POLAR COORDINATES

CONVERTING BETWEEN CARTESIANS AND POLARS

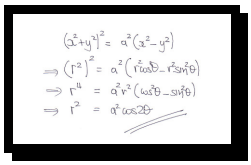
Question 1

A curve C has Cartesian equation

$$\left(x^2+y^2\right)^2=a^2\left(x^2-y^2\right),\ a\neq 0.$$

Determine a polar equation for C.

$$\boxed{r^2=a^2\cos 2\theta}$$



Question 2

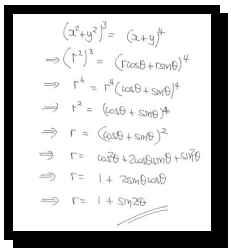
A curve C has Cartesian equation

$$\left(x^2+y^2\right)^3=\left(x+y\right)^4.$$

Show that a polar equation for C is given by

$$r=1+\sin 2\theta,\ r\geq 0.$$

$$\boxed{\text{proof}}$$



Question 3

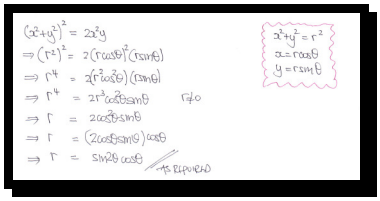
A curve C has Cartesian equation

$$\left(x^2+y^2\right)^2=2x^2y\,.$$

Show that a polar equation for C can be written as

$$r=\sin 2\theta\cos\theta\,.$$

proof



Question 4

A circle has Cartesian equation

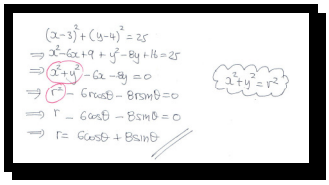
$$(x-3)^2+(y-4)^2=25\,.$$

Show that a polar equation for the circle is given by

$$r=A\cos\theta+B\sin\theta\,,$$

where A and B are constants.

$$r=6\cos\theta-4\sin\theta$$



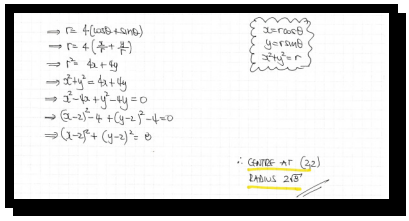
Question 5

A circle has polar equation

$$r = 4(\cos\theta + \sin\theta) \quad 0 \le \theta < 2\pi .$$

Determine the Cartesian coordinates of the centre of the circle and the length of its radius.

$$\boxed{(2,2),\ \text{radius} = \sqrt{8}}$$



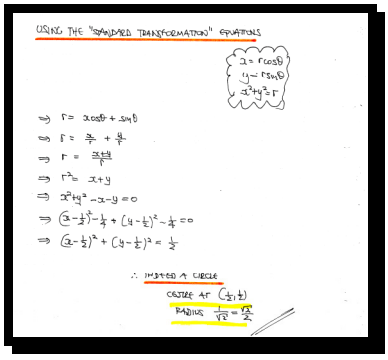
Question 6

Write the polar equation

$$r = \cos\theta + \sin\theta ,\ 0 \le \theta < 2\pi$$

in Cartesian form, and hence show that it represents a circle, further determining the coordinates of its centre and the size of its radius.

$$\boxed{\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 = \frac{1}{2}}$$



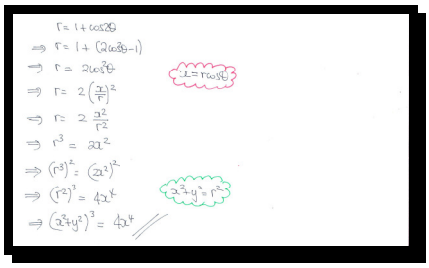
Question 7

A curve C has polar equation

$$r = 1 + \cos 2\theta .$$

Determine a Cartesian equation for C.

$$\boxed{\left(x^2 + y^2\right)^3 = 4x^4}$$



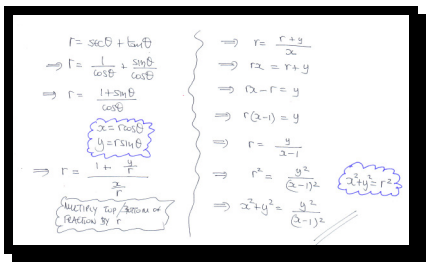
Question 8

A curve C has polar equation

$$r = \sec\theta + \tan\theta .$$

Determine a Cartesian equation for C.

$$\boxed{x^2 + y^2 = \frac{y^2}{(x-1)^2}}$$



Question 9

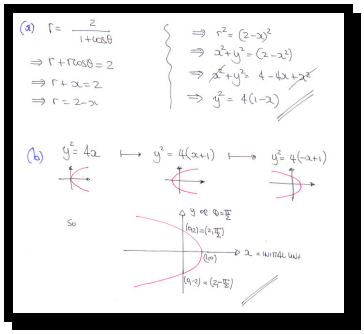
A curve C has polar equation

$$r = \frac{2}{1+\cos\theta}, \ 0 \le \theta < 2\pi .$$

a) Find a Cartesian equation for C.

b) Sketch the graph of C.

$$\boxed{y^2 = 4(1-x)}$$



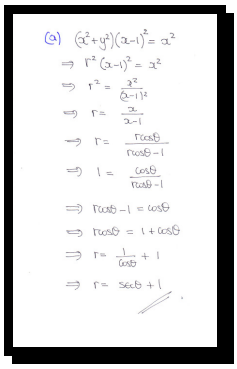
Question 10

The curve C has Cartesian equation

$$\left(x^2+y^2\right)(x-1)^2=x^2.$$

Find a polar equation of C in the form $r=f(\theta)$.

$$\boxed{r=1+\sec\theta}$$



Question 11

A curve C_1 has polar equation

$$r = 2\sin\theta\,,\ 0 \le \theta < 2\pi\,.$$

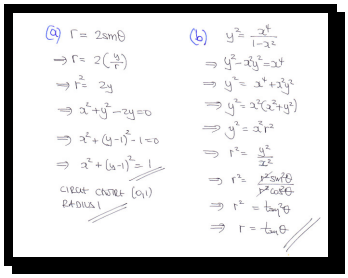
a) Find a Cartesian equation for C_1, and describe it geometrically.

A different curve C_2 has Cartesian equation

$$y^2 = \frac{x^4}{1-x^2}\,,\ x \neq \pm 1\,.$$

b) Find a polar equation for C_2, in the form $r = f(\theta)$.

$\boxed{x^2 + (y-1)^2 = 1}$, $\boxed{r = \tan\theta}$



Question 12

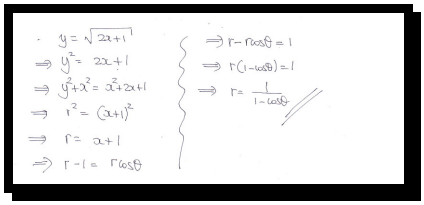
Show that the polar equation of the top half of the parabola with Cartesian equation

$$y=\sqrt{2x+1}\,,\ x\geq -\frac{1}{2}\,,$$

is given by the polar equation

$$r=\frac{1}{1-\cos\theta}\,,\ r\geq 0\,.$$

proof



Question 13

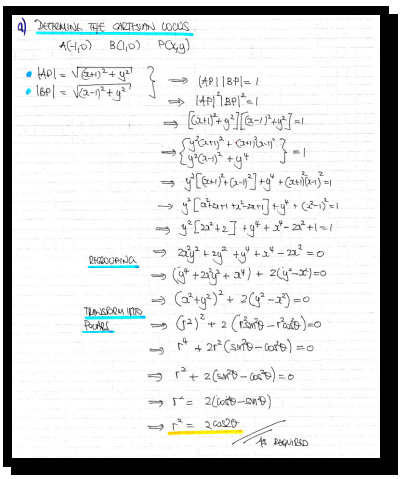
The points A and B have respective coordinates $(-1,0)$ and $(1,0)$. The locus of the point $P(x,y)$ traces a curve in such a way so that $|AP||BP|=1$.

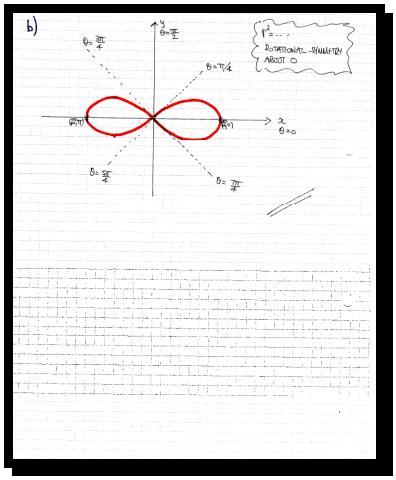
a) By forming a Cartesian equation of the locus of P, show that the polar equation of the curve is

$$r^2 = 2\cos 2\theta,\ 0 \le \theta < 2\pi .$$

b) Sketch the curve.

proof





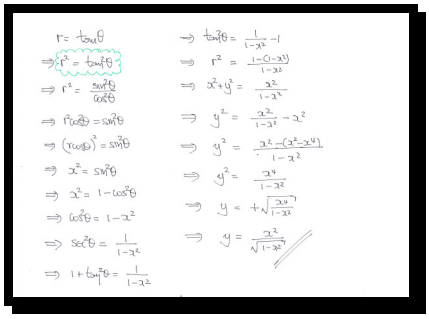
Question 14

The curve C has polar equation

$$r = \tan\theta\,,\ 0 \le \theta < \frac{\pi}{2}.$$

Find a Cartesian equation of C in the form $y = f(x)$.

$$\boxed{y = \frac{x^2}{\sqrt{1-x^2}}}$$



TANGENTS TO POLAR CURVES

Question 1

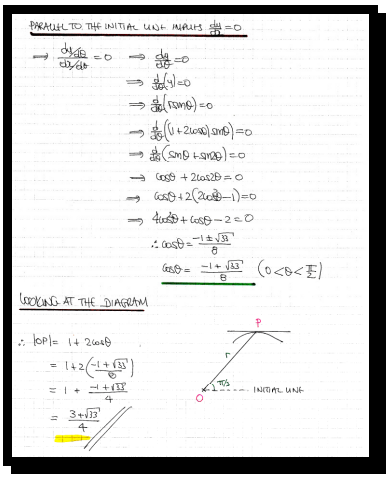
A Cardioid has polar equation

$$r = 1 + 2\cos\theta, \ 0 \le \theta \le \frac{\pi}{2}.$$

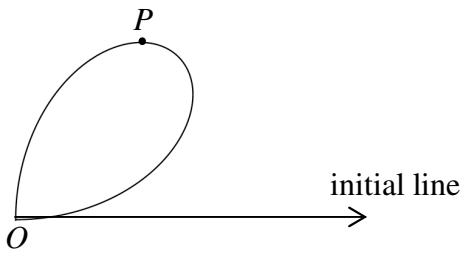
The point P lies on the Cardioid so that the tangent to the Cardioid at P is parallel to the initial line.

Determine the exact length of OP, where O is the pole.

$$\boxed{\frac{1}{4}\left(3+\sqrt{33}\right)}$$



Question 2



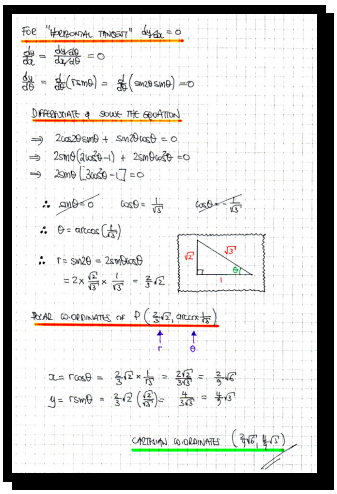
The figure above shows the polar curve with equation

$$r = \sin 2\theta\,,\ 0 \le \theta \le \frac{\pi}{2}.$$

The point P lies on the curve so that the tangent at P is parallel to the initial line.

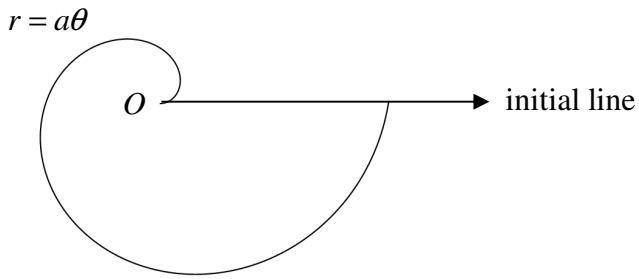
Find the **Cartesian** coordinates of P.

$$\boxed{\left(\frac{2}{9}\sqrt{6}, \frac{4}{9}\sqrt{3}\right)}$$



POLAR CURVE AREAS

Question 1



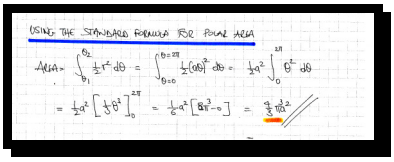
The figure above shows a spiral curve with polar equation

$$r = a\theta \, , \; 0 \leq \theta \leq 2\pi \, ,$$

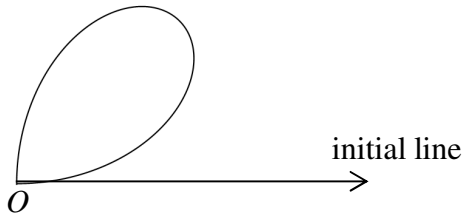
where a is a positive constant.

Find the area of the finite region bounded by the spiral and the initial line.

$$\boxed{\text{area} = \frac{4}{3} a^2 \pi^3}$$



Question 2

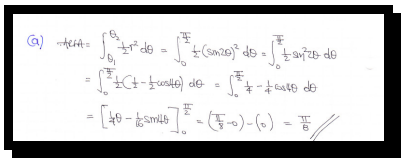


The figure above shows the polar curve C with equation

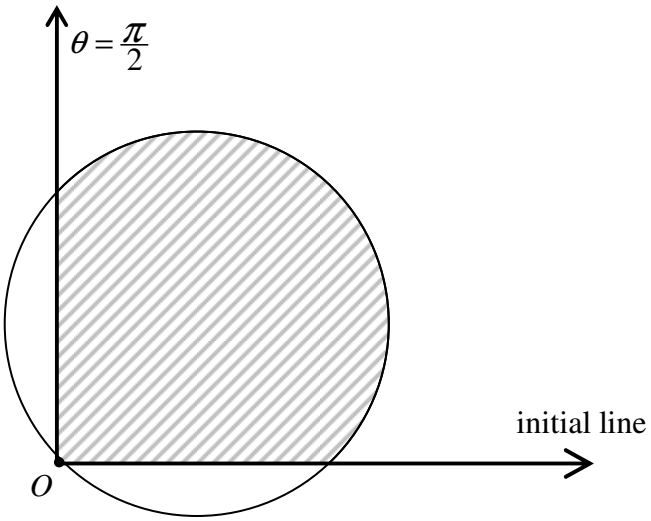
$$r = \sin 2\theta\,,\ 0 \le \theta \le \frac{\pi}{2}.$$

Find the exact value of the area enclosed by the curve.

$$\text{area} = \frac{\pi}{8}$$



Question 3

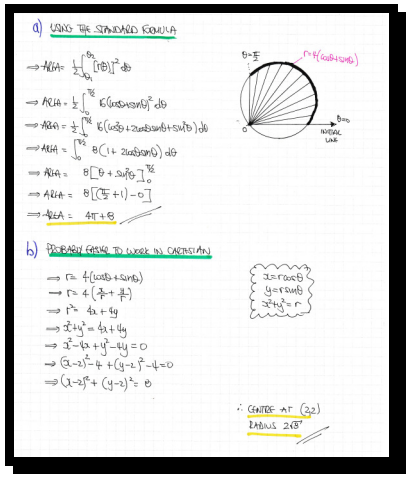


The figure above shows a circle with polar equation

$$r = 4\left(\cos\theta + \sin\theta\right) \quad 0 \le \theta < 2\pi \,.$$

Find the exact area of the shaded region bounded by the circle, the initial line and the half line $\theta = \frac{\pi}{2}$.

$$\boxed{\text{area} = 4\pi + 8}$$



Question 4

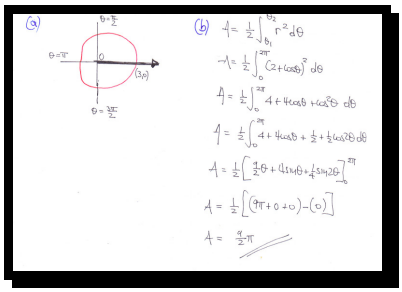
The polar curve C has equation

$$r = 2 + \cos\theta\,,\ 0 \leq \theta < 2\pi\,.$$

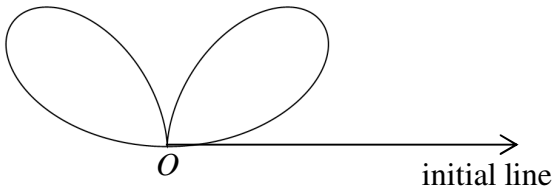
a) Sketch the graph of C.

b) Show that the area enclosed by the curve is $\dfrac{9}{2}\pi$.

proof



Question 5

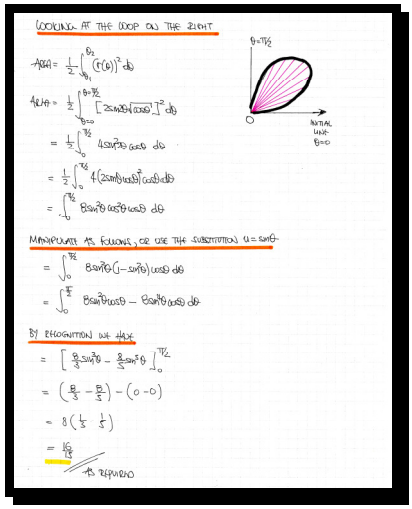


The figure above shows the polar curve C with equation

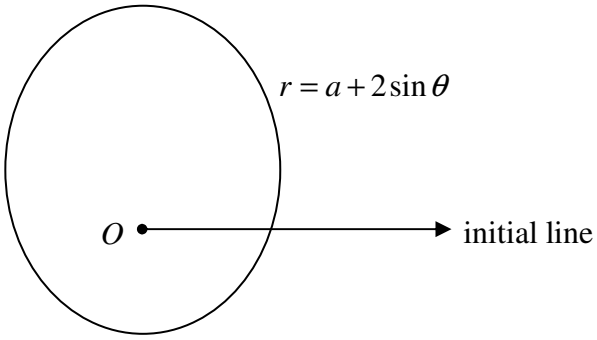
$$r = 2\sin 2\theta\sqrt{\cos\theta}\,,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$$

Show that the area enclosed by one of the two identical loops of the curve is $\frac{16}{15}$.

proof



Question 6



The diagram above shows the curve with polar equation

$$r = a + 2\sin\theta\,,\ 0 \le \theta < 2\pi\,,$$

where a is a positive constant.

Determine the value of a given that the area bounded by the curve is 38π.

$\boxed{a = 6}$

