FURTHER DIFFERENTIATION PRACTICE

Question 1

1. $y = \sinh 3x$

2. $y = \cosh 4x$

3. $y = 3\tanh 2x$

4. $y = 4\sinh\left(\frac{1}{2}x\right)$

5. $y = 3\coth 2x$

6. $y = 3\sinh^2 x$

7. $y = 4\cosh^3 2x$

8. $y = 4\operatorname{sech} 2x$

9. $y = 4\tanh^4 x$

10. $y = 5\operatorname{cosech} 3x$

11. $y = x\sinh x$

12. $y = x^3\cosh x$

13. $y = \mathrm{e}^x \sinh 2x$

14. $y = 4\mathrm{e}^{\cosh 2x}$

15. $y = 3\sinh^3\left(3x^3\right)$

16. $y = \sin\left(\sinh x\right)$

17. $y = \cosh\left(\cos x\right)$

18. $y = \dfrac{\sinh 2x}{x}$

19. $y = \dfrac{\ln\left(\cosh x\right)}{\sinh x}$

20. $y = \dfrac{\tanh x}{\sinh x}$

21. $y = \cosh\left(\mathrm{e}^{x} + x^{2}\right)$

22. $y = \sinh\left(\ln\left(x^{2} - 1\right)\right)$

23. $y = \cosh x \coth x$

24. $y = 2\,\mathrm{e}^{3x} \tanh 2x$

25. $y = \dfrac{\cosh x + \sinh x}{\mathrm{e}^{-x}}$

Question 2

1. $y = \operatorname{arsinh} 2x$

2. $y = \operatorname{arsinh} x^2$

3. $y = \operatorname{arsinh}(\sin x)$

4. $y = \operatorname{arcosh} 4x$

5. $y = \operatorname{arcosh} \sqrt{x}$

6. $y = \operatorname{arcosh}(x+1)$

7. $y = \operatorname{arsinh}(2x+1)$

8. $y = \operatorname{artanh}(1-4x)$

9. $y = x \operatorname{arcosh} 2x$

10. $y = (\operatorname{arsinh} x)^3$

11. $y = \operatorname{artanh}\left(\frac{1}{2}\mathrm{e}^x\right)$

12. $y = \operatorname{artanh}\left(\frac{x+1}{x-1}\right)$

Question 3

1. $y = \arcsin 3x$

2. $y = \arcsin x^2$

3. $y = \arctan 4x$

4. $y = \arccos x^2$

5. $y = \arctan \sqrt{x}$

6. $y = \arcsin\left(2x+3\right)$

7. $y = \arccos\left(\frac{2}{3}x\right)$

8. $y = \arctan\left(\frac{4}{3}x\right)$

9. $y = x \operatorname{arcosh} 2x$

10. $y = 2\left(\arcsin x\right)^4$

11. $y = \arctan\left(\frac{2}{x}\right)$

12. $y = \arcsin\left(\mathrm{e}^{2x}\right)$

13. $y = x^2 \arctan 2x$

14. $y = \arctan\left(\frac{x+1}{x-1}\right)$

Question 4

Prove that:

1. $\dfrac{d}{dx}\left(\dfrac{\tanh x}{\operatorname{sech} x}\right)=\cosh x$ (***)

2. $\dfrac{d}{dx}\left(\coth x\sinh x\right)=\sinh x$ (***)

3. $\dfrac{d}{dx}\left(\dfrac{\cosh x-\sinh x}{\mathrm{e}^{x}}\right)=-2\mathrm{e}^{-2x}$ (***)

4. $\dfrac{d}{dx}\left(\dfrac{1+\cosh x}{1-\cosh x}\right)=\coth\left(\dfrac{1}{2}x\right)\operatorname{cosech}^{2}\left(\dfrac{1}{2}x\right)$ (****)

5. $\dfrac{d}{dx}\left(2\arctan\left(x+\sqrt{x^{2}-1}\right)\right)=\dfrac{1}{x\sqrt{x^{2}-1}}$ (*****)

6. $\dfrac{d}{dx}\left(\arcsin\left(\dfrac{2x}{1+x^{2}}\right)\right)=-\dfrac{2}{1+x^{2}}$ (****)

7. $\dfrac{d}{dx}\left(\operatorname{artanh}\left(\dfrac{\cos x+1}{\cos x-1}\right)\right)=-\dfrac{1}{2}\tan x$ (****)

Question 1

1. $\dfrac{d}{dx}(\sinh 3x) = 3\cosh 3x$

2. $\dfrac{d}{dx}(\cosh 4x) = 4\sinh 4x$

3. $\dfrac{d}{dx}(3\tanh 2x) = 6\operatorname{sech}^2 2x$

4. $\dfrac{d}{dx}\left(4\sinh\left(\dfrac{1}{2}x\right)\right) = 2\cosh\left(\dfrac{1}{2}x\right)$

5. $\dfrac{d}{dx}(3\coth 2x) = -6\operatorname{cosech}^2 2x$

6. $\dfrac{d}{dx}(3\sinh^2 x) = 6\sinh x\cosh x$

7. $\dfrac{d}{dx}(4\cosh^3 2x) = 24\cosh^2 2x\sinh 2x$

8. $\dfrac{d}{dx}(4\operatorname{sech} 2x) = -8\operatorname{sech} 2x\tanh 2x$

9. $\dfrac{d}{dx}(4\tanh^4 x) = 16\tanh^3 x\operatorname{sech}^2 x$

10. $\dfrac{d}{dx}(5\operatorname{cosech} 3x) = -15\operatorname{cosech} 3x\coth 3x$

11. $\dfrac{d}{dx}(x\sinh x) = \sinh x + x\cosh x$

12. $\dfrac{d}{dx}(x^3\cosh x) = 3x^2\cosh x + x^3\sinh x$

13. $\dfrac{d}{dx}(\mathrm{e}^x\sinh 2x) = \mathrm{e}^x(\sinh 2x + 2\cosh 2x)$

14. $\dfrac{d}{dx}\left(4\mathrm{e}^{\cosh 2x}\right)=8\mathrm{e}^{\cosh 2x}\sinh 2x$

15. $\dfrac{d}{dx}\left(3\sinh^3\left(3x^3\right)\right)=81x^2\sinh^2\left(3x^3\right)\cosh\left(3x^3\right)$

16. $\dfrac{d}{dx}\left(\sin\left(\sinh x\right)\right)=\cosh x\cos\left(\sinh x\right)$

17. $\dfrac{d}{dx}\left(\cosh\left(\cos x\right)\right)=-\sin x\sinh\left(\cos x\right)$

18. $\dfrac{d}{dx}\left(\dfrac{\sinh 2x}{x}\right)=\dfrac{2x\cosh 2x-\sinh 2x}{x^2}$

19. $\dfrac{d}{dx}\left(\dfrac{\ln\left(\cosh x\right)}{\sinh x}\right)=\dfrac{\sinh^2 x-\cosh^2 x\ln\left(\cosh x\right)}{\cosh x\sinh^2 x}$

20. $\dfrac{d}{dx}\left(\dfrac{\tanh x}{\sinh x}\right)=\dfrac{\sinh x\operatorname{sech}^2 x-\tanh x\cosh x}{\sinh^2 x}=\dfrac{\operatorname{sech}^2 x-1}{\sinh x}$

21. $\dfrac{d}{dx}\left(\cosh\left(\mathrm{e}^x+x^2\right)\right)=\left(2x+\mathrm{e}^x\right)\sinh\left(\mathrm{e}^x+x^2\right)$

22. $\dfrac{d}{dx}\left(\sinh\left(\ln\left(x^2-1\right)\right)\right)=\dfrac{2x}{x^2-1}\cosh\left(\ln\left(x^2-1\right)\right)$

23. $\dfrac{d}{dx}\left(\cosh x\coth x\right)=\sinh x\coth x-\cosh x\operatorname{cosech}^2 x$

24. $\dfrac{d}{dx}\left(2\mathrm{e}^{3x}\tanh 2x\right)=2\mathrm{e}^{3x}\left(3\tanh 2x+2\operatorname{sech}^2 2x\right)$

25. $\dfrac{d}{dx}\left(\dfrac{\cosh x+\sinh x}{\mathrm{e}^{-x}}\right)=2\mathrm{e}^{2x}$

Question 2

1. $\dfrac{d}{dx}\left(\operatorname{arsinh} 2x\right)=\dfrac{2}{\sqrt{4x^2+1}}$

2. $\dfrac{d}{dx}\left(\operatorname{arsinh} x^2\right)=\dfrac{2x}{\sqrt{x^4+1}}$

3. $\dfrac{d}{dx}\left(\operatorname{arsinh}\left(\sin x\right)\right)=\dfrac{\cos x}{\sqrt{\sin^2 x+1}}$

4. $\dfrac{d}{dx}\left(\operatorname{arcosh} 4x\right)=\dfrac{4}{\sqrt{16x^2-1}}$

5. $\dfrac{d}{dx}\left(\operatorname{arcosh}\sqrt{x}\right)=\dfrac{1}{2\sqrt{x^2-x}}$

6. $\dfrac{d}{dx}\left(\operatorname{arcosh}\left(x+1\right)\right)=\dfrac{1}{\sqrt{x^2+2x}}$

7. $\dfrac{d}{dx}\left(\operatorname{arsinh}\left(2x+1\right)\right)=\dfrac{2}{\sqrt{4x^2+4x+2}}$

8. $\dfrac{d}{dx}\left(\operatorname{artanh}\left(1-4x\right)\right)=\dfrac{1}{4x^2-2x}$

9. $\dfrac{d}{dx}\left(x\operatorname{arcosh} 2x\right)=\operatorname{arcosh} 2x+\dfrac{2x}{\sqrt{4x^2-1}}$

10. $\dfrac{d}{dx}\left(\left(\operatorname{arsinh} x\right)^3\right)=\dfrac{3\left(\operatorname{arsinh} x\right)^2}{\sqrt{x^2+1}}$

11. $\dfrac{d}{dx}\left(\operatorname{artanh}\left(\dfrac{1}{2}\mathrm{e}^x\right)\right)=\dfrac{2\mathrm{e}^x}{4-\mathrm{e}^{2x}}$

12. $\dfrac{d}{dx}\left(\operatorname{artanh}\left(\dfrac{\cos x+1}{\cos x-1}\right)\right)=-\dfrac{1}{2}\tan x$

Question 3

1. $\dfrac{d}{dx}\left(\arcsin 3x\right)=\dfrac{3}{\sqrt{1-9x^2}}$

2. $\dfrac{d}{dx}\left(\arcsin x^2\right)=\dfrac{2x}{\sqrt{1-x^4}}$

3. $\dfrac{d}{dx}\left(\arctan 4x\right)=\dfrac{4}{16x^2+1}$

4. $\dfrac{d}{dx}\left(\arccos x^2\right)=-\dfrac{2x}{\sqrt{1-x^4}}$

5. $\dfrac{d}{dx}\left(\arctan\sqrt{x}\right)=\dfrac{1}{2\sqrt{x}\left(x+1\right)}$

6. $\dfrac{d}{dx}\left(\arcsin\left(2x+3\right)\right)=\dfrac{1}{\sqrt{-x^2-3x-2}}$

7. $\dfrac{d}{dx}\left(\arccos\left(\dfrac{2}{3}x\right)\right)=-\dfrac{2}{\sqrt{9-4x^2}}$

8. $\dfrac{d}{dx}\left(\arctan\left(\dfrac{4}{3}x\right)\right)=\dfrac{12}{16x^2+9}$

9. $\dfrac{d}{dx}\left(x\arccos 2x\right)=\arccos 2x-\dfrac{2x}{\sqrt{1-4x^2}}$

10. $\dfrac{d}{dx}\left(2\left(\arcsin x\right)^4\right)=\dfrac{8\left(\arcsin x\right)^3}{\sqrt{1-x^2}}$

11. $\dfrac{d}{dx}\left(\arctan\left(\dfrac{2}{x}\right)\right)=-\dfrac{2}{x^2+4}$

12. $\dfrac{d}{dx}\left(\arcsin\left(\mathrm{e}^{2x}\right)\right)=\dfrac{2\mathrm{e}^{2x}}{\sqrt{1-\mathrm{e}^{4x}}}$

13. $\dfrac{d}{dx}\left(x^2 \arctan 2x\right) = 2x \arctan 2x + \dfrac{2x^2}{4x^2+1}$

14. $\dfrac{d}{dx}\left(\arctan\left(\dfrac{x+1}{x-1}\right)\right) = -\dfrac{1}{x^2+1}$