RATIONAL FUNCTIONS

Question 1 (*+)**

The curves C_1 and C_2 have respective equations

$$y=\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0, \qquad \text{and} \qquad y=\frac{x}{x+2},\ x\in\mathbb{R},\ x\neq -2.$$

a) Sketch in the same set of axes the graph of C_1 and the graph of C_2.

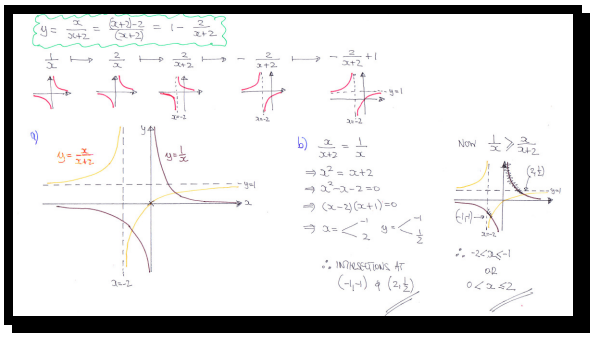
Indicate clearly in the sketch …

- … the equations of any asymptotes.
- … the coordinates of any intersections between each curve with the coordinate axes.

b) Find the coordinates of the points of intersection between the graph of C_1 and C_2, and hence solve the inequality

$$\frac{1}{x}\geq\frac{x}{x+2}.$$

$$\boxed{(-1,-1),\ \left(2,\tfrac{1}{2}\right)}, \qquad \boxed{-2<x\leq -1\ \cup\ 0<x\leq 2}$$



Question 2 (****)

The curve C has equation $y=f(x)$ given by

$$f(x)=\frac{x^2}{(x-1)(x-5)},\ x\in\mathbb{R},\ x\neq 1,\ x\neq 5.$$

a) State the equation of each of the two vertical asymptotes of the C.

b) Show clearly that $y=1$ is a horizontal asymptote to the curve.

c) Solve the equation $f(x)=1$.

The straight line with equation $y=k$, where k is a constant, touches C at two points.

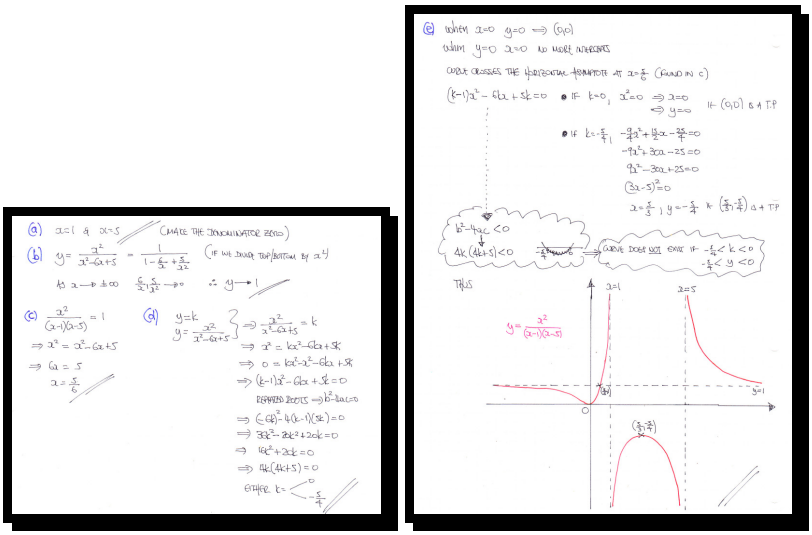
d) Find possible values the of k.

e) Sketch the graph of C.

Indicate clearly in the sketch

- the equations of the asymptotes
- the coordinates of any intersections of C with the coordinate axes.
- the coordinates of any turning points of C.

$\boxed{x=1,\ 5}$, $\boxed{x=\frac{5}{6}}$, $\boxed{k=-\frac{5}{4},\ 0}$



Question 3 (****)

The curve C has equation $y=f(x)$ given by

$$f(x)=\frac{x-4}{(x-5)(x-8)},\ x\in\mathbb{R},\ x\neq 5,\ x\neq 8.$$

a) State the equation of each of the two vertical asymptotes of the C.

b) Show clearly that $y=0$ is a horizontal asymptote to the curve.

The straight line with equation $y=k$, where k is a constant, touches C at two points.

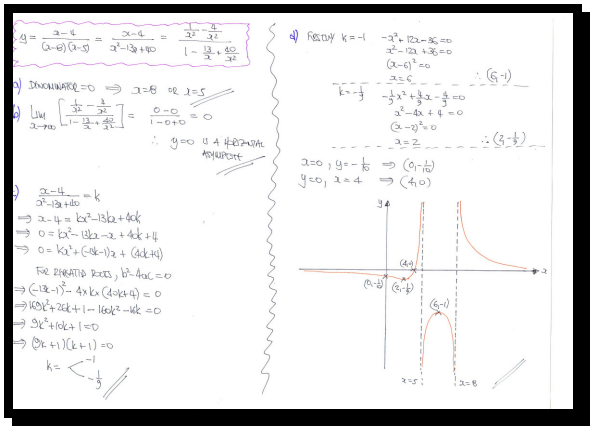
c) Find possible values the of k.

d) Sketch the graph of C.

Indicate clearly in the sketch

- the equations of the asymptotes
- the coordinates of any intersections of C with the coordinate axes.
- the coordinates of any turning points of C.

$$\boxed{x=5,\ 8},\quad \boxed{k=-1,\ -\frac{1}{9}}$$



Question 4 (****)

The curves C_1 and C_2 have respective equations

$$y=\frac{x+3}{x},\ x\in\mathbb{R},\ x\neq 0, \qquad \text{and} \qquad y=\frac{x}{2-x},\ x\in\mathbb{R},\ x\neq 2.$$

a) Sketch in the same set of axes the graph of C_1 and the graph of C_2.

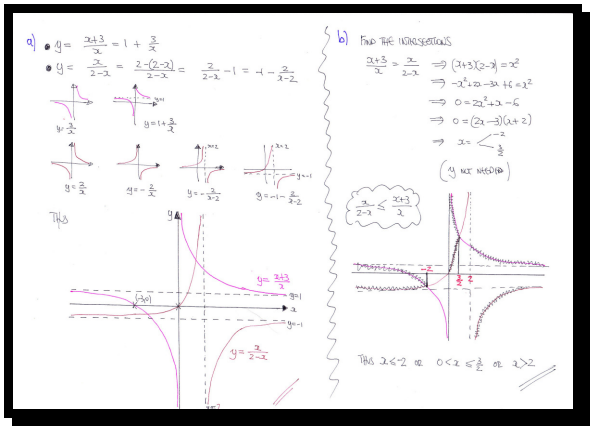
Indicate clearly in the sketch …

- … the equations of any asymptotes.
- … the coordinates of any intersections between each curve with the coordinate axes.

b) Use the sketch of part (a) to solve the inequality

$$\frac{x}{2-x}\leq\frac{x+3}{x}.$$

$$\boxed{x\leq -2\ \cup\ 0<x\leq\frac{3}{2}\ \cup\ x>2}$$



Question 5 (**+)**

A curve C has equation

$$h(x)=\frac{f(x)}{g(x)},\ x\in\mathbb{R},\ g(x)\neq 0.$$

It is further given that $f(x)$ is a quadratic function and $g(x)$ a linear function.

The asymptotes of C have equations $x=-1$ and $y=x+2$, and its graph is passing through the point $P(1,5)$.

a) Determine a simplified Cartesian equation for C.

b) Use a discriminant method to find the range of $h(x)$ and hence calculate the coordinates of the stationary points of C.

c) Sketch in separate diagrams, showing all the relevant details including asymptotic behaviour, the graph of …

i. … $y=h(x)$.

ii. … $y^2=h(x)$.

$$\boxed{h(x)=\frac{x^2+3x+6}{x+1}=x+2+\frac{4}{x+1}},\ \boxed{h(x)\le -3\cup h(x)\ge 5},\ \boxed{(1,5),\ (-3,-3)}$$

