ROOTS OF POLYNOMIAL EQUATIONS

QUADRATICS

Question 1 (***)

The quadratic equation

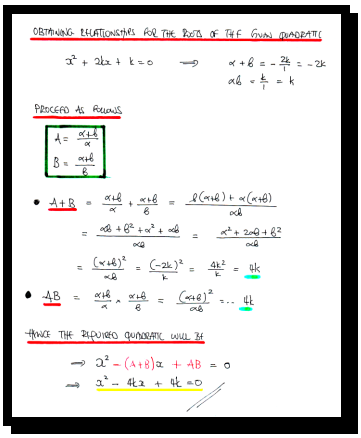
$$x^2+2kx+k=0,$$

where k is a non zero constant, has roots $x=\alpha$ and $x=\beta$.

Find a quadratic equation, in terms of k, whose roots are

$$\frac{\alpha+\beta}{\alpha} \quad \text{and} \quad \frac{\alpha+\beta}{\beta}.$$

$\boxed{}$, $\boxed{x^2-4kx+4k=0}$



Question 2 (***)

The two roots of the quadratic equation

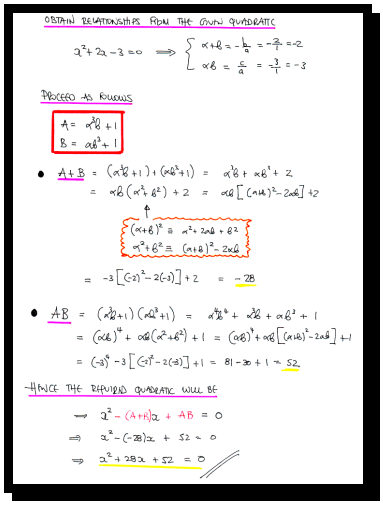
$$x^2+2x-3=0$$

are denoted, in the usual notation, as α and β .

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha^3\beta+1 \qquad \text{and} \qquad \alpha\beta^3+1.$$

$\boxed{}$, $\boxed{x^2+28x+52=0}$



Question 3 (***)

The roots of the quadratic equation

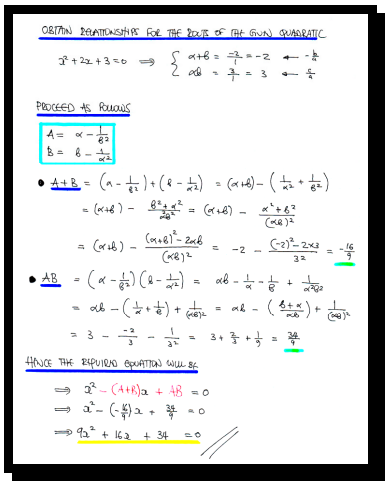
$$x^2+2x+3=0$$

are denoted, in the usual notation, as α and β .

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha-\frac{1}{\beta^2} \qquad \text{and} \qquad \beta-\frac{1}{\alpha^2}.$$

$\boxed{}$, $\boxed{9x^2+16x+34=0}$



Question 4 (***)

The roots of the quadratic equation

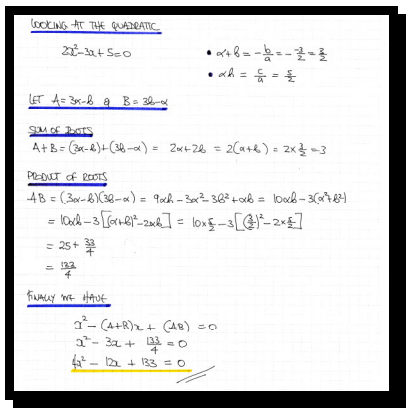
$$2x^2-3x+5=0$$

are denoted by α and β.

Find the quadratic equation, with integer coefficients, whose roots are

$$3\alpha-\beta \qquad \text{and} \qquad 3\beta-\alpha .$$

$\boxed{}$, $\boxed{4x^2-12x+133=0}$



Question 5 (***)

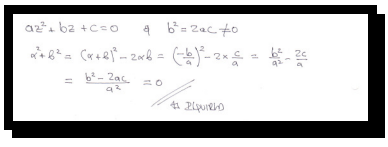
The roots of the equation

$$az^2+bz+c=0,$$

where a, b and c are real constants, are denoted by α and β.

Given that $b^2=2ac\neq 0$, show that $\alpha^2+\beta^2=0$.

proof



Question 6 **(***)**

The roots of the quadratic equation

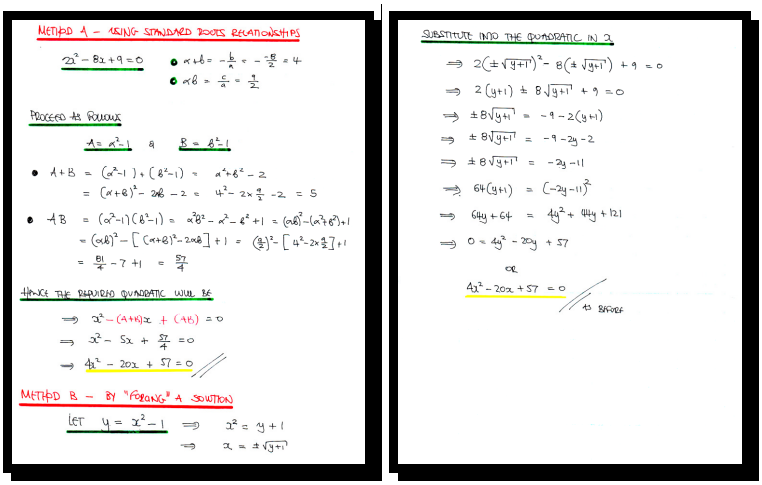
$$2x^2-8x+9=0$$

are denoted, in the usual notation, as α and β .

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha^2-1 \qquad \text{and} \qquad \beta^2-1.$$

$\square$, $\boxed{4x^2-20x+57=0}$



Question 7 (***)

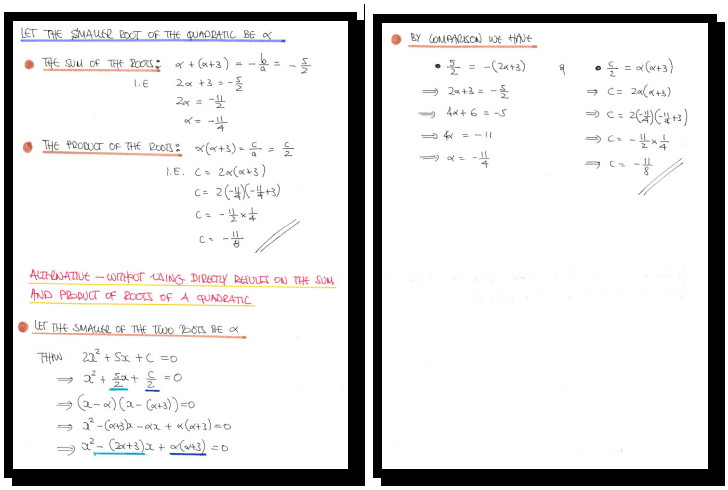
A curve has equation

$$y = 2x^2 + 5x + c,$$

where c is a non zero constant.

Given that the roots of the equation differ by 3, determine the value of c.

$\square$, $\boxed{c = -\frac{11}{8}}$



Question 8 (*+)**

The roots of the quadratic equation

$$2x^2-3x+5=0$$

are denoted by α and β.

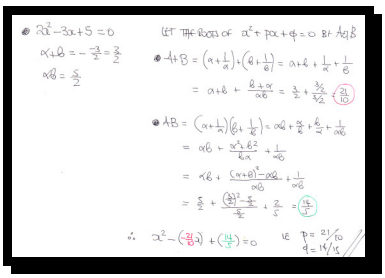
The roots of the quadratic equation

$$x^2+px+q=0,$$

where p and q are real constants, are denoted by $\alpha+\frac{1}{\alpha}$ and $\beta+\frac{1}{\beta}$.

Determine the value of p and the value of q.

$$\boxed{p=\frac{21}{10}}, \quad \boxed{q=\frac{14}{5}}$$



Question 9 (****)

Consider the quadratic equation

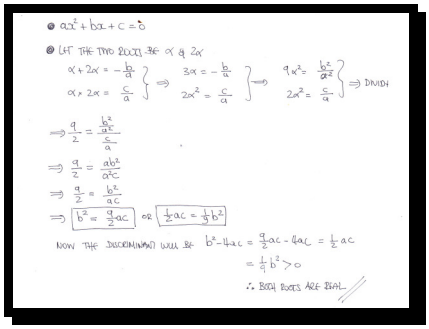
$$ax^2+bx+c=0,$$

where a, b and c are real constants.

One of the roots of this quadratic equation is double the other.

Show clearly that both roots must be real.

proof



Question 10 (****)

The roots of the quadratic equation

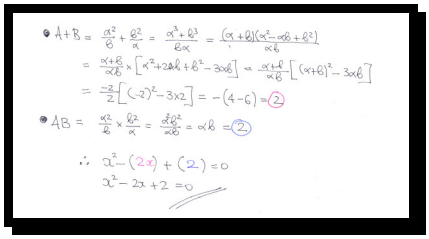
$$x^2+2x+2=0$$

are denoted by α and β .

Find the quadratic equation, with integer coefficients, whose roots are

$$\frac{\alpha^2}{\beta} \quad \text{and} \quad \frac{\beta^2}{\alpha}.$$

$$\boxed{x^2-2x+2=0}$$



Question 11 (****)

The roots of the quadratic equation

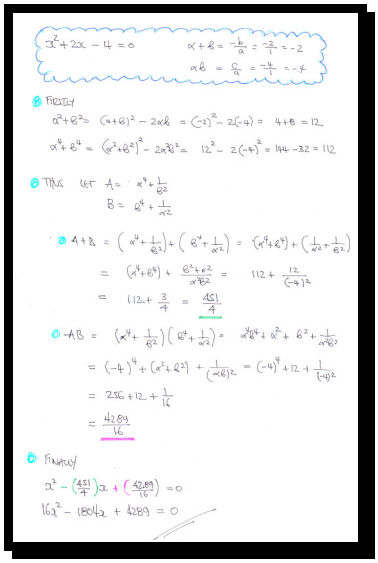
$$x^2+2x-4=0$$

are denoted by α and β .

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha^4+\frac{1}{\beta^2} \quad \text{and} \quad \beta^4+\frac{1}{\alpha^2}.$$

$$\boxed{16x^2-1804x+4289=0}$$



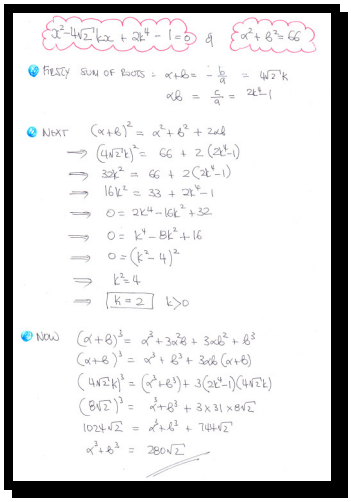
Question 12 (****)

$$x^2-4\sqrt{2}\,kx+2k^4-1=0\,.$$

The two roots of the above quadratic equation, where k is a constant, are denoted by α and β.

Given further that $\alpha^2+\beta^2=66$, determine the exact value of $\alpha^3+\beta^3$.

$$\boxed{\alpha^3+\beta^3=280\sqrt{2}}$$



Question 13 (**+)**

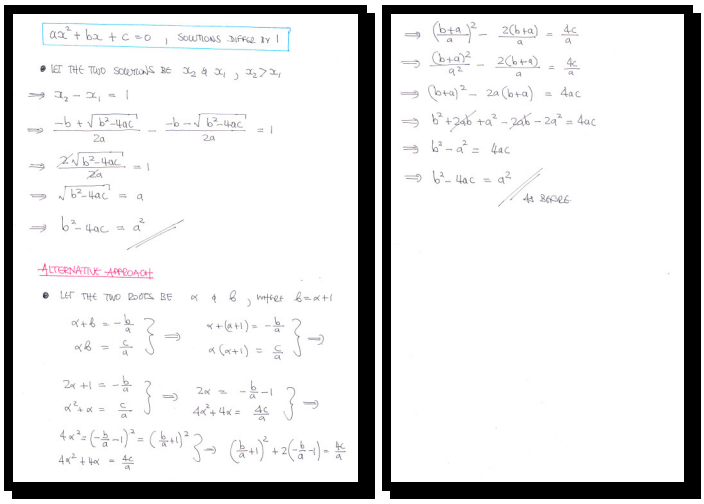
The quadratic equation

$$ax^2+bx+c=0\,,\ x\in\mathbb{R}\,,$$

where a, b and c are constants, $a\neq 0$, has real roots which differ by 1.

Determine a simplified relationship between a, b and c.

$$\boxed{b^2-4ac=a^2}$$



Question 14 (**+)**

The roots of the quadratic equation

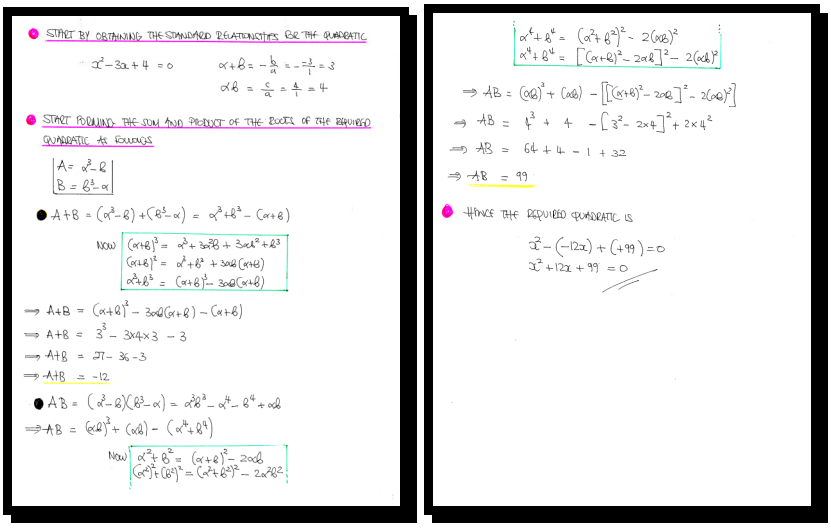
$$x^2-3x+4=0$$

are denoted by α and β .

Find the quadratic equation, with integer coefficients, whose roots are

$$\alpha^3-\beta \quad \text{and} \quad \beta^3-\alpha\,.$$

$\boxed{}$, $\boxed{x^2+12x+99=0}$



Question 15 (**+)**

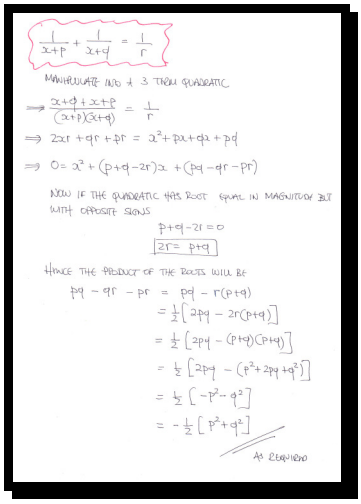
$$\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}, \qquad x\neq -p \;\; x\neq -q\,.$$

The roots of the above quadratic equation, where p, q and r are non zero constants, are equal in magnitude but opposite in sign.

Show that the product of these roots is

$$-\frac{1}{2}\left[p^2+q^2\right].$$

proof



Question 16 (**+)**

$$2x^2+kx+1=0\,.$$

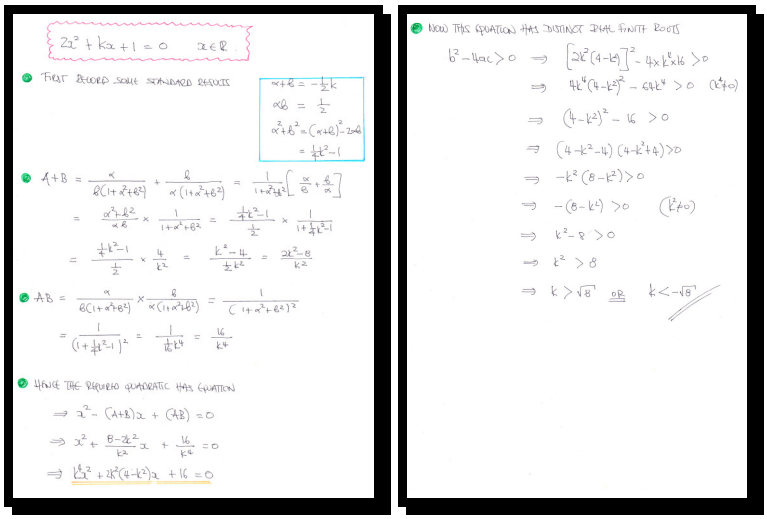
The roots of the above equation are α and β, where k is a non zero real constant.

Given further that the following two expressions

$$\frac{\alpha}{\beta\left(1+\alpha^2+\beta^2\right)} \quad \text{and} \quad \frac{\beta}{\alpha\left(1+\alpha^2+\beta^2\right)}$$

are real, finite and distinct, determine the range of the possible values of k.

$$\boxed{|k|>\sqrt{8}}$$



Question 17 (*****)

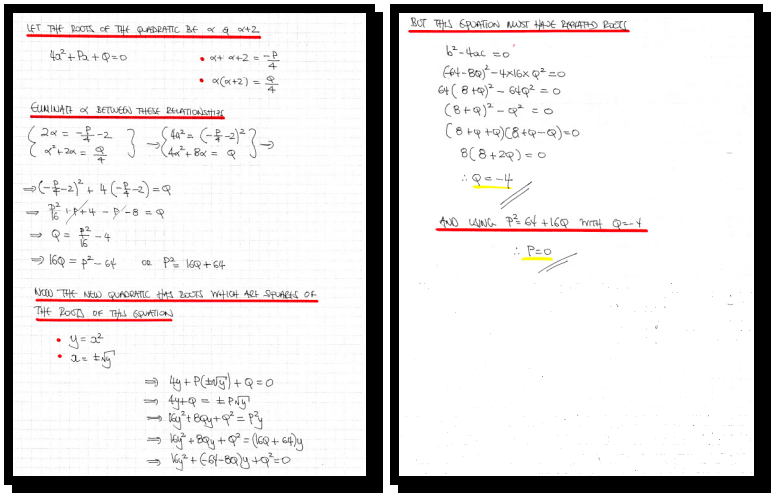
The quadratic equation

$$4x^2 + Px + Q = 0 ,$$

where P and Q are constants, has roots which differ by 2.

If another quadratic equation has repeated roots which are also the **squares of the roots** of the above given equation, find the value of P and the value of Q.

$\boxed{}$, $\boxed{P = 0}$, $\boxed{Q = -4}$



Question 18 (*****)

The quadratic equation

$$x^2-4x-2=0,$$

has roots α and β in the usual notation, where $\alpha>\beta$.

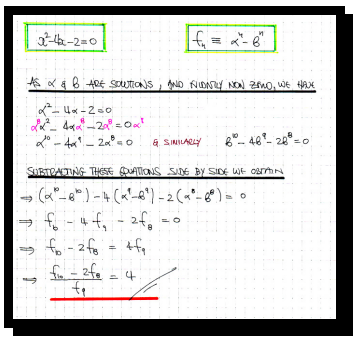
It is further given that

$$f_n\equiv\alpha^n-\beta^n.$$

Determine the value of

$$\frac{f_{10}-2f_8}{f_9}.$$

$\boxed{}$, $\boxed{4}$



Question 19 (*****)

The quadratic equation

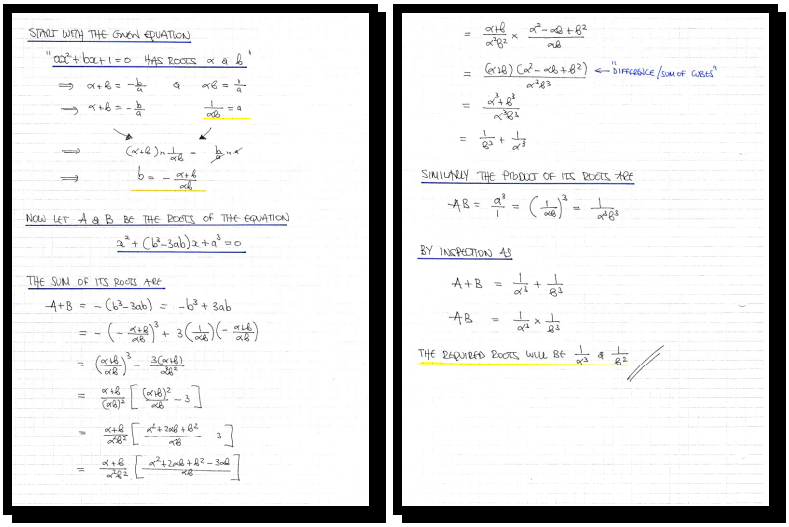
$$ax^2+bx+1=0, \qquad a\neq 0,$$

where a and b are constants, has roots α and β.

Find, in terms of α and β, the roots of the equation

$$x^2+\left(b^3-3ab\right)x+a^3=0.$$

$$\square, \quad \boxed{\frac{1}{\alpha^3}, \frac{1}{\beta^3}}$$



CUBICS

Question 1 **(**)**

$$x^3-6x^2+4x+12=0\,.$$

The three roots of the above cubic are denoted by α, β and γ.

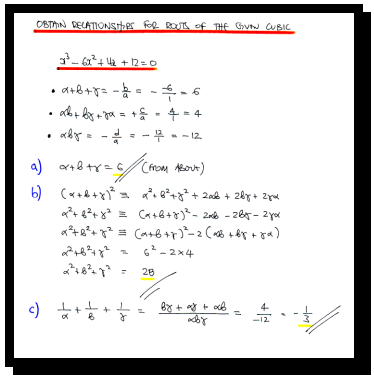
Find the value of …

a) … $\alpha+\beta+\gamma$.

b) … $\alpha^2+\beta^2+\gamma^2$.

c) … $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$.

$\boxed{}$, $\boxed{\alpha+\beta+\gamma=6}$, $\boxed{\alpha^2+\beta^2+\gamma^2=28}$, $\boxed{\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}=\dfrac{1}{3}}$



Question 2 (**)

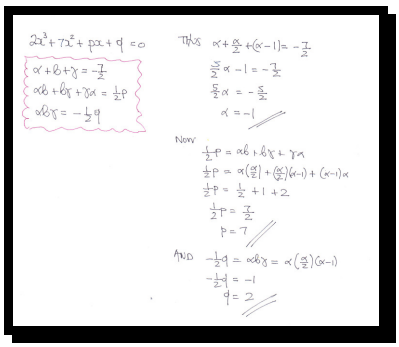
A cubic is given in terms of two constants p and q

$$2x^3+7x^2+px+q=0\,.$$

The three roots of the above cubic are α, $\frac{1}{2}\alpha$ and $(\alpha-1)$.

Find the value of α, p and q.

$\boxed{\alpha=-1}$, $\boxed{p=7}$, $\boxed{q=2}$



Question 3 (+)**

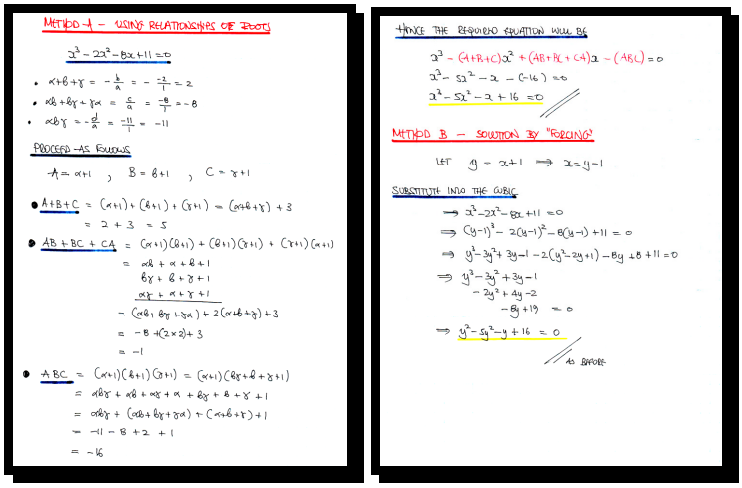
$$x^3-2x^2-8x+11=0.$$

The roots of the above cubic equation are α, β and γ.

Find a cubic equation, with integer coefficients, whose roots are

$$\alpha+1, \quad \beta+1, \quad \gamma+1.$$

$\square$, $\boxed{x^3-5x^2-x+16=0}$



Question 4 (+)**

The three roots of the cubic equation

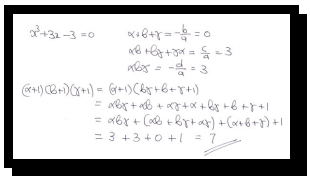
$$x^3+3x-3=0$$

are denoted in the usual notation by α, β and γ.

Find the value of

$$(\alpha+1)(\beta+1)(\gamma+1).$$

7



Question 5 (+)**

The roots of the cubic equation

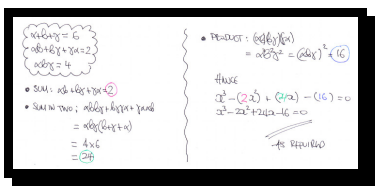
$$x^3-6x^2+2x-4=0$$

are denoted by α, β and γ.

Show that the equation of the cubic whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$ is given by

$$x^3-2x^2+24x-16=0.$$

proof



Question 6 (+)**

The two roots of the quadratic equation

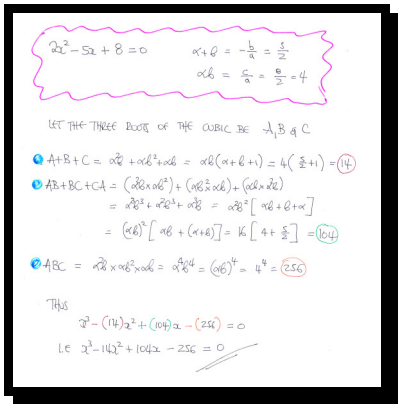
$$2x^2-5x+8=0,$$

are denoted by α and β.

Determine the cubic equation with integer coefficients whose three roots are

$$\alpha^2\beta,\ \alpha\beta^2 \text{ and } \alpha\beta.$$

$$\boxed{x^3-14x^2+104x-256=0}$$



Question 7 (+)**

$$x^3+bx^2+cx+d=0,$$

where b, c and d are real constants.

The three roots of the above cubic are denoted by α, β and γ.

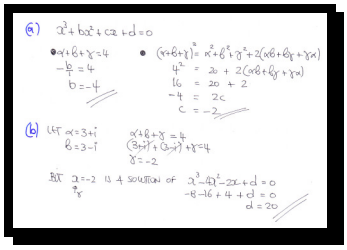
a) Given that

$$\alpha+\beta+\gamma=4 \quad \text{and} \quad \alpha^2+\beta^2+\gamma^2=20,$$

find the value of b and the value of c.

b) Given further that $\alpha=3+\mathrm{i}$, determine the value of d.

$\boxed{b=-4}$, $\boxed{c=-2}$, $\boxed{d=20}$



Question 8 **(**+)**

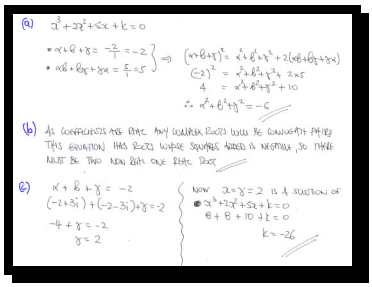
$$x^3+2x^2+5x+k=0\,.$$

The three roots of the above cubic are denoted by α, β and γ, where k is a real constant.

a) Find the value of $\alpha^2+\beta^2+\gamma^2$ and hence explain why this cubic has one real root and two non real roots.

b) Given that $x=-2+3\mathrm{i}$ is a root of the cubic show that $k=-26$.

$$\boxed{\alpha^2+\beta^2+\gamma^2=-6}$$



Question 9 **(**+)**

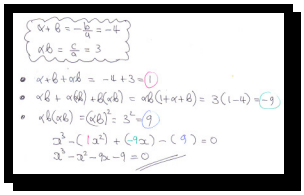
The roots of the quadratic equation

$$x^2+4x+3=0$$

are denoted, in the usual notation, as α and β.

Find the cubic equation, with integer coefficients, whose roots are α, β and $\alpha\beta$.

$$\boxed{x^3+x^2-9x-9=0}$$



Question 10 (***)

$$x^3 - x^2 + 3x + k = 0 .$$

The roots of the above cubic equation are denoted by α, β and γ, where k is a real constant.

a) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = -5 .$$

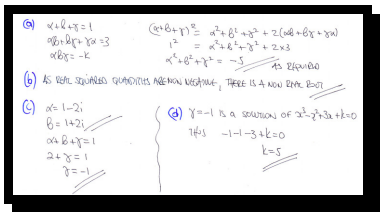
b) Explain why the cubic equation cannot possibly have 3 real roots.

It is further given that $\alpha = 1 - 2\mathrm{i}$.

c) Find the value of β and the value of γ.

d) Show that $k = 5$.

$\boxed{\beta = 1 + 2\mathrm{i}}$, $\boxed{\gamma = -1}$



Question 11 (***)

The roots of the quadratic equation

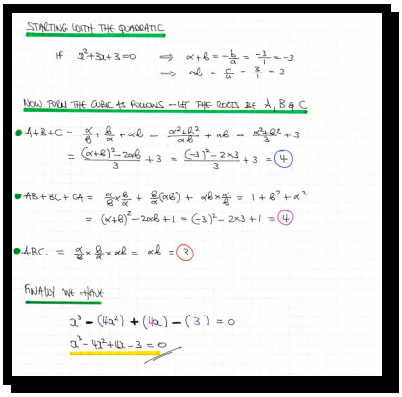
$$x^2+3x+3=0$$

are denoted by α and β.

Find the cubic equation, with integer coefficients, whose roots are

$$\frac{\alpha}{\beta},\ \frac{\beta}{\alpha} \text{ and } \alpha\beta .$$

$\boxed{}$, $\boxed{x^3-4x^2+4x-3=0}$



Question 12 (***)

The roots of the cubic equation

$$x^3 + px^2 + 74x + q = 0 ,$$

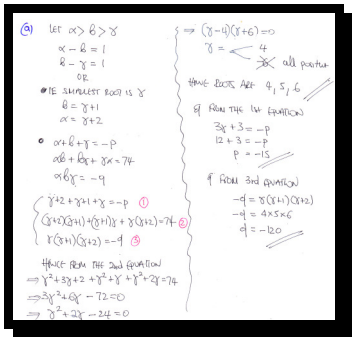
where p and q are constants, form an arithmetic sequence with common difference 1.

Given that all three roots are real and positive find in any order …

a) … the value of p and the value of q.

b) … the roots of the equation.

$$\boxed{p = -15,\ q = -120}, \quad \boxed{x = 4,5,6}$$



Question 13 (***)

The roots of the cubic equation

$$x^3 + px^2 + 56x + q = 0,$$

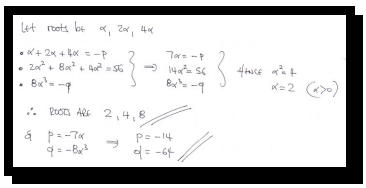
where p and q are constants, form a geometric sequence with common ratio 2.

Given that all three roots are real and positive find in any order …

a) … the value of p and the value of q.

b) … the roots of the equation.

$\boxed{p = -14,\ q = -64}$, $\boxed{x = 2, 4, 8}$



Question 14 (***)

The roots of the cubic equation

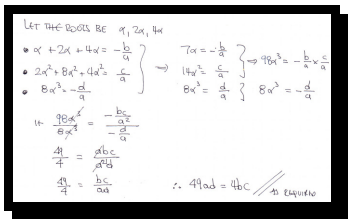
$$ax^3 + bx^2 + cx + d = 0 ,$$

where a, b, c and d are non zero constants, are the first three terms of a geometric sequence with common ratio 2.

Show clearly that

$$4bc = 49ad .$$

proof



Question 15 **(***)**

$$f(z)=z^3-(5+\mathrm{i})z^2+(9+4\mathrm{i})z+k(1+\mathrm{i}),\ z\in\mathbb{C},\ k\in\mathbb{R}$$

The roots of the equation $f(z)=0$ are denoted by α, β and γ.

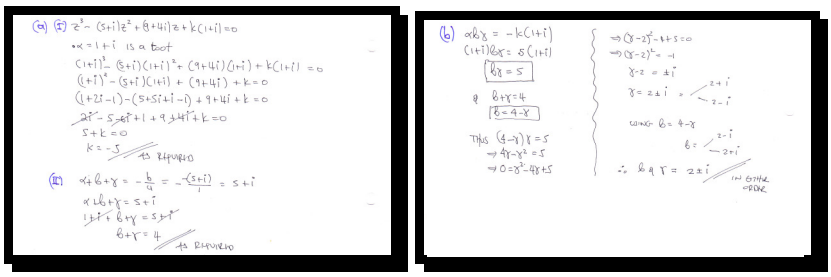
a) Given that $\alpha=1+\mathrm{i}$ show that …

i. … $k=-5$.

ii. … $\beta+\gamma=4$.

b) Hence find the value of β and the value of γ.

$$\boxed{\beta=2+\mathrm{i}},\ \boxed{\gamma=2-\mathrm{i}}$$



Question 16 (***)

$$z^3+pz+q=0,\ z\in\mathbb{C},\ p\in\mathbb{R},\ q\in\mathbb{R}.$$

The roots of the above equation are denoted by α, β and γ.

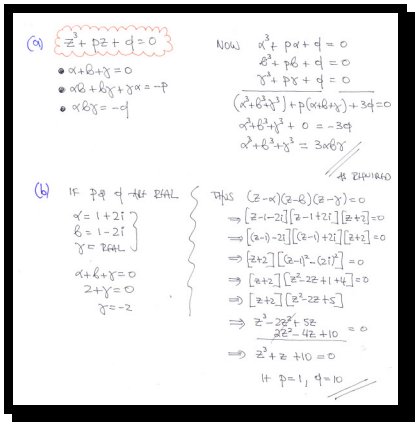
a) Show clearly that

$$\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma.$$

It is further given that $\alpha=1+2\mathrm{i}$.

b) Determine the value of p and the value of q.

$$\boxed{p=1},\ \boxed{q=10}$$



Question 17 (*+)**

The three solutions of the cubic equation

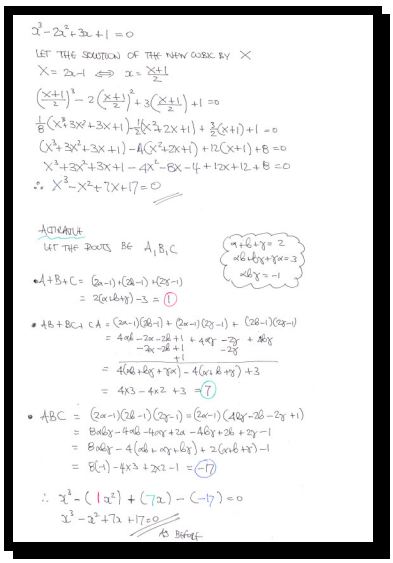
$$x^3-2x^2+3x+1=0 \quad x\in\mathbb{R},$$

are denoted by α, β and γ.

Find a cubic equation with integer coefficients whose solutions are

$$2\alpha-1, \quad 2\beta-1 \quad \text{and} \quad 2\gamma-1.$$

$$\boxed{x^3-x^2+7x+17=0}$$



Question 18 (*+)**

The roots of the cubic equation

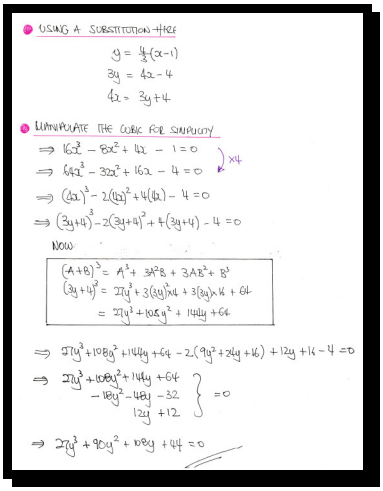
$$16x^3-8x^2+4x-1=0 \quad x\in\mathbb{R},$$

are denoted in the usual notation by α, β and γ.

Find a cubic equation, with integer coefficients, whose roots are

$$\frac{4}{3}(\alpha-1),\ \frac{4}{3}(\beta-1) \text{ and } \frac{4}{3}(\gamma-1).$$

$\square$, $\boxed{27x^3+90x^2+108x+44=0}$



Question 19 (*+)**

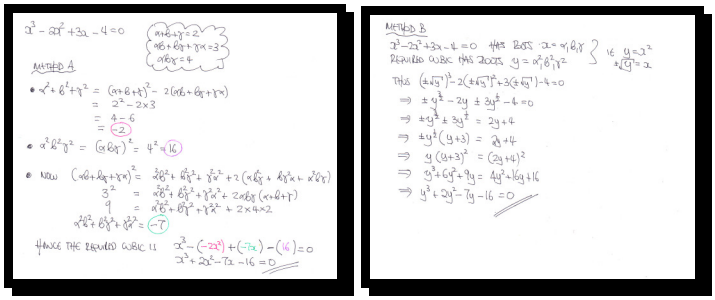
The roots of the equation

$$x^3-2x^2+3x-4=0,$$

are denoted in the usual notation by α, β and γ.

Find a cubic equation with integers coefficients whose roots are α^2, β^2 and γ^2.

$$\boxed{x^3+2x^2-7x-16=0}$$



Question 20 (*+)**

The roots of the equation

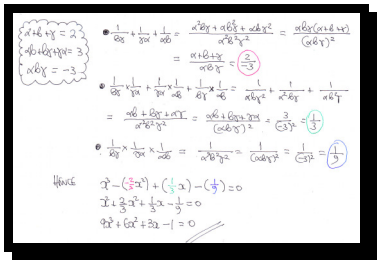
$$x^3 - 2x^2 + 3x + 3 = 0$$

are denoted by α, β and γ.

Find the cubic equation with integer coefficients whose roots are

$$\frac{1}{\beta\gamma}, \quad \frac{1}{\gamma\alpha} \quad \text{and} \quad \frac{1}{\alpha\beta}.$$

$$\boxed{9x^3 + 6x^2 + 3x - 1 = 0}$$



Question 21 (*+)**

The roots of the equation

$$x^3 + 2kx^2 - 27 = 0,$$

are α, β and $\alpha + \beta$, where k is a real constant.

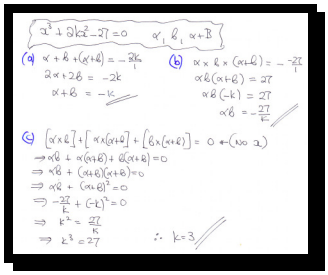
a) Find, in terms of k, the value of …

i. … $\alpha + \beta$

ii. … $\alpha\beta$

b) Use these results to show that $k = 3$.

$\boxed{\alpha + \beta = -k}$, $\boxed{\alpha\beta = -\frac{27}{k}}$



Question 22 (*+)**

The roots of the equation

$$x^3+2x^2+3x-4=0$$

are denoted by α, β and γ.

a) Show that that for all w, y and z

$$w^2+y^2+z^2\equiv(w+y+z)^2-2(wy+yz+zw).$$

Another cubic equation has roots A, B and C where

$$A=\frac{\beta\gamma}{\alpha},\ B=\frac{\gamma\alpha}{\beta}\text{ and }C=\frac{\alpha\beta}{\gamma}.$$

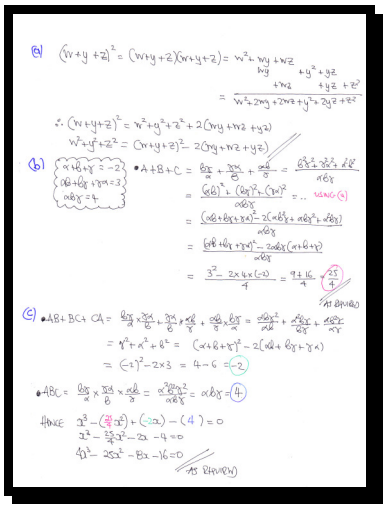
b) Show clearly that

$$A+B+C=\frac{25}{4}.$$

c) Show that the equation of the cubic whose roots are A, B and C is

$$4x^3-25x^2-8x-16=0.$$

proof



Question 23 **(***+)**

The cubic equation

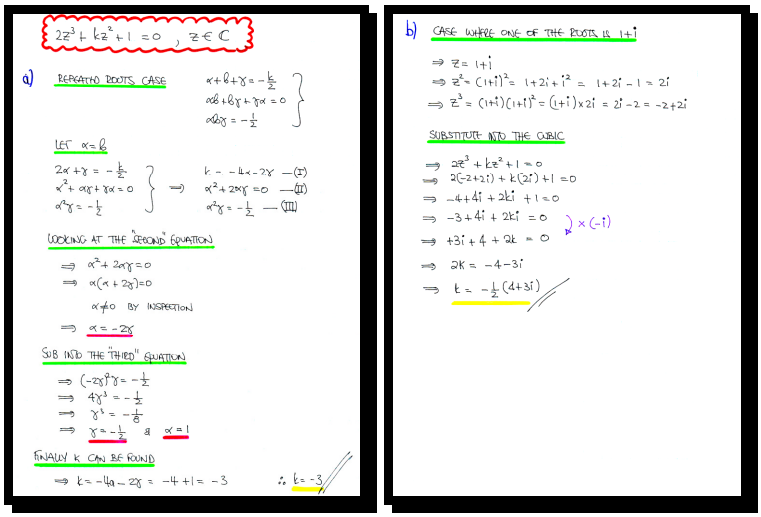
$$2z^3+kz^2+1=0\ ,\ z\in\mathbb{C},$$

where k is a non zero constant, is given.

a) If the above cubic has two identical roots, determine the value of k.

b) If **instead** one of the roots is $1+\mathrm{i}$, find the value of k in this case.

$\boxed{}$, $\boxed{k=-3}$, $\boxed{k=-\frac{1}{2}(4+3\mathrm{i})}$



Question 24 (*+)**

A cubic equation is given below as

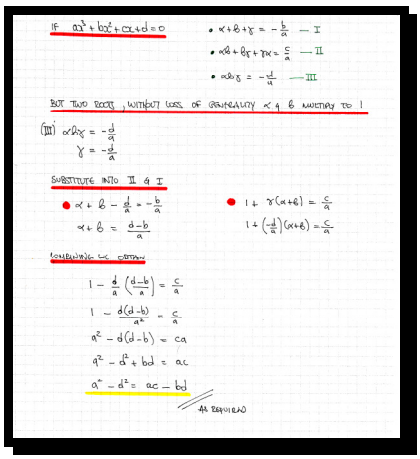
$$ax^3+bx^2+cx+d=0\,,$$

where a, b, c and d are non zero constants.

Given that the product of two of the three roots of above cubic equation is 1, show that

$$a^2-d^2=ac-bd\,.$$

$\boxed{}$, proof

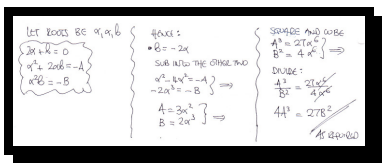


Question 25 (*+)**

If the cubic equation $x^3-Ax+B=0$, has two equal roots, show that

$$4A^3=27B^2\,.$$

proof



Question 26 (****)

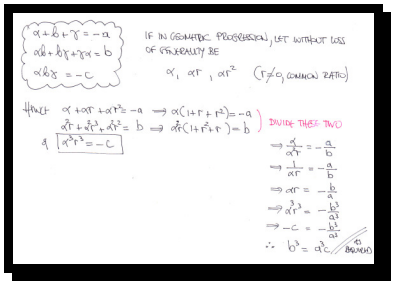
$$bx^3+bx^2+cx+d=0\,,$$

where a, b and c are non zero constants.

If the three roots of the above cubic equation are in geometric progression show that

$$b^3=ca^3\,.$$

proof



Question 27 (****)

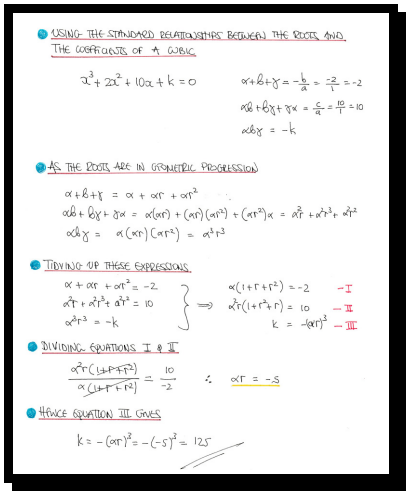
The three roots of the equation

$$x^3+2x^2+10x+k=0,$$

where k is a non zero constant, are in geometric progression.

Determine the value of k.

$\boxed{}$, $\boxed{k=125}$



Question 28 (****)

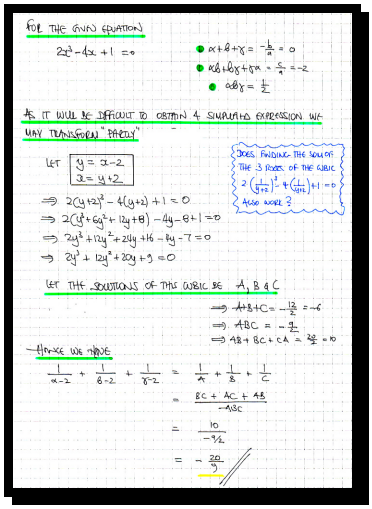
$$2x^3-4x+1=0\,.$$

The cubic equation shown above has three roots, denoted by α, β and γ.

Determine, as an exact simplified fraction, the value of

$$\frac{1}{\alpha-2}+\frac{1}{\beta-2}+\frac{1}{\gamma-2}\,.$$

$\square$, $\boxed{-\frac{20}{9}}$



Question 29 (****)

A cubic equation is given below as

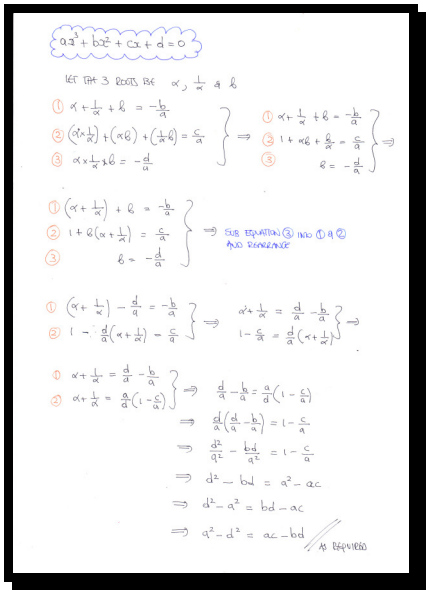
$$ax^3+bx^2+cx+d=0\,,$$

where a, b, c and d are non zero constants.

Given that two of the three roots of above cubic equation are reciprocals of one another show that

$$a^2-d^2=ac-bd\,.$$

proof



Question 30 **(****)**

$$x^3-2x^2+kx+10=0\,,\ k\neq 0$$

The roots of the above cubic equation are α, β and γ.

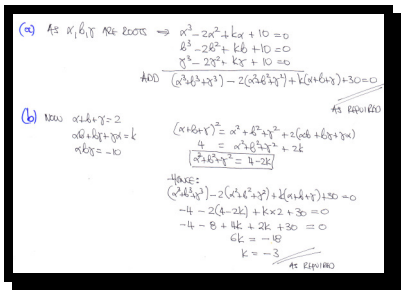
a) Show clearly that

$$\left(\alpha^3+\beta^3+\gamma^3\right)-2\left(\alpha^2+\beta^2+\gamma^2\right)+k\left(\alpha+\beta+\gamma\right)+30=0\,.$$

It is given that $\alpha^3+\beta^3+\gamma^3=-4$

b) Show further that $k=-3$.

proof



Question 31 (****)

The three roots of the equation

$$z^3+pz^2+qz+r=0\,,$$

where p, q and r are constants, are denoted by α, β and γ.

a) Given that

$$\alpha\beta+\beta\gamma+\gamma\alpha=-2+3\mathrm{i} \quad \text{and} \quad \alpha^2+\beta^2+\gamma^2=4-6\mathrm{i}\,,$$

determine the value of p and the value of q.

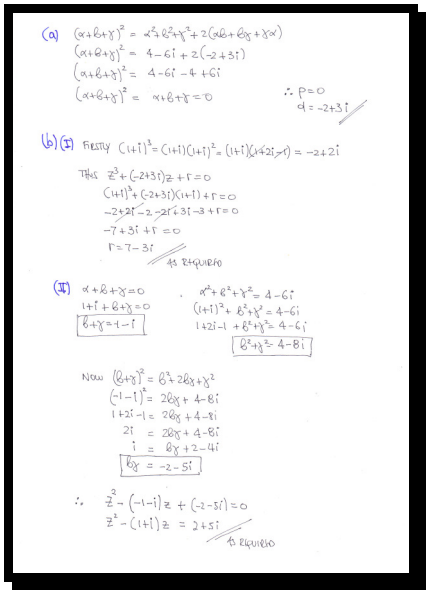
b) Given further that $\alpha=1+\mathrm{i}$, show that …

i. … $r=7-3\mathrm{i}$

ii. … β and γ are solutions of the equation

$$z^2-(1+\mathrm{i})z=2+5\mathrm{i}\,.$$

$\boxed{p=0}$, $\boxed{q=-2+3\mathrm{i}}$



Question 32 (****)

$$z^3+2z^2+k=0,$$

The roots of the above cubic equation, where k is a non zero constant, are denoted by α, β and γ.

a) Show that …

i. … $\alpha^2+\beta^2+\gamma^2=4$.

ii. … $\alpha^3+\beta^3+\gamma^3=-8-3k$.

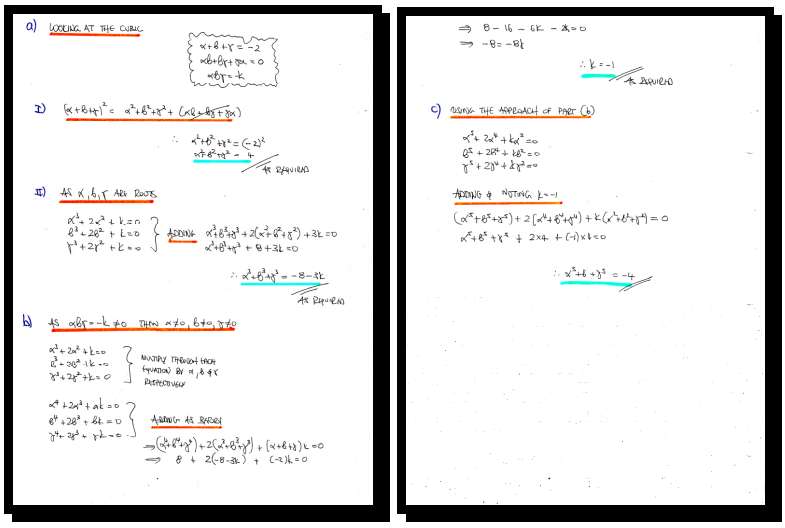
It is further given that $\alpha^4+\beta^4+\gamma^4=4$.

b) Show further that $k=-1$.

c) Determine the value of

$$\alpha^5+\beta^5+\gamma^5.$$

$\square$, $\boxed{\alpha^5+\beta^5+\gamma^5=-4}$



Question 33 (****)

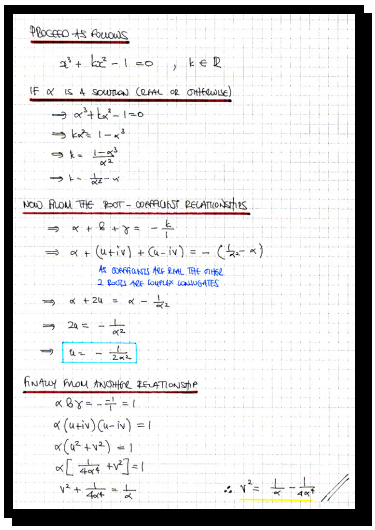
The cubic equation shown below has a real root α.

$$x^3+kx^2-1=0,$$

where k is a real constant.

Given that one of the complex roots of the equation is $u+\mathrm{i}v$, determine the value of v^2 in terms of α.

$\square$, $\boxed{v^2=\dfrac{1}{\alpha}-\dfrac{1}{4\alpha^4}}$



Question 34 **(****)**

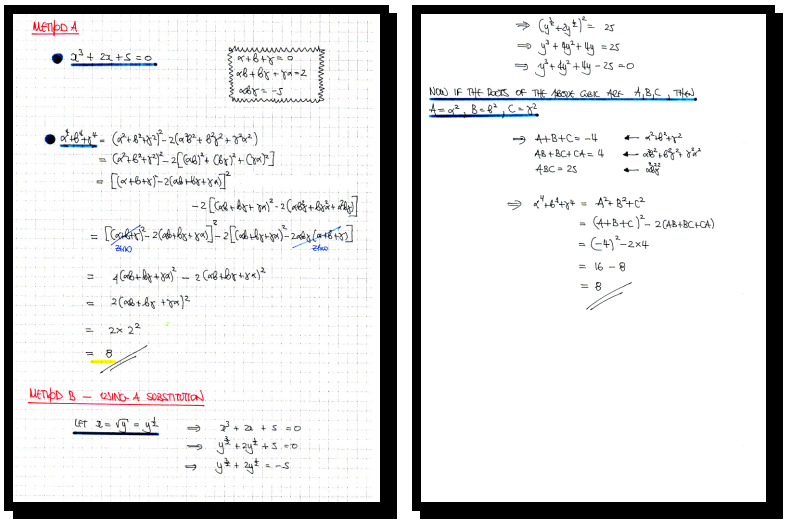
$$x^3+2x+5=0\,.$$

The cubic equation shown above has three roots, denoted by α, β and γ.

Determine the value of

$$\alpha^4+\beta^4+\gamma^4\,.$$

$\boxed{}$, $\boxed{8}$



Question 35 (****)

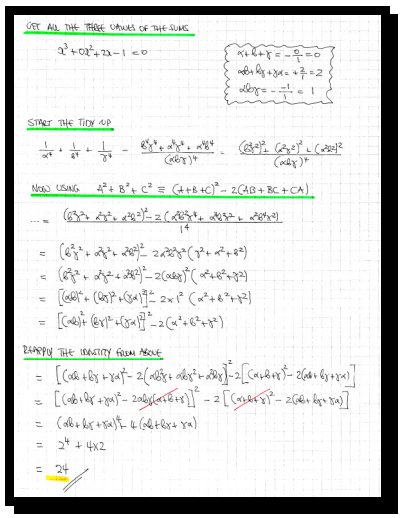
The three roots of the cubic equation

$$x^3+2x-1=0,$$

are denoted by α, β and γ.

Determine the exact value of $\dfrac{1}{\alpha^4}+\dfrac{1}{\beta^4}+\dfrac{1}{\gamma^4}$.

$\boxed{}$, $\boxed{24}$



Question 36 (**+)**

The roots of the cubic equation

$$x^3-4x^2+2x-5=0$$

are denoted in the usual notation by α, β and γ.

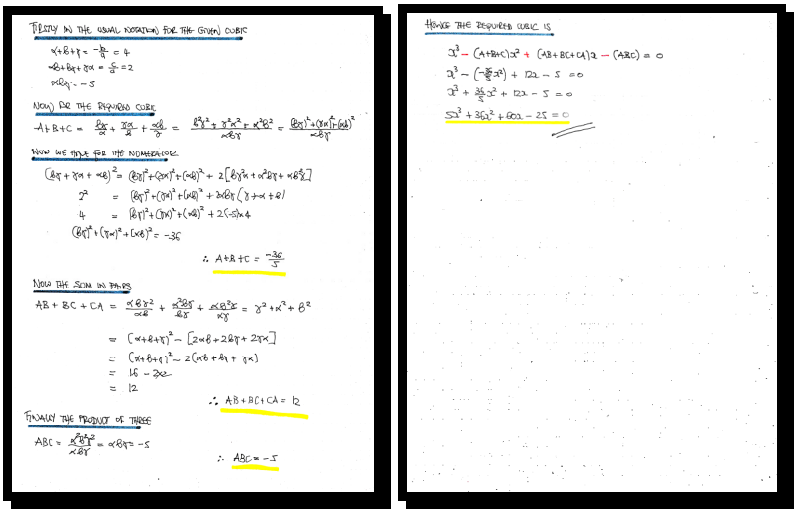
Show that the cubic equation whose roots are

$$\frac{\beta\gamma}{\alpha},\ \frac{\gamma\alpha}{\beta} \text{ and } \frac{\alpha\beta}{\gamma},$$

is given by

$$5x^3+36x^2+60x-25=0$$

$\boxed{}$, proof

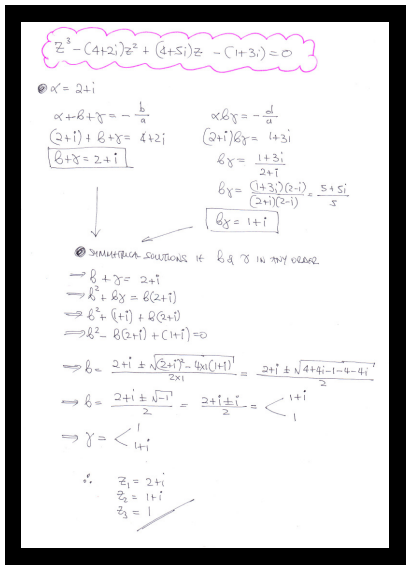


Question 37 (**+)**

$$z^3-(4+2\mathrm{i})z^2+(4+5\mathrm{i})z-(1+3\mathrm{i})=0\,,\ z\in\mathbb{C}\,.$$

Given that one of the solutions of the above cubic equation is $z=2+\mathrm{i}$, find the other two solutions.

$\boxed{}$, $\boxed{z=1}$, $\boxed{z=1+\mathrm{i}}$



Question 38 **(****+)**

The roots of the cubic equation

$$x^3-4x^2-3x-2=0$$

are denoted in the usual notation by α, β and γ.

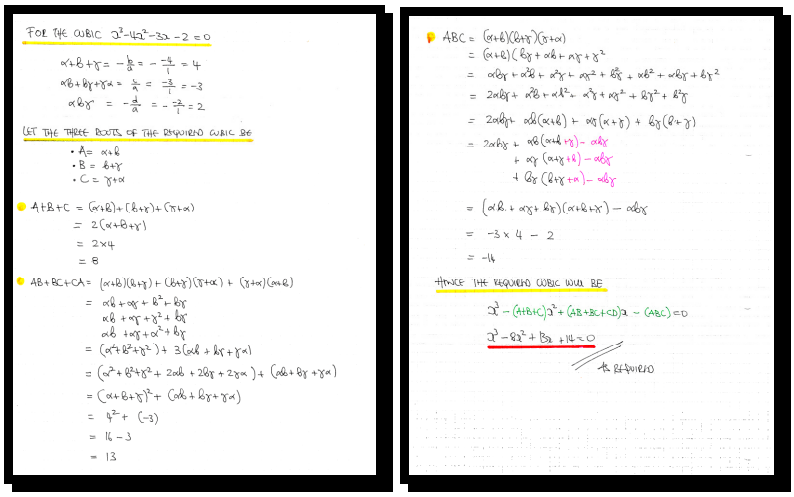
Show that the cubic equation whose roots are

$$\alpha+\beta, \quad \beta+\gamma \text{ and } \quad \gamma+\alpha,$$

is given by

$$x^3-8x^2+13x+14=0$$

, proof



Question 39 (*****)

A system of simultaneous equations is given below

$$\begin{aligned} x+y+z&=1\\ x^2+y^2+z^2&=21\\ x^3+y^3+z^3&=55\,. \end{aligned}$$

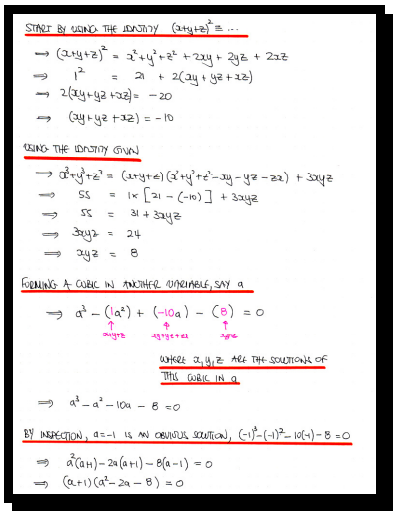
By forming an auxiliary cubic equation find the solution to the above system.

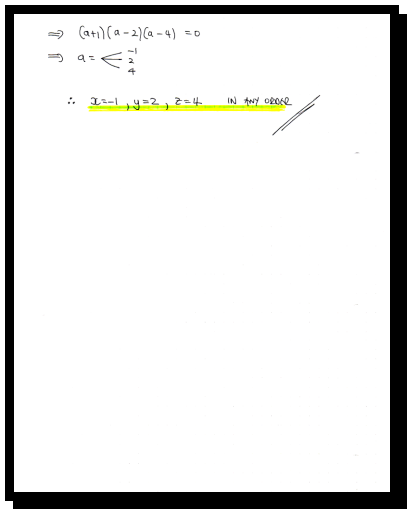
You may find the identity

$$x^3+y^3+z^3\equiv\left(x+y+z\right)\left(x^2+y^2+z^2-xy-yz-zx\right)+3xyz\,,$$

useful in this question.

$\boxed{}$, $\boxed{x,y,z=-2,-1,4 \text{ in any order}}$





Question 40 (*****)

The roots of the cubic equation

$$8x^3+12x^2+2x-3=0$$

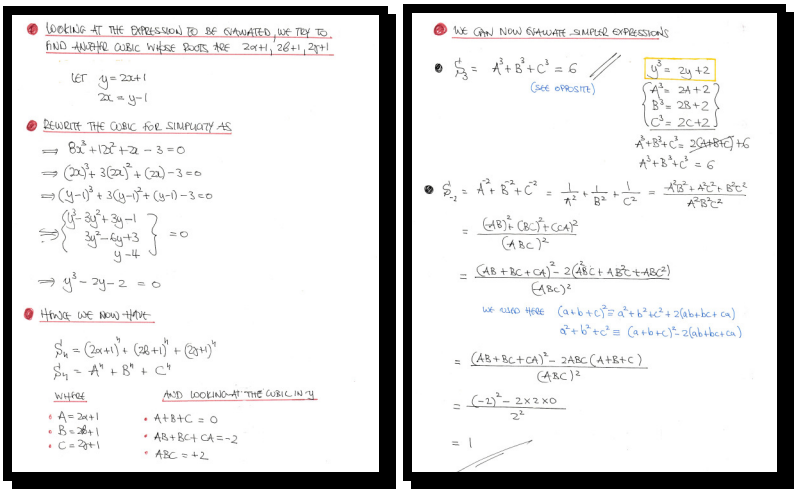
are denoted in the usual notation by α, β and γ.

An integer function S_n, is defined as

$$S_n \equiv (2\alpha+1)^n+(2\beta+1)^n+(2\gamma+1)^n, \quad n\in\mathbb{Z}.$$

Determine the value of S_3 and the value of S_{-2}.

$\boxed{}$, $\boxed{S_3=6}$, $\boxed{S_{-2}=1}$



QUARTICS