SERIES

79 EXAM QUESTIONS

SUMMATIONS BY FORMULAS

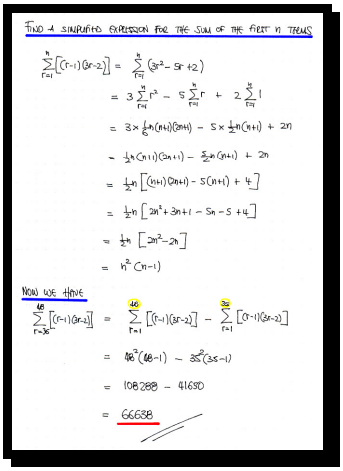
17 BASIC QUESTIONS

Question 1 **(**)**

Use standard results on summations to find the value of

$$\sum_{r=36}^{48}\big[(r-1)(3r-2)\big].$$

$\square$, $\boxed{66638}$



Question 2 **(**)**

Use standard results on summations to show that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+a)(n+b)(n+c),$$

where a, b, and c are positive integers to be found.

$\boxed{}$, $\boxed{a=1,\ b=2,\ c=7}$



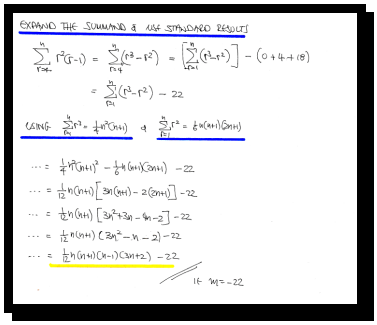
Question 3 (**)

Use standard results on summations to show that

$$\sum_{r=1}^{n}\left[r^2(r-1)\right] = \frac{1}{12}n(n-1)(n+1)(3n+2)+m,$$

where m is an integer to be found.

$\boxed{}$, $\boxed{m=-22}$



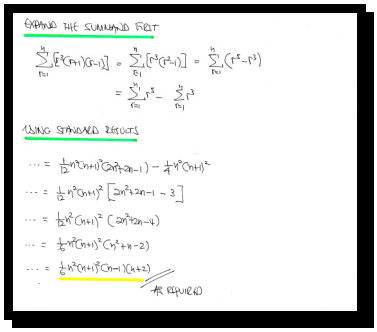
Question 4 (**)

Use standard results on summations to show that

$$\sum_{r=1}^{n}\left[r^3(r+1)(r-1)\right] = \frac{1}{6}n^2(n+1)^2(n-1)(n+2).$$

You may assume without proof that $\displaystyle\sum_{r=1}^{n} r^5 = \frac{1}{12}n^2(n+1)^2\left(2n^2+2n-1\right)$

[ ], [proof]

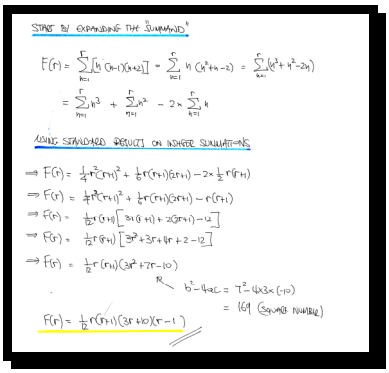


Question 5 (**)

$$F(r) \equiv \sum_{n=1}^{r}\left[n(n-1)(n+2)\right].$$

Use standard results on summations express $F(n)$ in fully factorized from.

$\boxed{}$, $\boxed{F(r)=\frac{1}{12}r(r+1)(r-1)(3r+10)}$

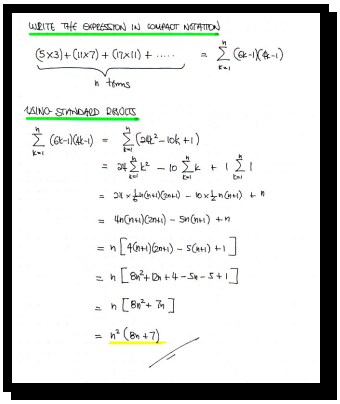


Question 6 (+)**

Find, in fully simplified factorized form, an expression for the sum of the first n terms of the following series.

$$(5\times 3)+(11\times 7)+(17\times 11)+(23\times 15)+...$$

$\boxed{}$, $\boxed{n^2(8n+7)}$



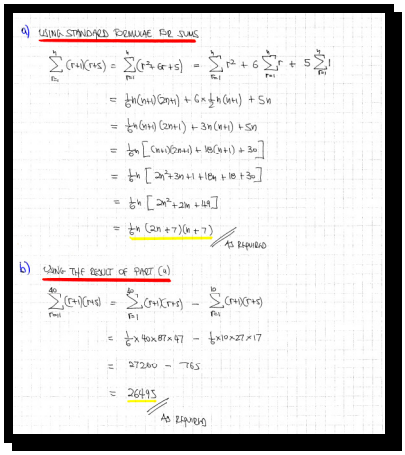
Question 7 (+)**

Show by using standard summation results that …

a) … $\displaystyle\sum_{r=1}^{n}(r+1)(r+5)=\frac{1}{6}n(n+7)(2n+7)$.

b) … $\displaystyle\sum_{r=11}^{40}(r+1)(r+5)=26495$.

[ ], [proof]



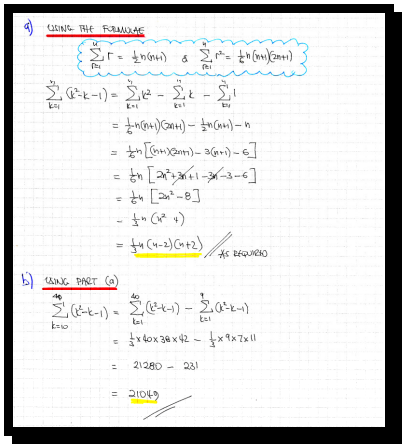
Question 8 **(**+)**

Show by using standard summation results that …

a) … $\displaystyle\sum_{k=1}^{n}\left(k^2-k-1\right)=\frac{1}{3}n\left(n+2\right)\left(n-2\right)$.

b) … $\displaystyle\sum_{k=10}^{40}\left(k^2-k-1\right)=21049$.

[ ], [proof]

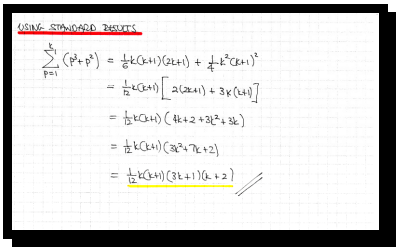


Question 9 **(**+)**

Find, in fully factorized form, an expression for the sum

$$\sum_{p=1}^{k}\left(p^3+p^2\right).$$

$\boxed{}$, $\boxed{\frac{1}{12}k(k+1)(k+2)(3k+1)}$

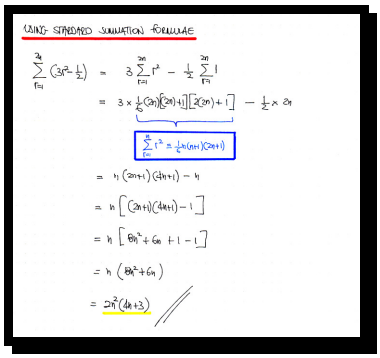


Question 10 **(**+)**

Find, in fully factorized form, an expression for the sum

$$\sum_{r=1}^{2n}\left(3r^2-\frac{1}{2}\right).$$

$\boxed{}$, $\boxed{2n^2(4n+3)}$

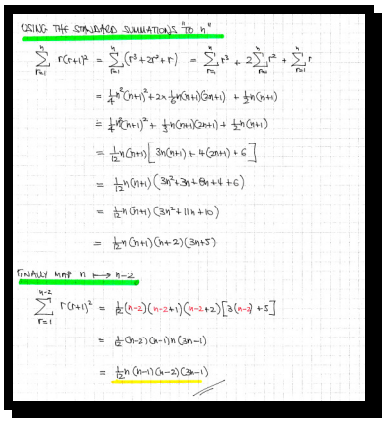


Question 11 (***)

Use standard results on summations to show that

$$\sum_{r=1}^{n-2} r(r+1)^2 = \frac{1}{12} n(n-1)(n-2)(3n-1) .$$

[ ], proof



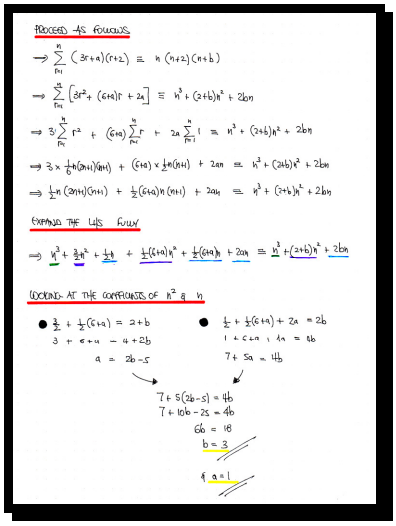
Question 12 (***)

It is given that

$$\sum_{r=1}^{n}\left[(3r+a)(r+2)\right] \equiv n(n+2)(n+b).$$

Determine the values of each of the constants a and b.

$\boxed{}$, $\boxed{a=1}$, $\boxed{b=3}$

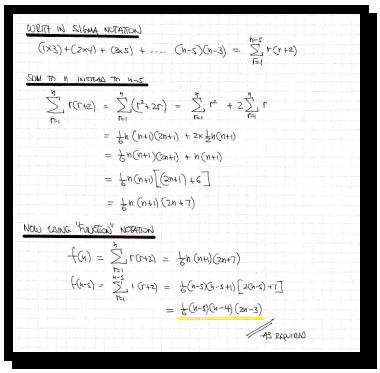


Question 13 **(***)**

Show clearly that

$$(1\times 3)+(2\times 4)+(3\times 5)+\ldots+(n-5)(n-3) \equiv \frac{1}{6}(n+6)(2n+11)(n+5).$$

[ ], proof

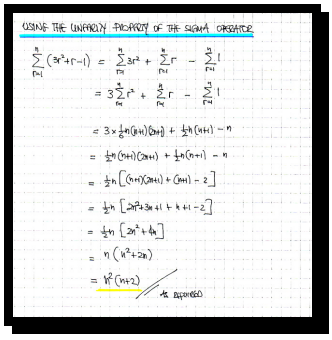


Question 14 **(***)**

Use standard results on summations to show that

$$\sum_{r=1}^{n}\left(3r^2+r-1\right) \equiv n^2(n+2).$$

[ ], proof

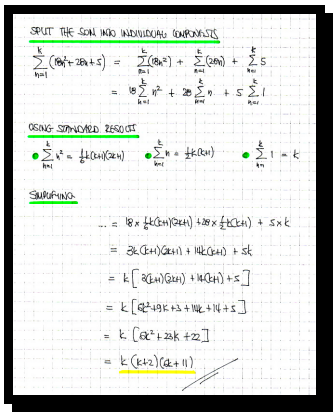


Question 15 **(***)**

Use standard results on summations to show that

$$\sum_{n=1}^{k}\left(18n^2+28n+5\right) \equiv k\left(k+2\right)\left(6k+11\right).$$

[ ], proof

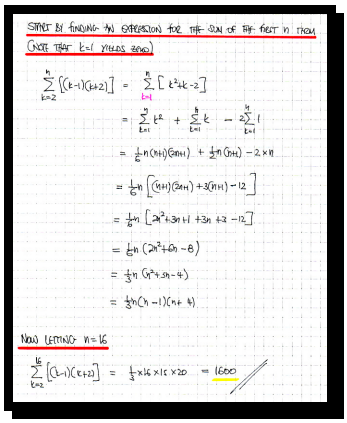


Question 16 **(***)**

Use standard results on summations to find the value of the following sum.

$$\sum_{k=2}^{16}\left[\left(k-1\right)\left(k+2\right)\right].$$

[ ], 1600



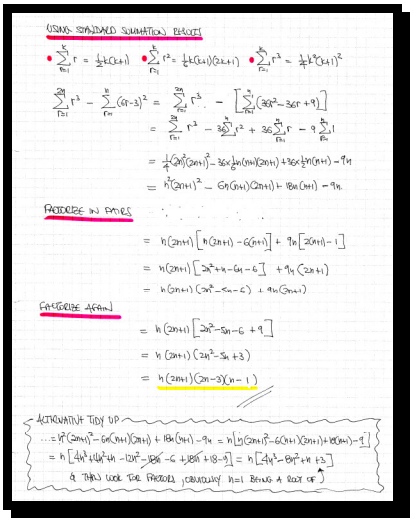
Question 17 **(***)**

Use standard results on summations to show that

$$\sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n} (6r-3)^2 \equiv f(n),$$

where $f(n)$ is written as a product of 4 linear factors.

$\square$, $\boxed{f(n)=n(n-1)(2n+1)(2n-3)}$



SUMMATIONS BY FORMULAS

15 STANDARD QUESTIONS

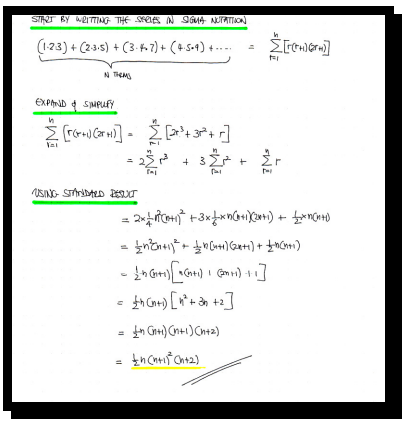
Question 1 **(***+)**

Find the sum of the first n terms of the series

$$1\cdot 2\cdot 3+2\cdot 3\cdot 5+3\cdot 4\cdot 7+4\cdot 5\cdot 9+\ldots .$$

Express the answer as a product of linear factors.

$\boxed{}$, proof

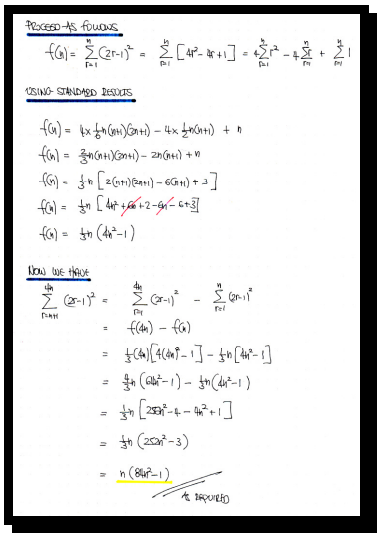


Question 2 (*+)**

By using standard results, show that

$$\sum_{r=n+1}^{4n}(2r-1)^2 \equiv n\left(84n^2-1\right).$$

[ ], [proof]

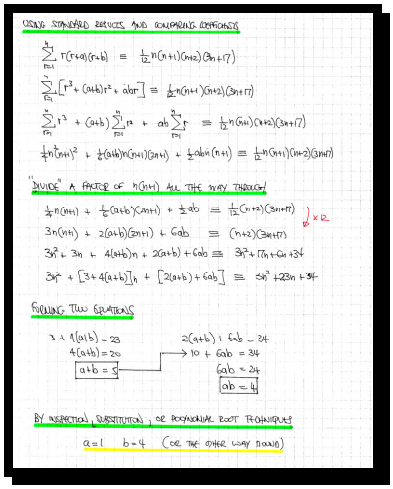


Question 3 **(***+)**

Determine the value of a and the value of b given that

$$\sum_{r=1}^{n} r(r+a)(r+b) \equiv \frac{1}{12} n(n+1)(n+2)(3n+17).$$

$\boxed{}$, $\boxed{a=1, \quad b=4 \quad \text{or the other way round}}$

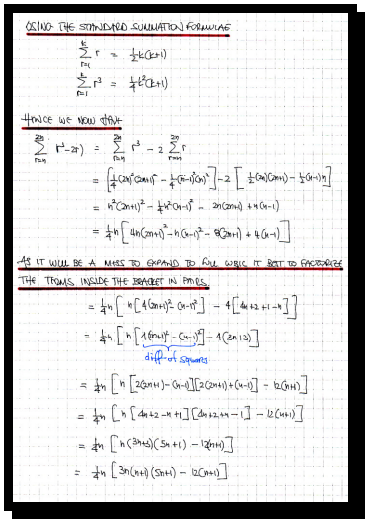


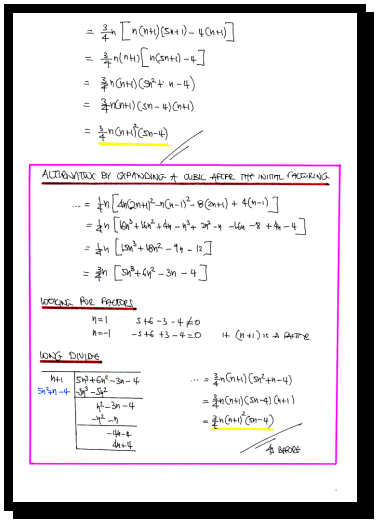
Question 4 (*+)**

Find, in fully factorized form, an expression for the following sum.

$$\sum_{r=n}^{2n}\left(r^3-2r\right).$$

$\square$, $\displaystyle\sum_{r=n}^{2n}\left(r^3-2r\right)=\frac{3}{4}n(5n-4)(n+1)^2$





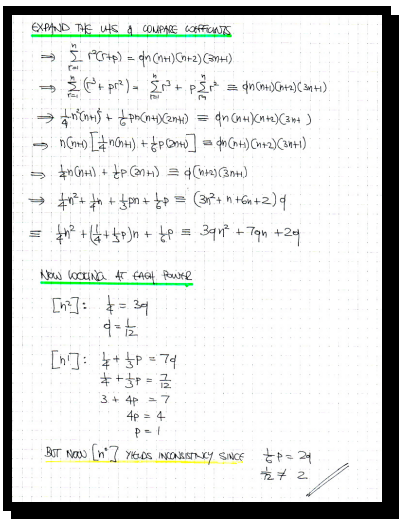
Question 5 (*+)**

It is thought that for some values of the constants p and q that

$$\sum_{r=1}^{n} r^2(r+p) \equiv n(n+1)(n+2)(3n+q).$$

Use a detailed method to show that there exist no such values of p and q.

$\boxed{}$, $\boxed{\text{proof}}$

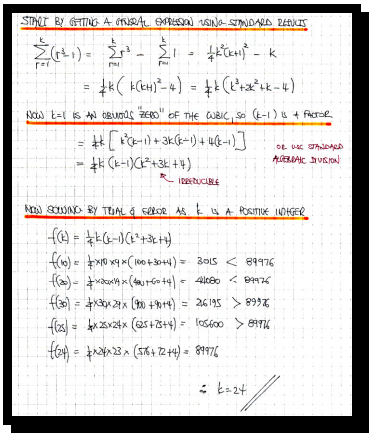


Question 6 (****)

Use standard results on summations to solve the following equation.

$$\sum_{r=1}^{k}\left(r^3-1\right)=89976\,.$$

$\boxed{}$, $\boxed{k=24}$



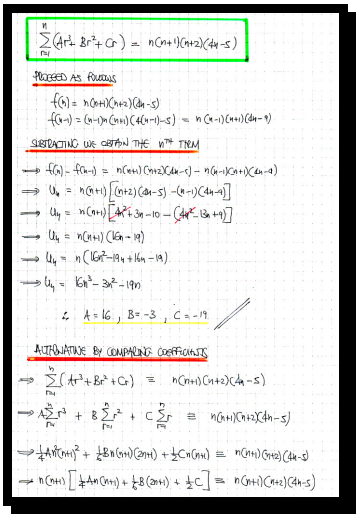
Question 7 **(****)**

It is given that

$$\sum_{r=1}^{n}\left(Ar^3+Br^2+Cr\right)=n(n+1)(n+2)(4n-5).$$

Use a detailed method to find the value of each of the integer constants, A, B and C.

$\boxed{}$, $\boxed{A=16}$, $\boxed{B=-3}$, $\boxed{C=-19}$

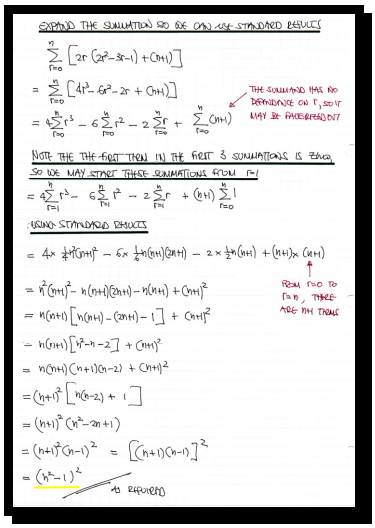


Question 8 (****)

Show by a detailed method that

$$\sum_{r=0}^{n}\left[2r\left(2r^2-3r-1\right)+n+1\right] \;=\; \left(n^2-1\right)^2 .$$

[ ] , [proof]



Question 9 **(****)**

The sum, S_n, of the first n terms of a series whose general term is denoted by u_n is given by the following expression.

$$S_n = n^2(n+1)(n+2).$$

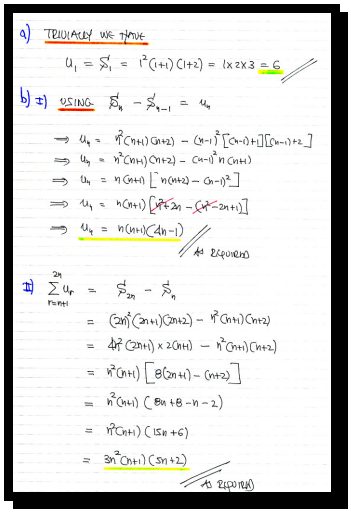
a) Find the first term of the series.

b) Show clearly that …

i. … $u_n = n(n+1)(4n-1)$

ii. … $\displaystyle\sum_{r=n+1}^{2n} u_r = 3n^2(n+1)(5n+2)$.

$\boxed{}$, $\boxed{u_1 = 6}$

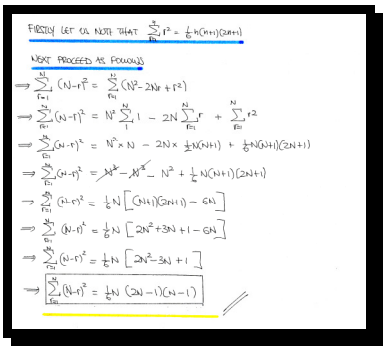


Question 10 (****)

Use standard summation results to prove that

$$\sum_{r=1}^{n}(n-r)^2 = \frac{1}{2}n(n-1)(2n-1).$$

[ ], [proof]

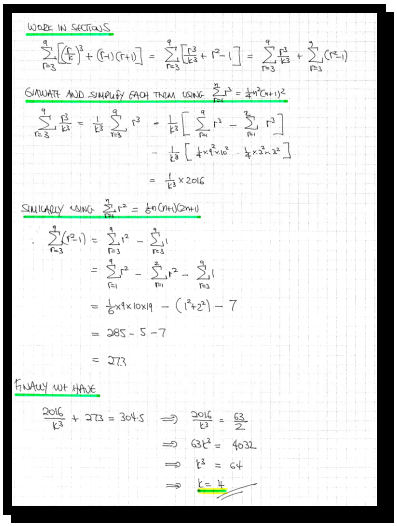


Question 11 (****)

Use standard results on summations to solve the following equation

$$\sum_{r=3}^{9}\left[\left(\frac{r}{k}\right)^3+(r-1)(r+1)\right]=304.5\,.$$

$\boxed{}$, $\boxed{k=4}$



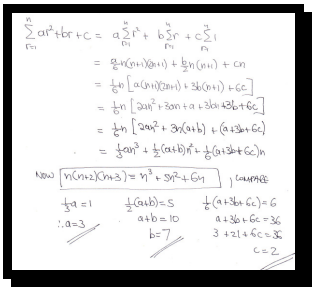
Question 12 (****)

$$\sum_{r=1}^{n}\left(ar^2+br+c\right)\equiv n^3+5n^2+6n\,,$$

where a, b and c are integer constants.

Determine the value of a, b and c.

$\boxed{a=3}$, $\boxed{b=7}$, $\boxed{c=2}$



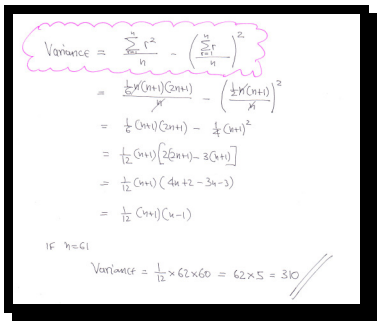
Question 13 (****)

The variance $\text{Var}(n)$ of the first n natural numbers is given by

$$\text{Var}(n)=\frac{1}{n}\sum_{r=1}^{n}r^2-\left[\frac{1}{n}\sum_{r=1}^{n}r\right]^2.$$

Determine a simplified expression for $\text{Var}(n)$ and hence evaluate $\text{Var}(61)$.

$$\boxed{\text{Var}(n)=\frac{1}{12}\left(n^2-1\right)},\ \boxed{\text{Var}(61)=310}$$



Question 14 (****)

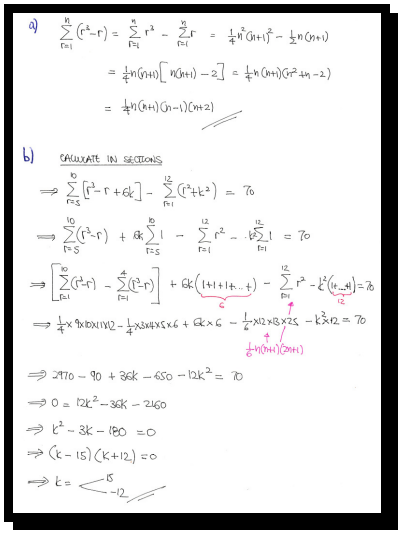
$$f(n)=\sum_{r=1}^{n}\left[r^3-r\right],\qquad n\in\mathbb{N}.$$

a) Use standard summation results to find a fully factorized expression for $f(n)$.

b) Hence solve the equation

$$\sum_{r=5}^{10}\left[r^3-r+6k\right]\ -\ \sum_{r=1}^{12}\left[r^2+k^2\right]\ =\ 70$$

$\square$, $\boxed{f(n)=\frac{1}{4}n(n-1)(n+1)(n+2)}$, $\boxed{k=-12,\quad k=15}$



Question 15 (****)

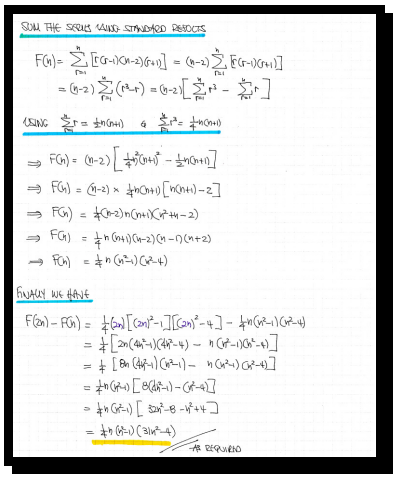
The function $F(n)$ is defined as

$$F(n)=\sum_{r=1}^{n}\left[r(r-1)(n-2)(r+1)\right] \quad n\in\mathbb{N}.$$

Show with detailed workings that

$$F(2n)-F(n)=\tfrac{1}{2}n\left(n^2-1\right)\left(31n^2-4\right).$$

$\boxed{}$, $\boxed{\text{proof}}$



SUMMATIONS BY FORMULAS

5 HARD QUESTIONS

Question 1 (**+)**

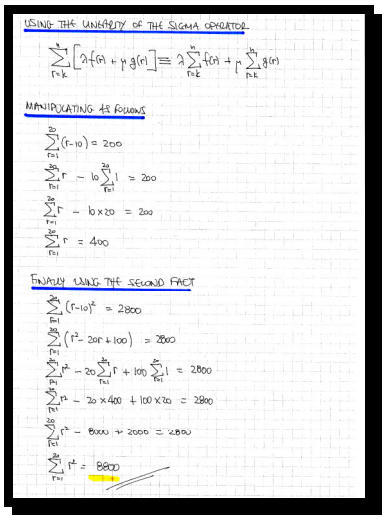
It is given that

$$\sum_{r=1}^{20}(r-10)=200 \quad \text{and} \quad \sum_{r=1}^{20}(r-10)^2=2800\,.$$

Find the value of

$$\sum_{r=1}^{20}r^2\,.$$

$\square$, $\boxed{\displaystyle\sum_{r=1}^{20}r^2=8800}$



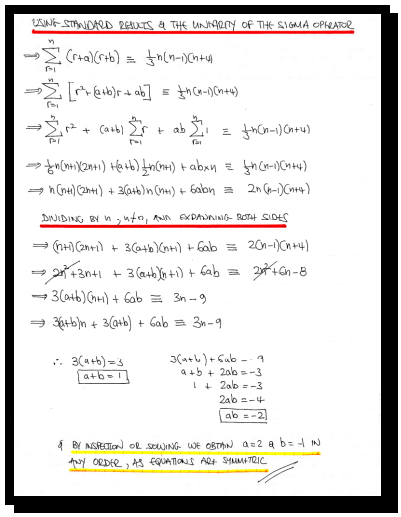
Question 2 (**+)**

$$\sum_{r=1}^{n}(r+a)(r+b)\equiv\frac{1}{3}n(n-1)(n+4),$$

where a and b are integer constants.

Use a clear algebraic method to determine the value of a and the value of b.

$\boxed{}$, $\boxed{2 \text{ and } -1 \text{ (in any order)}}$

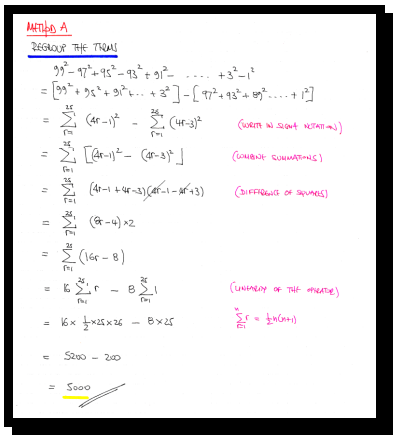


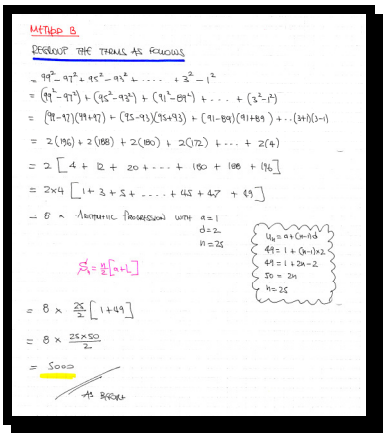
Question 3 (**+)**

By using an algebraic method, find the value of

$$99^2-97^2+95^2-93^2+...+3^2-1^2$$

$\square$, $\boxed{5000}$



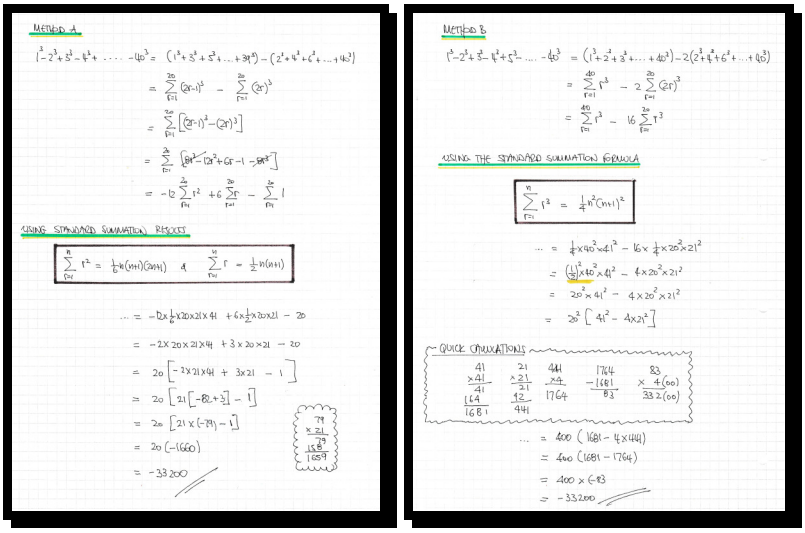


Question 4 **(****+)**

Show clearly that

$$1^3-2^3+3^3-4^3+\ldots-40^3 \quad = \quad -33200\,.$$

[ ], [proof]



Question 5 (**+)**

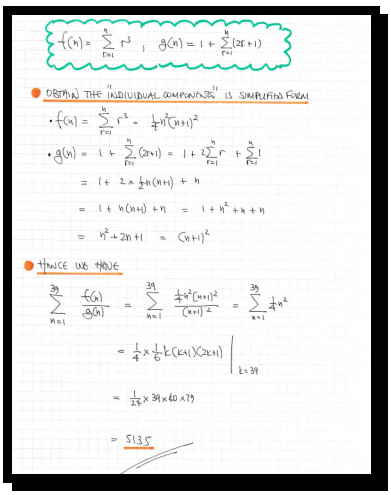
The positive integer functions f and g are defined as

$$f(n)=\sum_{r=1}^{n} r^3 \qquad \text{and} \qquad g(n)=1+\sum_{r=1}^{n}(2r+1).$$

Evaluate

$$\sum_{n=1}^{39}\left[\frac{f(n)}{g(n)}\right].$$

$\boxed{}$, $\boxed{5135}$



SUMMATIONS BY FORMULAS

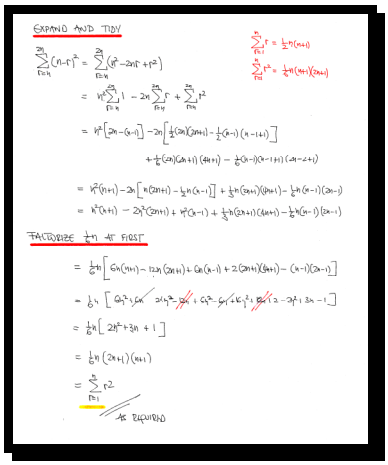
8 ENRICHMENT QUESTIONS

Question 1 (*****)

Use standard summation results to prove that

$$\sum_{r=n}^{2n}(n-r)^2 = \sum_{r=1}^{n} r^2 .$$

[ ], [proof]

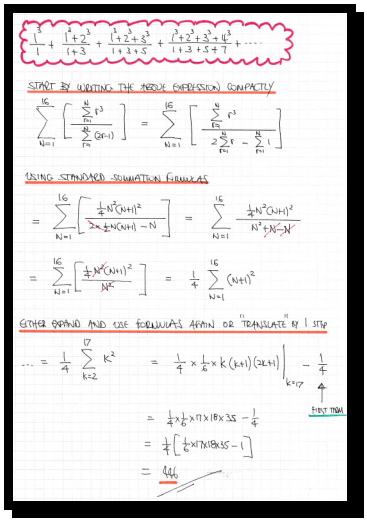


Question 2 **(*****)**

Find the sum of the first 16 terms of the following series.

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\frac{1^3+2^3+3^3+4^3}{1+3+5+7}+\ldots$$

$\boxed{}$, $\boxed{446}$



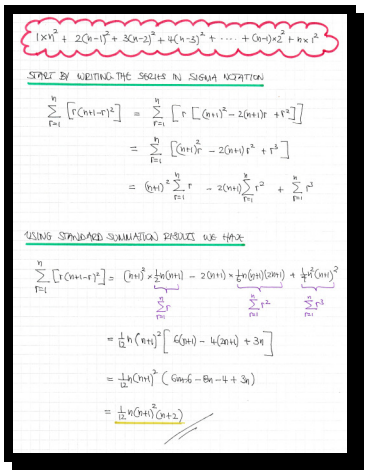
Question 3 **(*****)**

The function f is defined for $n \in \mathbb{N}$ as

$$f(n) \equiv 1\times n^2 + 2(n-1)^2 + 3(n-2)^2 + 4(n-3)^2 + \ldots + (n-1)\times 2^2 + n\times 1^2 .$$

Determine a simplified expression for the sum of $f(n)$, giving the final answer in fully factorized form.

$\square$, $\boxed{f(n) = \frac{1}{12} n(n+2)(n+1)^2}$

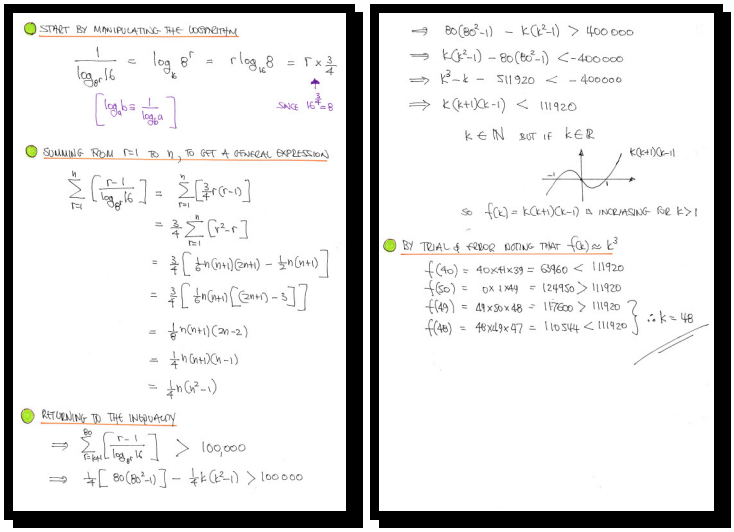


Question 4 **(*****)**

Use an algebraic method justifying each step, to find the greatest value of k, $k \in \mathbb{N}$, which satisfies the following inequality.

$$\sum_{r=k+1}^{80} \left[\frac{r-1}{\log_{8^r}(16)} \right] > 100\,000 .$$

$\square$, $\boxed{k = 48}$

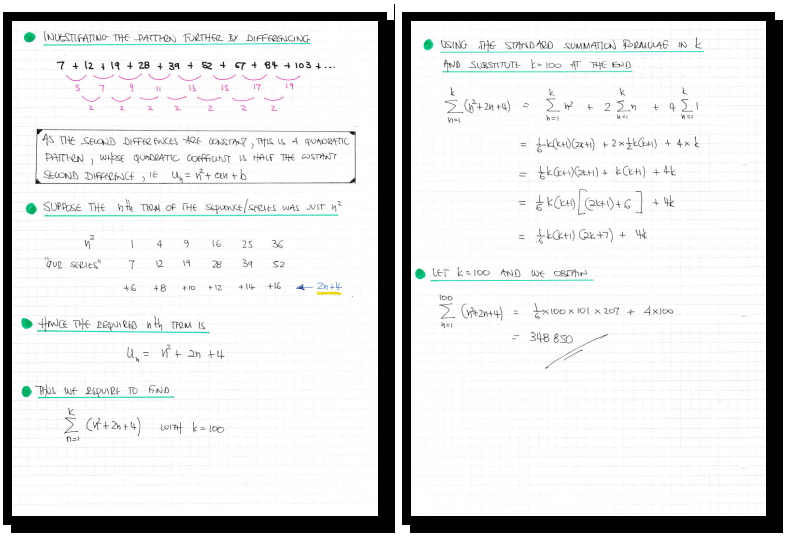


Question 5 **(*****)**

Use algebra to find the sum of the first 100 terms of the following sequence.

$$7,\ 12,\ 19,\ 28,\ 39,\ 52,\ 67,\ 84,\ 103,\ ...$$

$\square$, $\boxed{f(n)=\frac{1}{12}n(n+2)(n+1)^2}$



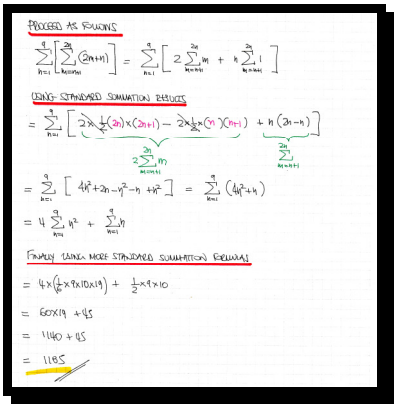
Question 6 (*****)

Evaluate the following expression

$$\sum_{n=1}^{9}\sum_{m=n+1}^{2n}\left[2m+n\right].$$

Detailed workings must be shown.

V, [ ], 1185



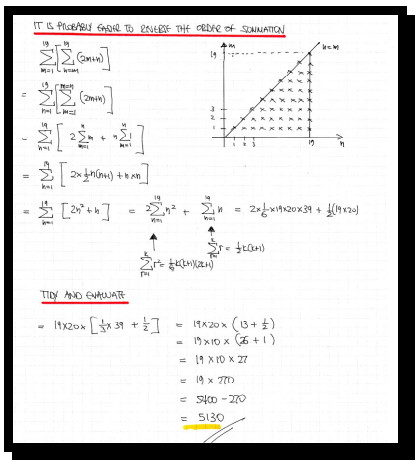
Question 7 (*****)

Evaluate the following expression

$$\sum_{m=1}^{19}\sum_{n=m}^{19}[2m+n].$$

You may find reversing the order of summation useful in this question

V, [ ], [5130]



Question 8 **(*****)**

The function f is defined as

$$f(n,y)\equiv\sum_{x=1}^{n}\frac{x^2y^x}{k},\quad n\in\mathbb{N},\ y\in\mathbb{R}$$

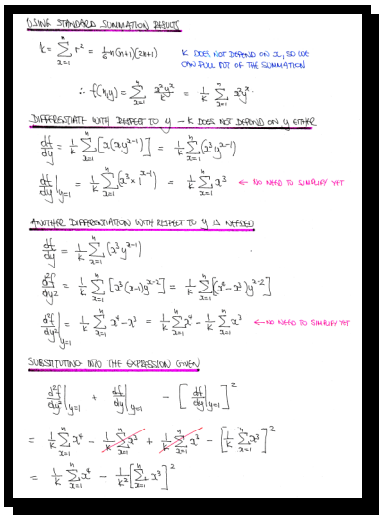
where $k=\sum_{r=1}^{n}r^2$.

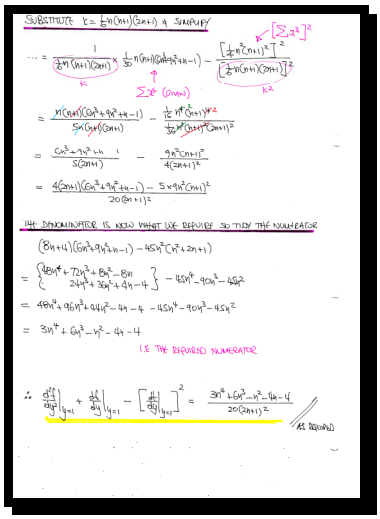
Use standard results on series to show that

$$\left.\frac{d^2f}{dy^2}\right|_{y=1}+\left.\frac{df}{dy}\right|_{y=1}-\left[\left.\frac{df}{dy}\right|_{y=1}\right]^2=\frac{3n^4+6n^3-n^2-4n-4}{20(2n+1)^2}.$$

You may assume without proof $\sum_{r=1}^{n}r^4=\frac{1}{30}n(n+1)\left(6n^3+9n^2+n-1\right)$.

[ ], [proof]





SUMMATIONS

METHOD OF DIFFERENCES

8 BASIC QUESTIONS

Question 1 (**)

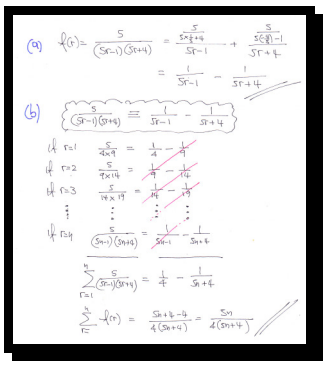
$$f(r)=\frac{5}{(5r-1)(5r+4)},\ r\in\mathbb{N}$$

a) Express $f(r)$ into partial fractions

b) Hence show that

$$\sum_{r=1}^{n} f(r)=\frac{5n}{4(5n+4)}.$$

$$\boxed{f(r)=\frac{1}{5r-1}-\frac{1}{5r+4}}$$



Question 2 (**)

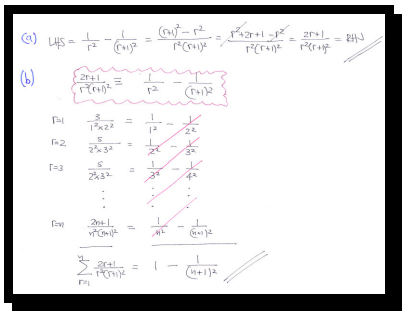
a) Show carefully that

$$\frac{1}{r^2}-\frac{1}{(r+1)^2}=\frac{2r+1}{r^2(r+1)^2}.$$

b) Hence use the method of differences to find

$$\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}.$$

$$\boxed{1-\frac{1}{(n+1)^2}}$$



Question 3 (**)

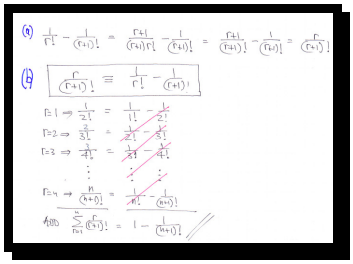
a) Show carefully that

$$\frac{1}{r!}-\frac{1}{(r+1)!}=\frac{r}{(r+1)!}.$$

b) Hence find

$$\sum_{r=1}^{n}\frac{r}{(r+1)!}.$$

$$\boxed{1-\frac{1}{(n+1)!}}$$



Question 4 **(**+)**

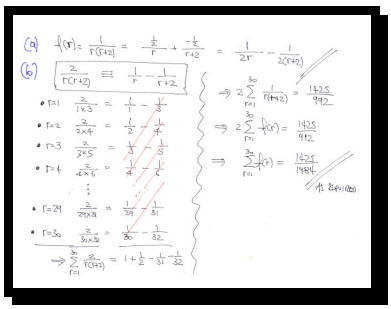
$$f(r)=\frac{1}{r(r+2)},\ r\in\mathbb{N}$$

a) Express $f(r)$ into partial fractions.

b) Hence show that

$$\sum_{r=1}^{30} f(r)=\frac{1425}{1984}.$$

$$\boxed{f(r)=\frac{1}{2r}-\frac{1}{2(r+2)}}$$



Question 5 **(**+)**

$$f(r)=\frac{2}{(r+1)(r+3)},\ r\in\mathbb{N}$$

a) Express $f(r)$ into partial fractions

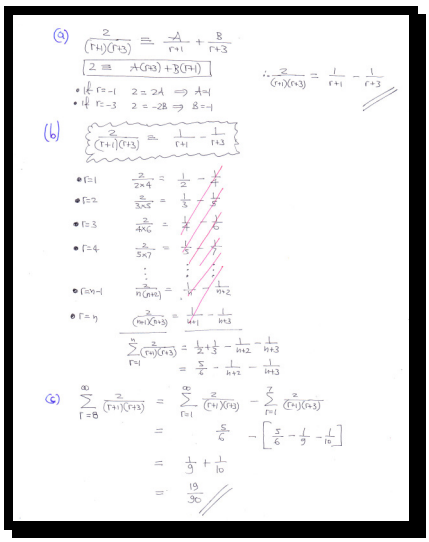
b) Use the method of differences to find

$$\sum_{r=1}^{n} f(r).$$

c) Hence evaluate

$$\sum_{r=8}^{\infty} f(r).$$

$$\boxed{f(r)=\frac{1}{r+1}-\frac{1}{r+3}},\ \boxed{\sum_{r=1}^{n} f(r)=\frac{5}{6}-\frac{1}{n+2}-\frac{1}{n+3}},\ \boxed{\sum_{r=8}^{\infty} f(r)=\frac{19}{90}}$$



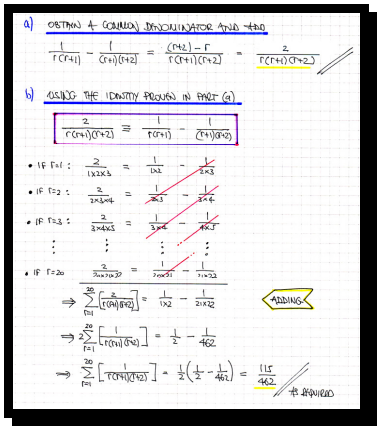
Question 6 **(***)**

a) Simplify $\dfrac{1}{r(r+1)}-\dfrac{1}{(r+1)(r+2)}$ into a single fraction.

b) Hence show that

$$\sum_{r=1}^{20}\left[\frac{1}{r(r+1)(r+2)}\right]=\frac{115}{462}.$$

$\square$, $\boxed{\dfrac{2}{r(r+1)(r+2)}}$



Question 7 **(***)**

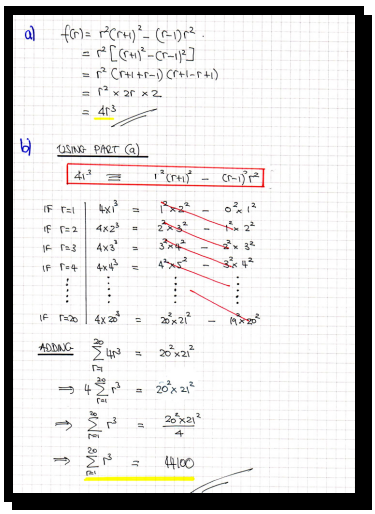
$$f(r)\equiv r^2(r+1)^2-(r-1)^2r^2,\ r\in\mathbb{N}.$$

a) Simplify $f(r)$ as far as possible.

b) Use the method of differences to show that

$$\sum_{r=1}^{20} r^3 = 44100.$$

$\boxed{}$, $\boxed{f(r)=4r^3}$



Question 8 **(***)**

$$f(r)=\frac{1}{r(r+2)}, \quad r\in\mathbb{N}.$$

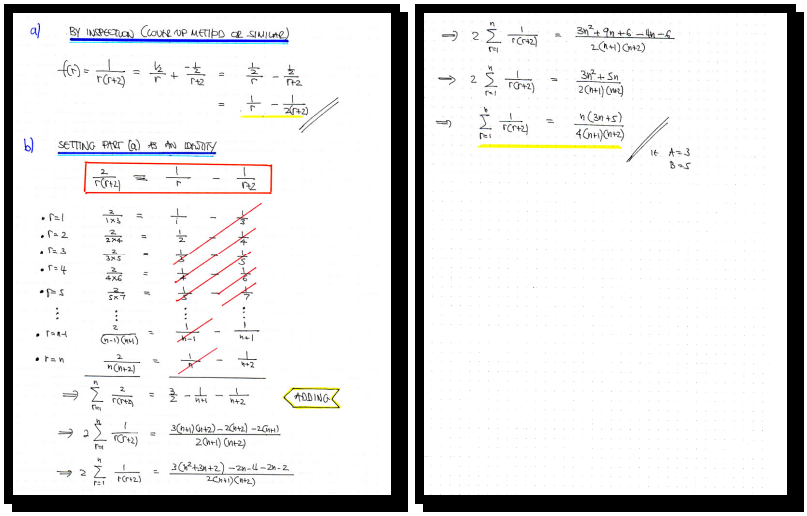
a) Express $f(r)$ in partial fractions.

b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} f(r)=\frac{n(An+B)}{4(n+1)(n+2)},$$

where A and B are constants to be found.

$\boxed{}$, $\boxed{A=3}$, $\boxed{B=5}$



SUMMATIONS

METHOD OF DIFFERENCES

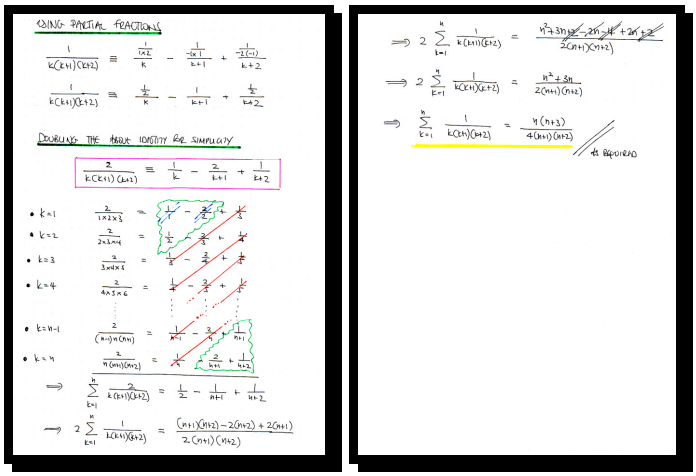
8 STANDARD QUESTIONS

Question 1 (*+)**

Use the method of differences to show that

$$\frac{1}{1\times 2\times 3}+\frac{1}{2\times 3\times 4}+\frac{1}{3\times 4\times 5}+\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}.$$

$\boxed{}$, proof



Question 2 **(***+)**

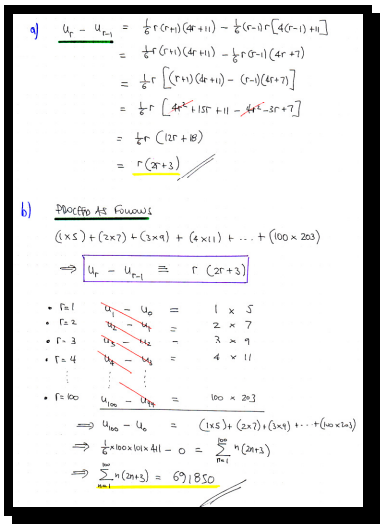
$$u_r = \frac{1}{6}r(r+1)(4r+11),\ r \in \mathbb{N}.$$

a) Simplify $u_r - u_{r-1}$ as far as possible.

b) By using the method of differences, or otherwise, find the sum of the first 100 terms of the following series.

$$(1\times 5)+(2\times 7)+(3\times 9)+(4\times 11)+\ldots$$

$\boxed{}$, $\boxed{r(2r+3)}$, $\boxed{691850}$



Question 3 **(***+)**

$$f(r)=\frac{1}{(r+1)(r-1)},\ r\in\mathbb{N}.$$

a) Express $f(r)$ into partial fractions.

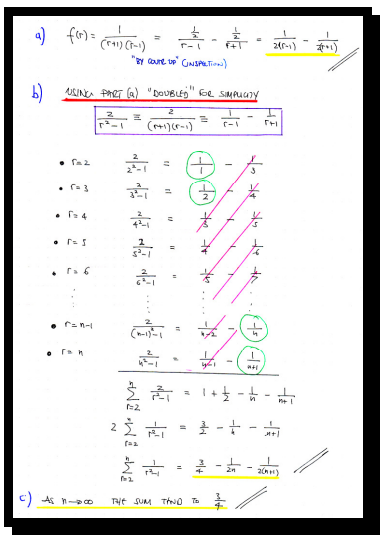
b) Hence show that

$$\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{1}{2n}-\frac{1}{2(n+1)}.$$

c) State the value of

$$\sum_{r=2}^{\infty}\frac{1}{r^2-1}$$

$\square$, $\boxed{f(r)=\frac{1}{2(r-1)}-\frac{1}{2(r+1)}}$, $\boxed{\frac{3}{4}}$



Question 4 **(***+)**

$$f(r)=\frac{2}{r(r+1)(r+2)},\ r\in\mathbb{N}.$$

a) Express $f(r)$ into partial fractions.

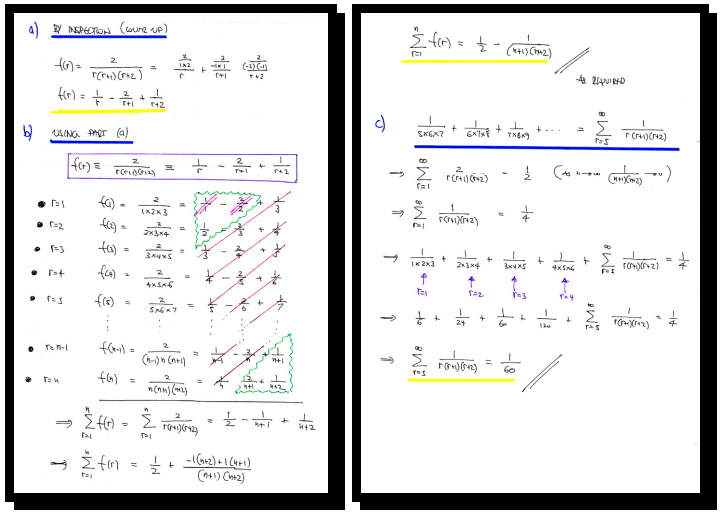
b) Hence show that

$$\sum_{r=1}^{n} f(r)=\frac{1}{2}-\frac{1}{(n+1)(n+2)}.$$

c) Find the value of the convergent infinite sum

$$\frac{1}{5\times6\times7}+\frac{1}{6\times7\times8}+\frac{1}{7\times8\times9}+\ldots$$

$\square$, $\boxed{f(r)=\frac{1}{r}-\frac{2}{r+1}+\frac{1}{r+2}}$, $\boxed{\frac{1}{60}}$

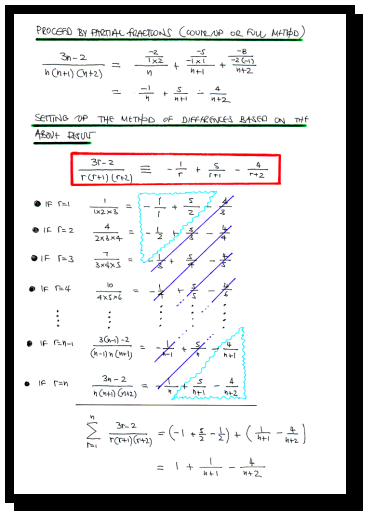


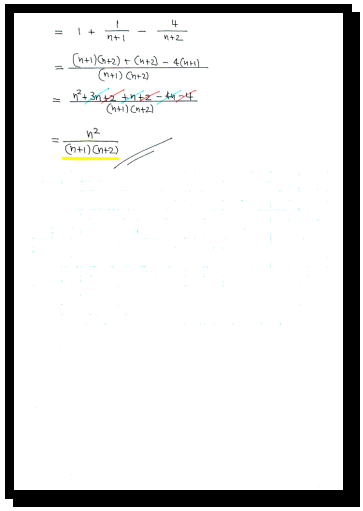
Question 5 **(***+)**

Use the method of differences to show that

$$\frac{1}{1\times 2\times 3}+\frac{4}{2\times 3\times 4}+\frac{7}{3\times 4\times 5}+\ldots+\frac{3n-2}{n(n+1)(n+2)} = \frac{n^2}{(n+1)(n+2)}$$

$\boxed{}$, proof





Question 6 **(***+)**

It is given that

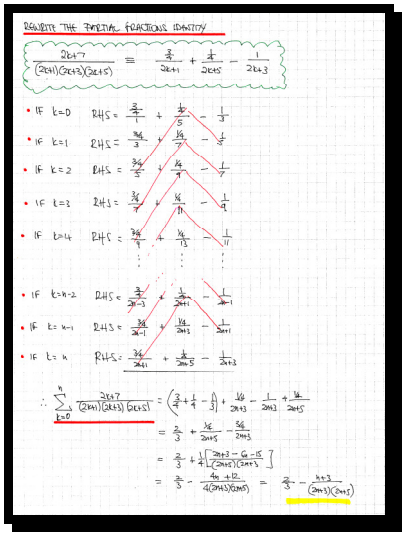
$$\frac{2k+7}{(2k+1)(2k+3)(2k+5)} \equiv \frac{3}{4(2k+1)} - \frac{1}{(2k+3)} + \frac{1}{4(2k+5)}.$$

Use the method of differences to find a simplified expression for

$$\frac{7}{1\times 3\times 5} + \frac{9}{3\times 5\times 7} + \frac{11}{5\times 7\times 9} + \ldots + \frac{2n+7}{(2n+1)(2n+3)(2n+5)}.$$

Give your answer in the form $\frac{2}{3} - f(n)$, where $f(n)$ is a single simplified fraction.

$\square$, $\boxed{f(n) = -\dfrac{n+3}{(2n+3)(2n+5)}}$



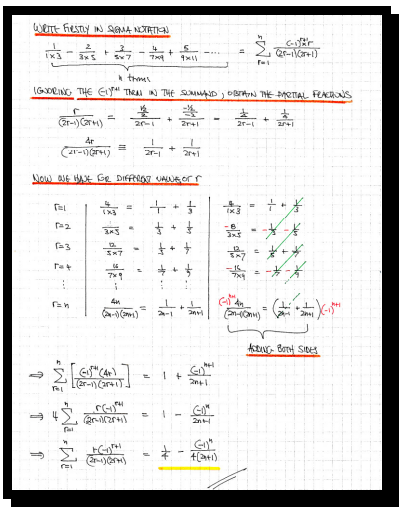
Question 7 **(****)**

Use the method of differences to find a simplified expression for the first n terms of the following series.

$$\frac{1}{1\times 3}+\frac{2}{3\times 5}+\frac{3}{5\times 7}+\frac{4}{7\times 9}+\ldots$$

Give your answer in the form $\frac{1}{4}-f(n)$, where $f(n)$ is a single simplified fraction.

$\square$, $\boxed{f(n)=\dfrac{(-1)^n}{4(2n+1)}}$



Question 8 **(****)**

$$f(r)=\frac{1}{\sqrt{r+2}+\sqrt{r}},\ r\geq 0.$$

a) Rationalize the denominator of $f(r)$.

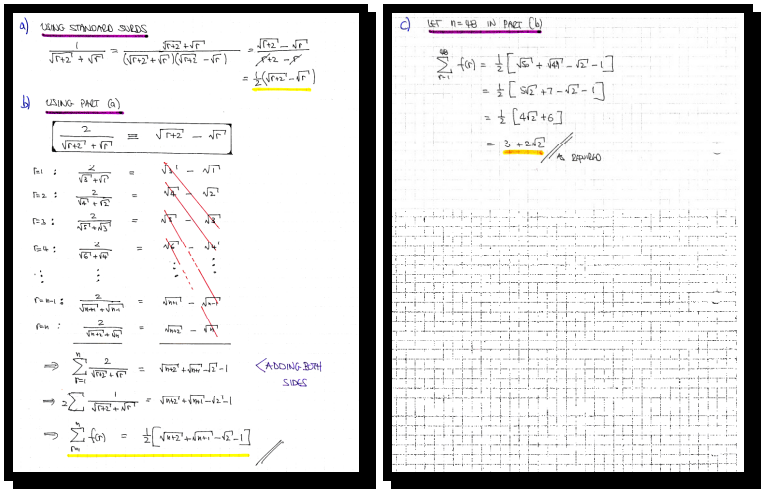
b) Find an expression for

$$\sum_{r=1}^{n} f(r).$$

c) Show clearly that

$$\sum_{r=1}^{48} f(r)=3+2\sqrt{2}$$

$\square$, $\boxed{f(r)=\frac{\sqrt{r+2}-\sqrt{r}}{2}}$, $\boxed{\sum_{r=1}^{n} f(r)=\frac{1}{2}\left(\sqrt{n+2}+\sqrt{n+1}-\sqrt{2}-1\right)}$



SUMMATIONS

METHOD OF DIFFERENCES

3 HARD QUESTIONS

Question 1 **(****+)**

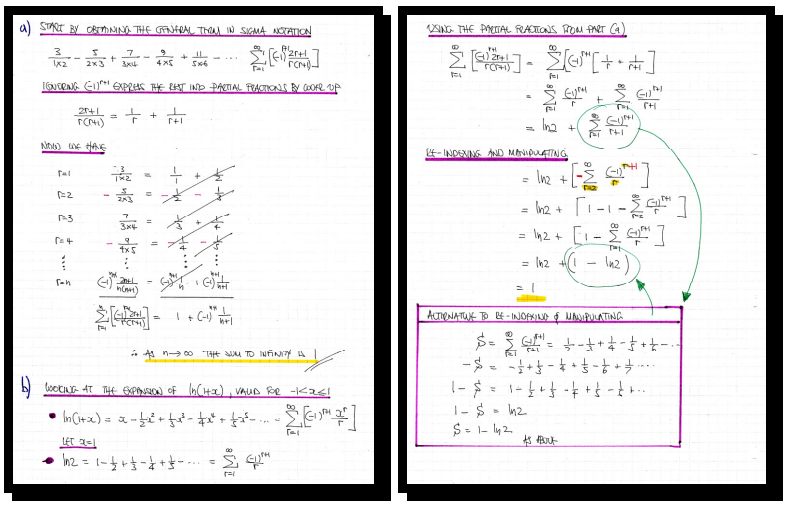
Consider the following infinite convergent series.

$$\frac{3}{1\times 2}-\frac{5}{2\times 3}+\frac{7}{3\times 4}-\frac{9}{4\times 5}+\frac{11}{5\times 6}-....$$

a) Use the method of differences, to find the sum of this series.

b) Verify the answer of part **(a)** by using a method based on the Maclaurin expansion of $\ln(1+x)$.

V, [ ], 1



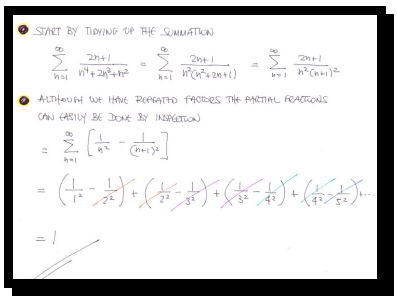
Question 2 (**+)**

Use partial fractions to sum the following series.

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^4+2n^3+n^2}.$$

You may assume that the series converges.

$\boxed{1}$



Question 3 (**+)**

It is given that

$$f(r)=\frac{6r^4+6r^3-ar^2-ar+1}{r(r+1)}, \quad r\in\mathbb{N},$$

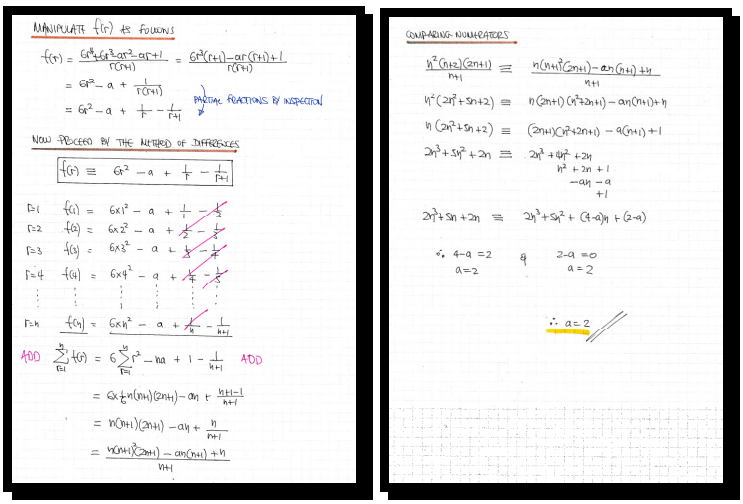
where a is a non zero constant.

It is further given that

$$\sum_{r=1}^{n} f(r) = \frac{n^2(n+2)(2n+1)}{n+1}.$$

Determine the value of a.

$\boxed{}$, $\boxed{a=2}$



SUMMATIONS

METHOD OF DIFFERENCES

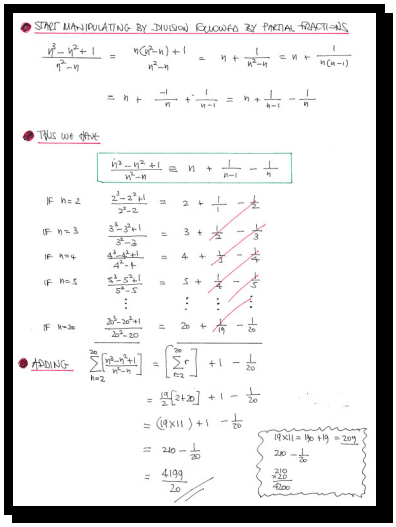
15 ENRICHMENT QUESTIONS

Question 1 **(*****)**

Determine the exact value of the following sum.

$$\sum_{n=2}^{20}\left[\frac{n^3-n^2+1}{n^2-n}\right].$$

$\square$, $\boxed{\frac{4199}{20}}$



Question 2 **(*****)**

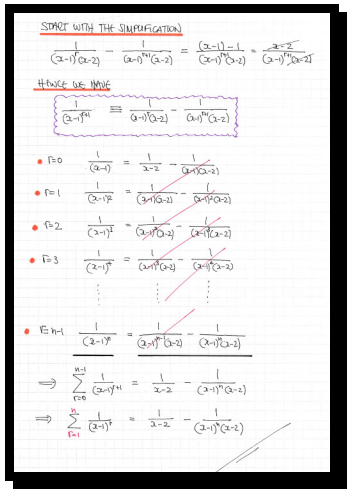
$$f(x,n)=\sum_{r=1}^{n}\left[\frac{1}{(x-1)^r}\right],\quad x\in\mathbb{R},\ n\in\mathbb{N}.$$

By observing the simplification of

$$\frac{1}{(x-2)(x-1)^r}-\frac{1}{(x-2)(x-1)^{r+1}}$$

find a simplified expression for $f(x,n)$.

$\square$, $\boxed{f(x,n)=\dfrac{1}{x-2}-\dfrac{1}{(x-2)(x-1)^n}}$

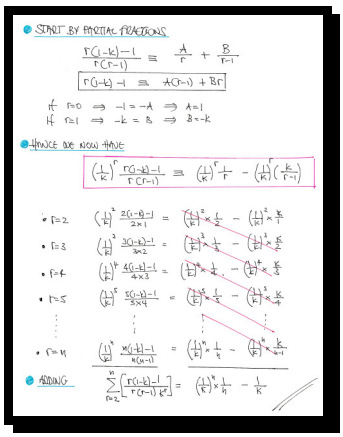


Question 3 **(*****)**

Determine, in terms of k and n, a simplified expression for

$$\sum_{r=2}^{n}\left[\frac{r(1-k)-1}{r(r-1)k^r}\right].$$

$\square$, $\boxed{\dfrac{1}{n}\left(\dfrac{1}{k}\right)^n-\dfrac{1}{k}}$

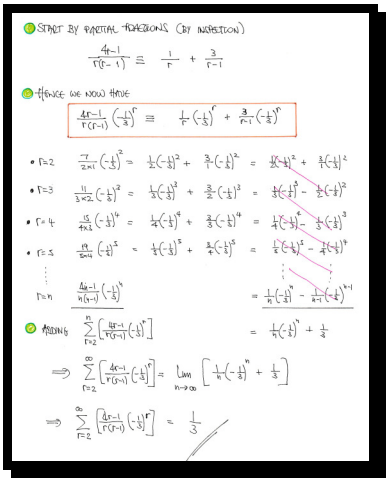


Question 4 (*****)

Determine the value of the following infinite convergent sum.

$$\sum_{r=2}^{\infty}\left[\frac{4r-1}{r(r-1)}\left(-\frac{1}{3}\right)^r\right].$$

$\square$, $\boxed{\frac{1}{3}}$

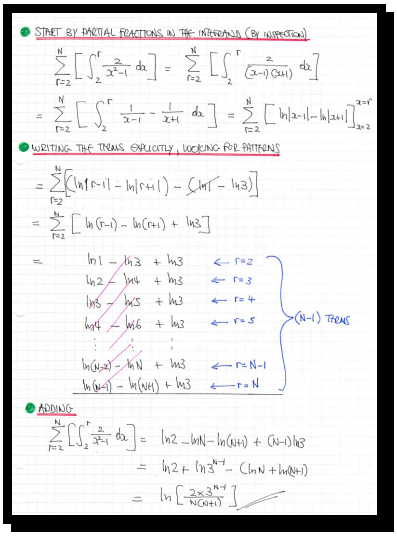


Question 5 (*****)

Determine a simplified expression, in the form $\ln\left[f(n)\right]$, for the following sum.

$$\sum_{r=2}^{N}\left[\int_{2}^{r}\frac{2}{x^2-1}\,dx\right].$$

$\square$, $\boxed{\ln\left[\dfrac{2\times 3^{N-1}}{N(N+1)}\right]}$



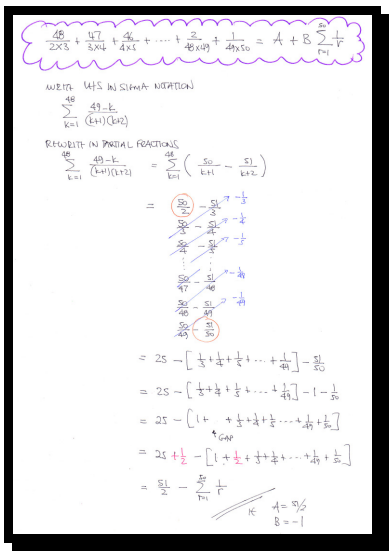
Question 6 **(*****)**

Show, by a detailed method, that

$$\frac{48}{2\times 3}+\frac{47}{3\times 4}+\frac{46}{4\times 5}\ldots+\frac{2}{48\times 49}+\frac{1}{49\times 50}=A+B\sum_{r=1}^{50}\frac{1}{r},$$

where A and B are constants to be found.

$\boxed{}$, $\boxed{A=\frac{51}{2},\quad B=-1}$

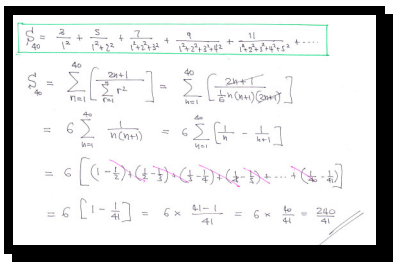


Question 7 (*****)

$$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\frac{9}{1^2+2^2+3^2+4^2}+\frac{11}{1^2+2^2+3^2+4^2+5^2}+\ldots,$$

Show, by a detailed method, that the sum of the first 40 terms of this series shown above is $\frac{240}{41}$.

$\boxed{}$, proof



Question 8 **(*****)**

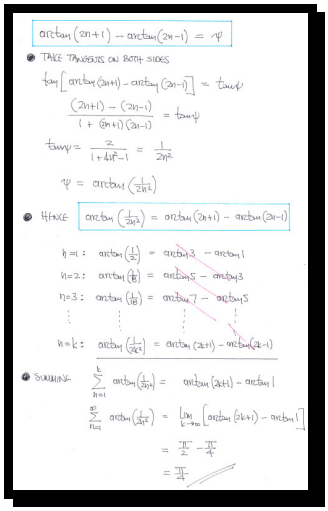
By considering the simplification of

$$\arctan\left(2n+1\right)-\arctan\left(2n-1\right),$$

determine the exact value of

$$\sum_{n=1}^{\infty}\left[\arctan\left(\frac{1}{2n^2}\right)\right].$$

$\square$, $\boxed{\frac{\pi}{4}}$



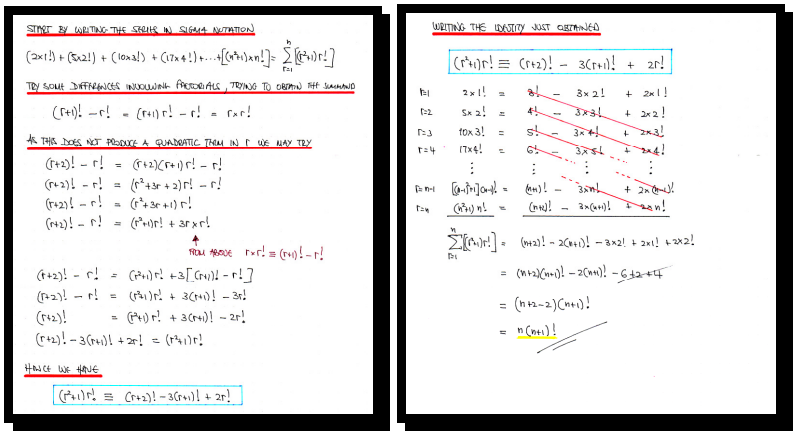
Question 9 **(*****)**

$$S_n=(2\times 1!)+(5\times 2!)+(10\times 3!)+(17\times 4!)+\ldots+\left(n^2+1\right)n!$$

Use an appropriate method to show that

$$S_n=n(n+1)!$$

[ ], proof



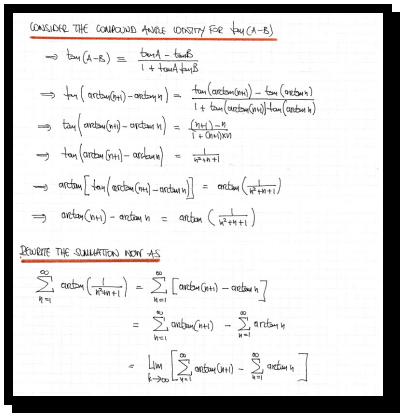
Question 10 (*****)

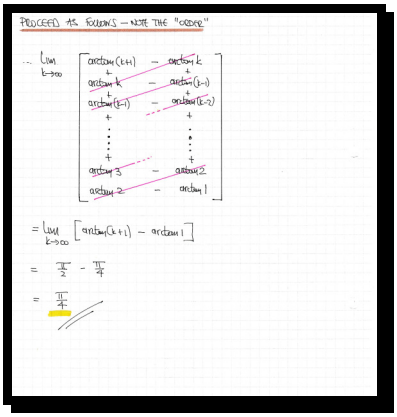
By considering the trigonometric identity for $\tan(A-B)$, with $A=\arctan(n+1)$ and $B=\arctan(n)$, sum the following series

$$\sum_{n=1}^{\infty}\arctan\left(\frac{1}{n^2+n+1}\right).$$

You may assume the series converges.

$\square$, $\square$, $\boxed{\frac{\pi}{4}}$





Question 11 (*****)

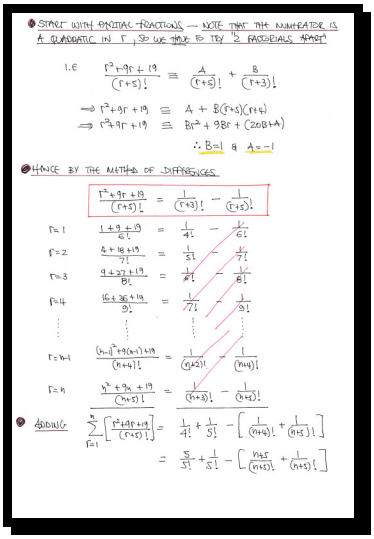
Determine, in terms of n, a simplified expression

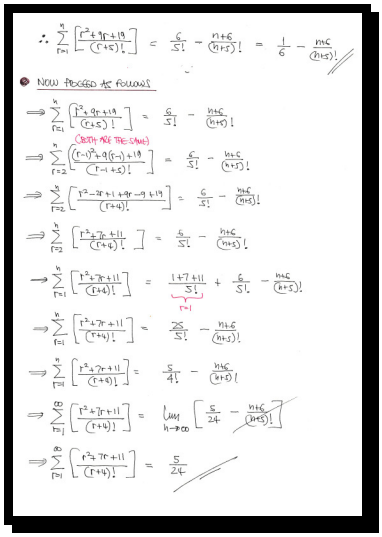
$$\sum_{r=1}^{n}\left[\frac{r^2+9r+19}{(r+5)!}\right],$$

and hence, or otherwise, deduce the value of

$$\sum_{r=1}^{\infty}\left[\frac{r^2+7r+11}{(r+4)!}\right].$$

$$\boxed{},\ \boxed{\sum_{r=1}^{n}\left[\frac{r^2+9r+19}{(r+5)!}\right]=\frac{1}{6}-\frac{n+5}{(n+5)!}},\ \boxed{\sum_{r=1}^{\infty}\left[\frac{r^2+7r+11}{(r+4)!}\right]=\frac{5}{24}}$$





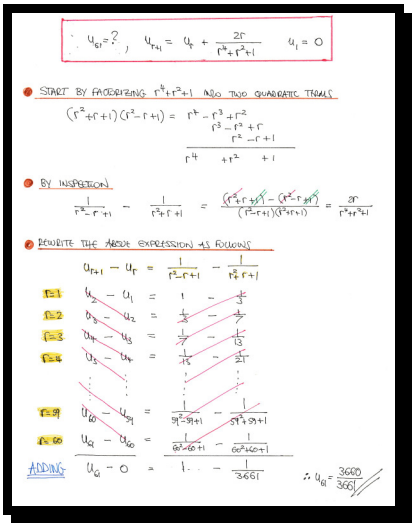
Question 12 (*****)

A sequence is defined as

$$u_{r+1}=u_r+\frac{2r}{r^4+r^2+1}, \qquad u_1=0, \quad r\in\mathbb{N}.$$

Determine the exact value of u_{61}.

$\square$, $\boxed{u_{61}=\dfrac{3660}{3661}}$



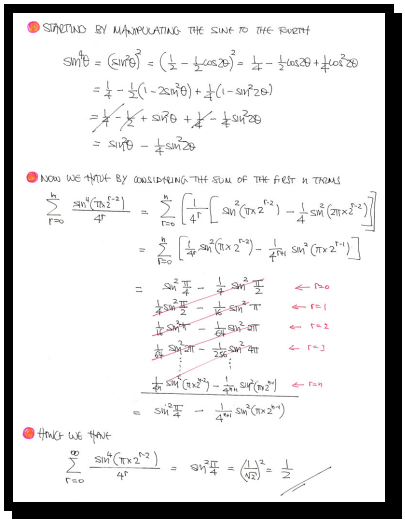
Question 13 (*****)

Find the value of

$$\sum_{r=0}^{\infty}\left[\frac{\sin^4\left(\pi\times 2^{r-2}\right)}{4^r}\right].$$

Hint: Express $\sin^4\theta$ in terms of $\sin^2\theta$ and $\sin^2 2\theta$ only.

$\square$, $\boxed{\frac{1}{2}}$

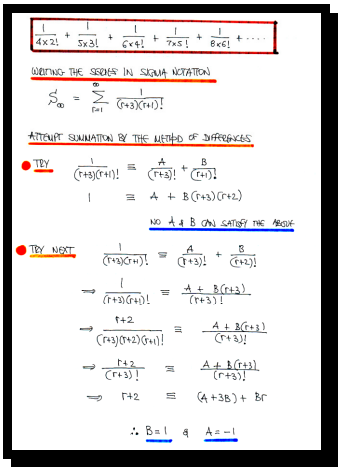


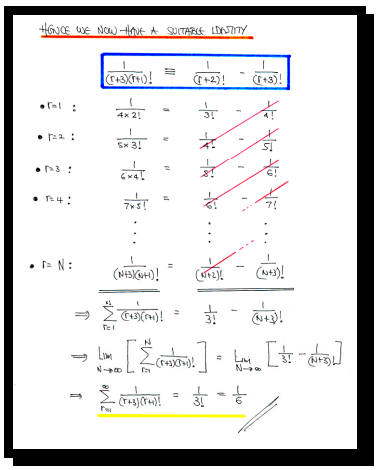
Question 14 (*****)

Find the sum to infinity of the following convergent series.

$$\frac{1}{4\times 2!}+\frac{1}{5\times 3!}+\frac{1}{6\times 4!}+\frac{1}{7\times 5!}+\frac{1}{8\times 6!}+...$$

$\square$, $\boxed{\frac{1}{6}}$





Question 15 (*****)

Evaluate the following expression

$$\sum_{k=1}^{\infty} \left[\sum_{r=1}^{k} r \right]^{-1} .$$

$\boxed{}$, $\boxed{2}$

