}$ and the point B has position vector $t\mathbf{i}+t\mathbf{j}+2t\mathbf{k}$.

a) Show clearly that

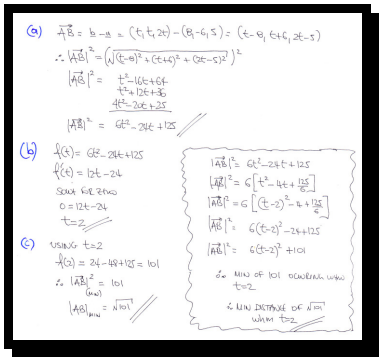
$$|AB|^2 = 6t^2 - 24t + 125 .$$

Let $f(t) = 6t^2 - 24t + 125$.

b) Find the value of t for which $f(t)$ takes a minimum value.

c) Hence determine the closest distance between A and B.

$\boxed{t=2}$, $\boxed{\sqrt{101}}$



Question 20 (***)

Relative to a fixed origin O, the points A and B have respective coordinates

$$(2,-3,3) \quad \text{and} \quad (5,1,b),$$

where b is a constant.

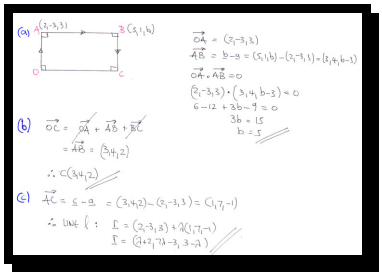
The point C is such so that $OABC$ is a rectangle, where O is the origin.

a) Show clearly that $b=5$.

b) Determine the position vector of C.

c) Find the vector equation of the straight line l that passes through A and C.

$$\boxed{C(3,4,2)}, \quad \boxed{\mathbf{r}=2\mathbf{i}-3\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}+7\mathbf{j}-\mathbf{k})}$$



Question 21 (***)

The following vectors are given

$$\mathbf{a}=6\mathbf{i}-3\mathbf{j}+2\mathbf{k} \quad \text{and} \quad \mathbf{b}=(4p+1)\mathbf{i}+(p-2)\mathbf{j}+\mathbf{k},$$

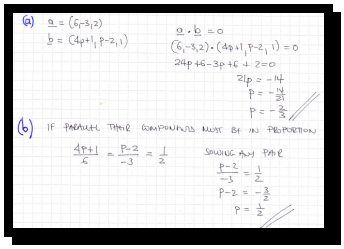
where p is a scalar constant.

Find the value of p if …

a) … $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

b) … $\mathbf{a}$ and $\mathbf{b}$ are parallel.

$$\boxed{p=-\frac{2}{3}}, \quad \boxed{p=\frac{1}{2}}$$



Question 22 (***)

$OABC$ is a parallelogram and the point M is the midpoint of AB.

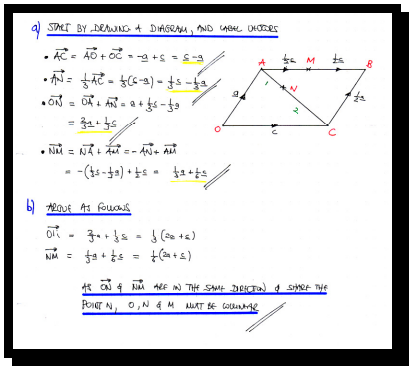
The point N lies on the diagonal AC so that $AN:NC=1:2$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OC}=\mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{AC}$, $\overrightarrow{AN}$, $\overrightarrow{ON}$ and $\overrightarrow{NM}$.

b) Deduce, showing your reasoning, that O, N and M are collinear.

$\boxed{}$, $\boxed{\overrightarrow{AC}=\mathbf{c}-\mathbf{a}}$, $\boxed{\overrightarrow{AN}=\frac{1}{3}\mathbf{c}-\frac{1}{3}\mathbf{a}}$, $\boxed{\overrightarrow{ON}=\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{c}}$, $\boxed{\overrightarrow{NM}=\frac{1}{3}\mathbf{a}+\frac{1}{6}\mathbf{c}}$



Question 23 (***)

The points with coordinates $A(1,4,3)$, $B(2,2,1)$ and $C(5,4,0)$ are given.

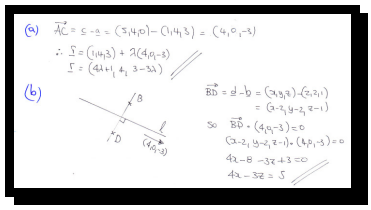
a) Find a vector equation of the straight line l, that passes through A and C.

The point $D(x,y,z)$ is such so that BD is perpendicular to l.

b) Show clearly that

$$4x-3z=5.$$

$$\boxed{\mathbf{r}=\mathbf{i}+4\mathbf{j}+3\mathbf{k}+\lambda(4\mathbf{i}-3\mathbf{k})}$$



Question 24 (***)

The straight line L_1 passes through the points $A(3,0,3)$ and $B(5,5,2)$.

The straight line L_2 has a vector equation given by

$$\mathbf{r}=\begin{pmatrix}5\\5\\2\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-3\end{pmatrix},$$

where μ is a scalar parameter.

a) Write down the coordinates of the point of intersection of L_1 and L_2.

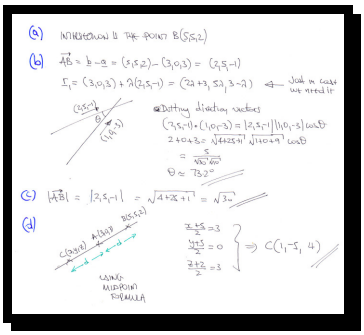
b) Find the size of the acute angle θ, between L_1 and L_2.

c) Calculate the distance AB.

The point C lies on L_1 so that the distance AB is equal to the distance AC.

d) Determine the coordinates of C.

$\boxed{P(5,5,2)}$, $\boxed{\theta\approx 73.2^\circ}$, $\boxed{|AB|=\sqrt{30}}$, $\boxed{C(1,-5,4)}$



Question 25 (***)

A tunnel is to be dug through a mountain in order to link two cities.

Digging at one end of the tunnel begins at the point with coordinates $(-3,-3,9)$ and continues in the direction $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$.

The digging at the other end of the tunnel starts at the point with coordinates $(-19,-7,-3)$ and continues in the direction $6\mathbf{i}+3\mathbf{j}+2\mathbf{k}$.

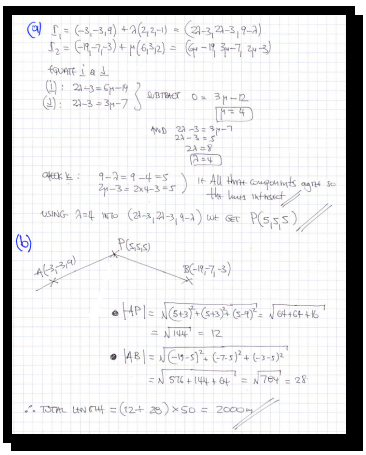
Both sections are assumed to be straight lines.

The coordinates are measured relative to a fixed origin O, where one unit is 50 metres.

a) Show that the two sections of the tunnel will eventually meet at a point P, and find the coordinates of this point.

b) Find the total length of the tunnel.

$\boxed{P(5,5,5)}$, $\boxed{\text{length = 2000 metres}}$



Question 26 (***)

Relative to a fixed origin O, the straight lines l_1 and l_2 have respective vector equations given by

$$\mathbf{r}_1 = \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 1 \\ p \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 9 \\ -2 \\ 5 \end{pmatrix},$$

where λ and μ are scalar parameters and p is a scalar constant.

The point T is the point of intersection between l_1 and l_2.

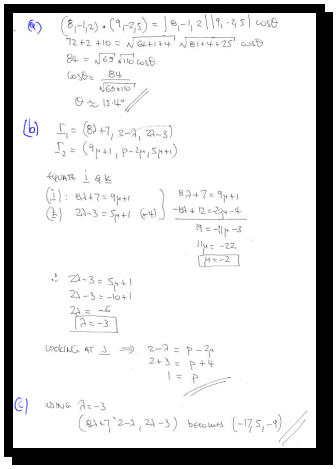
Find in any order …

a) … the size of acute angle between l_1 and l_2.

b) … the value of p.

c) … the coordinates of T.

$\boxed{\theta \approx 15.4^\circ}$, $\boxed{p = 1}$, $\boxed{T(-17, 5, -9)}$



Question 27 (***)

With respect to a fixed origin O, the points $A(2,6,5)$ and $B(5,0,-4)$ are given.

a) Find a vector equation of the straight line L_1, which passes through A and B.

The straight line L_2 has a vector equation

$$\mathbf{r}_2 = \begin{pmatrix} -4 \\ 4 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

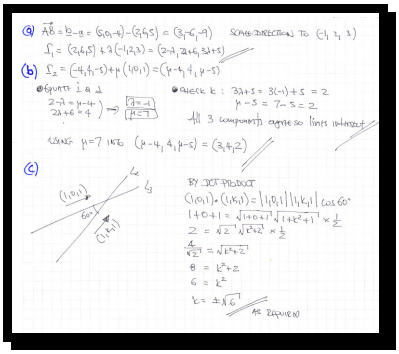
where μ is a scalar parameter.

b) Show clearly that L_1 and L_2 intersect, and find the coordinates of their point of intersection.

The straight line L_3 is in the direction $\begin{pmatrix} 1 \\ k \\ 1 \end{pmatrix}$.

c) Given the acute angle between L_2 and L_3 is $60°$, show clearly that $k = \pm\sqrt{6}$

$$\boxed{\mathbf{r}_1 = 2\mathbf{i} + 6\mathbf{j} + 5\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}, \quad \boxed{(3,4,2)}$$



Question 28 (***)

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i}+3\mathbf{j}+\lambda\left(2\mathbf{i}+\mathbf{j}+4\mathbf{k}\right)$$

$$\mathbf{r}_2 = 5\mathbf{i}+3\mathbf{j}+9\mathbf{k}+\mu\left(\mathbf{i}+2\mathbf{j}-\mathbf{k}\right),$$

where λ and μ are scalar parameters.

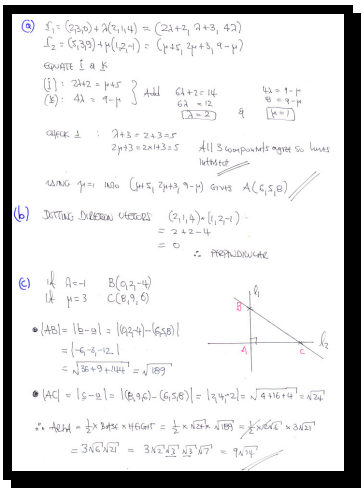
a) Show that l_1 and l_2 intersect at some point A, and find its coordinates.

b) Show further that l_1 and l_2 intersect at right angles.

The point B lies on l_1 where $\lambda=-1$ and the point C lies on l_2 where $\mu=3$.

c) Find the exact area of the triangle BAC.

$$\boxed{A\left(6,5,8\right)}, \quad \boxed{\text{area}=9\sqrt{14}}$$



Question 29 (***)

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 4\mathbf{i}+7\mathbf{j}+4\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j})$$

$$\mathbf{r}_2 = 8\mathbf{i}+5\mathbf{j}+2\mathbf{k}+\mu(\mathbf{i}-\mathbf{k}),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point A and find its coordinates.

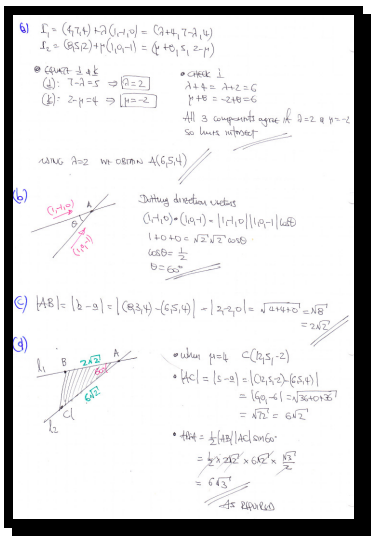
b) Calculate the acute angle between l_1 and l_2.

The point $B(8,3,4)$ lies on l_1 and the point C lies on l_2 where $\mu = 4$.

c) Find the distance AB.

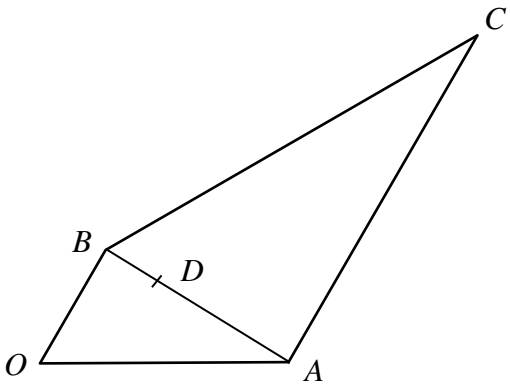
d) Show that the area of the triangle ABC is $6\sqrt{3}$ square units.

$\boxed{A(6,5,4)}$, $\boxed{60^\circ}$, $\boxed{|AB| = 2\sqrt{2}}$



Question 30 (***)

The figure below shows a trapezium $OBCA$ where OB is parallel to AC.



The point D lies on BA so that $BD:DA=1:2$.

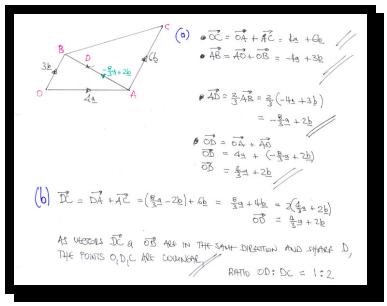
Let $\overrightarrow{OA}=4\mathbf{a}$, $\overrightarrow{OB}=3\mathbf{b}$ and $\overrightarrow{AC}=6\mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{OC}$, $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{OD}$.

b) Deduce, showing your reasoning, that O, D and C are collinear and state the ratio of $OD:DC$.

$$\boxed{\overrightarrow{OC}=4\mathbf{a}+6\mathbf{b}},\ \boxed{\overrightarrow{AB}=-4\mathbf{a}+3\mathbf{b}},\ \boxed{\overrightarrow{AD}=-\frac{8}{3}\mathbf{a}+2\mathbf{b}},\ \boxed{\overrightarrow{OD}=\frac{4}{3}\mathbf{a}+2\mathbf{b}},$$

$$\boxed{OD:DC=1:2}$$



Question 31 (***)

The points $A(5,1,3)$, $B(3,1,5)$, $C(5,3,5)$ and $D(4,0,3)$ are given.

a) Show that the triangle ABC is equilateral and find its area.

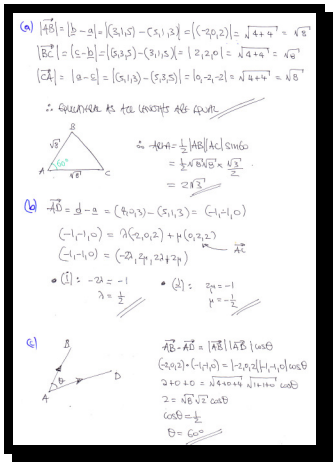
b) Show further that

$$\overrightarrow{AD}=\lambda\overrightarrow{AB}+\mu\overrightarrow{AC},$$

stating the exact values of the scalar constants λ and μ.

c) Find the size of the angle BAD.

$$\boxed{\text{area}=2\sqrt{3}},\quad \boxed{\lambda=\frac{1}{2},\mu=-\frac{1}{2}},\quad \boxed{\angle BAD=60^\circ}$$



Question 32 (***)

With respect to a fixed origin O, the straight lines l_1 and l_2 have respective vector equations given by

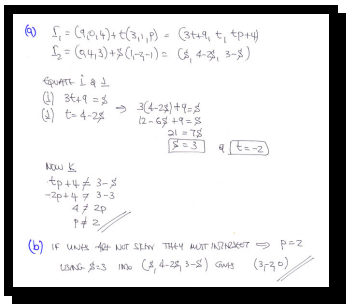
$$\mathbf{r}_1 = \begin{pmatrix} 9 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \\ p \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix},$$

where t and s are scalar parameters.

a) If l_1 and l_2 are skew find the value that p **cannot** take.

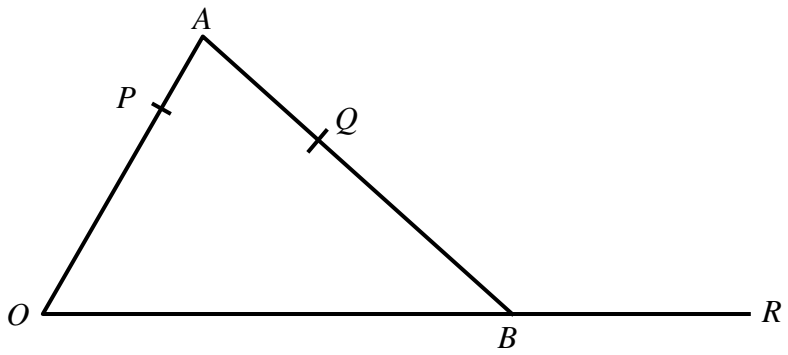
b) If l_1 and l_2 are not skew find the coordinates of their point of intersection.

$\boxed{p \neq 2}$, $\boxed{(3,-2,0)}$



Question 33 (***)

The figure below shows a triangle OAB.



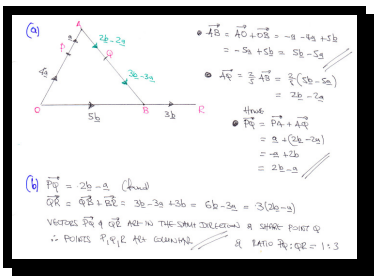
- The point P lies on OA so that $OP:PA=4:1$.
- The point Q lies on AB so that $AQ:QB=2:3$
- The side OB is extended to the point R so that $OB:BR=5:3$.

Let $\overrightarrow{PA}=\mathbf{a}$ and $\overrightarrow{OB}=5\mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AQ}$ and $\overrightarrow{PQ}$.

b) Deduce, showing your reasoning, that P, Q and R are collinear and state the ratio of $PQ:QR$.

$\boxed{\overrightarrow{AB}=5\mathbf{b}-5\mathbf{a}}$, $\boxed{\overrightarrow{AQ}=2\mathbf{b}-2\mathbf{a}}$, $\boxed{\overrightarrow{PQ}=2\mathbf{b}-\mathbf{a}}$, $\boxed{PQ:QR=1:3}$



Question 34 (***)

The straight lines l_1 and l_2 have respective vector equations

$$\mathbf{r}_1=\begin{pmatrix}6\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\4\\a\end{pmatrix}\quad\text{and}\quad\mathbf{r}_2=\begin{pmatrix}5\\8\\b\end{pmatrix}+\mu\begin{pmatrix}7\\1\\0\end{pmatrix},$$

where λ and μ are scalar parameters while a and b are positive constants.

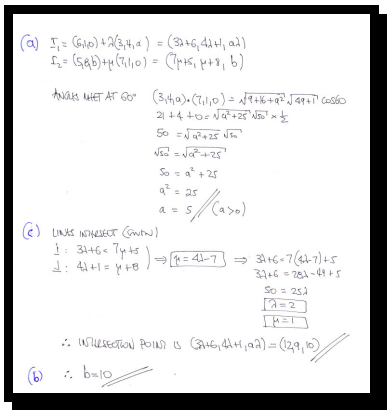
Given that l_1 and l_2 intersect at some point P, forming an angle of $60°$, determine in any order …

a) … the value of a.

b) … the value of b.

c) … the coordinates of P.

$\boxed{a=5}$, $\boxed{b=10}$, $\boxed{P(12,9,10)}$



Question 35 (***)

The points with coordinates $A(8,0,12)$ and $B(9,-2,14)$ are given.

a) Find the vector equation of the straight line l_1 that passes through A and B.

The straight line l_2 has equation

$$\mathbf{r}=\mathbf{i}+9\mathbf{j}+2\mathbf{k}+\mu(2\mathbf{i}+\mathbf{j}),$$

where μ is a scalar parameter.

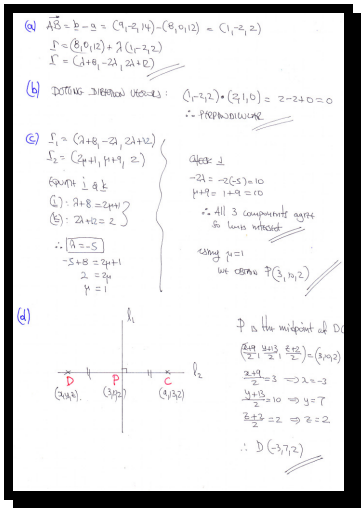
b) Show that l_1 and l_2 are perpendicular.

c) Show further that l_1 and l_2 intersect at some point P and state the coordinates of P.

The point $C(9,13,2)$ lies on l_2 and the point D is the reflection of C about l_1.

d) Determine the coordinates of D.

$\square$, $\boxed{\mathbf{r}_1=8\mathbf{i}+12\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})}$, $\boxed{P(3,10,2)}$, $\boxed{D(-3,7,2)}$



Question 36 (***)

Relative to a fixed origin O, the point A has position vector $7\mathbf{i}+4\mathbf{j}$ and the point B has position vector $-3\mathbf{j}+7\mathbf{k}$. The straight line L_1 passes through the points A and B.

a) Find a vector equation for L_1.

The straight line L_2 has a vector equation

$$\mathbf{r}=3\mathbf{i}-2\mathbf{j}-4\mathbf{k}+\mu\left(\mathbf{i}+2\mathbf{j}+3\mathbf{k}\right),$$

where μ is a scalar parameter.

b) Show that L_1 and L_2 intersect at some point C, and find its position vector.

c) Show further that L_1 and L_2 are perpendicular.

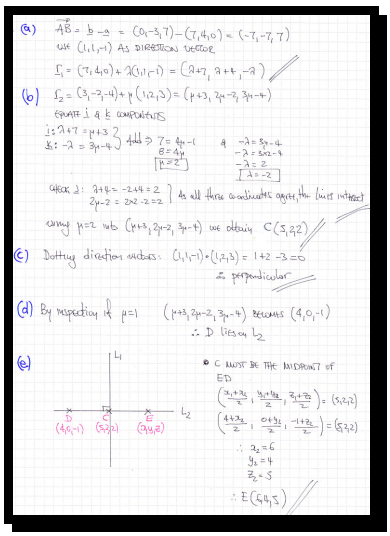
The point D has position vector $4\mathbf{i}-\mathbf{k}$.

d) Verify that D lies on L_2.

The point E is the image of D after reflection about L_1.

e) Find the position vector of E.

$\boxed{}$, $\boxed{\mathbf{r}=7\mathbf{i}+4\mathbf{j}+\lambda\left(\mathbf{i}+\mathbf{j}-\mathbf{k}\right)}$, $\boxed{\overrightarrow{OC}=5\mathbf{i}+2\mathbf{j}+2\mathbf{k}}$, $\boxed{\overrightarrow{OE}=6\mathbf{i}+4\mathbf{j}+5\mathbf{k}}$



Question 37 (*+)**

Relative to a fixed origin O, the points A and C have respective coordinates

$$(7,2,3) \quad \text{and} \quad (3,-2,1).$$

a) Find the vector $\overrightarrow{AC}$.

b) State the coordinates of the midpoint of AC.

The straight line l has vector equation

$$\mathbf{r}=\begin{pmatrix}5\\0\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-4\end{pmatrix},$$

where λ is a scalar parameter.

c) Show that $\overrightarrow{AC}$ is perpendicular to l.

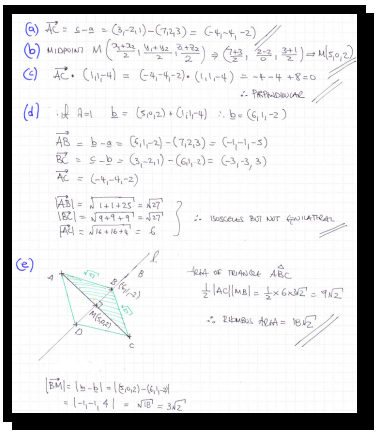
The point B lies on l, where $\lambda=1$.

d) Show further that the triangle ABC is isosceles but not equilateral.

The point D is such, so that $ABCD$ is a rhombus.

e) Show that the area of this rhombus is $18\sqrt{2}$ square units.

$\square$, $\boxed{\overrightarrow{AC}=-4\mathbf{i}-4\mathbf{j}-2\mathbf{k}}$, $\boxed{(5,0,2)}$



Question 38 (*+)**

Relative to a fixed origin O, the points A and B have respective position vectors

$$3\mathbf{i}-\mathbf{j}+2\mathbf{k} \quad \text{and} \quad -\mathbf{i}+\mathbf{j}+9\mathbf{k}\,.$$

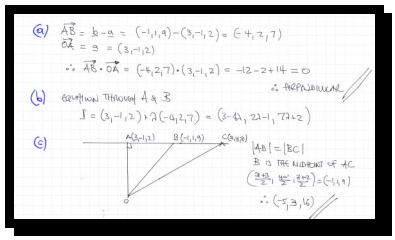
a) Show that $\overrightarrow{OA}$ and $\overrightarrow{AB}$ are perpendicular.

b) Find a vector equation of the straight line l, that passes through A and B.

The point C lies on l, so that the areas of the triangles OAB and OBC are equal.

c) Determine the position vector of C.

$\boxed{}$, $\boxed{\mathbf{r}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda\left(-4\mathbf{i}+2\mathbf{j}+7\mathbf{k}\right)}$, $\boxed{\overrightarrow{OC}=-5\mathbf{i}+3\mathbf{j}+16\mathbf{k}}$



Question 39 (*+)**

Relative to a fixed origin O, the points A, B and C have respective position vectors

$$\mathbf{i}+10\mathbf{k}\,, \quad 4\mathbf{i}+3\mathbf{j}+7\mathbf{k} \quad \text{and} \quad 8\mathbf{i}+7\mathbf{j}+3\mathbf{k}\,.$$

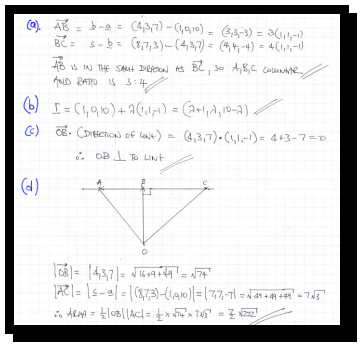
a) Show that A, B and C are collinear, and find the ratio $AB:BC$.

b) Find a vector equation for the straight line l that passes through A, B and C.

c) Show that OB is perpendicular to l.

d) Calculate the area of the triangle OAC.

$$\boxed{AB:BC=3:4}\,, \quad \boxed{\mathbf{r}=\mathbf{i}+10\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-\mathbf{k})}\,, \quad \boxed{\text{area}=\frac{7}{2}\sqrt{222}\approx 52.15}$$



Question 40 (*+)**

The points A and B have coordinates $(11,15,4)$ and $(13,23,7)$, respectively.

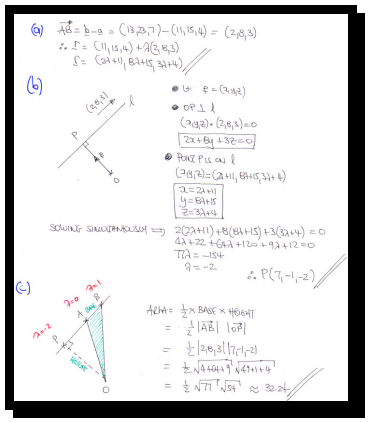
a) Find a vector equation for the straight line l that passes through A and B.

The point P lies on l, so that OP is perpendicular to l, where O is the origin.

b) Show, without verification, that the coordinates of P are $(7,-1,-2)$.

c) Calculate the area of the triangle OAB.

$\boxed{}$, $\boxed{\mathbf{r}=11\mathbf{i}+15\mathbf{j}+4\mathbf{k}+\lambda\left(2\mathbf{i}+8\mathbf{j}+3\mathbf{k}\right)}$, $\boxed{\text{area}=\frac{3}{2}\sqrt{462}\approx 32.24}$



Question 41 (*+)**

Relative to a fixed origin O, the position vectors of the points A, B, C and D are

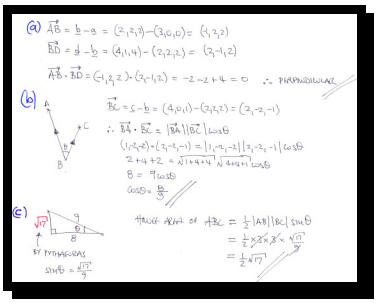
$$3\mathbf{i}\,, \quad 2\mathbf{i}+2\mathbf{j}+2\mathbf{k}\,, \quad 4\mathbf{i}+\mathbf{k} \quad \text{and} \quad 4\mathbf{i}+\mathbf{j}+4\mathbf{k}\,, \text{ respectively.}$$

a) Show that $\overrightarrow{AB}$ and $\overrightarrow{BD}$ are perpendicular.

b) Find the exact value of the cosine of the angle ABC.

c) Determine the exact value of the area of triangle ABC.

$$\boxed{\cos\left(\measuredangle ABC\right)=\frac{8}{9}}\,, \quad \boxed{\text{area}=\frac{1}{2}\sqrt{17}}$$



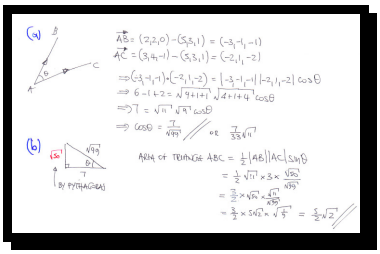
Question 42 (*+)**

Relative to a fixed origin O, the points A, B and C have respective coordinates $(5,3,1)$, $(2,2,0)$ and $(3,4,-1)$.

a) Find the exact value of the cosine of the angle BAC.

b) Show that the exact area of the triangle ABC is $\frac{5}{2}\sqrt{2}$.

$$\boxed{\cos\left(\measuredangle BAC\right)=\frac{7}{33}\sqrt{11}}$$



Question 43 (*+)**

The straight line l passes through the points $P(-5,9,-9)$ and $Q(a,b,11)$, where a and b are scalar constants.

The vector equation of l is given by

$$\mathbf{r}=\begin{pmatrix}1\\7\\c\end{pmatrix}+\lambda\begin{pmatrix}d\\-1\\2\end{pmatrix},$$

where c and d are scalar constants and λ is a scalar parameter.

a) Determine in any order the value of each the constants a, b, c and d.

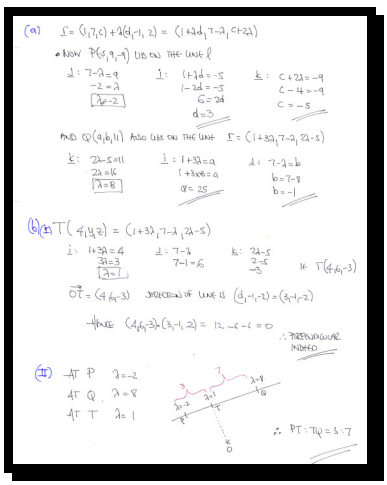
The point T with x coordinate 4 lies on l.

b) Show clearly that …

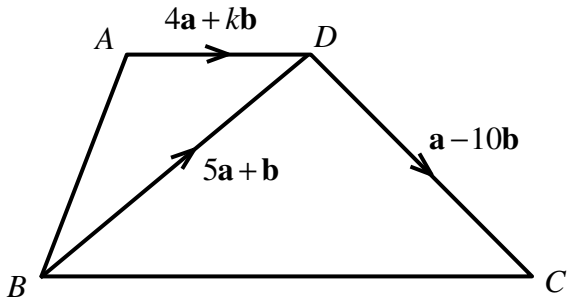
i. … OT is perpendicular to l, where O is the origin.

ii. … $PT:TQ=3:7$.

$\boxed{a=25}$, $\boxed{b=-1}$, $\boxed{c=-5}$, $\boxed{d=3}$



Question 44 (*+)**



The figure above shows a trapezium $ABCD$ where AD is parallel to BC.

The following information is given for this trapezium.

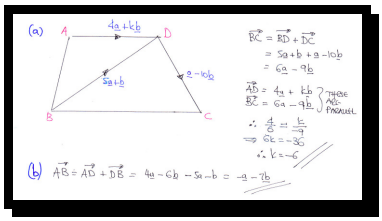
$$\overrightarrow{BD} = 5\mathbf{a} + \mathbf{b}, \qquad \overrightarrow{DC} = \mathbf{a} - 10\mathbf{b} \qquad \text{and} \qquad \overrightarrow{AD} = 4\mathbf{a} + k\mathbf{b},$$

where k is an integer.

a) Find the value of k.

b) Find a simplified expression for $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$\boxed{k = -6}, \quad \boxed{\overrightarrow{AB} = -\mathbf{a} - 7\mathbf{b}}$$



Question 45 (*+)**

The straight lines L_1 and L_2 have respective vector equations

$$\mathbf{r}_1=\begin{pmatrix}4\\-3\\3\end{pmatrix}+t\begin{pmatrix}5\\-1\\1\end{pmatrix} \quad \text{and} \quad \mathbf{r}_2=\begin{pmatrix}9\\8\\-2\end{pmatrix}+s\begin{pmatrix}-5\\-5\\2\end{pmatrix},$$

where t and s are scalar parameters.

a) Show that L_1 and L_2 intersect at some point P, and find its coordinates.

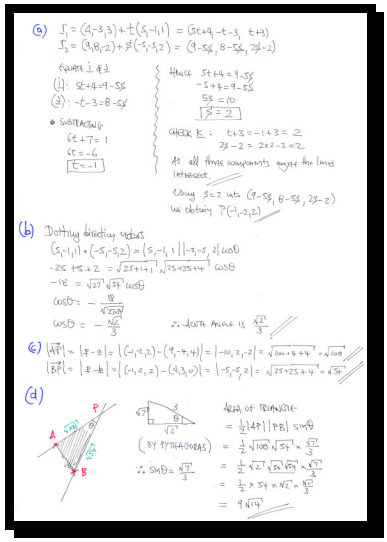
b) Find the exact value of the cosine of the acute angle θ, between L_1 and L_2.

The point $A(9,-4,4)$ lies on L_1 and the point $B(4,3,0)$ lies on L_2.

c) Find the distance of AP and the distance of BP.

d) Show the area of the triangle APB is $9\sqrt{14}$.

$$\boxed{P(-1,-2,2)}, \quad \boxed{\cos\theta=\frac{\sqrt{2}}{3}}, \quad \boxed{|AP|=6\sqrt{3}}, \quad \boxed{|BP|=3\sqrt{6}}$$



Question 46 (*+)**

With respect to a fixed origin O, the straight line l has vector equation

$$\mathbf{r}=\begin{pmatrix} a \\ b \\ 10 \end{pmatrix}+\lambda\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix},$$

where a and b are scalar constants and λ is a scalar parameter.

a) If l passes though the point $P(7,3,6)$, find the value of a and the value of b.

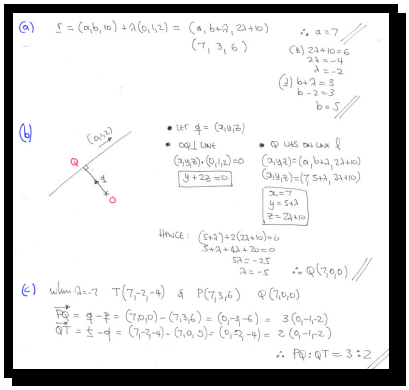
The point Q lies on l so that OQ is perpendicular to l.

b) Find the coordinates of Q.

The point T lies on l where $\lambda=-7$.

c) Find the ratio $PQ:QT$.

$\boxed{}$, $\boxed{a=7}$, $\boxed{b=5}$, $\boxed{Q(7,0,0)}$, $\boxed{PQ:QT=3:2}$



Question 47 (*+)**

Relative to a fixed origin O, the straight lines l and m have respective vector equations

$$\mathbf{r}_1=\begin{pmatrix}6\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\0\\a\end{pmatrix} \quad \text{and} \quad \mathbf{r}_2=\begin{pmatrix}4\\-3\\7\end{pmatrix}+s\begin{pmatrix}2\\2\\-5\end{pmatrix},$$

where t and s are scalar parameters, and a is a constant.

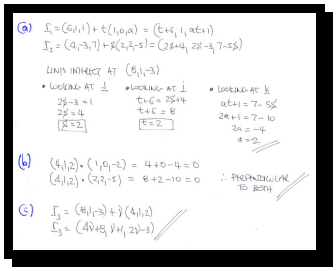
l and m intersect at the point $P(8,1,-3)$.

a) Find the value of a.

b) Show that the vector $\begin{pmatrix}4\\1\\2\end{pmatrix}$ is perpendicular to both l and m.

c) Determine a vector equation of the straight line n, such that all three straight lines intersect, with the n being perpendicular to both l and m.

$\boxed{a=-2}$, $\boxed{\mathbf{r}=8\mathbf{i}+\mathbf{j}-3\mathbf{k}+\nu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})}$



Question 48 (*+)**

With respect to a fixed origin O, the straight line l has vector equation

$$\mathbf{r}=\begin{pmatrix}24\\6\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\2\end{pmatrix},$$

where λ is a scalar parameter.

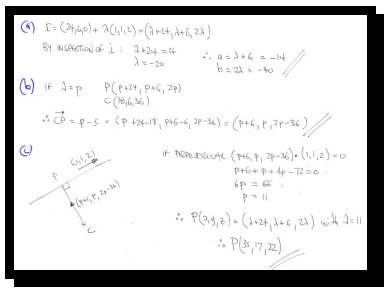
a) If the point $D(4,a,b)$ lies on l, find the value of a and the value of b.

The point P lies on l where $\lambda=p$, and the point C has coordinates $(18,6,36)$.

b) Show that $\overrightarrow{CP}=\begin{pmatrix}p+6\\p\\2p-36\end{pmatrix}$.

c) Given further that $\overrightarrow{CP}$ is perpendicular to l, find the coordinates of P.

$\boxed{a=-14}$, $\boxed{b=-40}$, $\boxed{P(35,17,22)}$



Question 49 (*+)**

The straight line l has the following vector equation

$$\mathbf{r}=-2\mathbf{i}-12\mathbf{j}-9\mathbf{k}+\lambda\left(\mathbf{i}+3\mathbf{j}+2\mathbf{k}\right),$$

where λ is a scalar parameter.

The point $P\left(a,b,3\right)$ lies on l.

a) Find the value of each of the scalar constants a and b.

The point O represents a fixed origin.

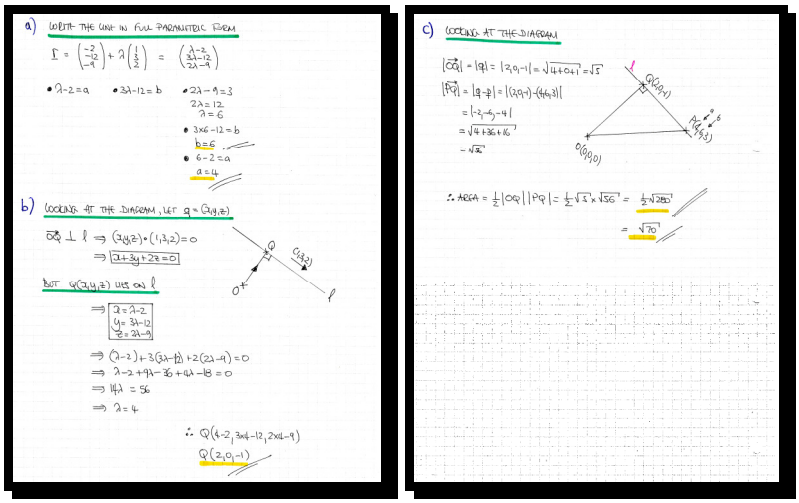
The point Q lies on l, so that OQ is perpendicular to l.

b) Show that the coordinates of Q are $\left(2,0,-1\right)$.

You may not verify this fact by using the coordinates of Q.

c) Find the exact area of the triangle OPQ.

$\boxed{}$, $\boxed{a=4}$, $\boxed{b=6}$, $\boxed{\text{area}=\sqrt{70}}$



Question 50 (*+)**

The straight line L has the vector equation

$$\mathbf{r}=\begin{pmatrix}3\\-4\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\-2\end{pmatrix},$$

where λ is a scalar parameter.

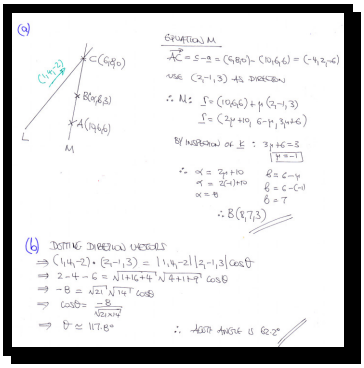
The straight line M passes through the points with coordinates $A(10,6,6)$ and $B(\alpha,\beta,3)$, where α and β are scalar constants.

L and M intersect at the point $C(6,8,0)$.

a) Find the coordinates of B.

b) Calculate the acute angle between L and M.

$\boxed{P(8,7,3)}$, $\boxed{62.2^\circ}$



Question 51 (*+)**

The straight line l_1 passes through the points $A(2,8,1)$ and $B(2,4,3)$.

a) Find a vector equation for l_1, in terms of a scalar parameter λ.

The straight line l_2 has a vector equation

$$\mathbf{r}=\begin{pmatrix}5\\2\\a\end{pmatrix}+\mu\begin{pmatrix}1\\b\\-1\end{pmatrix},$$

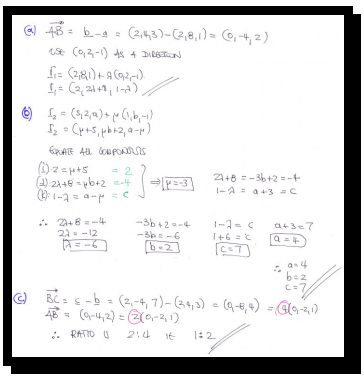
where a and b are scalar constants, and μ is a scalar parameter.

The point $C(2,-4,c)$, where c is a scalar constant, is the point of intersection between l_1 and l_2.

b) Find the value of each of the scalar constants a, b and c.

c) Determine the ratio $AB:BC$.

$$\boxed{\mathbf{r}_1=2\mathbf{i}+8\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{j}-\mathbf{k})},\quad \boxed{a=4},\quad \boxed{b=2},\quad \boxed{c=7},\quad \boxed{AB:BC=1:2}$$



Question 52 (*+)**

The straight lines L_1 and L_2 have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 12 \\ 7 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 0 \\ 1 \\ 21 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix},$$

where λ and μ are scalar parameters.

a) Show that L_1 and L_2 intersect at the point P, and find its coordinates .

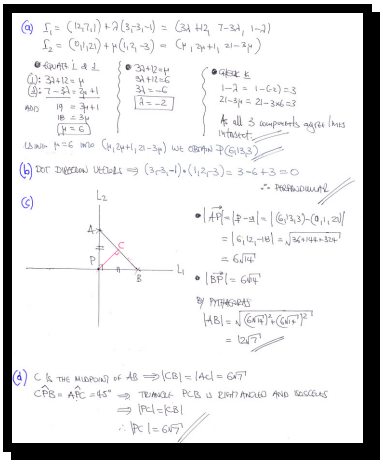
b) Show further that L_1 and L_2 are perpendicular to each other.

The point $A(0,1,21)$ lies on L_2 and the point B lies on L_1 so that $|\overrightarrow{AP}| = |\overrightarrow{PB}|$.

c) Find the distance AB.

d) Hence state the shortest distance of P from the line through A and B.

$\boxed{P(6,13,3)}$, $\boxed{|AB| = 12\sqrt{7}}$, $\boxed{6\sqrt{7}}$



Question 53 **(***+)**

The straight lines l_1 and l_2 have the following Cartesian equations

$$l_1: \quad x-a=\frac{y+4}{-4}=\frac{z}{-2}$$

$$l_2: \quad \frac{x-a}{2}=\frac{y+1}{-5}=\frac{z-1}{-3}$$

where a is a scalar constant.

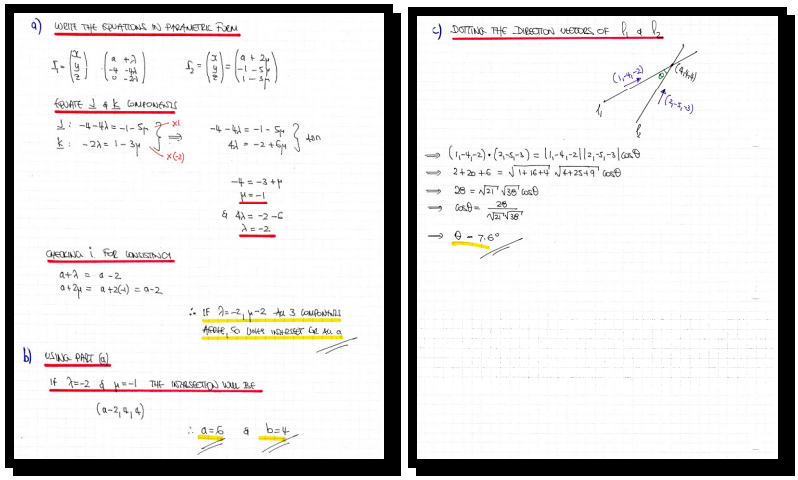
a) Show that l_1 and l_2 intersect at for all values of a.

The intersection point of l_1 and l_2 has coordinates (b,b,b), where b is a scalar constant.

b) Find the value of a and the value of b.

c) Calculate the acute angle formed by l_1 and l_2.

$\boxed{}$, $\boxed{a=6}$, $\boxed{b=4}$, $\boxed{\theta\approx 7.6^\circ}$



Question 54 (*+)**

OAB is a triangle with the point P being the midpoint of OB and the point Q being the midpoint of AB.

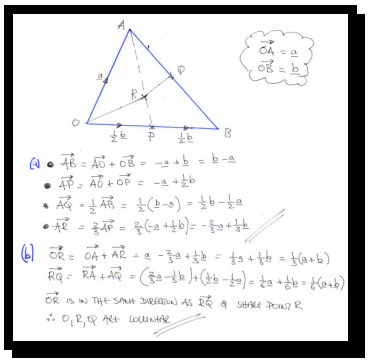
The point R is such so that $\overrightarrow{AR}=\frac{2}{3}\overrightarrow{AP}$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AP}$, $\overrightarrow{AQ}$ and $\overrightarrow{AR}$.

b) By finding simplified expressions, in terms $\mathbf{a}$ and $\mathbf{b}$, for two more suitable vectors, show that the points O, R and Q are collinear.

$$\boxed{\overrightarrow{AB}=\mathbf{b}-\mathbf{a}},\ \boxed{\overrightarrow{AP}=\frac{1}{2}\mathbf{b}-\mathbf{a}},\ \boxed{\overrightarrow{AQ}=\frac{1}{2}\mathbf{b}-\frac{1}{2}\mathbf{a}},\ \boxed{\overrightarrow{AR}=\frac{1}{3}\mathbf{b}-\frac{2}{3}\mathbf{a}}$$



Question 55 (*+)**

The points $A(1,1,2)$, $B(2,1,5)$, $C(4,0,1)$ and D form the parallelogram $ABCD$, where the above coordinates are measured relative to a fixed origin.

a) Find the coordinates of D.

The points E, B and D are collinear, so that B is the midpoint of ED.

b) Determine the coordinates of E.

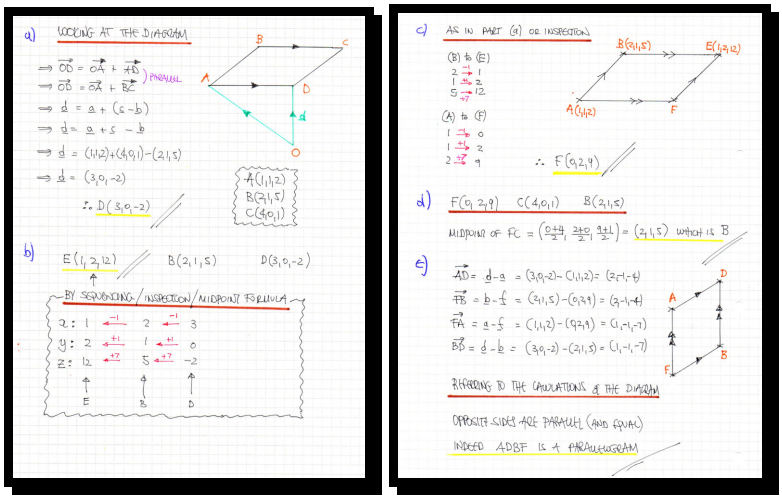
The point F is such so that $ABEF$ is also a parallelogram.

c) Find the coordinates of F.

d) Show that B is the midpoint of FC.

e) Prove that $ADBF$ is another parallelogram.

$\boxed{}$, $\boxed{D(3,0,-2)}$, $\boxed{E(1,2,12)}$, $\boxed{F(0,2,9)}$



Question 56 (*+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i}+\mathbf{j}+8\mathbf{k}+\lambda\left(\mathbf{j}-2\mathbf{k}\right)$$

$$\mathbf{r}_2 = \mathbf{i}+\mu\left(a\mathbf{i}+b\mathbf{j}+2\mathbf{k}\right),$$

where λ and μ are scalar parameters, and a and b are scalar constants.

l_1 and l_2 intersect at right angles at the point P.

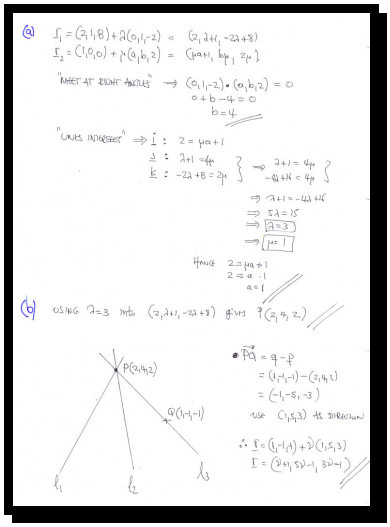
a) Find the value of a and the value of b.

b) Determine the coordinates of P.

The straight line l_3 passes through the point $Q\left(1,-1,-1\right)$.

c) Find a vector equation for l_3, given that all three lines intersect at the same point.

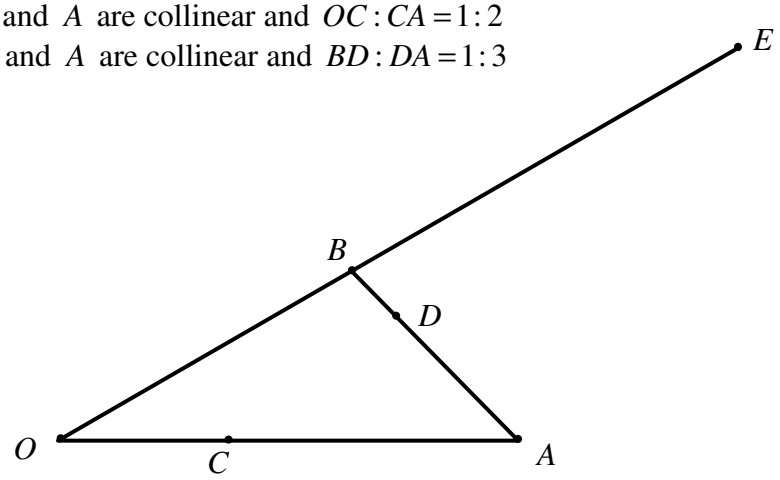
$$\boxed{a=1,\ b=4},\quad \boxed{P\left(2,4,2\right)},\quad \boxed{\mathbf{r}=\mathbf{i}-\mathbf{j}-\mathbf{k}+\nu\left(\mathbf{i}+5\mathbf{j}+3\mathbf{k}\right)}$$



Question 57 (*+)**

The figure below shows the points O, C, A, D, B and E, which are related as follows.

- O, B and E are collinear and $OB:BE=1:2$
- O, C and A are collinear and $OC:CA=1:2$
- B, D and A are collinear and $BD:DA=1:3$



Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

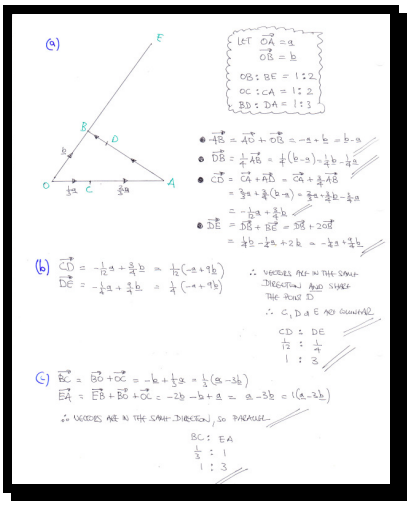
a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{DB}$, $\overrightarrow{CD}$ and $\overrightarrow{DE}$.

b) Show that the points C, D and E are collinear, and find the ratio $CD:DE$.

c) Show further that BC is parallel to EA, and find the ratio $BC:EA$.

$\boxed{\overrightarrow{AB}=\mathbf{b}-\mathbf{a}}$, $\boxed{\overrightarrow{DB}=\frac{1}{4}\mathbf{b}-\frac{1}{4}\mathbf{a}}$, $\boxed{\overrightarrow{CD}=-\frac{1}{12}\mathbf{a}+\frac{3}{4}\mathbf{b}}$, $\boxed{\overrightarrow{DE}=-\frac{1}{4}\mathbf{a}+\frac{9}{4}\mathbf{b}}$,

$\boxed{CD:DE=1:3}$, $\boxed{BC:EA=1:3}$



Question 58 (*+)**

With respect to a fixed origin O, the following points are given

$$A(2,2,5),\ B(12,7,0),\ C(0,0,1) \text{ and } D(9,k,4),$$

where k is a scalar constant.

a) Find the vector equation of the straight line l_1 that passes through A and B.

The straight line l_2 passes through C and D, and intersects l_1 at the point P.

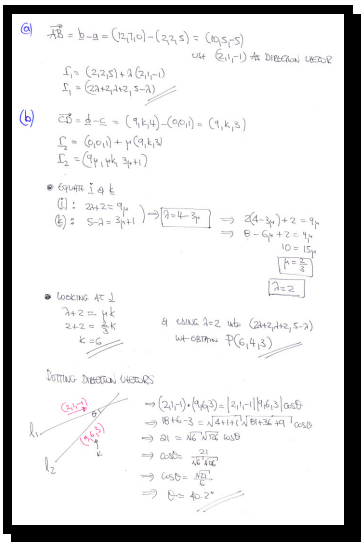
b) Determine in any order …

i. … the coordinates of P.

ii. … the value of k.

iii. … the acute angle between l_1 and l_2.

$$\boxed{},\ \boxed{\mathbf{r}=2\mathbf{i}+2\mathbf{j}+5\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}-\mathbf{k})},\ \boxed{P(6,4,3)},\ \boxed{k=6},\ \boxed{40.2^\circ}$$



Question 59 (****)

Two submarines S_1 and S_2, are travelling through the ocean.

They both appeared on the radar screen of a tracking station at the same time. The distances are measured in hundreds of metres and the time t, in seconds, is measured from the instant they were both observed on the radar screen of the tracking station.

The coordinates of S_1 and S_2, relative to a fixed origin O, are given by

$$S_1: \ \mathbf{r}_1 = (2t-4)\mathbf{i}+(t-15)\mathbf{j}+(t+5)\mathbf{k}$$

$$S_2: \ \mathbf{r}_2 = 10\mathbf{i}+(-2t+6)\mathbf{j}+(2t-2)\mathbf{k}$$

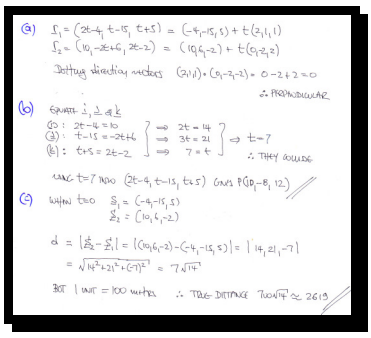
a) Show that S_1 and S_2 are travelling in perpendicular directions to each other.

Suppose that S_1 and S_2 continue to travel according to the above vector equations.

b) Show further that S_1 and S_2, will eventually collide at some point P, and further determine the coordinates of P.

c) Calculate, to the nearest metre, the distance between S_1 and S_2, when they were first observed by the tracking station.

$\boxed{P(10,-8,12)}$, $\boxed{\text{distance} = 700\sqrt{14} \approx 2619\,\text{m}}$



Question 60 (*+)**

The straight line l passes through the points with coordinates

$$A(-1,-4,8) \quad \text{and} \quad B(1,-2,5).$$

a) Find a vector equation of l.

The origin is denoted by O.

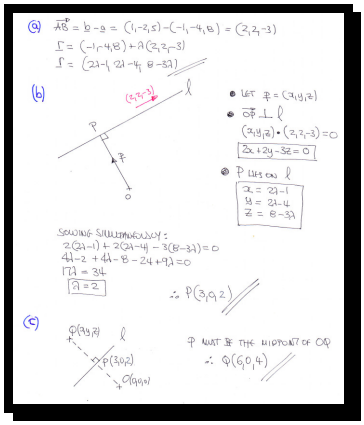
The point P lies on l, so that OP is perpendicular to l.

b) Determine the coordinates of P.

The point Q is the reflection of O, about l.

c) State the coordinates of Q.

$$\boxed{\mathbf{r}=-\mathbf{i}-4\mathbf{j}+8\mathbf{k}+\lambda(2\mathbf{i}+2\mathbf{j}-3\mathbf{k})}, \quad \boxed{P(3,0,2)}, \quad \boxed{Q(6,0,4)}$$



Question 61 (*+)**

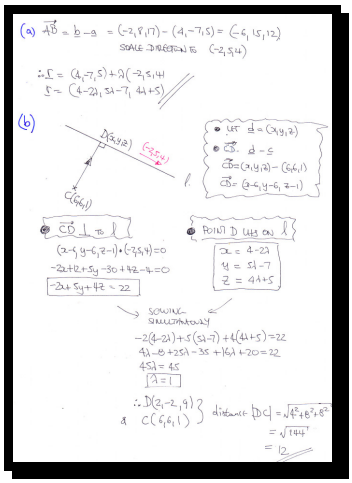
The points A and B have coordinates $(4,-7,5)$ and $(-2,8,17)$, respectively.

a) Find the equation of the straight line l, which passes through A and B.

The point C has coordinates $(6,6,1)$.

b) Find the shortest distance from C to l.

$\boxed{\mathbf{r}=4\mathbf{i}-7\mathbf{j}+5\mathbf{k}+\lambda(-2\mathbf{i}+5\mathbf{j}+4\mathbf{k})}$, $\boxed{\text{shortest distance}=12}$



Question 62 (*+)**

Relative to a fixed origin O, the points A, B and C have respective coordinates $(-2,5,13)$, $(1,1,1)$ and $(3,5,5)$.

a) Determine the size of the angle ABC.

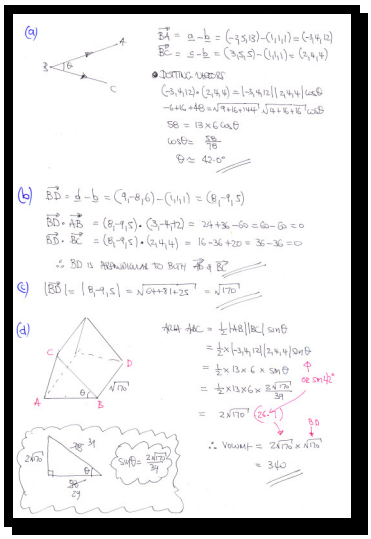
The point D has coordinates $(9,-8,6)$.

b) Show that BD is perpendicular to both AB and BC.

c) Find the distance BD.

d) Calculate the volume of the right triangular prism with base the triangle ABC and height BD.

$\boxed{42.0^\circ}$, $\boxed{|BD|=\sqrt{170}}$, $\boxed{340 \text{ cubic units}}$



Question 63 (*+)**

The points with coordinates $A(3,0,3)$ and $B(4,-1,5)$ are given.

a) Find a vector equation of the straight line l_1 that passes through A and B.

The straight line l_2 has equation

$$\mathbf{r}=5\mathbf{i}+10\mathbf{j}+4\mathbf{k}+\mu(\mathbf{i}+3\mathbf{j}+\mathbf{k}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 are perpendicular.

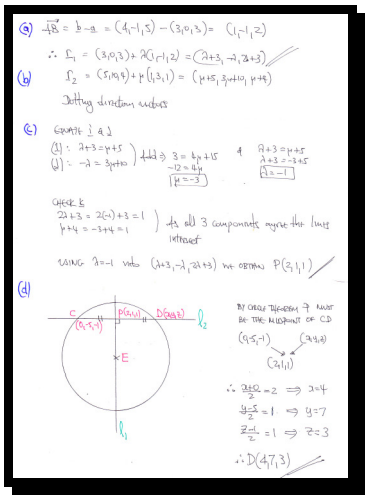
c) Show further that l_1 and l_2 intersect at some point P and find its coordinates.

The point E is on the l_1.

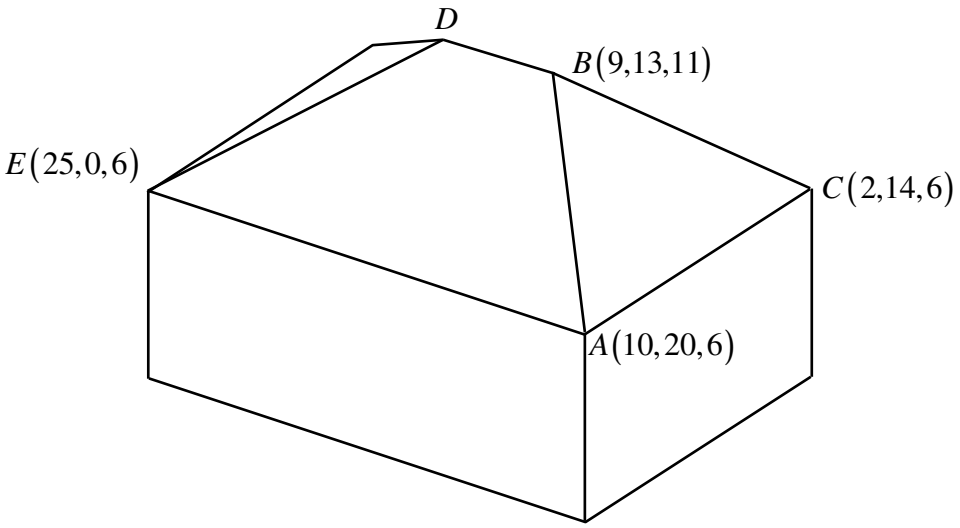
A circle with centre at E is drawn so that it cuts l_2 at the points C and D.

d) Given that the coordinates of C are $(0,-5,-1)$, find the coordinates of D.

$\boxed{}$, $\boxed{\mathbf{r}_1=3\mathbf{i}+3\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+2\mathbf{k})}$, $\boxed{P(2,1,1)}$, $\boxed{D(4,7,3)}$



Question 64 (****)



The figure above shows a solid, modelling a house with a standard slanted roof, where all the distances are measured in metres. With respect to a fixed origin, the coordinates of some of the vertices of the solid are marked in the diagram.

a) Find a vector equation of AE.

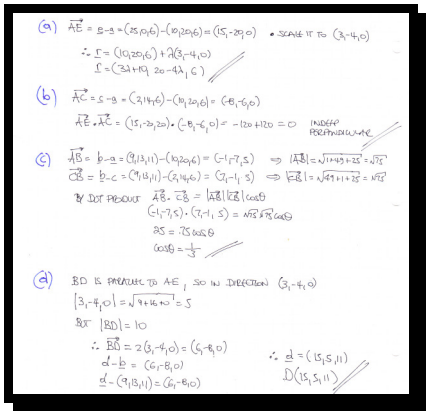
b) Show that AE is perpendicular to AC.

c) Find the cosine of the angle ABC.

The straight line BD is parallel to AE. The length of BD is 10 metres.

d) Determine the coordinates of D.

$\boxed{}$, $\boxed{\mathbf{r}=10\mathbf{i}+20\mathbf{j}+6\mathbf{k}+\lambda(3\mathbf{i}-4\mathbf{j})}$, $\boxed{\cos(\angle ABC)=\frac{1}{3}}$, $\boxed{D(15,5,11)}$



Question 65 (****)

Relative to a fixed origin O the following position vectors are given.

$$\overrightarrow{OA}=\begin{pmatrix}0\\8\\3\end{pmatrix} \quad \text{and} \quad \overrightarrow{OB}=\begin{pmatrix}1\\13\\1\end{pmatrix}.$$

a) Find a vector equation for the line straight l_1 which passes through A and B.

The straight line l_2 has vector equation

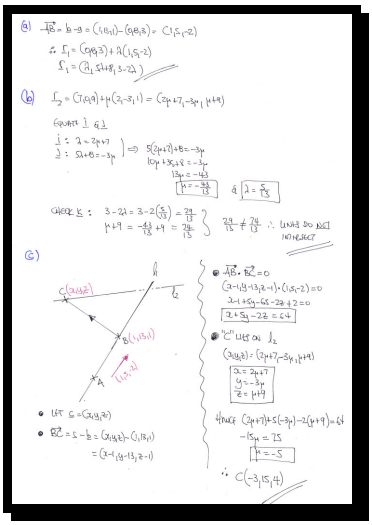
$$\mathbf{r}=\begin{pmatrix}7\\0\\9\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\1\end{pmatrix},$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 do not intersect.

c) Find the position vector of C, given it lies on l_2 and $\angle ABC=90°$.

$\square$, $\boxed{\mathbf{r}=8\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}+5\mathbf{j}-2\mathbf{k})}$, $\boxed{C(-3,15,4)}$



Question 66 (****)

The straight line L_1 passes through the points A and B, whose respective position vectors relative to a fixed origin O are

$$\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}.$$

a) Find a vector equation for L_1.

The angle ABC is denoted by θ, where C is the point with position vector $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$.

b) Show clearly that $\cos\theta = \frac{1}{3}$.

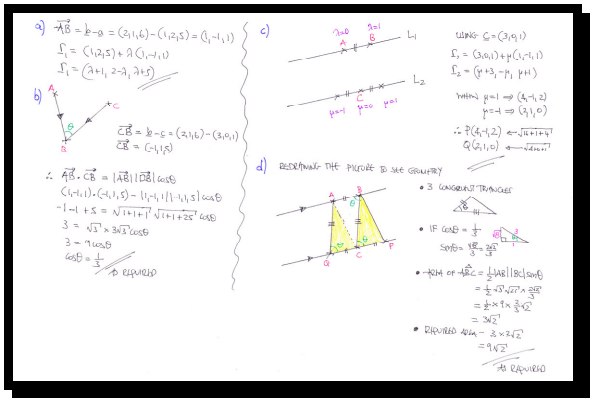
The straight line L_2 passes through C and is parallel to L_1.

The points P and Q both lie on L_2 so that $|AB| = |CP| = |CQ|$.

c) Determine the position vector of P and the position vector of Q, given that P is furthest away from O.

d) Show further that the area of the quadrilateral $ABPQ$ is $9\sqrt{2}$.

$$\boxed{\mathbf{r}_1 = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})}, \quad \boxed{\overrightarrow{OP} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k}}, \quad \boxed{\overrightarrow{OQ} = 2\mathbf{i} + \mathbf{j}}$$



Question 67 (****)

The points A and B have position vectors $9\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $9\mathbf{i}+4\mathbf{j}+\mathbf{k}$, respectively.

a) Find a vector equation of the straight line l_1 that passes through A and B.

The straight line l_2 has the vector equation

$$\mathbf{r}_2 = 6\mathbf{i}+3\mathbf{j}-4\mathbf{k}+\mu(\mathbf{i}+\mathbf{j}-\mathbf{k}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection.

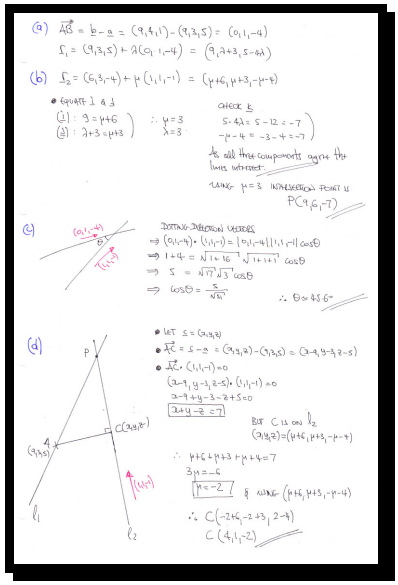
c) Find the acute angle between l_1 and l_2.

The point C lies on l_2 in such a position so that is closest to A.

d) Show that the position vector of C is given by

$$\mathbf{c} = 4\mathbf{i}+\mathbf{j}-2\mathbf{k}.$$

$\boxed{\mathbf{r}_1 = 9\mathbf{i}+3\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{j}-4\mathbf{k})}$, $\boxed{9\mathbf{i}+6\mathbf{j}-7\mathbf{k}}$, $\boxed{45.6°}$



Question 68 (****)

With respect to a fixed origin O, the points with coordinates $A(2,3,5)$, $B(6,-1,5)$, $C(9,2,2)$ and $D(5,6,2)$ are given.

Prove that $ABCD$ is a rectangle and show that its area is $12\sqrt{6}$ square units.

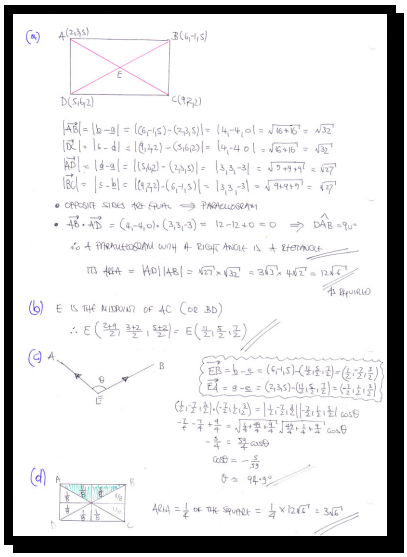
The diagonals of the rectangle intersect at the point E.

a) Find the coordinates of E.

b) Find the size of the angle BEA.

c) State the exact area of the triangle BEA.

$$\boxed{E\left(\frac{11}{2},\frac{5}{2},\frac{7}{2}\right)}, \quad \boxed{\measuredangle BEA = 94.9°}, \quad \boxed{\text{area}\left(\triangle BEA\right) = 3\sqrt{6}}$$

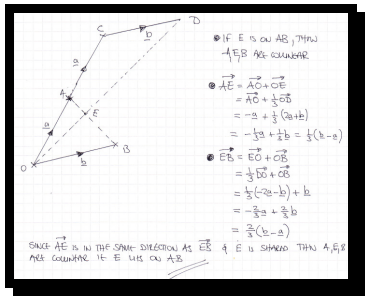


Question 69 (****)

Let $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$, $\overrightarrow{OC}=2\mathbf{a}$ and $\overrightarrow{OD}=2\mathbf{a}+\mathbf{b}$.

If $\overrightarrow{OE}=\dfrac{1}{3}\overrightarrow{OD}$ prove that the point E lies on the straight line AB.

proof



Question 70 (****)

The point A has position vector $-5\mathbf{j}+7\mathbf{k}$.

a) Find a vector equation of the straight line l that passes through A and is parallel to the vector $\mathbf{i}+3\mathbf{j}-\mathbf{k}$.

The point P lies on l so that OP is perpendicular to l, where O is the origin.

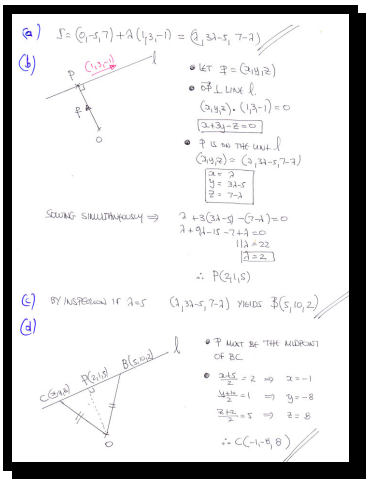
b) Determine the coordinates of P.

c) Show that the point $B(5,10,2)$ lies on l.

The point C is on l so that $|OB|=|OC|$.

d) Find the coordinates of C.

$$\boxed{\mathbf{r}=-5\mathbf{j}+7\mathbf{k}+\lambda(\mathbf{i}+3\mathbf{j}-\mathbf{k})}\,,\quad \boxed{P(2,1,5)}\,,\quad \boxed{C(-1,-8,8)}$$



Question 71 (****)

Relative to a fixed origin O, the straight lines l_1 and l_2 have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 2 \\ a \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} b \\ 2 \\ 14 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -4 \\ 6 \end{pmatrix},$$

where λ and μ are scalar parameters, and a and b are constants.

l_1 and l_2 intersect at the point P, whose z coordinate is 8.

a) Find the coordinates of the point P.

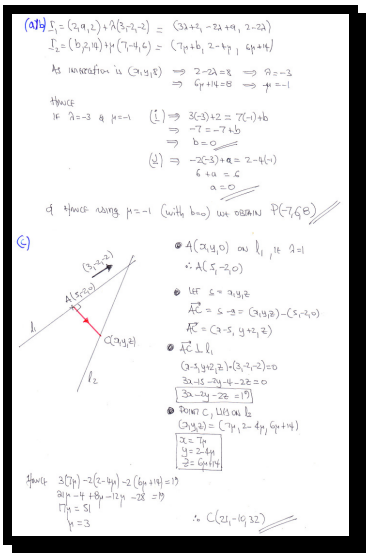
b) Show that the value of both a and b, is zero.

The point A, whose z coordinate is zero, lies on l_1.

The point C lies on l_2, so that AC is perpendicular to l_1.

c) Determine the coordinates of C.

$$\boxed{P(-7,6,8)}, \quad \boxed{C(21,-10,32)}$$



Question 72 (****)

The straight line l passes through the points A and C whose respective coordinates are $(-2,7,9)$ and $(8,-3,-1)$.

a) Find a vector equation for l.

The point $E(2,p,q)$ lies on l and the point B has coordinates $(-4,1,1)$.

b) Determine the value of p and the value of q.

c) Show that BE is perpendicular to l.

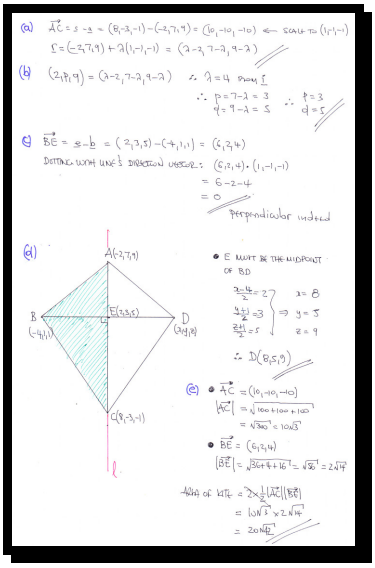
The point D is such, so that $ABCD$ is a kite with $\angle ABC = \angle ADC$.

Determine …

d) … the coordinates of D.

e) … the area of the kite $ABCD$.

$\boxed{}$, $\boxed{\mathbf{r}=-2\mathbf{i}+7\mathbf{j}+9\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}-\mathbf{k})}$, $\boxed{p=3 \text{ and } q=5}$, $\boxed{D(8,5,9)}$, $\boxed{20\sqrt{42}}$



Question 73 (****)

The straight line l passes through the points with coordinates $(4,-1,1)$ and $(-1,4,6)$.

a) Determine a vector equation of l.

The points C and D have coordinates $(4,-2,-3)$ and $(p,q,-1)$, respectively.

The midpoint of CD is the point M, where M lies on l.

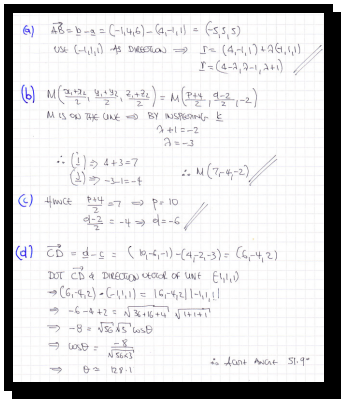
Find in any order …

b) … the coordinates of M.

c) … the value of p and the value of q.

d) … the size of the acute angle θ, between CD and l.

$\boxed{}$, $\boxed{\mathbf{r}=4\mathbf{i}-\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}+\mathbf{j}+\mathbf{k})}$, $\boxed{M(7,-4,-2)}$, $\boxed{p=10,\ q=-6}$, $\boxed{\theta\approx 51.9^\circ}$



Question 74 (****)

Relative to a fixed origin O, the straight lines L and M have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 4 \\ 10 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 0 \\ 14 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix},$$

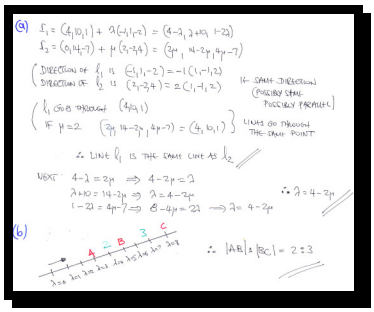
where λ and μ are scalar parameters.

a) Show that L and M represent the same straight line and find a linear relationship between λ and μ, giving the answer in the form $\lambda = f(\mu)$.

The points A, B and C lie on L, where $\lambda = 3$, $\lambda = 5$ and $\lambda = 8$ respectively.

b) State the ratio $AB : BC$.

$\boxed{}$, $\boxed{\lambda = 4 - 2\mu}$, $\boxed{AB : BC = 2 : 3}$



Question 75 (****)

Relative to a fixed origin O, the points A and B have respective position vectors

$$\mathbf{a}=2\mathbf{i}+4\mathbf{j}+6\mathbf{k} \quad \text{and} \quad \mathbf{b}=4\mathbf{i}+4\mathbf{j}-4\mathbf{k}\,.$$

a) Find the position vector of the point C, given that $\mathbf{c}=\mathbf{a}+\mathbf{b}$.

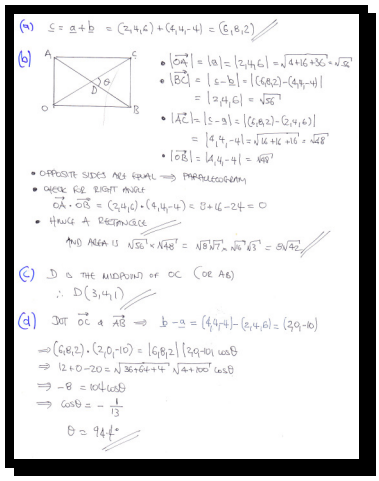
b) Show that $OACB$ is a rectangle, and calculate its area.

The diagonals of the rectangle $OACB$, OC and AB, meet at the point D.

c) State the position vector of D.

d) Calculate the size of the angle BDC.

$$\boxed{\mathbf{c}=6\mathbf{i}+8\mathbf{j}+2\mathbf{k}}\,, \quad \boxed{\text{area}=8\sqrt{42}}\,, \quad \boxed{\mathbf{d}=3\mathbf{i}+4\mathbf{j}+\mathbf{k}}\,, \quad \boxed{94.4^\circ}$$



Question 76 (****)

With respect to a fixed origin O, the points A and B have coordinates $(5,-1,-1)$ and $(1,-5,7)$, respectively.

a) Find a vector equation of the straight line l which passes through A and B.

The point C has coordinates $(4,-2,1)$.

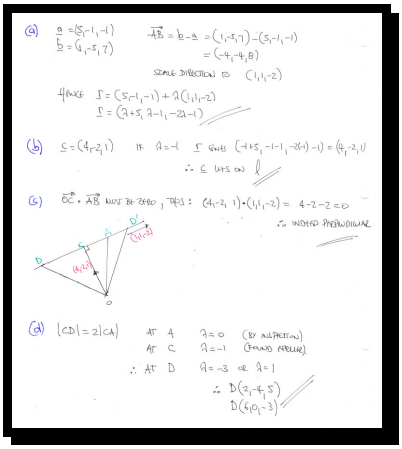
b) Show that C lies on l.

c) Show further that $\overrightarrow{OC}$ is perpendicular to l.

The point D lie on l so that $\left|\overrightarrow{CD}\right| = 2\left|\overrightarrow{CA}\right|$.

d) Find the **two** possible sets for the coordinates of D.

$\boxed{}$, $\boxed{\mathbf{r} = 5\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k})}$, $\boxed{D(2,-4,5) \text{ or } D(6,0,-3)}$



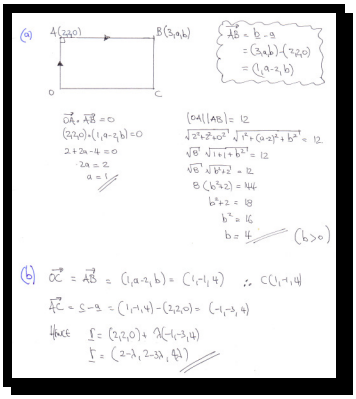
Question 77 (****)

$OABC$ is a rectangle, with $A(2,2,0)$, $B(3,a,b)$, where a and b are positive constants and O is a fixed origin.

a) Given that the area of $OABC$ is 12 square units determine the value of a and the value of b.

b) Find a vector equation of the straight line l that passes through A and C.

$$\boxed{a=1}\,,\quad \boxed{b=4}\,,\quad \boxed{\mathbf{r}=2\mathbf{i}+2\mathbf{j}+\lambda\left(-\mathbf{i}-3\mathbf{j}+4\mathbf{k}\right)}$$



Question 78 (****)

With respect to a fixed origin O, the points A and B have coordinates $(1,5,4)$ and $(3,4,5)$, respectively.

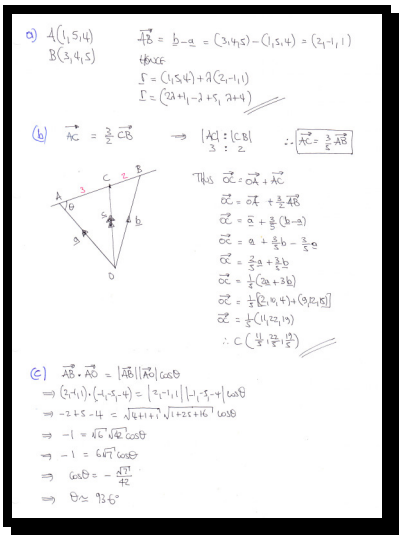
a) Find a vector equation of the straight line l that passes through A and B.

The point C lie on l so that $\overrightarrow{AC}=\frac{1}{2}\overrightarrow{CB}$.

b) Determine the coordinates of C.

c) Calculate the size of the angle OAC.

$$\mathbf{r}=\mathbf{i}+5\mathbf{j}+4\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k}), \quad C\left(\frac{11}{5},\frac{22}{5},\frac{19}{5}\right), \quad 93.6°$$



Question 79 (****)

Relative to a fixed origin O, the points A and B have position vectors $8\mathbf{i}+5\mathbf{j}+7\mathbf{k}$ and $8\mathbf{i}+6\mathbf{j}+3\mathbf{k}$, respectively.

a) Find a vector equation of the straight line l_1 which passes through A and B.

The straight line l_2 has the vector equation

$$\mathbf{r}_2 = 5\mathbf{i}+5\mathbf{j}-2\mathbf{k}+\mu(\mathbf{i}+\mathbf{j}-\mathbf{k}),$$

where μ is a scalar parameter.

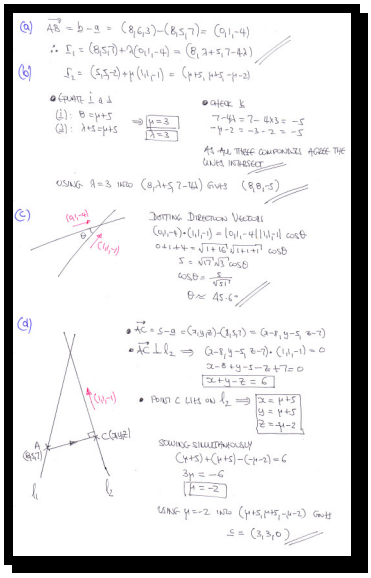
b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection.

c) Calculate the acute angle between l_1 and l_2.

The point C lies on l_2 so that C is as close as possible to A.

d) Find the position vector of C.

$\boxed{\mathbf{r}_1 = 8\mathbf{i}+5\mathbf{j}+7\mathbf{k}+\lambda(\mathbf{j}-4\mathbf{k})}$, $\boxed{8\mathbf{i}+8\mathbf{j}-5\mathbf{k}}$, $\boxed{45.6°}$, $\boxed{\overrightarrow{OC}=3\mathbf{i}+3\mathbf{j}}$



Question 80 (****)

Relative to a fixed origin O, the position vectors of three points A, B and C are

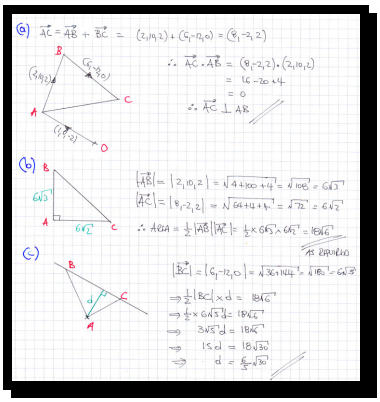
$$\overrightarrow{OA}=\mathbf{i}-2\mathbf{k}, \quad \overrightarrow{AB}=2\mathbf{i}+10\mathbf{j}+2\mathbf{k} \quad \text{and} \quad \overrightarrow{BC}=6\mathbf{i}-12\mathbf{j}.$$

a) Show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{AB}$.

b) Show further that the area of the triangle ABC is $18\sqrt{6}$.

c) Hence, or otherwise, determine the shortest distance of A from the straight line through B and C.

$\square$, $\boxed{\text{distance}=\frac{6}{5}\sqrt{30}}$



Question 81 (****)

The straight line l_1 passes through the points $A(6,2,0)$ and $B(5,0,5)$.

a) Find a vector equation of l_1.

The straight line l_2 has vector equation

$$\mathbf{r}_2 = \begin{pmatrix} -7 \\ 6 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 0 \\ 2 \end{pmatrix},$$

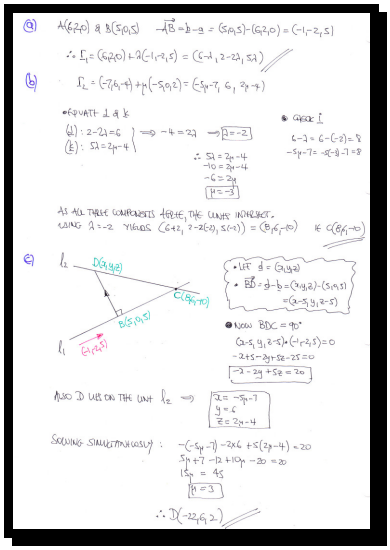
where μ is a scalar parameter.

b) Show that l_1 and l_2 intersect at some point C, and find its coordinates.

The point D lies on l_2 so that $DBC = 90°$.

c) Determine the coordinates of D.

$\boxed{}$, $\boxed{\mathbf{r} = 6\mathbf{i} + 2\mathbf{j} + \lambda(-5\mathbf{i} + 2\mathbf{k})}$, $\boxed{C(8,6,-10)}$, $\boxed{D(-22,6,2)}$



Question 82 (****)

The straight lines L_1 and L_2 have vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 2 \\ 10 \\ 14 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} a \\ 8 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ b \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters, and a and b are scalar constants.

L_1 and L_2 intersect at the point P whose z coordinate is 6, and the acute angle between L_1 and L_2, is θ.

a) Determine the coordinates of P.

b) Find the value of a and the value of b.

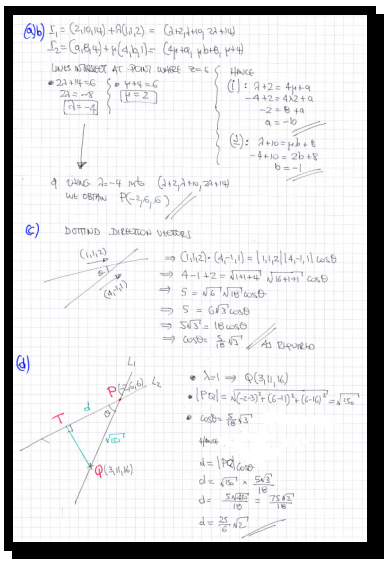
c) Show that $\cos\theta = \frac{5}{18}\sqrt{3}$.

The point Q lies on L_1 where $\lambda = 1$.

The point T lies on L_2 so that $\overrightarrow{QT}$ is perpendicular to L_2.

d) Determine the exact distance PT.

$\square$, $\boxed{P(-2,6,6)}$, $\boxed{a=-10, \quad b=-1}$, $\boxed{|PT| = \frac{25}{6}\sqrt{2}}$



Question 83 (****)

The points A and B have coordinates $(3,-1,2)$ and $(2,0,2)$, respectively.

a) Find a vector equation of the straight line l_1 that passes through A and B.

The straight line l_2 has equation

$$\mathbf{r}=2\mathbf{i}+\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}-\mathbf{k}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 intersect at some point P and find its coordinates.

c) Verify that the point $C(9,1,-6)$ lies on l_2.

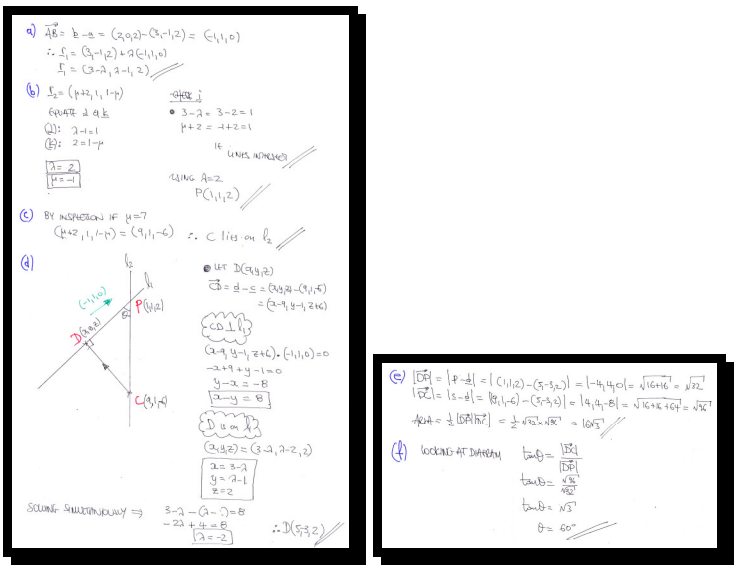
The point D lies on l_1 so that CD is perpendicular to l_1.

d) Determine the coordinates of D.

e) Calculate the area of the triangle PDC.

f) Deduce the acute angle between l_1 and l_2.

$$\boxed{\mathbf{r}_1=2\mathbf{i}+2\mathbf{k}+\lambda(-\mathbf{i}+\mathbf{j})},\ \boxed{P(1,1,2)},\ \boxed{D(5,-3,2)},\ \boxed{\text{area}=16\sqrt{3}},\ \boxed{60^\circ}$$



Question 84 (****)

Relative to a fixed origin O, the points A and B have respective position vectors

$$2\mathbf{i}+10\mathbf{j}+2\mathbf{k} \quad \text{and} \quad 2\mathbf{i}+\mathbf{j}+2\mathbf{k}.$$

The angle AOB is θ.

a) Show that $\sin\theta=\dfrac{\sqrt{6}}{3}$.

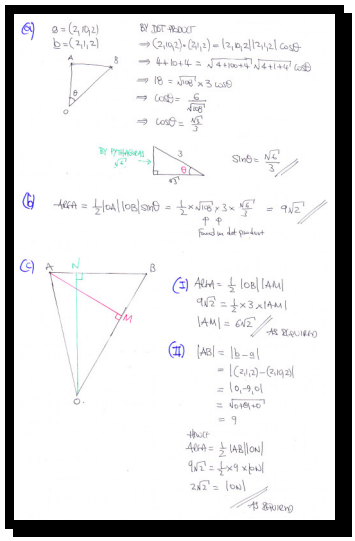
b) Calculate the exact area of the triangle AOB.

c) Show further that the shortest distance of …

i. … A from the straight line OB is $6\sqrt{2}$,

ii. … the straight line AB from O is $2\sqrt{2}$.

$\square$, $\boxed{\text{area}=9\sqrt{2}}$



Question 85 (****)

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 2\mathbf{i}+\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-\mathbf{k})$$

$$\mathbf{r}_2 = 2\mathbf{i}+\mathbf{j}+5\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-\mathbf{k})$$

where λ and μ are scalar parameters.

The point A is the intersection of l_1 and l_2.

The point $B(b,1,-1)$, where b is a scalar constant, lies on l_1.

The point $D(4,d,3)$, where d is a scalar constant, lies on l_2.

a) Find the value of b and the value of d.

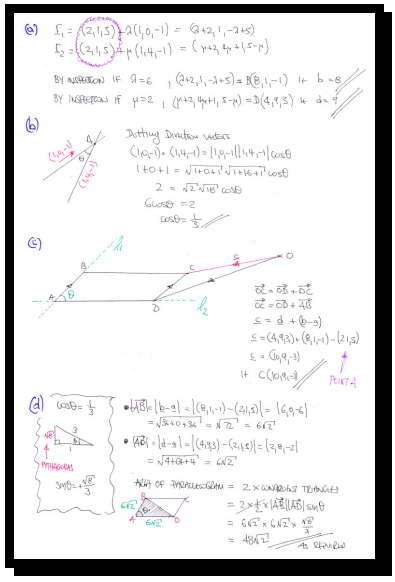
b) Calculate the cosine of θ, where θ is the acute angle formed by l_1 and l_2.

The point C is such so that $ABCD$ is a parallelogram.

c) Determine the coordinates of C.

d) Show that the area of the parallelogram $ABCD$ is $48\sqrt{2}$ square units.

$\boxed{}$, $\boxed{b=8,\ \ d=9}$, $\boxed{\cos\theta=\frac{1}{3}}$, $\boxed{C(10,9,-3)}$



Question 86 (****)

The straight line L_1 passes through the point $A(5,-2,1)$ and is parallel to the vector

$$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}.$$

a) Find a vector equation for L_1, in terms of a scalar parameter λ.

The straight line L_2 has a vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},$$

where μ is scalar parameter.

b) Show that the lines intersect at some point P, and find its coordinates .

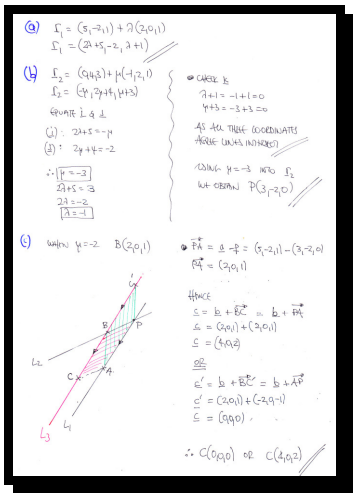
The point B lies on L_2 where $\mu = -2$.

The point C lies on a straight line which is parallel to L_1 and passes through B.

The points A, B, C and P are vertices of a parallelogram.

c) Show that one of the possible positions for C is the origin O and find the coordinates of the other possible position for C.

$\boxed{}$, $\boxed{\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{k})}$, $\boxed{P(3,-2,0)}$, $\boxed{C(4,0,2)}$



Question 87 (****)

The straight line L has vector equation

$$\mathbf{r}=\mathbf{i}-2\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-\mathbf{k}),$$

where λ is a scalar parameter.

The point A has position vector $\mathbf{i}+\mathbf{j}-\mathbf{k}$.

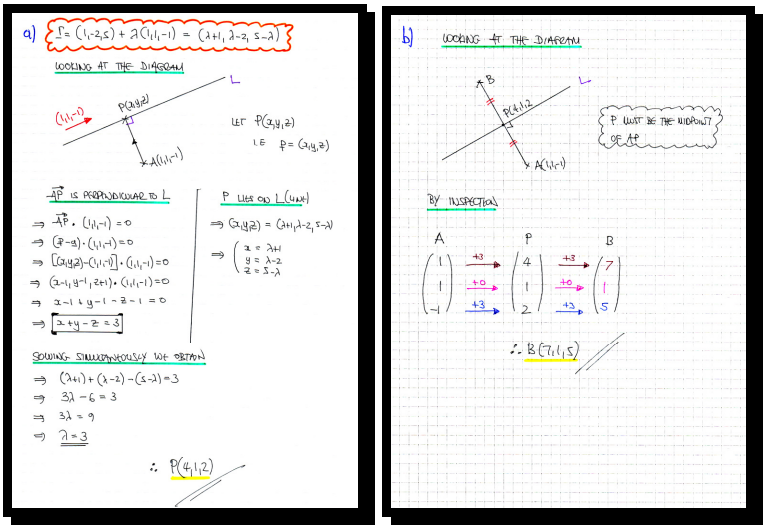
The point P lies on L so that AP is perpendicular to L.

a) Find the position vector of P.

The point B is the reflection of A about L.

b) Determine the position vector of B.

$\boxed{}$, $\boxed{\overrightarrow{OP}=4\mathbf{i}+\mathbf{j}+2\mathbf{k}}$, $\boxed{\overrightarrow{OB}=7\mathbf{i}+\mathbf{j}+5\mathbf{k}}$



Question 88 (****)

All the position vectors and coordinates in this question are measured from a fixed origin O.

The point P lies on the straight line l with vector equation

$$\mathbf{r}=\mathbf{i}-3\mathbf{k}+\lambda\left(2\mathbf{i}+3\mathbf{j}+5\mathbf{k}\right),$$

where λ is a scalar parameter.

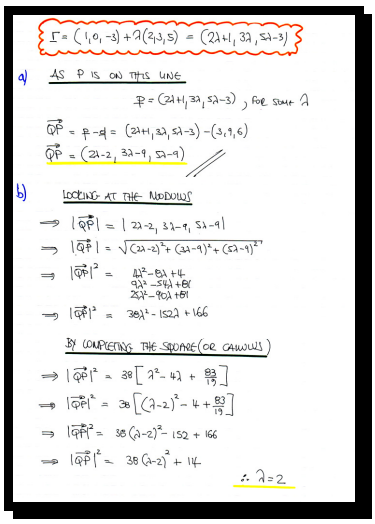
The point Q has position vector $3\mathbf{i}+9\mathbf{j}+6\mathbf{k}$.

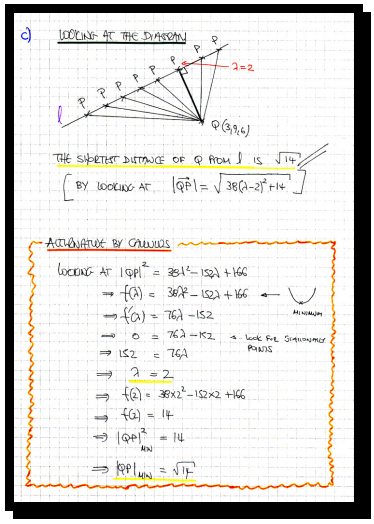
a) Determine, in terms of λ, an expression for the vector $\overrightarrow{QP}$.

b) By considering $\left|\overrightarrow{QP}\right|^2$, find the value of λ which makes $\left|\overrightarrow{QP}\right|$ minimum.

c) Hence, or otherwise, find the shortest of distance of Q from l.

$$\boxed{},\quad \boxed{\overrightarrow{QP}=(2\lambda-2)\mathbf{i}+(3\lambda-9)\mathbf{j}+(5\lambda-9)\mathbf{k}},\quad \boxed{\lambda=2},\quad \boxed{\sqrt{14}}$$





Question 89 (****)

The points $A(-1,4)$, $B(2,3)$ and $C(8,1)$ lie on the x-y plane, where O is the origin.

a) Show that A, B and C are collinear.

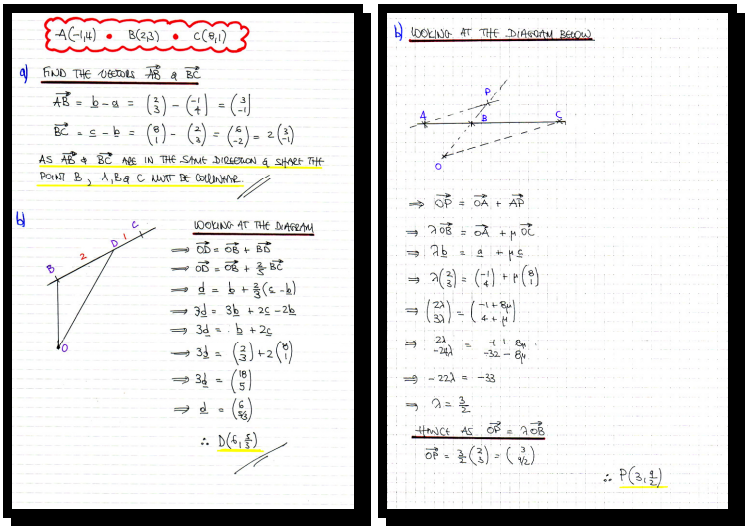
The point D lies on BC so that $\overrightarrow{BD}:\overrightarrow{BC}=2:3$.

b) Find the coordinates of D.

The straight line OB is extended to the point P, so that $\overrightarrow{AP}$ is parallel to $\overrightarrow{OC}$.

c) Determine the coordinates of P.

$\boxed{}$, $\boxed{D\left(6,\frac{5}{3}\right)}$, $\boxed{P\left(3,\frac{9}{2}\right)}$



Question 90 (****)

Relative to a fixed origin O, the points A, B and C have respective position vectors

$$-6\mathbf{i}-5\mathbf{j}-21\mathbf{k}, \quad 8\mathbf{i}+9\mathbf{j} \quad \text{and} \quad u\mathbf{i}-3\mathbf{j}+v\mathbf{k},$$

where u and v are scalar constants.

A, B and C lie on the straight line l.

a) Find a vector equation of l.

b) Determine the value of u and the value of v.

c) Calculate the distance AB.

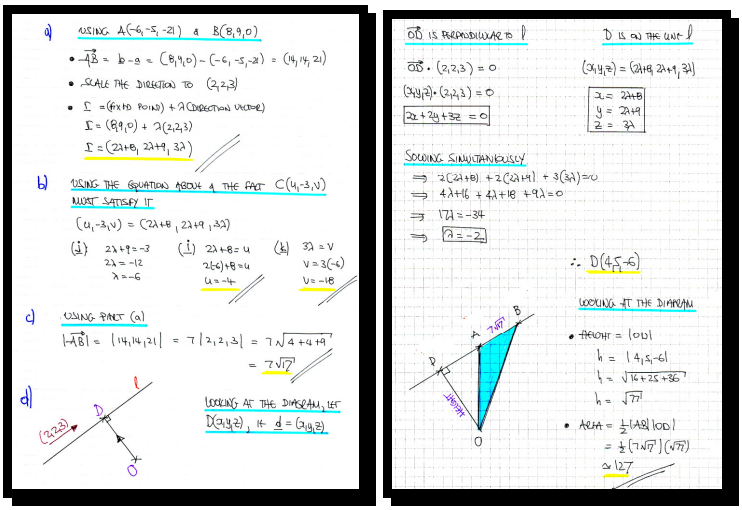
The point D lies on l so that $\overrightarrow{OD}$ is perpendicular to l.

d) Determine the position vector of D.

e) Calculate, correct to three significant figures, the area of the triangle OAB.

$$\boxed{}, \quad \boxed{\mathbf{r}=8\mathbf{i}+9\mathbf{j}+\lambda\left(2\mathbf{i}+2\mathbf{j}+3\mathbf{k}\right)}, \quad \boxed{u=-4}, \quad \boxed{v=-18}, \quad \boxed{|AB|=7\sqrt{17}},$$

$$\boxed{\mathbf{d}=4\mathbf{i}+5\mathbf{j}-6\mathbf{k}}, \quad \boxed{\text{area}\approx 127}$$



Question 91 (****)

The position vectors and coordinates in this question are relative to a fixed origin O.

The straight lines l_1, l_2 and l_3 have the following vector equations

$$\mathbf{r}_1 = 10\mathbf{i}+6\mathbf{j}+9\mathbf{k}+\lambda\left(3\mathbf{i}+\mathbf{j}+4\mathbf{k}\right)$$

$$\mathbf{r}_2 = -4\mathbf{j}+13\mathbf{k}+\mu\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)$$

$$\mathbf{r}_3 = -3\mathbf{i}-4\mathbf{k}+\nu\left(4\mathbf{i}+3\mathbf{j}+\mathbf{k}\right),$$

where λ, μ and ν are scalar parameters.

a) Show that l_1 and l_2 intersect at some point A, and find its coordinates.

b) Verify that $B\left(5,6,-2\right)$ lies on both l_2 and l_3.

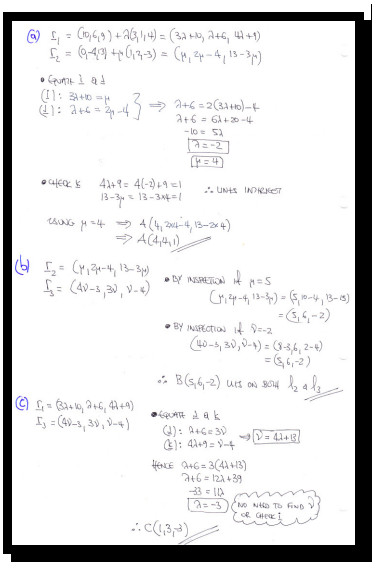
The point C is the intersection of l_1 and l_3.

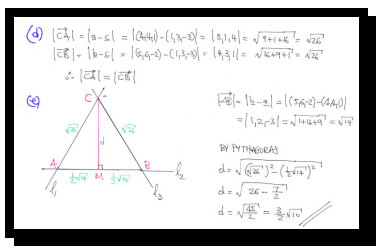
c) Find the coordinates of C.

d) Show that $\left|CA\right|=\left|CB\right|$.

e) Hence calculate the shortest distance of C from l_2.

$$\boxed{A\left(4,4,1\right)}\,,\quad \boxed{C\left(1,3,-3\right)}\,,\quad \boxed{\text{distance}=\frac{3}{2}\sqrt{10}}$$





Question 92 (****)

The position vectors and coordinates in this question are relative to a fixed origin O.

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 3\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda\left(4\mathbf{i}+4\mathbf{j}+3\mathbf{k}\right)$$

$$\mathbf{r}_2 = 9\mathbf{i}+\mu\left(\mathbf{i}-3\mathbf{j}+a\mathbf{k}\right)$$

where λ and μ are scalar parameters, and a is a scalar constant.

The point A is the intersection between l_1 and l_2, and the acute angle between them is denoted by θ.

a) Find in any order …

i. … the value of a.

ii. … the coordinates of A.

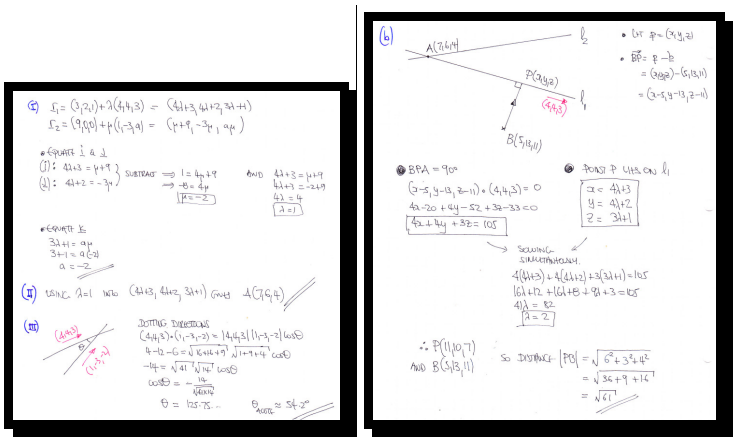
iii. … the value of θ.

The point B has coordinates $(5,13,11)$.

The point P lies on l_1 so that the angle APB is $90°$.

b) Calculate the distance BP.

$$\boxed{a=-2}\,,\quad \boxed{A(7,6,4)}\,,\quad \boxed{\theta=54.2°}\,,\quad \boxed{|BP|=\sqrt{61}}$$



Question 93 (****)

The position vectors and coordinates in this question are relative to a fixed origin O.

The points A and B have respective position vectors

$$2\mathbf{i}+3\mathbf{j} \quad \text{and} \quad 6\mathbf{i}-2\mathbf{j}+3\mathbf{k}\,.$$

a) Find a vector equation of the straight line l_1 that passes through A and B.

The straight line l_2 has vector equation

$$\mathbf{r}=5\mathbf{i}-3\mathbf{j}+6\mathbf{k}+\mu\left(\mathbf{i}-2\mathbf{j}+2\mathbf{k}\right),$$

where μ is a scalar parameter.

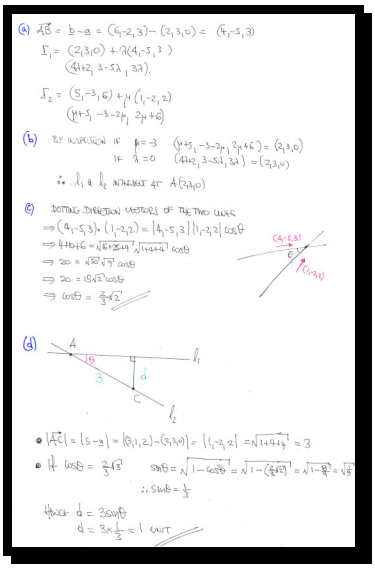
b) Show that l_1 and l_2 intersect at the point A.

c) Find the exact value of $\cos\theta$, where θ is the acute angle between l_1 and l_2.

The point C with position vector $3\mathbf{i}+\mathbf{j}+2\mathbf{k}$ lies on l_2.

d) Show that the shortest distance from C to l_1 is exactly one unit.

$$\boxed{\mathbf{r}=2\mathbf{i}+3\mathbf{j}+\lambda\left(4\mathbf{i}-5\mathbf{j}+3\mathbf{k}\right)}\,,\quad \boxed{\cos\theta=\frac{2}{3}\sqrt{2}}$$



Question 94 (****)

The position vectors and coordinates in this question are relative to a fixed origin O.

The straight line l has vector equation

$$\mathbf{r}=\begin{pmatrix}10\\7\\7\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\2\end{pmatrix},$$

where λ is a scalar parameter.

The point $P(14,15,15)$ lies on l and the point A has coordinates $(5,1,2)$.

a) Calculate the size of the acute angle that AP makes with l.

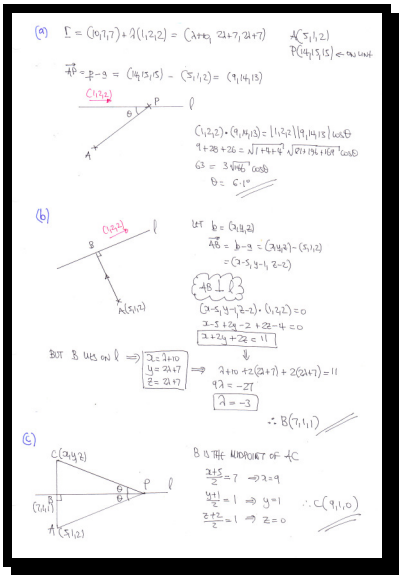
The point B lies on l so that $ABP=90°$.

b) Determine the coordinates of B.

The point C is such so that l is the angle bisector of APC.

c) Find a set of the possible coordinates of C.

$\boxed{\theta\approx 6.1°}$, $\boxed{B(7,1,1)}$, $\boxed{C(9,1,0)}$



Question 95 (****)

The position vectors and coordinates in this question are relative to a fixed origin O.

The points A, B and C have coordinates $(0,0,8)$, $(2,6,4)$ and $(8,8,0)$, respectively.

The point D is such so that $ABCD$ is a parallelogram.

The angle ABC is θ.

a) Determine the coordinates of D.

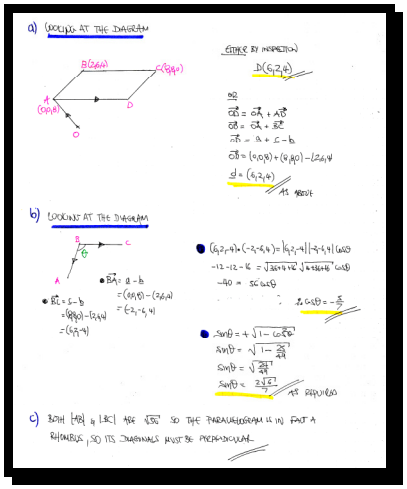
b) Use the scalar product to find an exact value for $\cos\theta$ and hence show

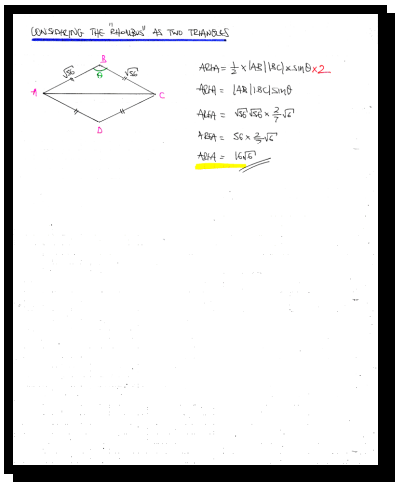
$$\sin\theta=\frac{2}{7}\sqrt{6}\,.$$

c) Explain, with reference to the calculations of part **(b)**, why AC must be perpendicular to BD.

d) Show that the area of the parallelogram is $16\sqrt{6}$.

$\boxed{}$, $\boxed{D(6,2,4)}$, $\boxed{\cos\theta=-\frac{5}{7}}$





Question 96 (****)

The position vectors and coordinates in this question are relative to a fixed origin O.

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 5\mathbf{i}+3\mathbf{j}+6\mathbf{k}+\lambda\left(2\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$$

$$\mathbf{r}_2 = -\mathbf{i}+5\mathbf{j}+a\mathbf{k}+\mu\left(\mathbf{i}-2\mathbf{j}-2\mathbf{k}\right),$$

where λ and μ are scalar parameters, and a is a scalar constant.

The point A lies on both l_1 and l_2.

a) Find the value of a and the coordinates of A.

The point $P(11, p, 12)$, where p is a scalar constant, lies on l_1.

The point $Q(q, -9, -8)$, where q is a scalar constant, lies on l_2.

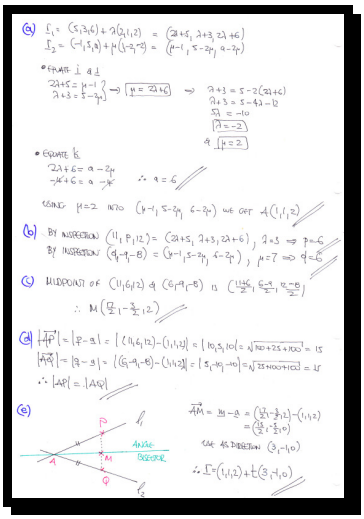
b) Find the value of p and the value of q.

c) Determine the coordinates of the midpoint of PQ.

d) Show that $|AP| = |AQ|$.

e) Hence, or otherwise, find a vector equation of the angle bisector of $\angle PAQ$.

$\square$, $\boxed{a=6}$, $\boxed{A(1,1,2)}$, $\boxed{p=6,\ q=6}$, $\boxed{M\left(\frac{17}{2},-\frac{3}{2},2\right)}$, $\boxed{\mathbf{r}=\mathbf{i}+\mathbf{j}+2\mathbf{k}+t\left(3\mathbf{i}-\mathbf{j}\right)}$



Question 97 (****)

The points with coordinates $A(4,0,-4)$ and $B(5,-1,-6)$ lie on the line L, where the point O is a fixed origin.

a) Find a vector equation of the line L.

b) Find the distance between the points A and B.

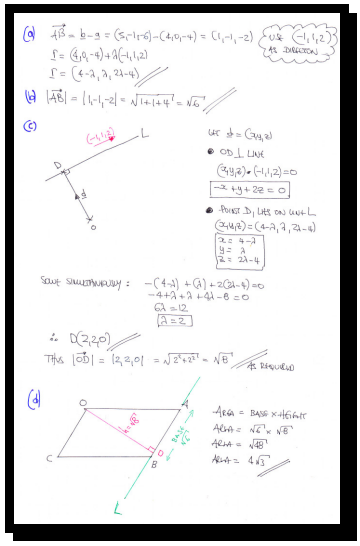
The point D lies on the line L, so that OD is perpendicular to L.

c) Find the coordinates of the point D, and hence show that $\left|\overrightarrow{OD}\right|=\sqrt{8}$

The point C is such so that $OABC$ is parallelogram.

d) Find the exact area of the parallelogram $OABC$.

$$\boxed{\mathbf{r}=4\mathbf{i}-4\mathbf{k}+\lambda(-\mathbf{i}+\mathbf{j}+2\mathbf{k})}\,,\quad \boxed{|AB|=\sqrt{6}}\,,\quad \boxed{D(2,2,0)}\,,\quad \boxed{\text{area}=4\sqrt{3}}$$



Question 98 (****)

The lines l_1 passes through the point $A(2,2,-2)$ and has direction vector $\mathbf{i}+\mathbf{j}+2\mathbf{k}$.

a) Find a vector equation for l_1.

The line l_2 has the vector equation

$$\mathbf{r}_2 = 7\mathbf{i}+\mathbf{j}-2\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-\mathbf{k}),$$

where μ is a scalar parameter.

The lines intersect at the point B and the acute angle between the two lines is θ.

b) Find the coordinates of B.

c) Show that $\cos\theta = \dfrac{1}{6}$.

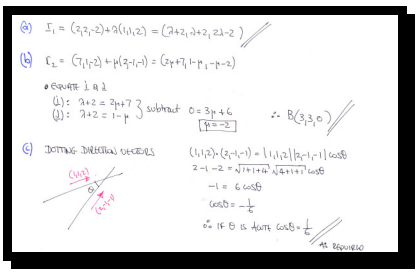
The point $P(15,-3,-6)$ lies on l_2 and the point Q lies on l_1 so that $\angle PQB = 90°$.

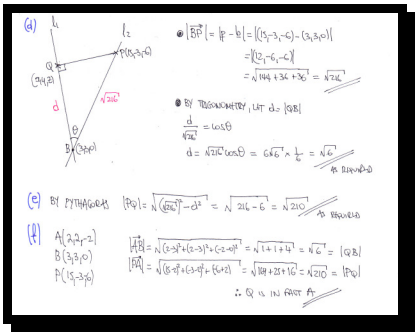
d) Find $\left|\overrightarrow{BP}\right|$ and show that $\left|\overrightarrow{BQ}\right| = \sqrt{6}$.

e) Show that $\left|\overrightarrow{PQ}\right| = \sqrt{210}$.

f) Verify that the point Q is in fact the same the point as A.

$$\boxed{\mathbf{r}_1 = 2\mathbf{i}+2\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}+2\mathbf{k})}, \quad \boxed{B(3,3,0)} \quad \boxed{\left|\overrightarrow{BP}\right| = \sqrt{216} = 6\sqrt{6}}$$





Question 99 (****)

With respect to a fixed origin, the points A, B and C have coordinates $(-2,-4,6)$, $(-16,1,4)$ and $(4,8,-6)$, respectively.

a) Find a vector equation for the line L_1, through the points A and B.

b) Find a vector equation for the line L_2, that passes through the point C and is parallel to the vector $p\mathbf{i}+q\mathbf{j}-4\mathbf{k}$, where p and q are scalar constants.

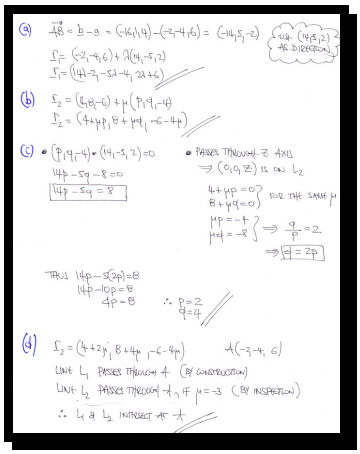
The line L_2 passes through the z axis, and is perpendicular to L_1.

c) Find the values of p and q.

d) Verify that L_1 and L_2 lines intersect at the point A.

$$\boxed{\mathbf{r}_1=-2\mathbf{i}-4\mathbf{j}+6\mathbf{k}+\lambda\left(14\mathbf{i}-5\mathbf{j}+2\mathbf{k}\right)}\,,\quad \boxed{\mathbf{r}_2=4\mathbf{i}+8\mathbf{j}-6\mathbf{k}+\lambda\left(p\mathbf{i}+q\mathbf{j}-4\mathbf{k}\right)}\,,$$

$$\boxed{p=2,\quad q=4}$$



Question 100 (****)

The straight lines L and M have the following vector equations

$$L:\ \mathbf{r}_1=(3\lambda+3)\mathbf{i}+(4-4\lambda)\mathbf{j}+2\lambda\mathbf{k}$$

$$M:\ \mathbf{r}_2=(3\mu+12)\mathbf{i}+(20-4\mu)\mathbf{j}+(2\mu+4)\mathbf{k},$$

where λ and μ are scalar parameters.

a) Show that L and M are parallel.

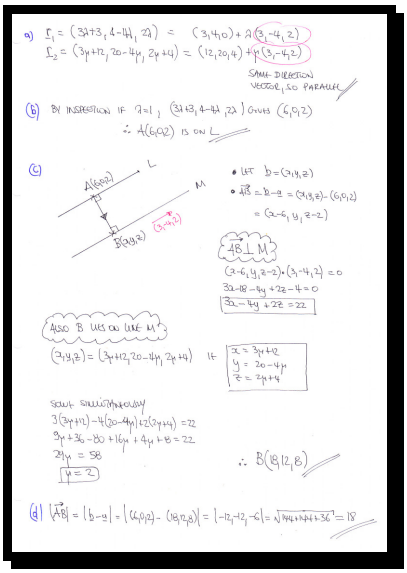
b) Show further that $A(6,0,2)$ lies on L.

The point B lies on M so that AB is perpendicular to M.

c) Find the coordinates of B.

d) Hence determine the shortest distance between L and M.

$\boxed{B(18,12,8)}$, $\boxed{\text{distance}=18\text{ units}}$



Question 101 (****)

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = -2\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_2 = 8\mathbf{i} - 5\mathbf{j} + 26\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ and μ are scalar parameters.

a) Given l_1 and l_2 intersect at the point P, find the coordinates of P.

b) Show l_1 and l_2 are perpendicular.

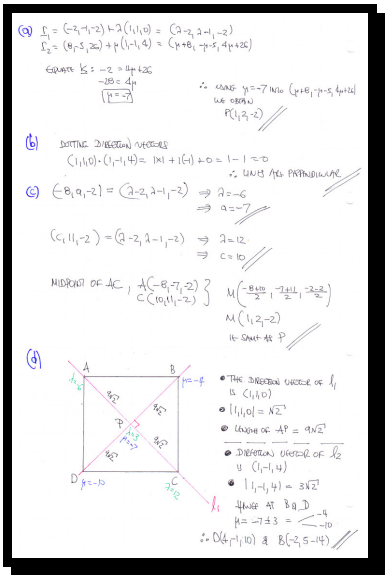
The points $A(-8, a, -2)$ and $C(c, 11, -2)$ lie on l_1.

c) Find the value of each of the constants a and c, and show further that P is the midpoint of AC.

The quadrilateral $ABCD$ is a square.

d) Determine the coordinates of the points B and D.

$\boxed{P(1,2,-2)}$, $\boxed{a=-7}$, $\boxed{c=10}$, $\boxed{B(4,-1,10) \text{ \& } D(-2,5,-14) \text{ in any order}}$



Question 102 (****)

Relative to a fixed origin, the points P and Q have position vectors $9\mathbf{j}-2\mathbf{k}$ and $7\mathbf{i}-8\mathbf{j}+11\mathbf{k}$, respectively.

a) Determine the distance between the points P and Q.

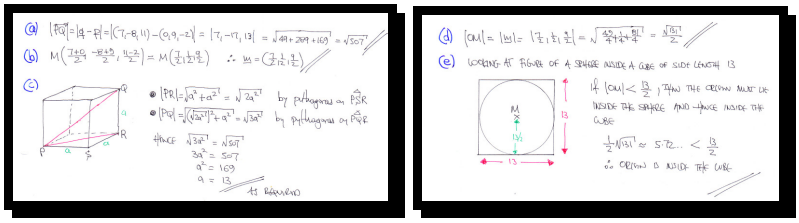
b) Find the position vector of the point M, where M is the midpoint of PQ.

The points P and Q are vertices of a cube, so that PQ is one of the longest diagonals of the cube.

c) Show that the length of one of the sides of the cube is 13 units.

d) Show that the origin O lies inside the cube.

$\boxed{}$, $\boxed{|PQ|=\sqrt{507}}$, $\boxed{\overrightarrow{OM}=\frac{7}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{9}{2}\mathbf{k}}$



Question 103 (****)

The points $A(7,a,5)$ and $B(b,1,12)$ lie on the straight line L, with vector equation

$$\mathbf{r}=19\mathbf{i}-2\mathbf{j}-9\mathbf{k}+\lambda(6\mathbf{i}-\mathbf{j}-7\mathbf{k}),$$

where λ is a scalar parameter.

a) Find the value of a and the value of b.

The point C has coordinates $(-3,19,10)$ and M is the midpoint of BC.

b) Determine the coordinates of M.

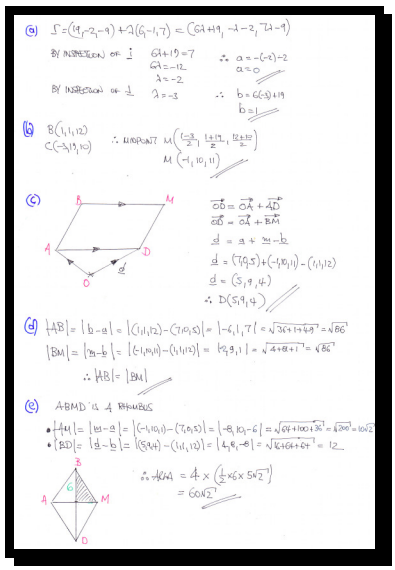
The point D is such so that $ABMD$ is a parallelogram.

c) Find the coordinates of D.

d) Show that $|AB|=|BM|$.

e) Find the exact area of the parallelogram $ABMD$.

$$\boxed{a=0,\ b=1},\ \boxed{A(-1,10,11)},\ \boxed{D(5,9,4)},\ \boxed{\text{area}=60\sqrt{2}}$$



Question 104 (****)

The straight line L_1 passes through the points $A(1,-2,5)$ and $B(4,-3,3)$.

a) Find a vector equation for L_1.

The straight line L_2 has a vector equation

$$\mathbf{r}=\begin{pmatrix}8\\p\\q\end{pmatrix}+\mu\begin{pmatrix}1\\4\\-1\end{pmatrix},$$

where μ is a scalar parameter, and p and q are scalar constants.

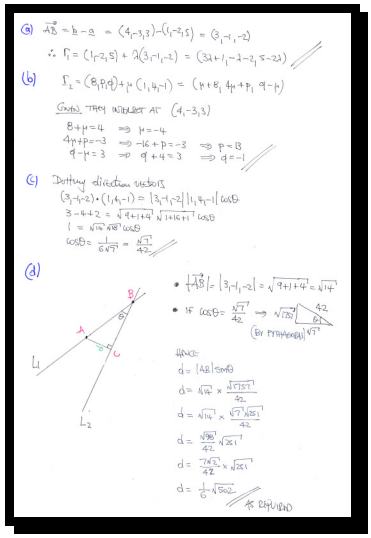
b) Given that L_1 and L_2 intersect at B, find the value of p and the value of q.

c) Find the cosine of the acute angle θ, between L_1 and L_2.

The point C is on L_2, so that AC is perpendicular to L_2.

d) Show that the length of AC is $\dfrac{\sqrt{502}}{6}$.

$\boxed{\mathbf{r}=\mathbf{i}-2\mathbf{j}+5\mathbf{k}+\lambda(3\mathbf{i}-\mathbf{j}-2\mathbf{k})}$, $\boxed{p=13,\ q=-1}$, $\boxed{\cos\theta=\dfrac{1}{42}\sqrt{7}}$



Question 105 (****)

With respect to a fixed origin O, the variable points A and B have the following position vectors

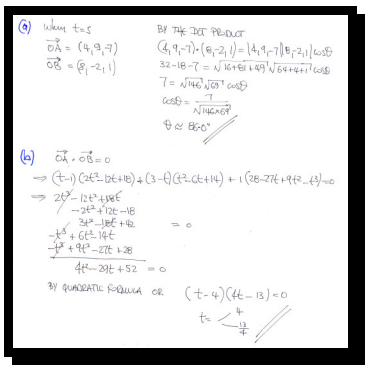
$$\overrightarrow{OA}=\begin{pmatrix} t-1 \\ t^2-6t+14 \\ 28-27t+9t^2-t^3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB}=\begin{pmatrix} 2t^2-12t+18 \\ 3-t \\ 1 \end{pmatrix},$$

where t is a scalar parameter.

a) Calculate the angle AOB when $t=5$

b) Find the values of t for which the angle AOB is a right angle.

$\boxed{\theta \approx 86.0}$, $\boxed{t=4 \text{ or } t=\frac{13}{4}}$



Question 106 (****)

The points A and B have coordinates $(7,13,14)$ and $(15,19,15)$, respectively.

a) Find a vector equation of the straight line l_1 which passes through A and B.

The line l_2 has vector equation

$$\mathbf{r}=5\mathbf{i}-8\mathbf{j}-9\mathbf{k}+\mu\left(-2\mathbf{i}+5\mathbf{j}+3\mathbf{k}\right),$$

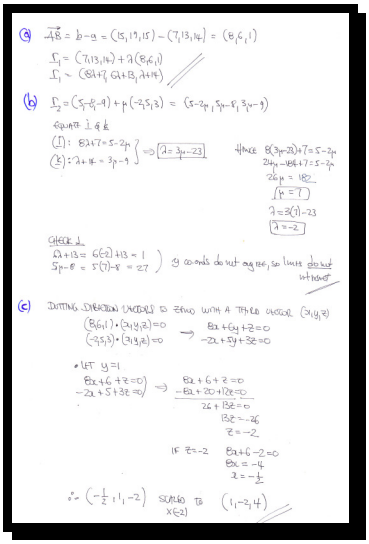
where μ is a scalar parameter.

b) Show that l_1 and l_2 do not intersect.

c) Find a vector with integer components in their simplest proportions, which is perpendicular to both lines.

[*you may not use the cross product for this part*]

$$\boxed{\mathbf{r}=7\mathbf{i}+13\mathbf{j}+14\mathbf{k}+\lambda\left(8\mathbf{i}+6\mathbf{j}+\mathbf{k}\right)}\,,\quad \boxed{\mathbf{i}-2\mathbf{j}+4\mathbf{k}}$$



Question 107 (****)

The straight lines l_1 and l_2 have respective vector equations

$$\mathbf{r}_1 = 5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}+\mathbf{j}+\mathbf{k}\right) \quad \text{and} \quad \mathbf{r}_2 = -3\mathbf{i}+4\mathbf{j}+8\mathbf{k}+\mu\left(2\mathbf{i}-\mathbf{j}-3\mathbf{k}\right),$$

where λ and μ are scalar parameters.

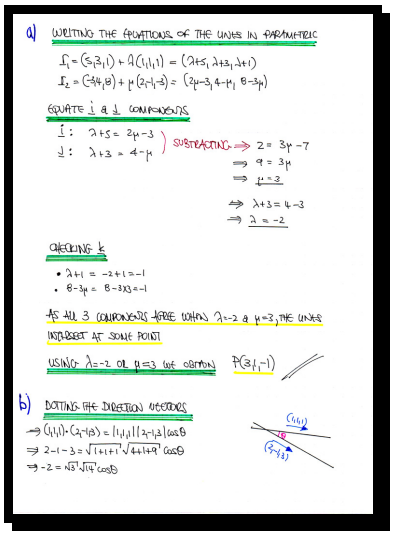
a) Show that l_1 and l_2 intersect at some point P, further finding its coordinates.

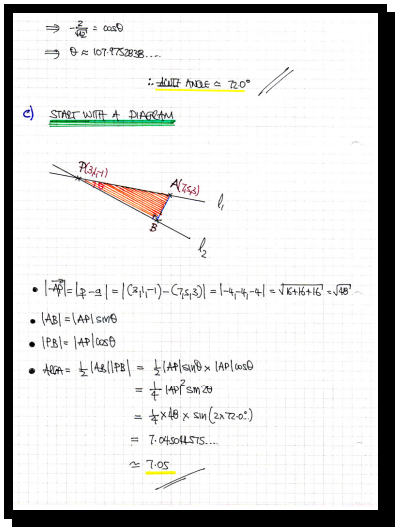
b) Calculate the acute angle between l_1 and l_2.

The point $A(7,5,3)$ lies on l_1 and the point B lies on l_2, such that the straight line AB is perpendicular to l_2.

c) Determine the area of the triangle ABP.

$$\boxed{}, \quad \boxed{P(3,1,-1)}, \quad \boxed{72.0^\circ}, \quad \boxed{\text{area}\left(ABC\right)=\frac{8}{7}\sqrt{38}\approx 7.05}$$





Question 108 (****)

The straight line L passes through the points $B(1,4,0)$ and $D(2,2,6)$.

a) Find a vector equation of L.

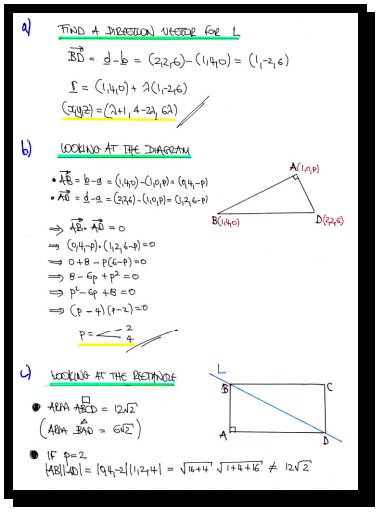
The point $A(1,0,p)$, where p is a scalar constant, is such that $\angle BAD = 90°$.

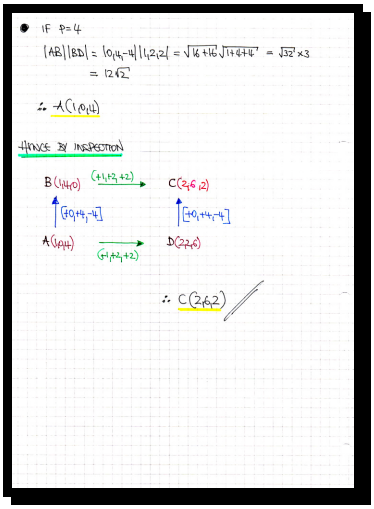
b) Find the possible values of p.

The rectangle $ABCD$ has an area of $12\sqrt{2}$ square units.

c) Find the coordinates of C.

$\boxed{}$, $\boxed{\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} - 2\mathbf{j} + 6\mathbf{k})}$, $\boxed{p = 2, 4}$, $\boxed{C(2,6,2)}$





Question 109 **(****)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 12\mathbf{i}+7\mathbf{j}+3\mathbf{k}+\lambda\left(2\mathbf{i}+\mathbf{j}+\mathbf{k}\right)$$

$$\mathbf{r}_2 = \mathbf{i}+3\mathbf{j}+\mu\left(3\mathbf{i}-\mathbf{k}\right),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point A, further finding its coordinates.

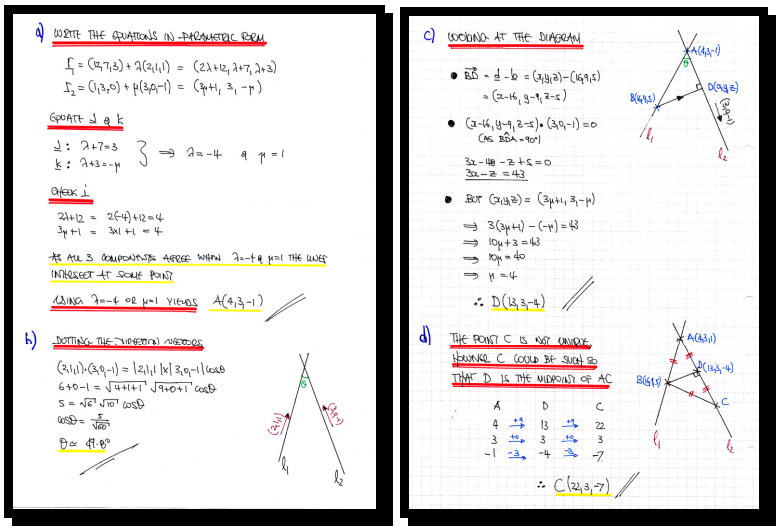
b) Calculate the acute angle between l_1 and l_2.

The point $B(16,9,5)$ lies on l_1 and the point D lies on l_2.

c) If BD is perpendicular to l_2 find the coordinates of D.

d) Find the coordinates of a point C so that the triangle ABC is isosceles.

$\boxed{}$, $\boxed{A(4,3,-1)}$, $\boxed{49.8°}$, $\boxed{D(13,3,-4)}$, $\boxed{C(22,3,-7)}$



Question 110 (****)

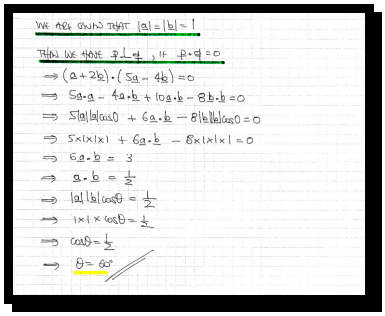
The vectors $\mathbf{p}$ and $\mathbf{q}$ are defined as

$$\mathbf{p}=\mathbf{a}+2\mathbf{b} \qquad \text{and} \qquad \mathbf{q}=5\mathbf{a}-4\mathbf{b}$$

where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors.

Given that $\mathbf{p}$ and $\mathbf{q}$ are perpendicular, determine the acute angle between $\mathbf{a}$ and $\mathbf{b}$.

$\boxed{}$, $\boxed{\theta=60^\circ}$



Question 111 (****)

The straight lines l_1 and l_2 have the following Cartesian equations

$$l_1: \quad \frac{x-8}{1}=\frac{y+1}{-1}=\frac{z-2}{1}.$$

$$l_2: \quad \frac{x-3}{-1}=\frac{y-4}{1}=\frac{z-1}{1}.$$

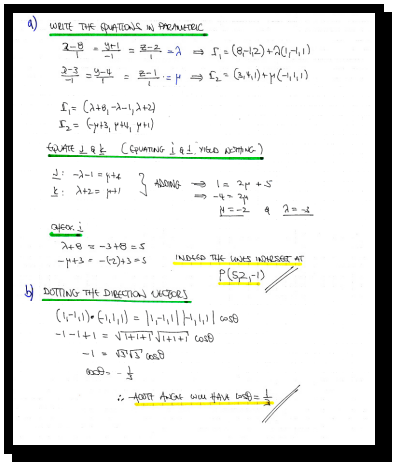
a) Show that l_1 and l_2 intersect at some point P, and find its coordinates.

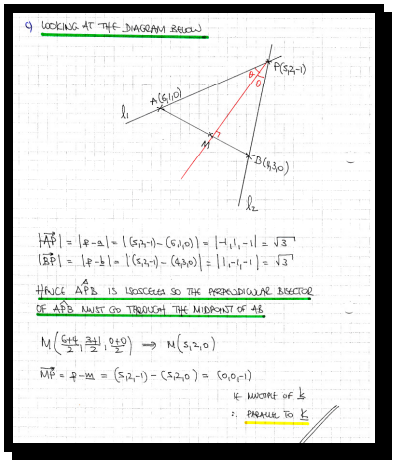
b) Find the exact value of $\cos\theta$, where θ is the acute angle formed by l_1 and l_2.

The point $A(6,1,0)$ lies on l_1 and the point $B(4,3,0)$ lies on l_2.

c) By considering $|AP|$ and $|BP|$ show further that the angle bisector of $\angle APB$ is parallel to the vector $\mathbf{k}$.

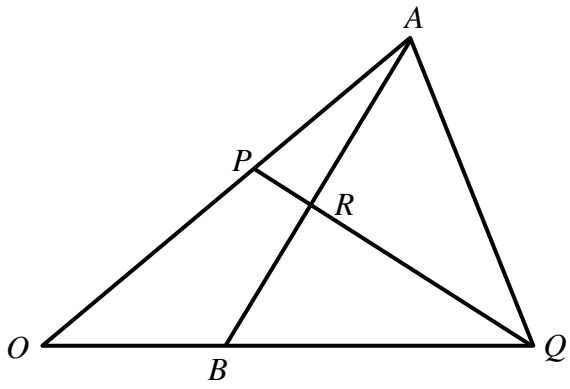
$\boxed{}$, $\boxed{P(5,2,-1)}$, $\boxed{\cos\theta=\frac{1}{3}}$





Question 112 (**+)**

The figure below shows a triangle OAQ.



- The point P lies on OA so that $OP:OA=3:5$.
- The point B lies on OQ so that $OB:BQ=1:2$.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

a) Given that $\overrightarrow{AR}=h\overrightarrow{AB}$, where h is a scalar parameter with $0<h<1$, show that

$$\overrightarrow{OR}=(1-h)\mathbf{a}+h\mathbf{b}.$$

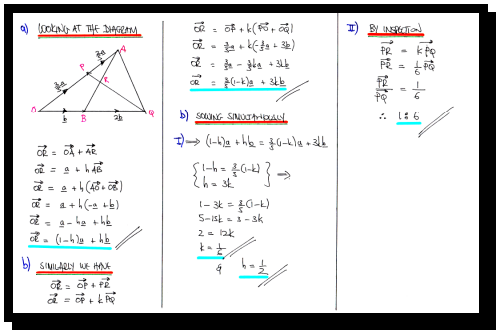
b) Given further that $\overrightarrow{PR}=k\overrightarrow{PQ}$, where k is a scalar parameter with $0<k<1$, find a similar expression for $\overrightarrow{OR}$ in terms of k, $\mathbf{a}$, $\mathbf{b}$.

c) Determine …

i. … the value of k and the value of h.

ii. … the ratio of $\overrightarrow{PR}:\overrightarrow{PQ}$.

$\square$, $\boxed{\overrightarrow{OR}=\frac{3}{5}(1-k)\mathbf{a}+k\mathbf{b}}$, $\boxed{k=\frac{1}{6}}$, $\boxed{h=\frac{1}{2}}$, $\boxed{PR:PQ=1:6}$



Question 113 (**+)**

The points A and B have coordinates $(4,0,2)$ and $(7,0,-1)$, respectively.

a) Find the vector $\overrightarrow{AB}$.

The straight line l has vector equation

$$\mathbf{r}=-3\mathbf{i}-4\mathbf{j}+\mathbf{k}+\lambda(7\mathbf{i}+4\mathbf{j}+\mathbf{k}),$$

where λ is a scalar parameter.

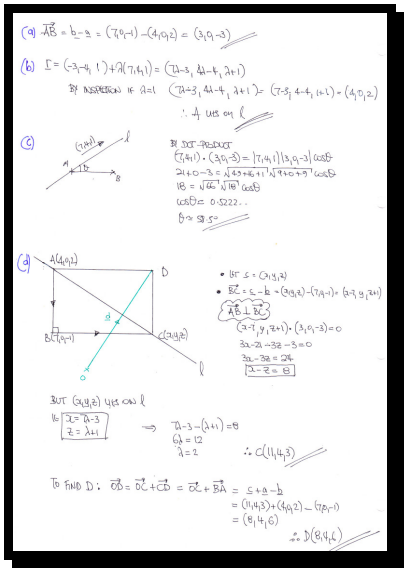
b) Show that A lies on l.

c) Calculate the acute angle between $\overrightarrow{AB}$ and l.

The point C lies on l so that $ABCD$ is a rectangle.

d) Find the coordinates of D.

$\boxed{\overrightarrow{AB}=3\mathbf{i}-3\mathbf{k}}$, $\boxed{\theta\approx 58.5^\circ}$, $\boxed{D(8,4,6)}$



Question 114 (**+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 3\mathbf{j}+\mathbf{k}+\lambda\left(2\mathbf{i}-\mathbf{j}+2\mathbf{k}\right)$$

$$\mathbf{r}_2 = 2\mathbf{i}+7\mathbf{k}+\mu\left(\mathbf{i}-\mathbf{j}+2\mathbf{k}\right),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point P and find its position vector.

The points A and C lie on l_1 and the points B and D lie on l_2.

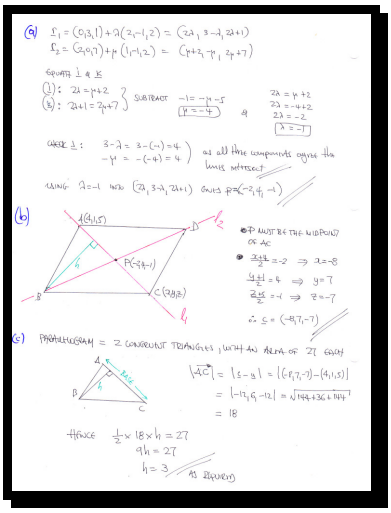
The point A has position vector $4\mathbf{i}+\mathbf{j}+5\mathbf{k}$.

The quadrilateral $ABCD$ is a parallelogram with an area of 54 square units.

b) State the position vector of the point C.

c) Show that the distance of the point B from l_1 is 3 units.

$\boxed{\mathbf{p}=-2\mathbf{i}+4\mathbf{j}-\mathbf{k}}$, $\boxed{\mathbf{c}=-8\mathbf{i}+7\mathbf{j}-7\mathbf{k}}$



Question 115 (**+)**

With respect to a fixed origin O, the points A and B have respective position vectors

$$\mathbf{i}+11\mathbf{j}+3\mathbf{k} \quad \text{and} \quad 11\mathbf{i}-4\mathbf{j}+8\mathbf{k}.$$

a) Find a vector equation for the straight line l, which passes through A and B.

The point C is the point on l closest to the origin O.

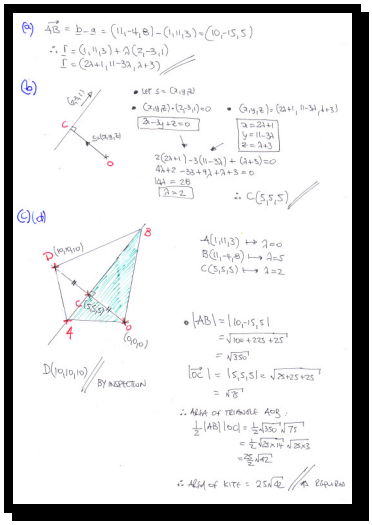
b) Determine the position vector of C.

The point D is the reflection of O about l.

c) State the position vector of D.

d) Show that the area of the kite $OADB$ is $25\sqrt{42}$ square units.

$$\boxed{\mathbf{r}=\mathbf{i}+11\mathbf{j}+3\mathbf{k}+\lambda(2\mathbf{i}-3\mathbf{j}+\mathbf{k})}, \quad \boxed{\overrightarrow{OC}=5\mathbf{i}+5\mathbf{j}+5\mathbf{k}}, \quad \boxed{\overrightarrow{OD}=10\mathbf{i}+10\mathbf{j}+10\mathbf{k}}$$



Question 116 (**+)**

Relative to a fixed origin O, the points $A(3t-19, 2t-14, 28-t)$, $B(t+1, t-2, 5t)$ and $C(2t-11, 10-t, 2t+4)$, represent the coordinates of the paths of three helicopters, where t represents the time in minutes after a certain instant.

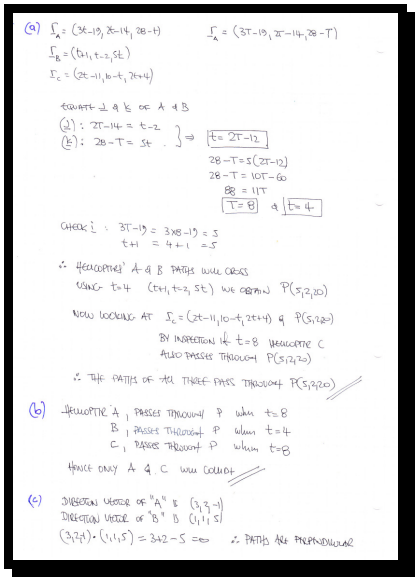
All distances are in kilometres with the coordinates axes Ox, Oy, Oz oriented due east, due north and vertically upwards, respectively.

a) Show that all three helicopters pass through a point P and find its coordinates.

b) Explain why only two of the helicopters will collide at the point P if they maintain their courses as described in this problem.

c) Show that the paths of A and B are perpendicular.

$$\boxed{P(5,2,20)}$$


(a) $\underline{r}_A = (3t-19, 2t-14, 28-t)$ $\underline{r}_B = (t+1, t-2, 5t)$
$\underline{r}_C = (2t-11, 10-t, 2t+4)$
EQUATE $\perp$ & $\parallel$ OF A & B
(i): $2t-14 = t-2$ (ii): $28-T = 5t$ $\Rightarrow$ $t = 2T-12$
$28-T = 5(2T-12)$
$28-T = 10T-60$
$88 = 11T$
$T = 8$ & $t = 4$
CHECK $\parallel$: $3T-19 = 3\times8-19 = 5$
$t+1 = 4+1 = 5$
$\therefore$ HELICOPTERS' A & B PATHS WILL CROSS
WHEN $t=4$ $(t+1, t-2, 5t)$ WE OBTAIN $P(5,2,20)$
NOW LOOKING AT $\underline{r}_C = (2t-11, 10-t, 2t+4)$ & $P(5,2,20)$
BY INSPECTION $\parallel$ $t=8$ HELICOPTER C ALSO PASSES THROUGH $P(5,2,20)$
$\therefore$ THE PATHS OF ALL THREE PASS THROUGH $P(5,2,20)$
(b) HELICOPTER A, PASSES THROUGH P WHEN $t=8$
B, PASSES THROUGH P WHEN $t=4$
C, PASSES THROUGH P WHEN $t=8$
HENCE ONLY A & C WILL COLLIDE
(c) DIRECTION VECTOR OF "A" IS $(3,2,-1)$
DIRECTION VECTOR OF "B" IS $(1,1,5)$
$(3,2,-1)\cdot(1,1,5) = 3+2-5 = 0$ $\therefore$ PATHS ARE PERPENDICULAR

Question 117 (**+)**

The straight lines l_1 and l_2, where λ and μ are scalar parameters, have the following vector equations

$$\mathbf{r}_1 = (\lambda+2)\mathbf{i}+(2\lambda+6)\mathbf{j}+(-\lambda-1)\mathbf{k}$$

$$\mathbf{r}_2 = (2\mu-4)\mathbf{i}+(4-\mu)\mathbf{j}+(3-\mu)\mathbf{k}\,.$$

l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

a) Find in any order…

i. … the coordinates of A.

ii. … the exact value of $\cos\theta$.

The point B lies on l_1 and the point C lies on l_2. The triangle ABC is isosceles with $|AB|=|AC|=6\sqrt{6}$.

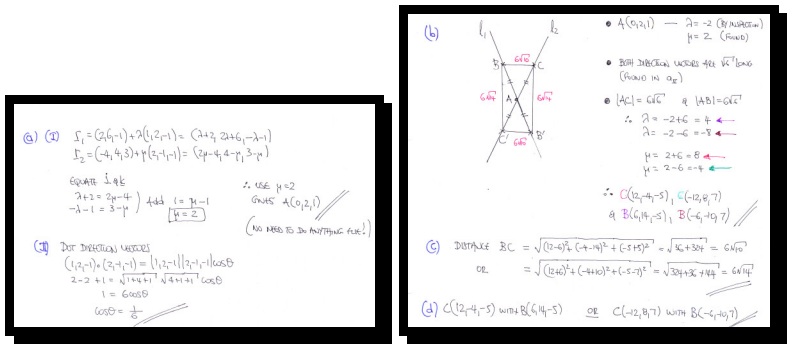
b) Find the two possible sets of coordinates for the points B and C.

c) Show that either $|BC|=6\sqrt{14}$ or $|BC|=6\sqrt{10}$

In the triangle ABC the angle BAC is acute.

a) In the triangle ABC, determine the two possible pairings for the coordinates of the point B and the corresponding coordinates of the point C.

$$\boxed{A(0,2,1)},\ \boxed{\cos\theta=\frac{1}{6}},\ \boxed{B_1(-6,-10,7)\quad\text{or}\quad B_2(6,14,-5)},$$
$$\boxed{C_1(-12,8,7)\quad\text{or}\quad C_2(12,-4,-5)},\ \boxed{B_1\ \&\ C_1\quad\text{or}\quad B_2\ \&\ C_2}$$



Question 118 (**+)**

Relative to a fixed origin O, the points A, B, C and D have coordinates $(7,6,2)$, $(12,10,5)$, $(1,-4,-8)$ and $(11,4,-2)$, respectively.

a) Find the vector equation of the straight line l_1 which passes through the point A and B and the vector equation of the straight line l_2 which passes through the point C and D.

b) Explain why l_1 and l_2 do not intersect.

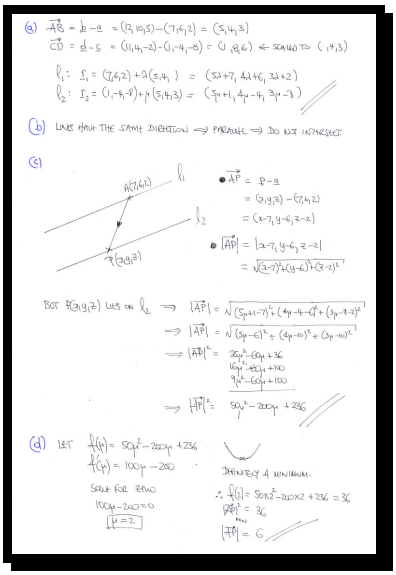
The point P lies on l_2.

c) Find an expression for $\left|\overrightarrow{AP}\right|^2$, in terms of μ.

d) Calculate the distance between l_1 and l_2.

$\boxed{\mathbf{r}_1 = 7\mathbf{i}+6\mathbf{j}+2\mathbf{k}+\lambda\left(5\mathbf{i}+4\mathbf{j}+3\mathbf{k}\right)}$, $\boxed{\mathbf{r}_2 = \mathbf{i}-4\mathbf{j}-8\mathbf{k}+\mu\left(5\mathbf{i}+4\mathbf{j}+3\mathbf{k}\right)}$,

$\boxed{\left|\overrightarrow{PA}\right|^2 = 50\mu^2-200\mu+236}$, $\boxed{6 \text{ units}}$



Question 119 (**+)**

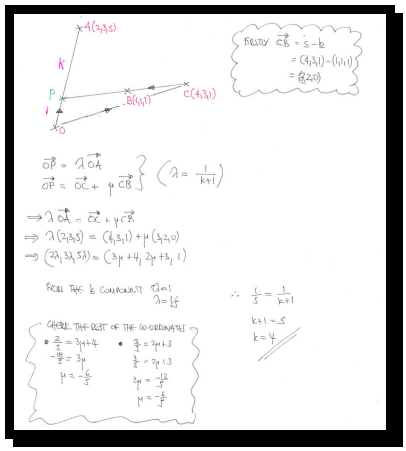
Relative to a fixed origin O, the points A, B and C have coordinates $(2,3,5)$, $(1,1,1)$ and $(4,3,1)$, respectively.

The line segment CB is extended to the point P.

It is further given that P lies on the line segment OA so that $|OP|:|PA| = 1:k$.

Determine the value of k.

$$\boxed{k=4}$$



Question 120 (**+)**

The straight line l_1 passes through the points with coordinates $A(-2,3,4)$ and $B(8,-1,14)$.

a) Find a vector equation for l_1.

The straight line l_2 has vector equation

$$\mathbf{r}=\mathbf{i}+5\mathbf{j}-5\mathbf{k}+\mu(\mathbf{i}-2\mathbf{j}+7\mathbf{k}),$$

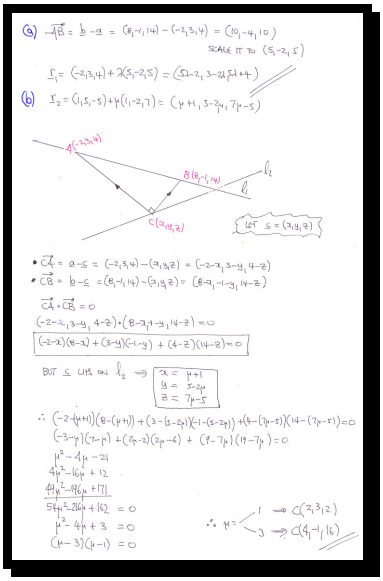
where μ is a scalar parameter.

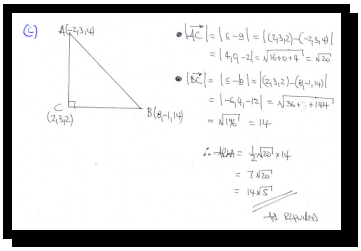
The point C lies on l_2 so that AC is perpendicular to BC.

b) Show that one possible position for the point C has coordinates $(2,3,2)$ and find the other.

c) Assuming further that C has coordinates $(2,3,2)$, show that the area of the triangle ABC is $14\sqrt{5}$ square units.

$\mathbf{r}_1=2\mathbf{i}-3\mathbf{j}+4\mathbf{k}+\lambda(5\mathbf{i}-2\mathbf{j}+5\mathbf{k})$, $(4,-1,16)$





Question 121 (**+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 13\mathbf{i} - 5\mathbf{j} + 8\mathbf{k} + \lambda\left(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\right)$$

$$\mathbf{r}_2 = -5\mathbf{i} - 4\mathbf{j} + 8\mathbf{k} + \mu\left(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}\right),$$

where λ and μ are scalar parameters.

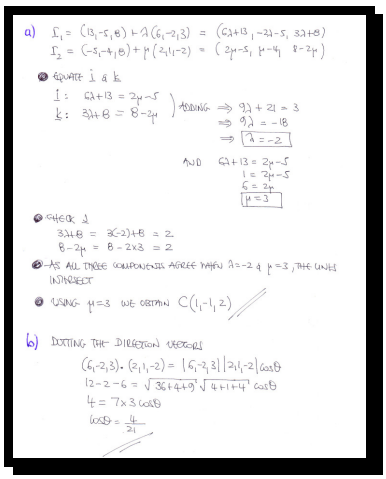
a) Show that l_1 and l_2 intersect at some point C and find its coordinates.

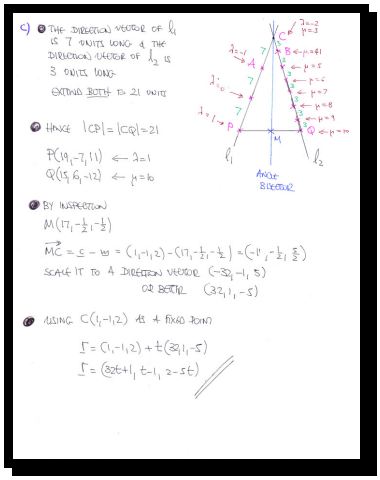
b) Find the cosine of the acute angle between l_1 and l_2.

The point A lies on l_1 where $\lambda = -1$ and the point B lies on l_2 where $\mu = 4$.

c) Determine a vector equation of the angle bisector of $\angle ACB$.

$\square$, $\boxed{C\left(1,-1,2\right)}$, $\boxed{\cos\theta = \frac{4}{21}}$, $\boxed{\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + t\left(32\mathbf{i} + \mathbf{j} - 5\mathbf{k}\right)}$





Question 122 (**+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 9\mathbf{i}+7\mathbf{j}+11\mathbf{k}+\lambda\left(4\mathbf{i}+3\mathbf{j}+5\mathbf{k}\right)$$

$$\mathbf{r}_2 = -2\mathbf{i}+5\mathbf{j}-4\mathbf{k}+\mu\left(3\mathbf{i}-4\mathbf{j}+a\mathbf{k}\right),$$

where λ and μ are scalar parameters and a is a scalar constant.

The point A is the intersection of l_1 and l_2.

b) Find in any order …

i. … the value of a.

ii. … the coordinates of A.

The acute angle between l_1 and l_2 is θ.

c) Show that $\theta = 60°$.

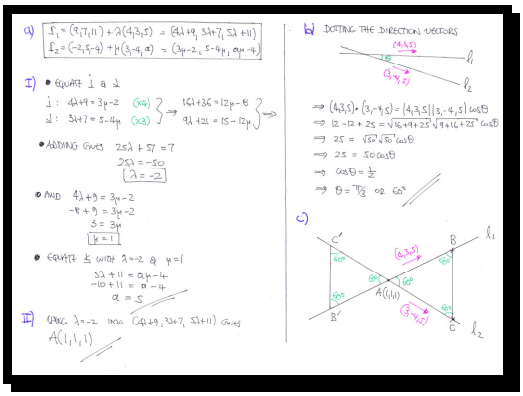
The point B lies on l_1 and the point C lies on l_2.

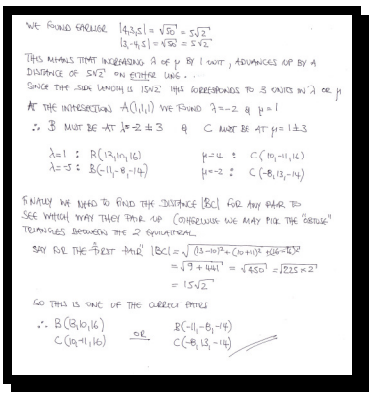
The triangle ABC is equilateral with sides of length $15\sqrt{2}$.

d) Find the **two** possible pairings for the coordinates of B and C.

$\boxed{}$, $\boxed{a=5}$, $\boxed{A(1,1,1)}$,

$\boxed{B(13,10,16)\ \&\ C(10,-11,16) \text{ or } B(-11,-8,-14)\ \&\ C(-8,13,-14)}$





Question 123 (**+)**

The points $A(3,2,14)$, $B(0,1,13)$ and $C(5,6,8)$ are defined with respect to a fixed origin O.

a) Show that the cosine of the angle ABC is $\dfrac{3}{\sqrt{33}}$.

The straight line L passes through A and it is parallel to the vector $\overrightarrow{BC}$.

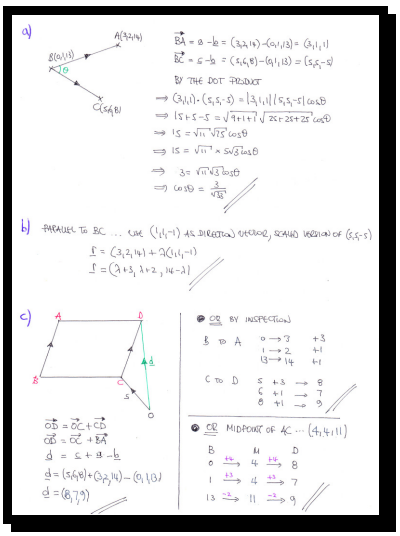
b) Find a vector equation of L.

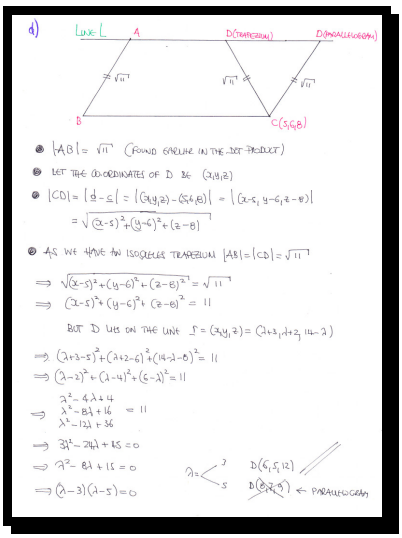
The point D lies on L so that $ABCD$ is a parallelogram.

c) Find the coordinates of D.

d) If instead $ABCD$ is an isosceles trapezium and the point D still lies on L, determine the new coordinates of D.

$\boxed{}$, $\boxed{\mathbf{r}=3\mathbf{i}+2\mathbf{j}+14\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-\mathbf{k})}$, $\boxed{D(8,7,9)}$, $\boxed{D(6,5,11)}$





Question 124 (**+)**

With respect to a fixed origin O, the point A and the point B have position vectors $\mathbf{i}-7\mathbf{j}+5\mathbf{k}$ and $-9\mathbf{j}+6\mathbf{k}$, respectively.

a) Find a vector equation of the straight line l which passes through A and B.

A variable vector is defined as

$$\mathbf{p}=(p+6)\mathbf{i}+(2p+3)\mathbf{j}-p\mathbf{k},$$

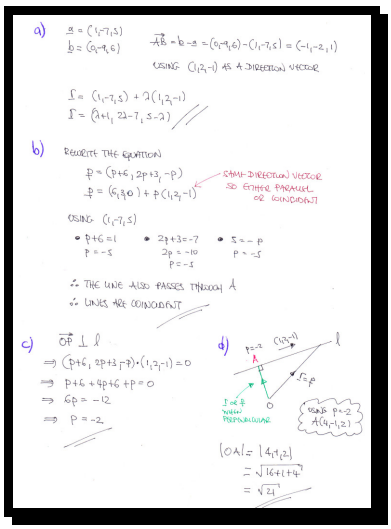
where p is a scalar parameter.

b) Show that for all values of p, the point P with position vector $\mathbf{p}$, lies on l.

c) Determine the value of p for which $\overrightarrow{OP}$ is perpendicular to l.

d) Hence, or otherwise, find the shortest distance of l from the origin O.

$\boxed{}$, $\boxed{\mathbf{r}=\mathbf{i}-7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}-\mathbf{k})}$, $\boxed{p=-2}$, $\boxed{\text{shortest distance}=\sqrt{21}}$



Question 125 (**+)**

The points with coordinates $A(7,6,10)$, $B(6,5,6)$ and $C(1,0,4)$ are the vertices of the parallelogram $ABCD$.

a) Find …

i. … the coordinates of D.

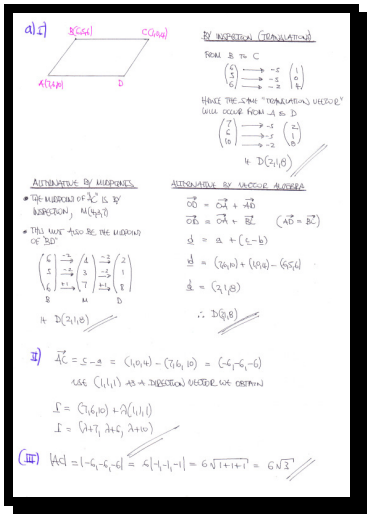
ii. … a vector equation of the straight line l which passes through the points A and C.

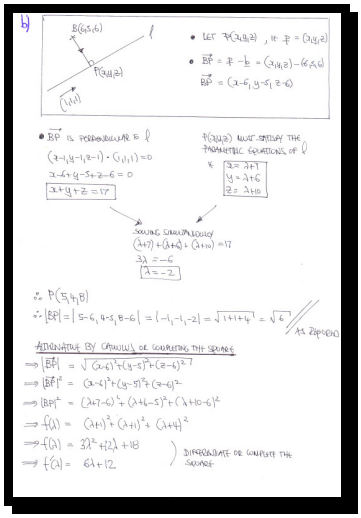
iii. … the distance AC.

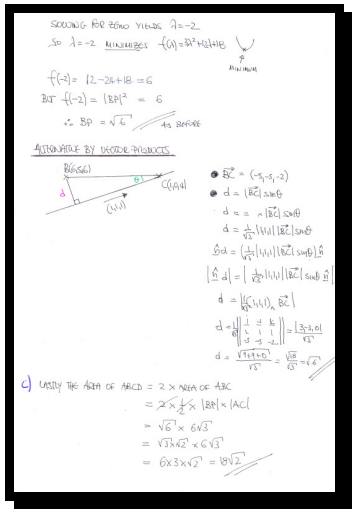
b) Show that the shortest distance of l from B is $\sqrt{6}$ units.

c) Hence find the exact area of the parallelogram $ABCD$.

$\boxed{}$, $\boxed{D(2,1,8)}$, $\boxed{\mathbf{r}=7\mathbf{i}+6\mathbf{j}+10\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}+\mathbf{k})}$, $\boxed{|AC|=6\sqrt{3}}$, $\boxed{18\sqrt{2}}$







Question 126 (**+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = \mathbf{i} - 5\mathbf{j} + \lambda(4\mathbf{j} - \mathbf{k})$$

$$\mathbf{r}_2 = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters.

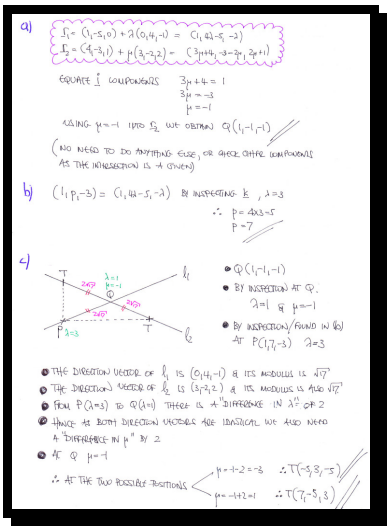
a) Given that l_1 and l_2 intersect at some point Q, find the position vector of Q.

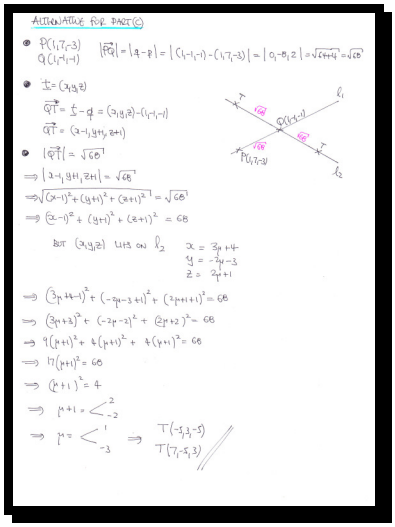
b) Given further that the point P lies on l_1 and has position vector $\mathbf{i} + p\mathbf{j} - 3\mathbf{k}$, find the value of p.

The point T lies on l_2 so that $\left|\overrightarrow{PQ}\right| = \left|\overrightarrow{QT}\right|$.

c) Determine the two possible position vectors for T.

$\boxed{}$, $\boxed{\mathbf{q} = \mathbf{i} - \mathbf{j} - \mathbf{k}}$, $\boxed{p = 7}$, $\boxed{\mathbf{t} = -5\mathbf{i} + 3\mathbf{j} - 5\mathbf{k} \text{ or } \mathbf{t} = 7\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}}$





Question 127 (**+)**

The point P has position vector $2\mathbf{i}+2\mathbf{j}+21\mathbf{k}$.

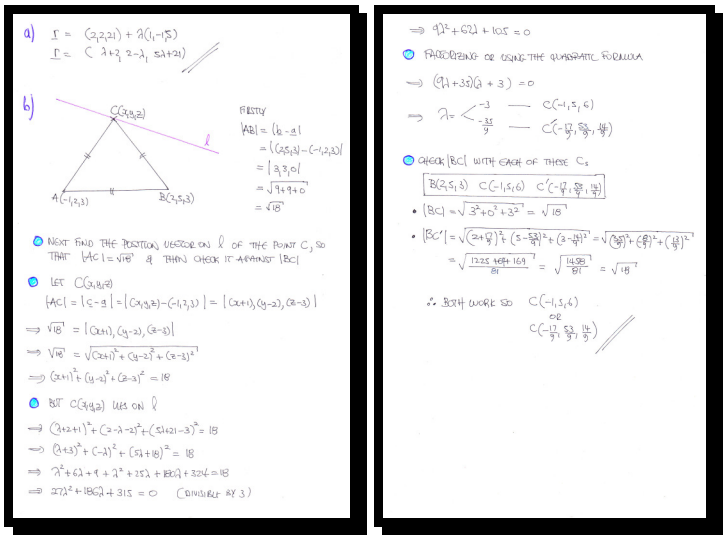
a) Find the vector equation of the straight line l which passes through P and is parallel to the vector $\mathbf{i}-\mathbf{j}+5\mathbf{k}$.

The points A and B have coordinates $(-1,2,3)$ and $(2,5,3)$, respectively.

The point C lies on l so that the triangle ABC is equilateral.

b) Find the two possible position vectors for C.

$\boxed{}$, $\boxed{\mathbf{r}=2\mathbf{i}+2\mathbf{j}+21\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+5\mathbf{k})}$, $\boxed{\mathbf{c}=-\mathbf{i}+5\mathbf{j}+6\mathbf{k} \quad \text{or} \quad \mathbf{c}=-\frac{17}{9}\mathbf{i}+\frac{53}{9}\mathbf{j}+\frac{14}{9}\mathbf{k}}$



Question 128 (**+)**

The quadrilateral $ABCD$ is a rectangle with the vertex A having coordinates $(2,1,2)$.

The diagonals of the rectangle intersect at the point with coordinates $(7,0,4)$.

a) Find the coordinates of the point C.

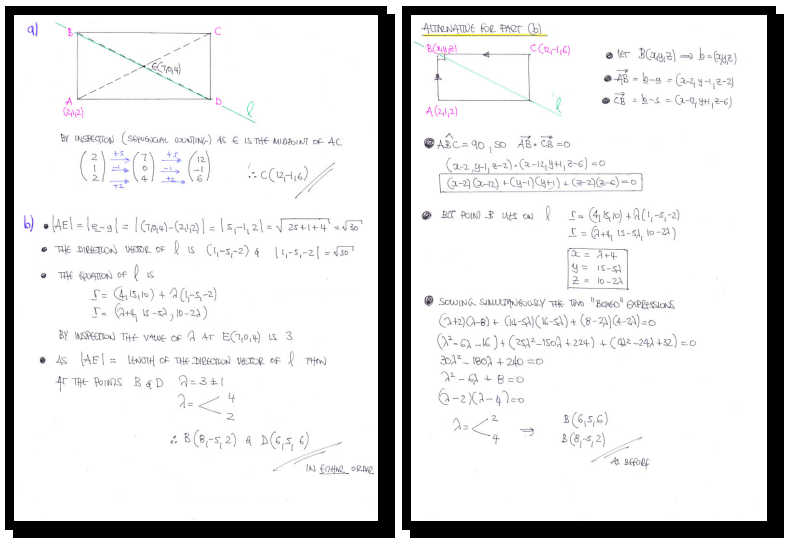
The points B and D both lie on the straight line with vector equation

$$\mathbf{r}=4\mathbf{i}+15\mathbf{j}+10\mathbf{k}+\lambda(\mathbf{i}-5\mathbf{j}-2\mathbf{k}),$$

where λ is a scalar parameter.

b) Determine the coordinates of B and D.

$\boxed{}$, $\boxed{C(12,-1,6)}$, $\boxed{B(6,5,6),\ D(8,-5,2)\text{ in any order}}$



Question 129 (**+)**

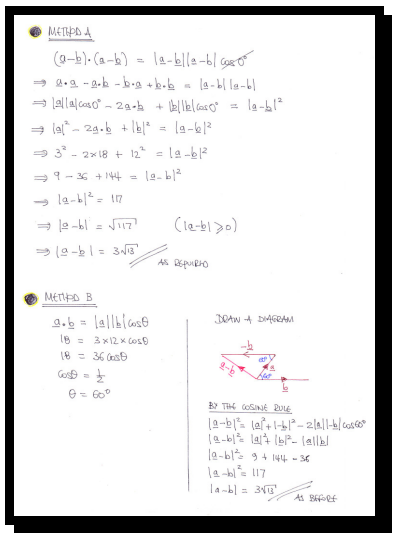
The vectors $\mathbf{a}$ and $\mathbf{b}$ are such so that

$$|\mathbf{a}|=3\,,\ |\mathbf{b}|=12 \quad \text{and} \quad \mathbf{a}\cdot\mathbf{b}=18\ .$$

Show clearly that

$$|\mathbf{a}-\mathbf{b}|=3\sqrt{13}\,.$$

[ ], [proof]



Question 130 (**+)**

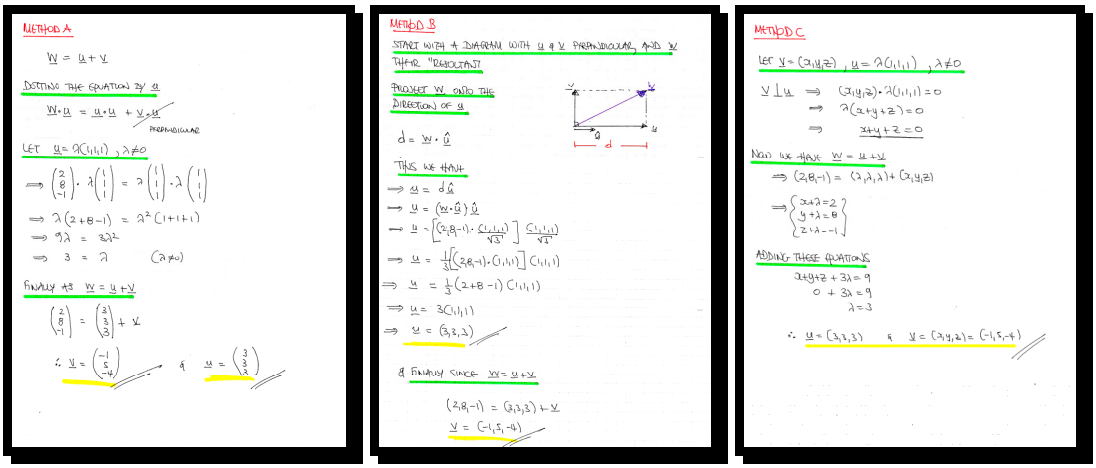
It is given that

$$\mathbf{w}=\mathbf{u}+\mathbf{v},$$

where $\mathbf{w}=2\mathbf{i}+8\mathbf{j}-\mathbf{k}$.

Given further that $\mathbf{u}$ is in the direction $\mathbf{i}+\mathbf{j}+\mathbf{k}$, and the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular to one another, determine $\mathbf{u}$ and $\mathbf{v}$ in component form.

$\boxed{}$, $\boxed{\mathbf{u}=3\mathbf{i}+3\mathbf{j}+3\mathbf{k}}$, $\boxed{\mathbf{v}=-\mathbf{i}+5\mathbf{j}-4\mathbf{k}}$



Question 131 (**+)**

The points $A(3,3,2)$, $B(6,4,3)$ and $C(5,1,4)$ are referred with respect to a fixed origin O. The point M is the midpoint of AC.

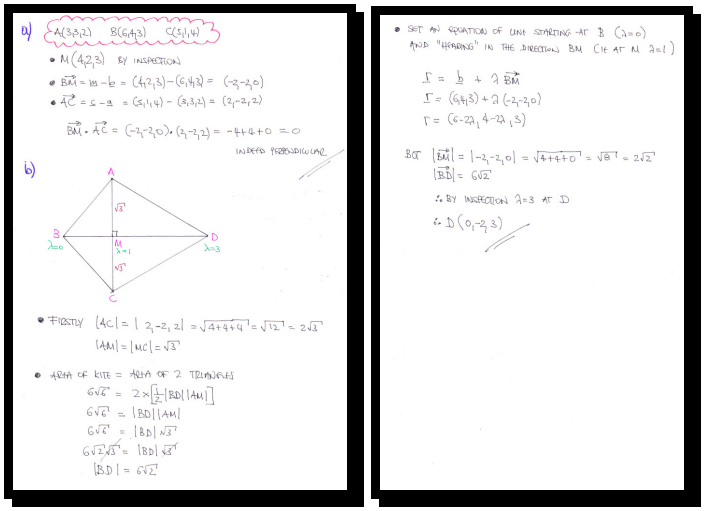
a) Show that $\overrightarrow{BM}$ is perpendicular to $\overrightarrow{AC}$.

The point D is such so that $ABCD$ is a kite with an area of $6\sqrt{6}$.

The straight line BD is a line of symmetry for the kite $ABCD$.

b) Find the coordinates of D.

$\boxed{}$, $\boxed{D(0,-2,3)}$



Question 132 **(****+)**

Relative to a fixed origin O, the straight lines l and m have vector equations

$$\mathbf{r}_1=\begin{pmatrix}p\\4\\5\end{pmatrix}+t\begin{pmatrix}q\\-1\\2\end{pmatrix}\qquad\text{and}\qquad\mathbf{r}_2=\begin{pmatrix}9\\0\\16\end{pmatrix}+s\begin{pmatrix}1\\-2\\7\end{pmatrix},$$

where t and s are scalar parameters, and p and q are scalar constants.

The point A is the intersection of l and m, and the cosine of acute angle θ between l and m is $\frac{1}{3}\sqrt{6}$.

a) Find the value of p and the value of q, given that q is a positive integer.

b) Determine the coordinates of A.

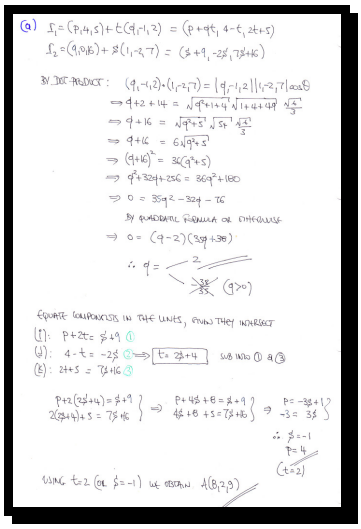
The point B has coordinates $B(12,5,9)$.

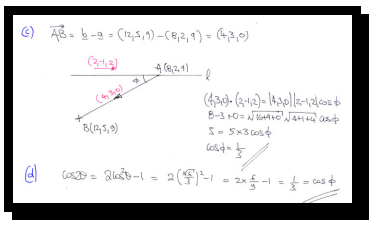
c) Find the cosine of the acute angle φ between AB and l.

d) Hence show, without the use of any calculating aid, that

$$\varphi=2\theta.$$

$$\boxed{p=0},\ \boxed{q=2},\ \boxed{A(8,2,9)},\ \boxed{\cos\varphi=\frac{1}{3}}$$





Question 133 (**+)**

The straight lines l_1 and l_2 have the vector equations given below

$$\mathbf{r}_1 = 3\mathbf{i}+\mathbf{j}+7\mathbf{k}+\lambda\left(2\mathbf{i}-\mathbf{j}+3\mathbf{k}\right),$$

$$\mathbf{r}_2 = 3\mathbf{i}+3\mathbf{k}+\mu\left(2\mathbf{i}-2\mathbf{j}-\mathbf{k}\right),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point P and find its coordinates.

The points A and C lie on l_1 and the points B and D lie on l_2, such that $ABCD$ forms a parallelogram.

The point A has coordinates $(7,-1,13)$.

b) Find …

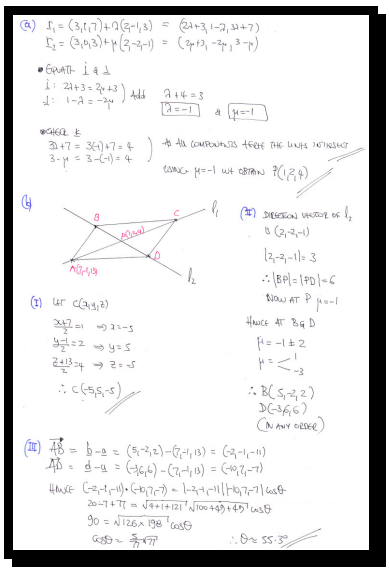
i. … the coordinates of C.

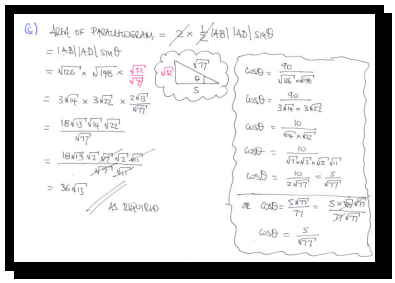
ii. … the coordinates of B and D, given further that $|BD|=12$.

iii. … the angle $\angle BAD$.

c) Show that the exact area of the parallelogram $ABCD$ is $36\sqrt{13}$.

$\boxed{P(1,2,4)}$, $\boxed{C(-5,5,-5)}$, $\boxed{B(5,-2,2),\ D(-3,6,6) \text{ in any order}}$, $\boxed{\angle BAD \approx 55.3^\circ}$





Question 134 (**+)**

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 7\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda\left(\mathbf{i}-\mathbf{j}+a\mathbf{k}\right)$$

$$\mathbf{r}_2 = 3\mathbf{i}+b\mathbf{j}+5\mathbf{k}+\mu\left(\mathbf{i}-\mathbf{k}\right),$$

where λ and μ are scalar parameters, and a and b are scalar constants.

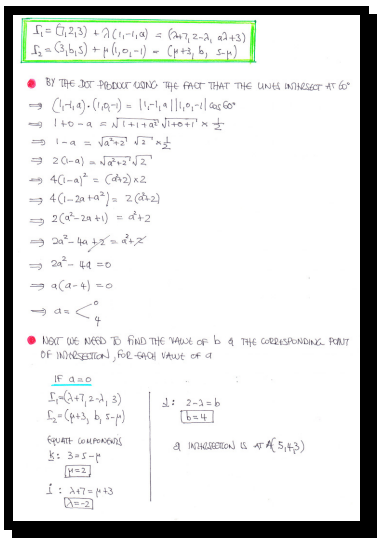
It is further given that the point A is the intersection of l_1 and l_2, and the acute angle between l_1 and l_2 is $60°$.

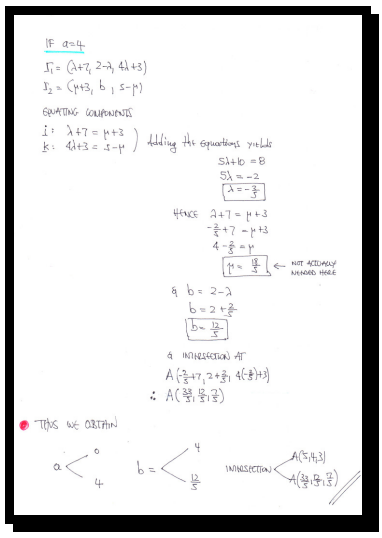
Find in any order …

… the two possible pairings for the value of a and the value b.

… the possible coordinates of A for each possible pair of a and b.

$\square$, $\boxed{a=0 \text{ with } b=4 \text{ and } A(5,4,3)}$, $\boxed{a=4 \text{ with } b=\frac{12}{5} \text{ and } A\left(\frac{33}{5},\frac{12}{5},\frac{7}{5}\right)}$





Question 135 (**+)**

The point A has position vector $-\mathbf{i}+7\mathbf{j}-\mathbf{k}$.

a) Find the vector equation of the straight line l_1 which passes through A and is parallel to the vector $3\mathbf{i}-2\mathbf{j}+2\mathbf{k}$.

The straight line l_2 has equation

$$\mathbf{r}_2 = 9\mathbf{i}-9\mathbf{j}+8\mathbf{k}+\mu\left(3\mathbf{i}-3\mathbf{j}+4\mathbf{k}\right),$$

where μ is a scalar parameter.

b) Show that …

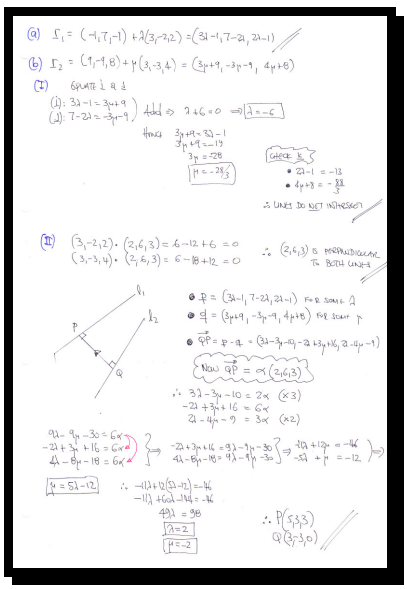
i. … l_1 and l_2 do not intersect.

ii. … the vector $2\mathbf{i}+6\mathbf{j}+3\mathbf{k}$ is perpendicular to both l_1 and l_2.

The point P lies on l_1 and the point Q lies on l_2 so that the distance PQ is least.

c) Find the coordinates of P and Q.

$$\boxed{\mathbf{r}_1 = -\mathbf{i}+7\mathbf{j}-\mathbf{k}+\lambda\left(3\mathbf{i}-2\mathbf{j}+2\mathbf{k}\right)}, \quad \boxed{P(5,3,3) \ \ \& \ \ Q(3,-3,0)}$$



Question 136 (**+)**

With respect to a fixed origin O, the straight lines L_1 and L_2 have respective vector equations

$$\mathbf{r}_1=\begin{pmatrix}-1\\5\\-1\end{pmatrix}+t\begin{pmatrix}1\\-2\\1\end{pmatrix} \quad \text{and} \quad \mathbf{r}_2=\begin{pmatrix}-3\\1\\5\end{pmatrix}+s\begin{pmatrix}1\\0\\-1\end{pmatrix},$$

where t and s are scalar parameters.

The points A and C lie on L_1 and L_2, where $t=0$ and $s=0$, respectively.

a) Find $\left|\overrightarrow{AC}\right|$, in exact surd form.

b) Show that L_1 and L_2 intersect at some point B and find its coordinates.

c) Find the size of the angle θ, between L_1 and L_2.

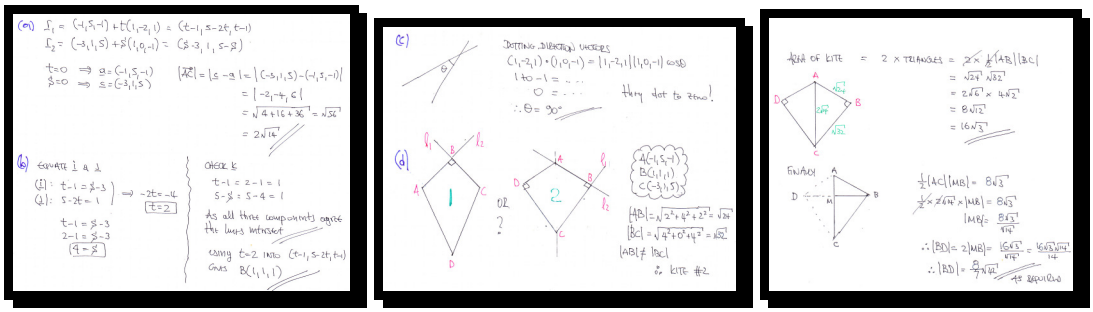
The point D is such so that $ABCD$ is a kite.

d) Show further that …

i. … the area of the kite is $16\sqrt{3}$ square units.

ii. … the length of BD is $\frac{8}{7}\sqrt{42}$.

$$\boxed{\left|\overrightarrow{AC}\right|=2\sqrt{14}}\,, \quad \boxed{B(1,1,1)}\,, \quad \boxed{\theta=90^\circ}$$



Question 137 (**+)**

A person standing at a fixed origin O observes an insect taking off from a point A on level horizontal ground. The position vector of the insect $\mathbf{r}$ metres, t seconds after taking off, is given by

$$\mathbf{r}=(t+1)\mathbf{i}+\left(2t+\frac{1}{2}\right)\mathbf{j}+2t\mathbf{k}\,.$$

All distances are in metres and the coordinates axes Ox, Oy, Oz are oriented due east, due north and vertically upwards, respectively.

a) Find …

i. … the bearing of the insect's flight path.

ii. … the angle between the flight path and the horizontal ground.

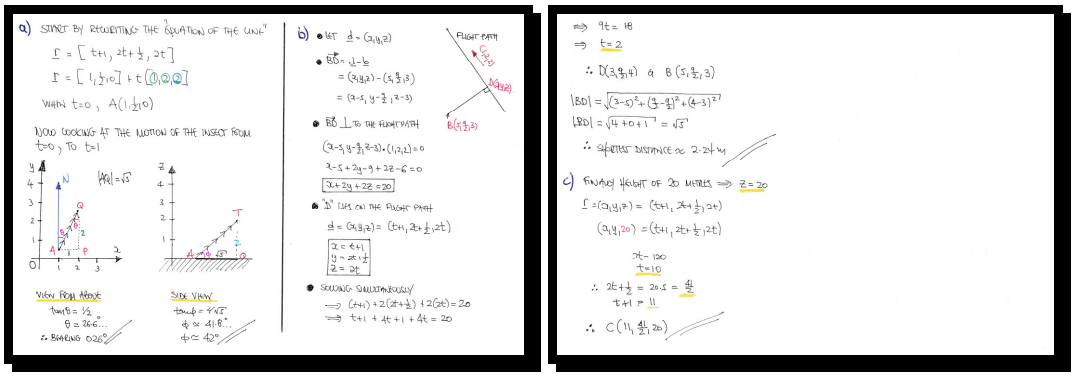
The roof top of a garden shed is located at $B\left(5,\frac{9}{2},3\right)$.

b) Calculate the shortest distance between the insect's path and the point B.

When the insect reaches a height of 20 metres above the ground, at the point C, the insect gets eaten by a bird.

c) Determine the coordinates of C.

$\boxed{}$, $\boxed{\text{bearing}\approx 027^\circ}$, $\boxed{\theta\approx 42^\circ}$, $\boxed{\sqrt{5}}$, $\boxed{C\left(11,\frac{41}{2},20\right)}$



Question 138 (**+)**

With respect to a fixed origin O, the points with coordinates $A(4,3,-1)$, $B(5,1,2)$, $C(2,0,3)$ and $D(4,2,-1)$ are given.

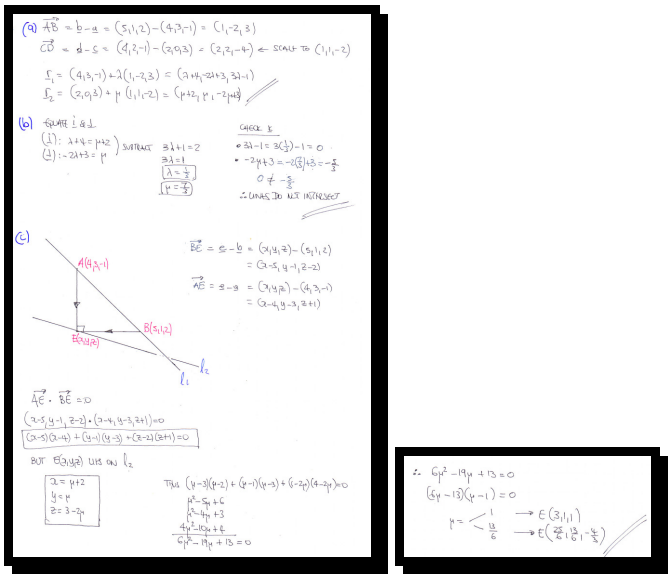
a) Find the vector equation of the line l_1 which passes through A and B, and the vector equation of the line l_2 which passes through C and D.

b) Show that l_1 and l_2 do not intersect.

The point E is on l_2 so that $\angle AEB = 90°$.

c) Show that one possible position for E has coordinates $\left(\frac{25}{6},\frac{13}{6},-\frac{4}{3}\right)$ and find the coordinates of the other possible position.

$$\boxed{\mathbf{r}_1 = 4\mathbf{i}+3\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+3\mathbf{k})}, \quad \boxed{\mathbf{r}_2 = 2\mathbf{i}+3\mathbf{k}+\mu(\mathbf{i}+\mathbf{j}-2\mathbf{k})}, \quad \boxed{E(3,1,1)}$$



Question 139 (**+)**

Relative to a fixed origin O, the points A and B have respectively position vectors $\mathbf{i}-3\mathbf{j}-9\mathbf{k}$ and $-4\mathbf{j}-10\mathbf{k}$.

a) Find the vector equation of the straight line l_1 which passes through A and B.

The straight line l_2 has the vector equation

$$\mathbf{r}_2 = 6\mathbf{i}+\mathbf{k}+\mu\left(-\mathbf{i}+p\mathbf{j}+q\mathbf{k}\right),$$

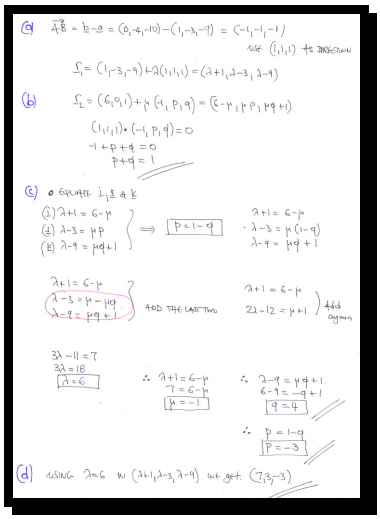
where μ is a scalar parameter, and p and q are scalar constants.

b) Given that l_1 and l_2 are perpendicular, write an equation in terms of p and q.

c) Given further that l_1 and l_2 intersect, find the value of p and the value of q.

d) Determine the position vector of the point of intersection of l_1 and l_2.

$$\boxed{\mathbf{r}_1 = \mathbf{i}-3\mathbf{j}-9\mathbf{k}+\lambda\left(\mathbf{i}+\mathbf{j}+\mathbf{k}\right)}, \quad \boxed{p+q=1}, \quad \boxed{p=-3,\ q=4}, \quad \boxed{7\mathbf{i}+3\mathbf{j}-3\mathbf{k}}$$



Question 140 (**+)**

The straight lines l_1 and l_2 have respective vector equations

$$\mathbf{r}_1 = 3\lambda\mathbf{i} + (6-2\lambda)\mathbf{j} + (2\lambda+1)\mathbf{k}$$

$$\mathbf{r}_2 = (3\mu+10)\mathbf{i} + (-3\mu-10)\mathbf{j} + (4\mu+10)\mathbf{k}$$

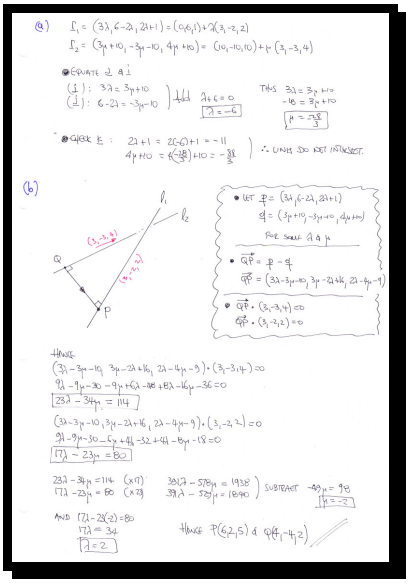
where λ and μ are scalar parameters.

a) Show that l_1 and l_2 do not intersect.

The point P lies on l_1 and the point Q lies on l_2 so that the distance PQ is least.

b) Find the coordinates of P and the coordinates of Q.

$$\boxed{P(6,2,5) \quad \& \quad Q(4,-4,2)}$$



Question 141 (**+)**

Relative to a fixed origin O, the straight lines l_1 and l_2 have the following respective vector equations

$$\mathbf{r}_1 = \begin{pmatrix} 8 \\ q \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ p \end{pmatrix} \qquad \text{and} \qquad \mathbf{r}_2 = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters, and p and q are scalar constants.

a) Given that l_1 and l_2 are perpendicular, determine the value of p.

The point D is the intersection of l_1 and l_2

b) Find the value of q and the coordinates of D.

Another straight line l_3 intersects **both** l_1 and l_2, and is also perpendicular to **both** l_1 and l_2.

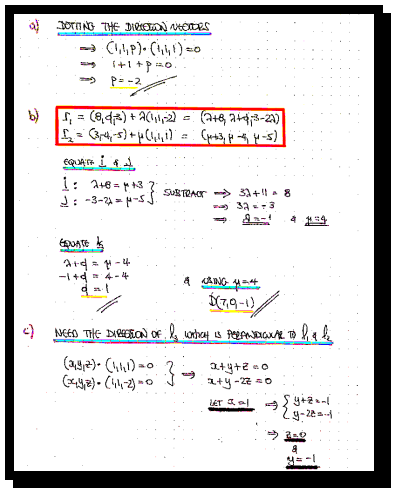
c) Find a vector equation for l_3.

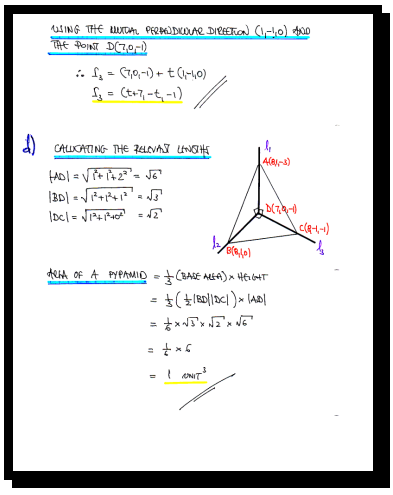
You may not use the vector (cross) product in this part

The points $A(8,1,-3)$, $B(8,1,0)$ and $C(8,-1,-1)$ lie on l_1, l_2 and l_3, respectively.

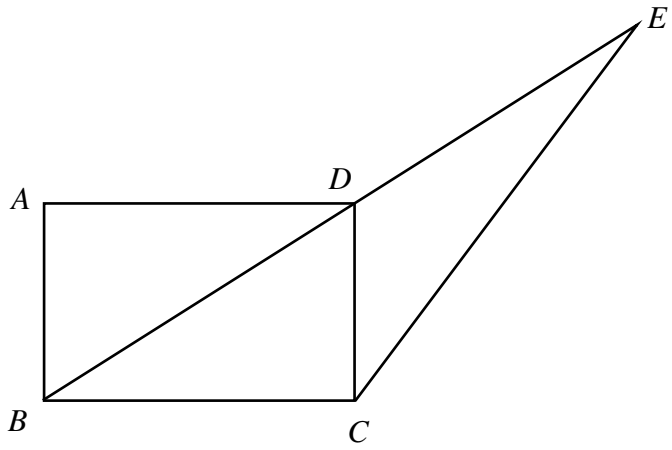
d) Show that the volume of the triangle based pyramid with vertices at A, B, C and D is 1 cubic unit.

$\boxed{}$, $\boxed{p=-2}$, $\boxed{q=1}$, $\boxed{D(7,0,-1)}$, $\boxed{\mathbf{r}=7\mathbf{i}-\mathbf{k}+\nu(\mathbf{i}-\mathbf{j})}$





Question 142 (*****)

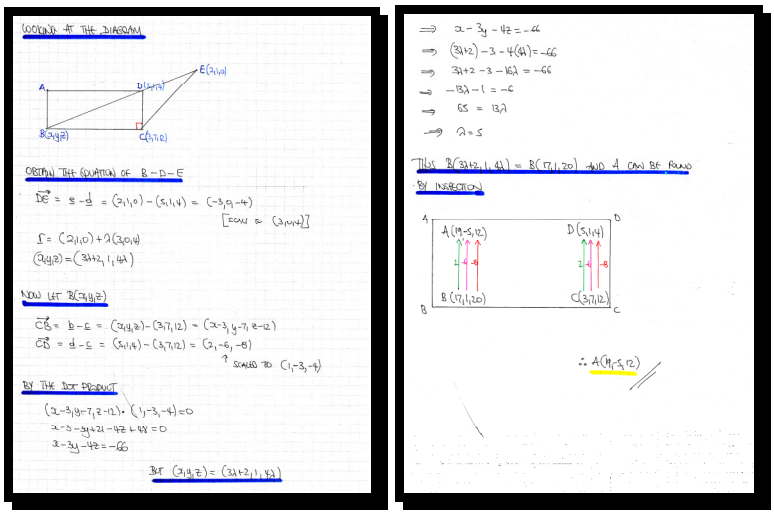


The figure above shows the rectangle $ABCD$, where $C(3,7,12)$ and $D(5,1,4)$.

The point $E(2,1,0)$ is such so that BDE and EC are straight lines.

Use vector methods to determine the coordinates of A.

$\boxed{}$, $\boxed{A(19,-5,12)}$



Question 143 (*****)

The coordinates in this question are relative to a fixed origin O at $(0,0,0)$.

The straight line l_1 has vector equation

$$\mathbf{i}+3\mathbf{j}+5\mathbf{k}+\lambda(-\mathbf{i}+3\mathbf{j}+\mathbf{k}),$$

where λ is a scalar parameter.

The straight line l_2 passes thought the point with coordinates $(6,0,6)$ and is in the direction $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$.

a) Verify that $A(4,3,5)$ is the intersection of l_1 and l_2, and show further that $B(12,-9,9)$ lies on l_2.

The point $C(6,-3,3)$ lies on l_1.

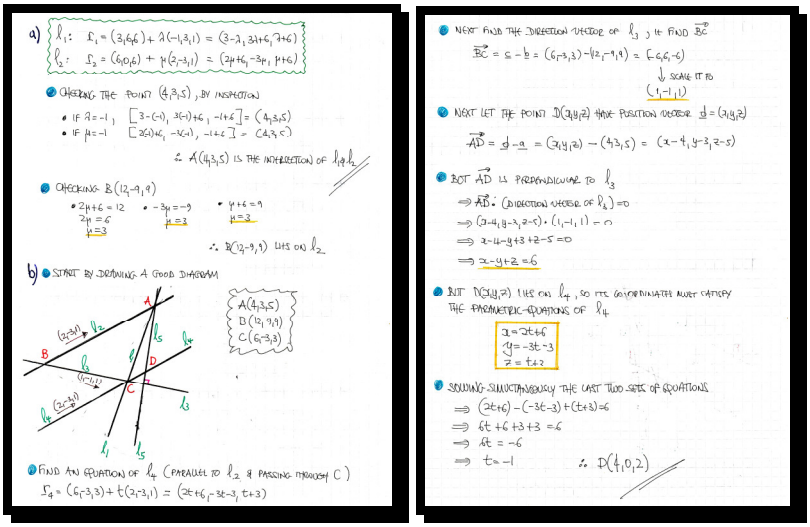
The straight line l_3 passes through B and C.

The straight line l_4 is parallel to l_2 and passes through C.

The straight line l_5 is perpendicular to l_3 and passes through A.

b) Given that l_4 and l_5 intersect at the point D, find the coordinates of D.

$\square$, $\boxed{D(4,0,2)}$



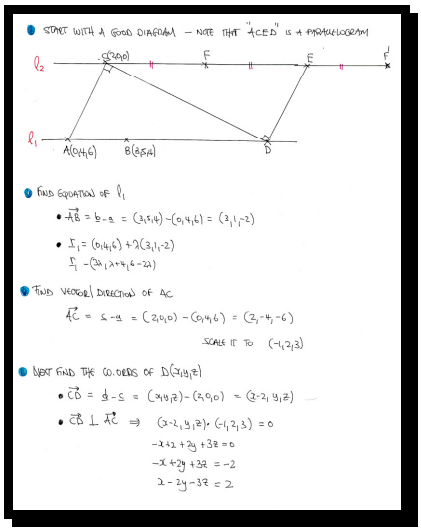
Question 144 (*****)

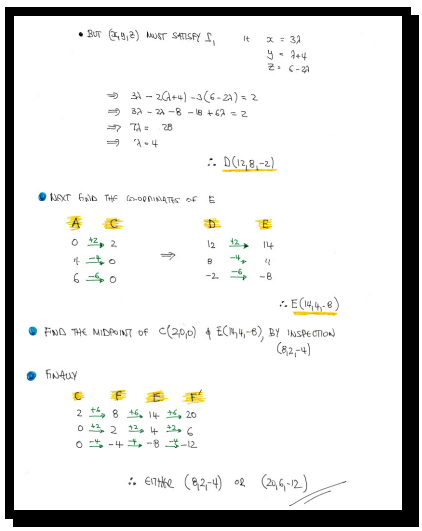
Relative to a fixed origin O at $(0,0,0)$ the points A, B and C have coordinates $(0,4,6)$, $(3,5,4)$ and $(2,0,0)$, respectively.

- The straight line l_1 passes through A and B.
- The straight line l_2 passes through C and is parallel to l_1.
- The point D lies on l_1 so that $\angle ACD = 90°$.
- The point E lies on l_2 so that $\angle CDE = 90°$.
- The point F lies on l_2 so that $|EC| = 2|EF|$.

Determine the coordinates of the possible positions of F.

$\boxed{\quad}$, $\boxed{F(8,2,-4) \cup F(20,6,-12)}$





Question 145 (*****)

With respect to a fixed origin O, the points A, B and C have position vectors

$$\mathbf{a}=\begin{pmatrix}0\\5\\2\end{pmatrix},\ \mathbf{b}=\begin{pmatrix}8\\2\\7\end{pmatrix} \text{ and } \mathbf{c}=\begin{pmatrix}11\\0\\1\end{pmatrix}.$$

a) Determine the volume of the cube, with vertices the points A, B and C.

The points P, Q and R are vertices of a different cube, so that

$$\overrightarrow{PQ}=\begin{pmatrix}0\\1\\7\end{pmatrix} \text{ and } \overrightarrow{PR}=\begin{pmatrix}k\\4\\3\end{pmatrix},$$

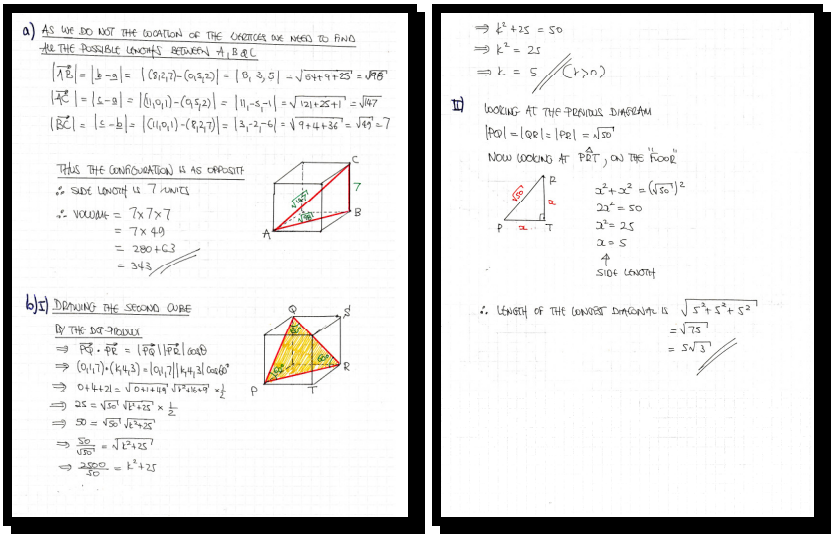
where k is a positive constant.

b) Given that $\angle QPR=60°$, determine …

i. … the value of k.

ii. … the length of the diagonal of the second cube.

$\boxed{}$, $\boxed{\text{volume}=343}$, $\boxed{k=5}$, $\boxed{\sqrt{75}=5\sqrt{3}}$



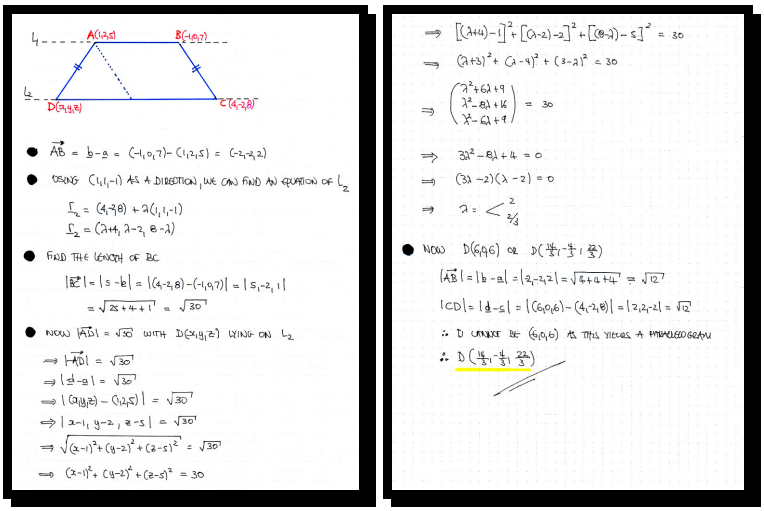
Question 146 (*****)

Relative to a fixed origin O at $(0,0,0)$ the points A, B and C have coordinates $(1,2,5)$, $(-1,0,7)$ and $(4,-2,8)$, respectively.

The point D is such so that $ABCD$ is an isosceles trapezium with $|BC|=|AD|$.

Determine the coordinates of D.

$\square$, $\boxed{D\left(\frac{14}{3},-\frac{4}{3},\frac{22}{3}\right)}$

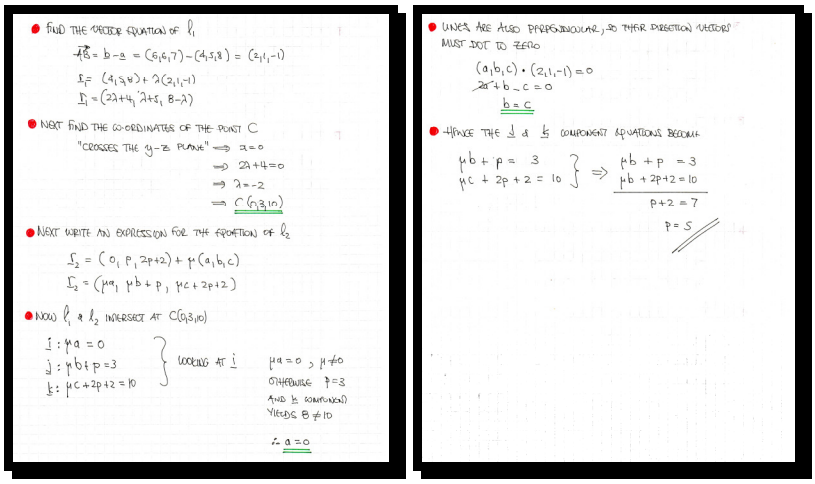


Question 147 (*****)

Relative to a fixed origin O, the points A and B have position vectors $4\mathbf{i}+5\mathbf{j}+8\mathbf{k}$ and $6\mathbf{i}+6\mathbf{j}+7\mathbf{k}$, respectively. The straight line l_1 passes through A and B, and crosses the $y-z$ plane at the point C. The straight line l_2 passes through the point D with position vector $p\mathbf{j}+(2p+2)\mathbf{k}$, where p is a scalar constant.

Given that l_1 and l_2 are perpendicular, and intersect at C, find the value of p.

$\boxed{}$, $\boxed{p=5}$



Question 148 (*****)

The points A and B, have respective position vectors $\mathbf{a}$ and $\mathbf{b}$, relative to a fixed origin O.

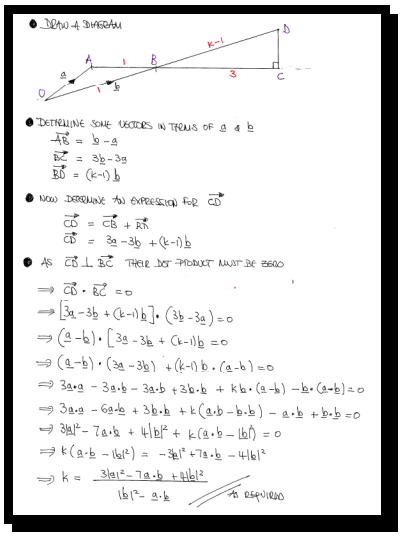
The point C lies on AB produced such that $|AB| : |AC| = 1 : 4$

The point D lies on OB produced such that $|OB| : |OD| = 1 : k$, where $|OB| : |OD| = 1 : k$ is a scalar constant.

Given that AB is perpendicular to CD show that

$$k = \frac{3|\mathbf{a}|^2 - 7\mathbf{a}\cdot\mathbf{b} + 4|\mathbf{b}|^2}{|\mathbf{b}|^2 - \mathbf{a}\cdot\mathbf{b}}.$$

, proof



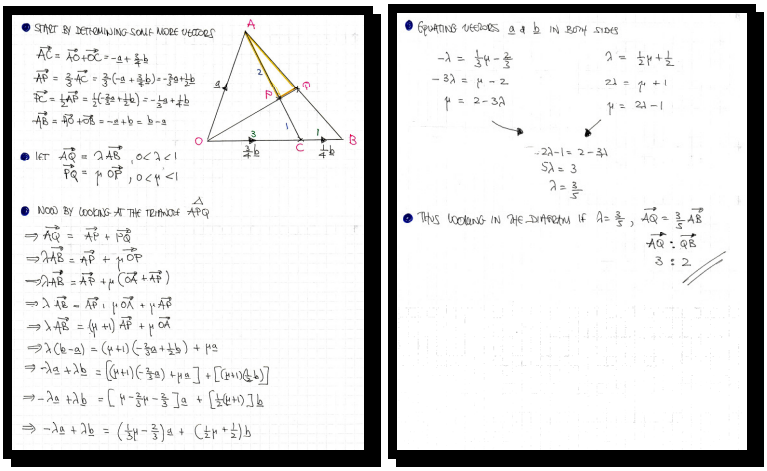
Question 149 (*****)

OAB is a triangle and $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

- The point C lies on OB so that $OC:CB=3:1$.
- The point P lies on AC so that $AP:PC=2:1$.
- The point Q lies on AB so that O, P and Q are collinear.

Determine the ratio $AQ:QB$.

$\boxed{}$, $\boxed{AQ:QB=3:2}$



Question 150 (*****)

Relative to a fixed origin O, the straight lines l_1 and l_2 have vector equations

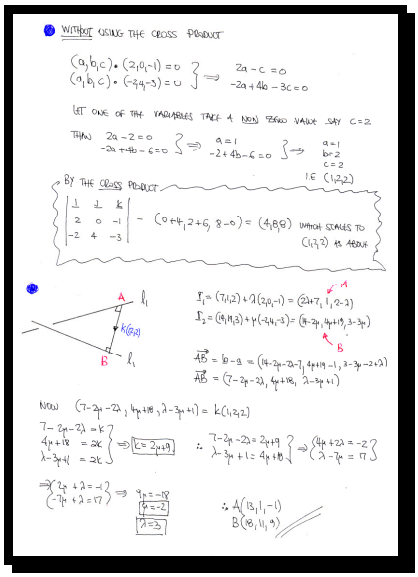
$$\mathbf{r}_1 = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 14 \\ 19 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix},$$

where λ and μ are scalar parameters.

The point A lies on l_1 and the point B lies on l_2, so that the distance AB is least.

Find the coordinates of A and the coordinates of B.

$\square$, $\boxed{A(13,1,-1)}$, $\boxed{B(18,11,9)}$



Question 151 (*****)

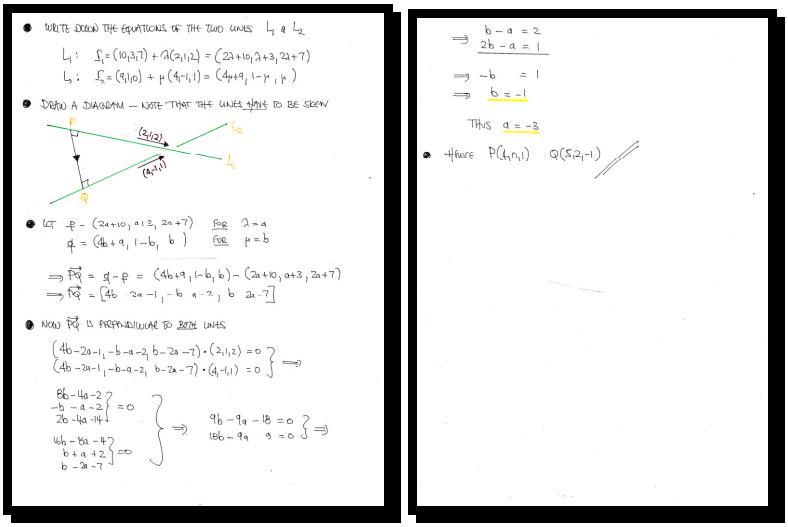
The point P lies on the straight line L_1, which is parallel to the vector $2\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and passes through the point with coordinates $(10,3,7)$, relative to an origin at $(0,0,0)$.

The point Q lies on another straight line L_2, which is in the direction of the vector $4\mathbf{i}-\mathbf{j}+\mathbf{k}$ and passes through the point with coordinates $(9,1,0)$.

The straight line L_3 is perpendicular to both L_1 and L_2, and meets L_1 and L_2 at the points P and Q, respectively.

Find the coordinates of P and Q.

$\boxed{}$, $\boxed{P(4,0,1),\ Q(5,2,-1)}$



Question 152 (*****)

The straight line l_1, where λ is a scalar parameter, has vector equation

$$\mathbf{r}=10\mathbf{i}+8\mathbf{j}+5\mathbf{k}+\lambda\left(\mathbf{i}+\mathbf{j}+\mathbf{k}\right).$$

The points $A(4,1,3)$ and $B(6,5,-3)$ lie on the straight line l_2.

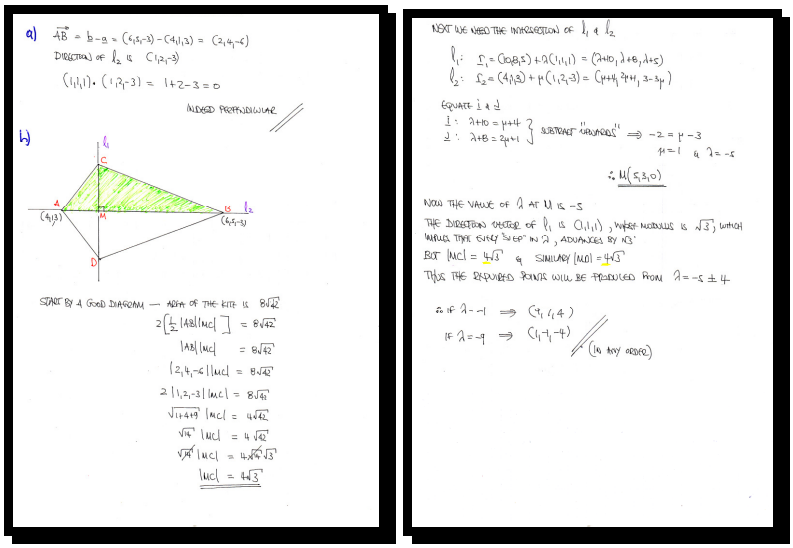
a) Given that l_1 and l_2 lie on the same plane, show that l_1 is perpendicular to l_2.

The points C and D lie on l_1 so that the resulting quadrilateral $ACBD$ is a kite, whose line of symmetry is l_2.

b) Given further that the area of the kite is $8\sqrt{42}$ square units, determine the possible coordinates of the points C and D.

$\boxed{}$, $\boxed{\mathbf{r}=4\mathbf{i}+\mathbf{j}+3\mathbf{k}+\mu\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)}$, $\boxed{|CD|=8\sqrt{3}}$,

$\boxed{C(9,7,4)\ \&\ D(1,-1,-4)\ \text{in any order}}$



Question 153 (*****)

The straight line L_1 passes through the points A and B, whose respective position vectors relative to a fixed origin O are

$$\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}.$$

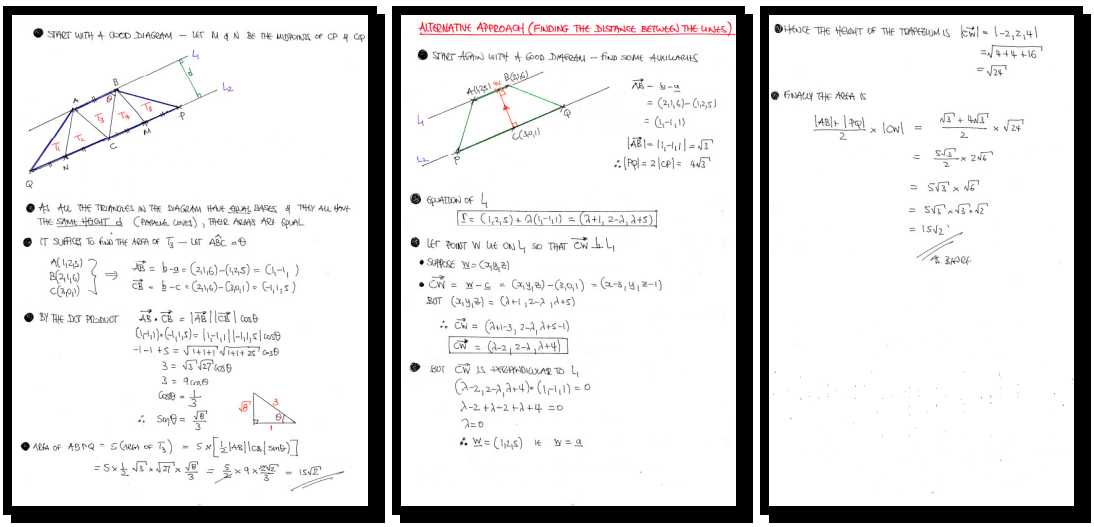
The point C has position vector $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$.

The straight line L_2 passes through C and is parallel to L_1.

The points P and Q both lie on L_2 so that $|CP| = |CQ| = 2|AB|$.

Find the area of the quadrilateral with vertices at A, B, P and Q.

$\square$, $\boxed{15\sqrt{2}}$



Question 154 (*****)

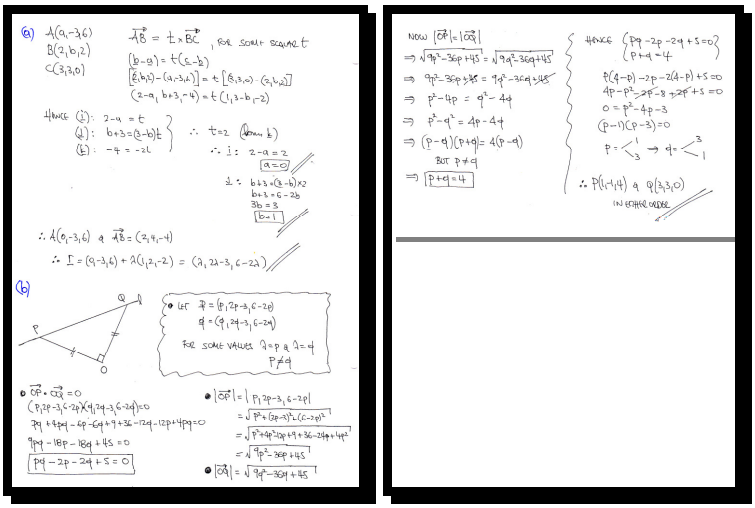
Relative to a fixed origin O, the straight line l passes through the points $A(a,-3,6)$, $B(2,b,2)$ and $C(3,3,0)$, where a and b are constants.

a) Find the value of a and the value of b, and hence find a vector equation of l.

The points P and Q lie on the l so that $|OP|=|OQ|$ and $\angle POQ=90°$.

b) Find the coordinates of P and the coordinates of Q.

$\boxed{}$, $\boxed{a=0,\,b=1}$, $\boxed{\mathbf{r}=-3\mathbf{j}+6\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}-2\mathbf{k})}$, $\boxed{(1,-1,4)\ \&\ (3,3,0)}$



Question 155 (*****)

The straight line l has vector equation

$$\mathbf{r}=3\mathbf{i}+3\mathbf{k}+\lambda\left(\mathbf{i}+2\mathbf{j}-2\mathbf{k}\right),$$

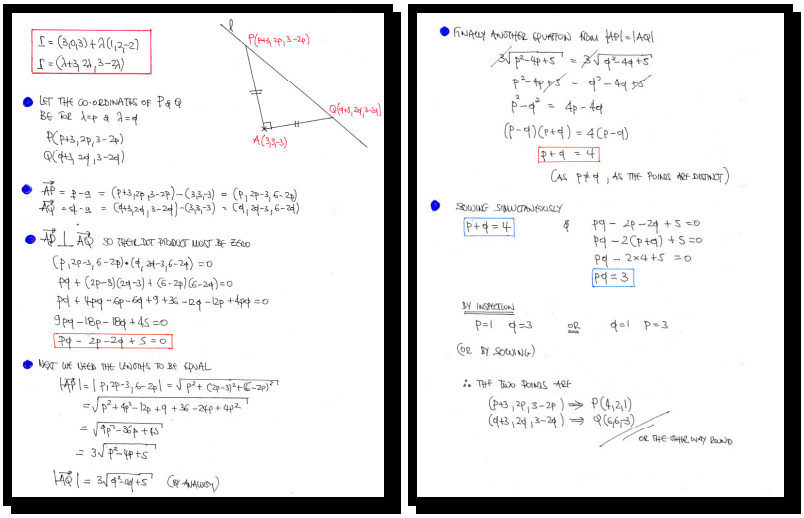
where λ is a scalar parameter.

The point A has coordinates $(3,3,-3)$, relative to a fixed origin O.

The points P and Q lie on the l so that $|AP|=|AQ|$.

Given further that $\angle PAQ=90°$, find the coordinates of P and the coordinates of Q.

$\boxed{}$, $\boxed{(4,2,1)\ \&\ (6,6,-3)}$



Question 156 (*****)

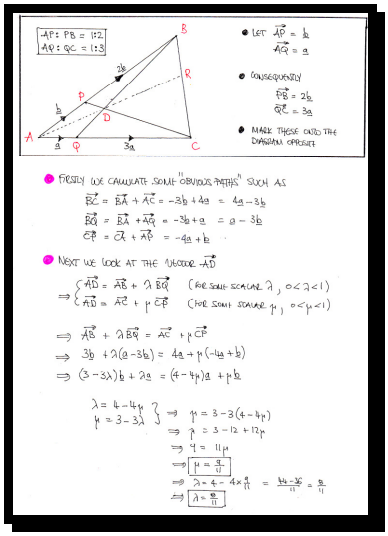
In the acute triangle ABC the following information is given.

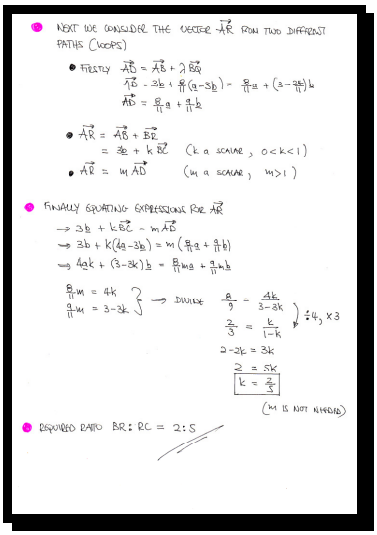
- The point P lies on AB so that $AP:PB=1:2$.
- The point Q lies on AC so that $AQ:QC=1:3$.
- The point D is the intersection of CP and BQ.

The straight line through A and D is extended so that it meets BC at the point R.

Determine the ratio $BR:RC$.

$\boxed{}$, $\boxed{BR:RC=2:5}$





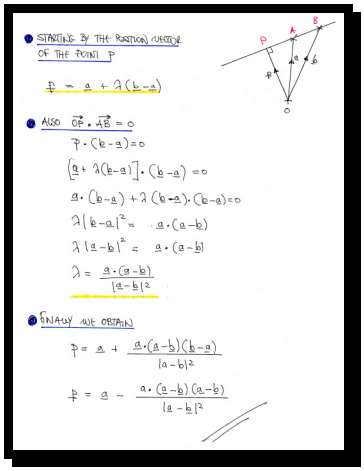
Question 157 (*****)

Relative to a fixed origin O, the position vectors of two points A and B are denoted by $\mathbf{a}$ and $\mathbf{b}$. The point P is the foot of the perpendicular from O to the straight line through A and B.

Show that if $\mathbf{p}$ denotes the position vector of P, then

$$\mathbf{p}=\mathbf{a}-\frac{\mathbf{a}\cdot(\mathbf{a}-\mathbf{b})(\mathbf{a}-\mathbf{b})}{|\mathbf{a}-\mathbf{b}|^2}.$$

, proof



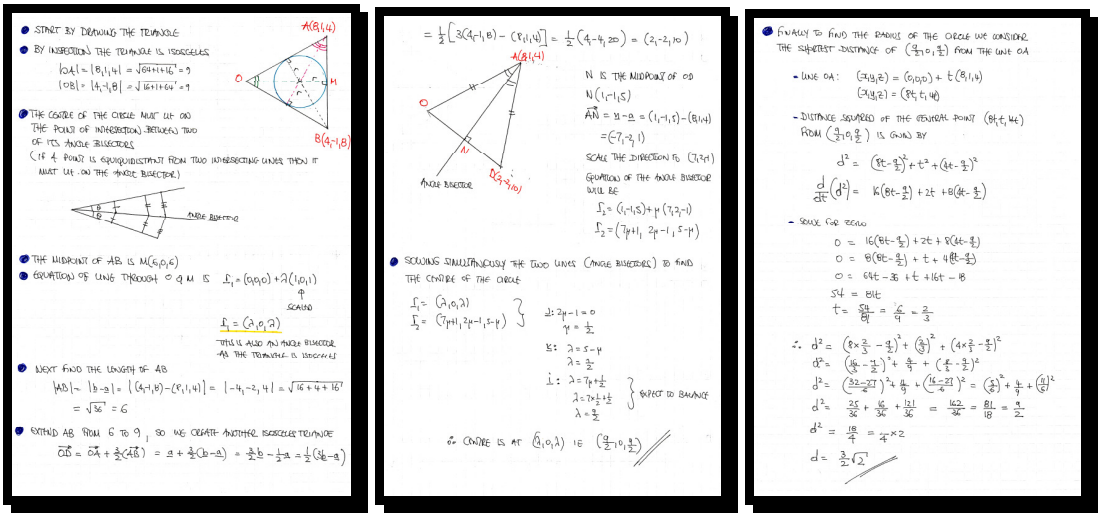
Question 158 (*****)

Relative to a fixed origin O located at the point with coordinates $(0,0,0)$, the points $A(8,1,4)$ and $B(4,-1,8)$ are given.

A circle, with centre at the point P and radius r, is drawn so that the three sides of the triangle OAB are tangents to this circle.

Determine the coordinates of P and the exact value of r.

$\square$, $\boxed{P\left(\frac{9}{2},0,\frac{9}{2}\right)}$, $\boxed{r=\frac{3}{2}\sqrt{2}}$



Question 159 (*****)

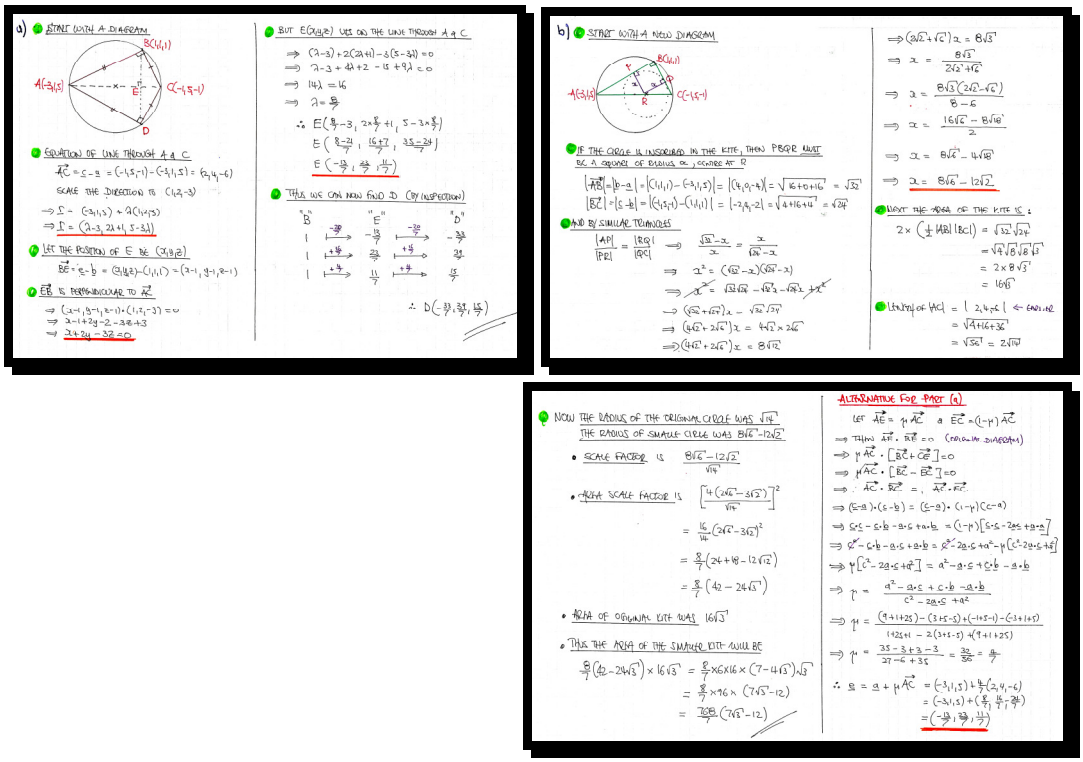
The points $A(-3,1,5)$, $B(1,1,1)$ and $C(-1,5,-1)$ are three of the vertices of the kite $ABCD$, which is circumscribed by a circle.

a) Given that $|AB|=|AD|$ and $|BC|=|DC|$, find the exact coordinates of D.

A smaller circle is circumscribed by the kite, and a smaller kite similar to $ABCD$ is circumscribed by the smaller circle.

b) Determine in exact form the area of the smaller kite.

$\square$, $\boxed{D\left(-\frac{33}{7},\frac{39}{7},\frac{15}{7}\right)}$, $\boxed{\text{area}=\frac{768\left(7\sqrt{3}-12\right)}{7}}$

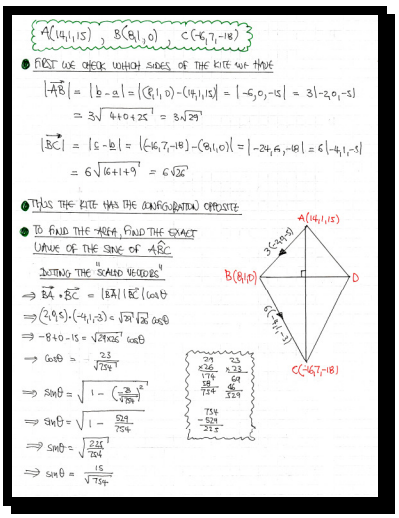


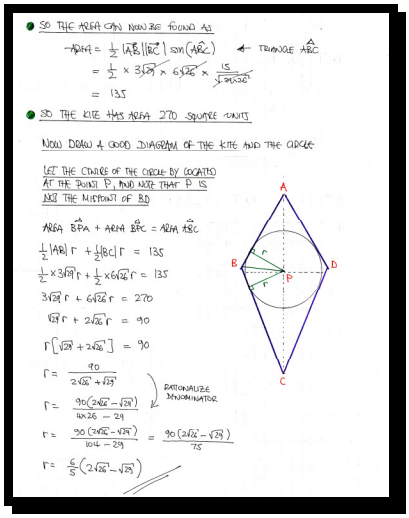
Question 160 (*****)

The points $A(14,1,15)$, $B(8,1,0)$ and $C(-16,7,-18)$ are three of the vertices of the kite $ABCD$. A circle of radius r is circumscribed by the kite.

Find the area of the kite and hence or otherwise determine, in exact simplified surd form, the value of r.

$\boxed{}$, $\boxed{\text{area} = 270}$, $\boxed{r = \frac{6}{5}\left(2\sqrt{26} - \sqrt{29}\right)}$

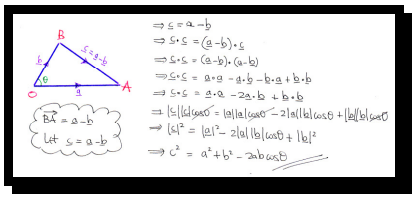




Question 161 (*****)

Use a vector method involving the scalar product to prove the validity of the Cosine Rule.

[ ], proof



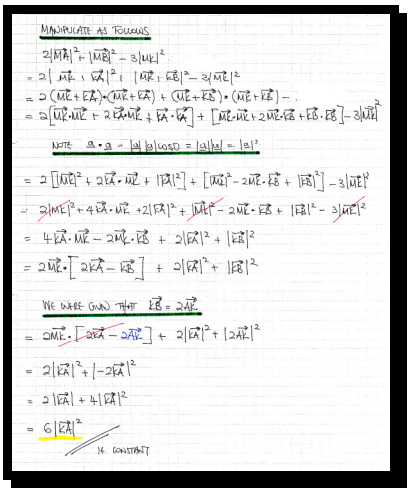
Question 162 (*****)

Three points in space A, B and K are such so that $\overrightarrow{KB} = 2\overrightarrow{AK}$.

Prove that if M is a fourth distinct arbitrary point in space, then

$$2\left|\overrightarrow{MA}\right|^2 + \left|\overrightarrow{MB}\right|^2 - 3\left|\overrightarrow{MK}\right|^2 = \text{constant}.$$

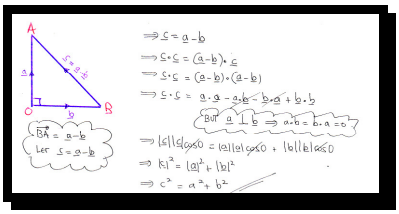
[ ], proof



Question 163 (*****)

Use a vector method involving the scalar product to prove the validity of Pythagoras' Theorem.

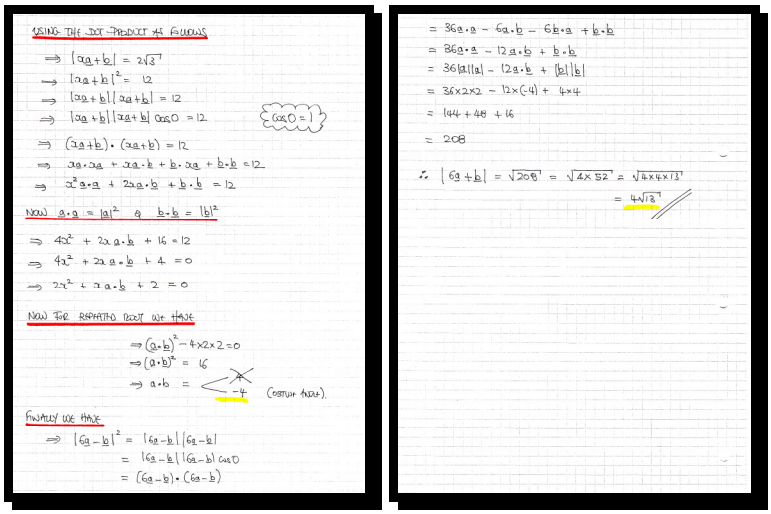
$\boxed{}$, $\boxed{\text{proof}}$



Question 164 (*****)

Find the modulus of $6\mathbf{a} - \mathbf{b}$, given that the equation $|x\mathbf{a} + \mathbf{b}| = 2\sqrt{3}$ has repeated roots in x, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

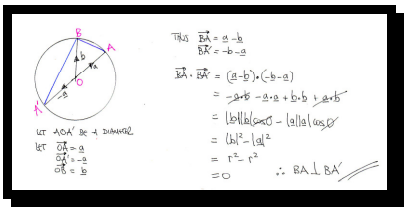
$\boxed{}$, $\boxed{4\sqrt{13}}$



Question 165 (*****)

Use a vector method involving the scalar product to prove that an inscribed angle in a circle which corresponds to a diameter is always a right angle.

[ ], [proof]



Question 166 (*****)

The vertices of the triangle OAB have coordinates $A(6,-18,-6)$, $B(7,-1,3)$, where O is a fixed origin.

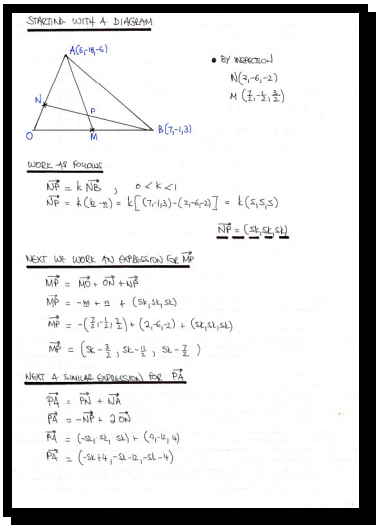
The point N lies on OA so that $ON : NA = 1:2$.

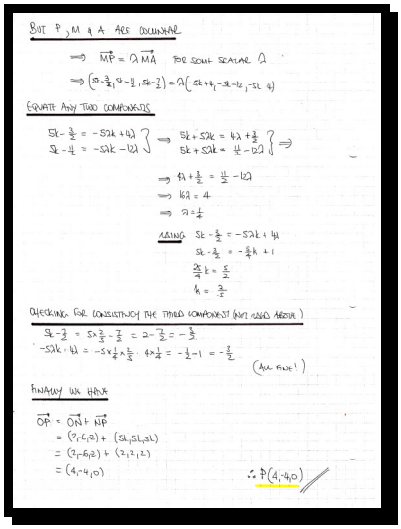
The point M is the midpoint of OB.

The point P is the intersection of AM and BN.

By using vector methods, or otherwise, determine the coordinates of P.

[ ], [$P(4,-4,0)$]





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