NUMERICAL METHODS

for

O.D.E.s

1^{st} order O.D.E.s

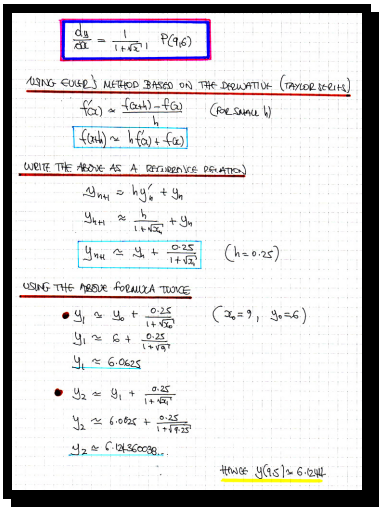
Question 1 (**)

The curve with equation $y=f(x)$, passes through the point $(9,6)$ and satisfies

$$\frac{dy}{dx}=\frac{1}{1+\sqrt{x}},\ x\geq 0.$$

Use Euler's method, with a step of 0.25, to find, correct to 4 decimal places, the value of y at $x=9.5$.

$\boxed{}$, $\boxed{y(9.5)\approx 6.1244}$



Question 2 (**)

The curve with equation $y=f(x)$, passes through the point $(1,1)$ and satisfies the following differential equation.

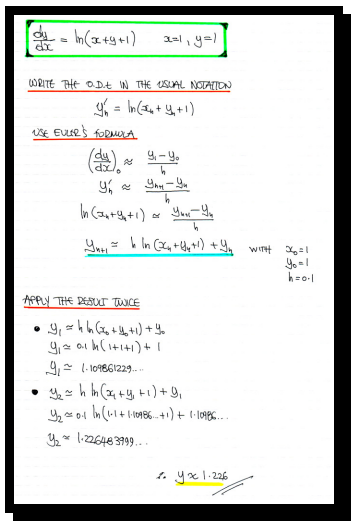
$$\frac{dy}{dx}=\ln(x+y+1),\ x+y>-1.$$

Use the approximation

$$\left(\frac{dy}{dx}\right)_0\approx\frac{y_1-y_0}{h},$$

with $h=0.1$, to find, correct to 3 decimal places, the value of y at $x=1.2$.

$\boxed{}$, $\boxed{y(1.2)\approx 1.226}$



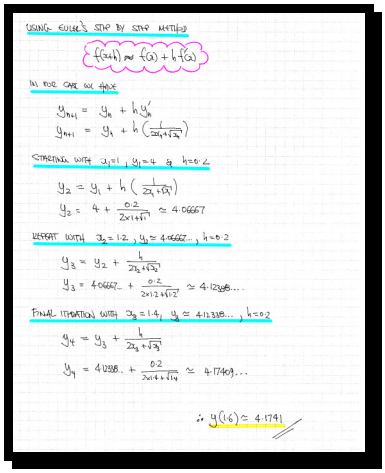
Question 3 (**)

The curve with equation $y=f(x)$, passes through the point $(1,4)$ and satisfies

$$\frac{dy}{dx}=\frac{1}{2x+\sqrt{x}}\,,\quad x\geq 0\,.$$

Use Euler's method, with a step of 0.2, to find, correct to 4 decimal places, the value of y at $x=1.6$.

$\boxed{}$, $\boxed{y(1.6)\approx 1.1741}$



Question 4 (**)

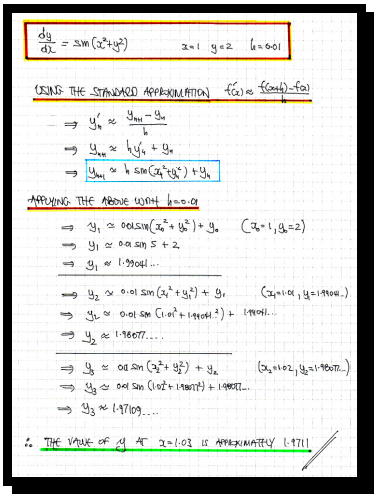
$$\frac{dy}{dx}=\sin\left(x^2+y^2\right),\quad y(1)=2\,.$$

Use, in the standard notation, the approximation

$$y_n'\approx\frac{y_{n+1}-y_n}{h},$$

with $h=0.01$, to find, correct to 4 decimal places, the value of y at $x=1.03$.

$\boxed{}$, $\boxed{y(1.03)\approx 1.9711}$



Question 5 (**)

$$\frac{dy}{dx}=4x^2-y^2\,,\quad y(1)=0\,.$$

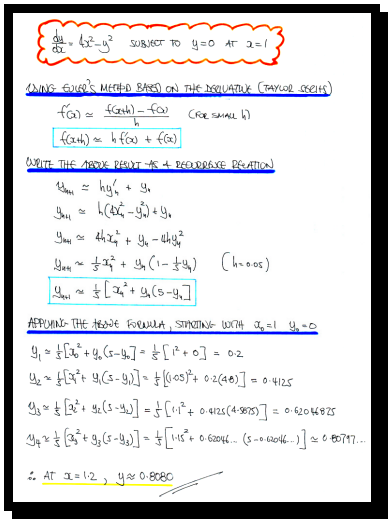
Use the approximation

$$f'(x)\approx\frac{f(x+h)-f(x)}{h}\,,$$

with $h=0.05$, to find, correct to 4 decimal places, the value of y at $x=1.2$.

No credit will be given for solving the differential equation analytically.

$\square$, $\boxed{y(1.2)\approx 0.8080}$



Question 6 (+)**

The curve with equation $y=f(x)$, passes through the point $(1,0)$ and satisfies the following differential equation.

$$\frac{dy}{dx}=x+\ln x\,,\ x>0\,.$$

Use the approximation

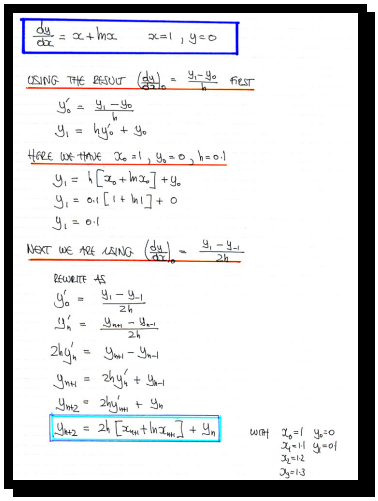
$$\left(\frac{dy}{dx}\right)_0\approx\frac{y_1-y_0}{h},\ h=0.1\,,$$

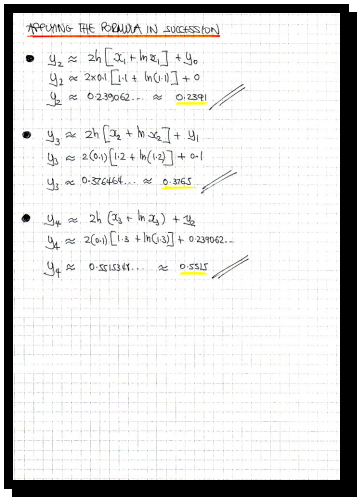
to find the value of y at $x=1.1$, and use this answer with the approximation

$$\left(\frac{dy}{dx}\right)_0\approx\frac{y_1-y_{-1}}{2h},\ h=0.1\,,$$

to find, correct to 3 decimal places, the value of y at $x=1.2$, $x=1.3$ and $x=1.4$.

$\boxed{}$, $\boxed{y(1.1)=0.1}$, $\boxed{y(1.2)\approx 0.2391}$, $\boxed{y(1.3)\approx 0.3765}$, $\boxed{y(1.4)\approx 0.5515}$





Question 7 (+)**

The curve with equation $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = x + y + y^2, \quad y(0.9) = 3.75, \quad y(1) = 4$$

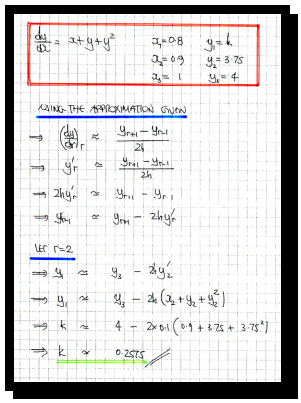
Using, in the standard notation, the approximation

$$\left(\frac{dy}{dx}\right)_r \approx \frac{y_{r+1} - y_{r-1}}{2h},$$

with $h = 0.1$, the value of y at $x = 0.8$ was estimated to k.

Determine the value of k.

$\boxed{}$, $\boxed{k \approx 0.2575}$



Question 8 (+)**

$$\frac{dy}{dx}=x^2-y^2,\ y(3)=2.$$

Use the approximation

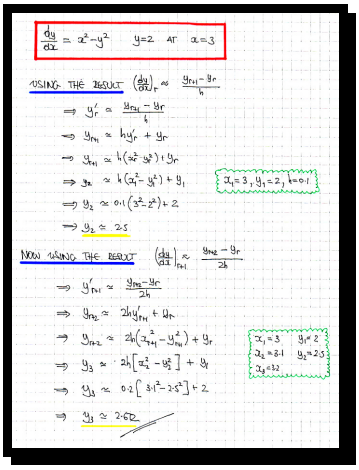
$$\left(\frac{dy}{dx}\right)_r\approx\frac{y_{r+1}-y_r}{h},\ h=0.1,$$

to find the value of y at $x=2.1$, and use this answer with the approximation

$$\left(\frac{dy}{dx}\right)_{r+1}\approx\frac{y_{r+2}-y_r}{2h},\ h=0.1,$$

to find, correct to 3 decimal places, the value of y at $x=2.2$, $x=1.3$ and $x=1.4$.

$\square$, $\boxed{y(2.2)=2.672}$



Question 9 (+)**

$$\frac{dy}{dx}=xy\,,\quad y(0)=2\,.$$

Use the approximation

$$f'(x)\approx\frac{f(x+h)-f(x)}{h},\ h=0.1$$

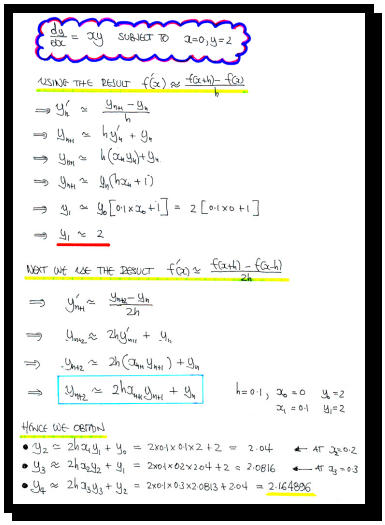
to find the value of y at $x=0.1$, and use this answer with the approximation

$$f'(x)\approx\frac{f(x+h)-f(x-h)}{2h},\ h=0.1\,,$$

to find the value of y at $x=0.4$.

No credit will be given for solving the differential equation analytically.

$\boxed{}$, $\boxed{y(0.1)=2}$, $\boxed{y(0.4)\approx 2.1649}$



Question 10 (+)**

The curve with equation $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = \frac{e^{x+y}}{3x+y+k}, \quad y(0) = 0,$$

where k is a positive constant.

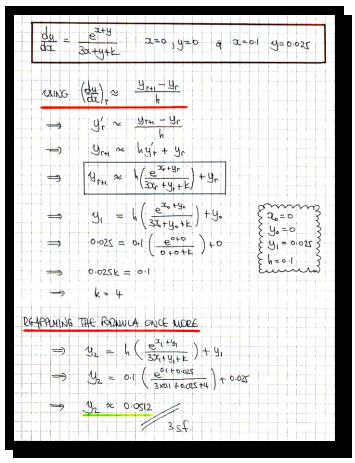
Using, in the standard notation, the approximation

$$\left(\frac{dy}{dx}\right)_r \approx \frac{y_{r+1} - y_r}{h},$$

with $h = 0.1$, the value of y at $x = 0.1$ was estimated to 0.025.

Use the approximation formula given above to find, correct to 3 significant figures, the value of y at $x = 0.2$.

$k = 4$, $\boxed{y(1.2) \approx 0.0512}$



Question 11 (+)**

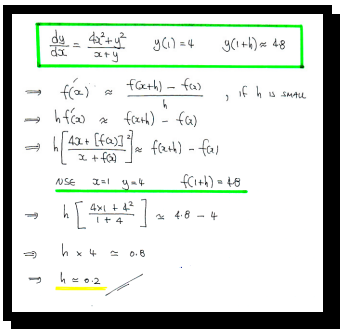
$$\frac{dy}{dx}=\frac{4x^2+y^2}{x+y}, \quad y(1)=4.$$

Use the approximation

$$f'(x)\approx\frac{f(x+h)-f(x)}{h},$$

with h to be found, given further that $f(1+h)\approx 4.8$.

$\boxed{}$, $\boxed{h=0.2}$



Question 12 (***)

$$\frac{dy}{dx}=\mathrm{e}^{x}-y^{2},\quad y(0)=0.$$

a) Use, in the standard notation, the approximation

$$y_{n+1}\approx h\,y_n'+y_n,$$

with $h=0.1$, to find the approximate value of y at $x=0.1$.

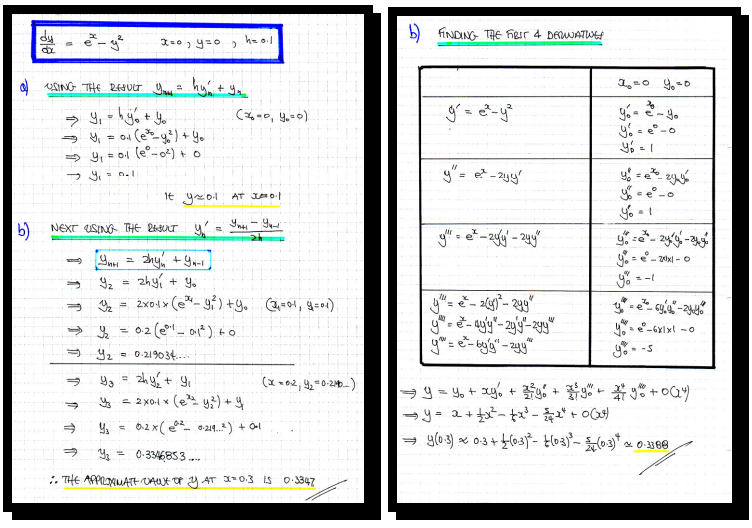
b) Use the answer of part **(a)** and the approximation

$$y_n'\approx\frac{y_{n+1}-y_{n-1}}{2h},$$

with $h=0.1$, to find, correct to 4 decimal places, the approximate value of y at $x=0.3$.

c) By differentiating the differential equation given, determine the first four **non zero** terms in the infinite series expansion of y in ascending powers of x, and use it to find, correct to 4 decimal places, another approximation for the value of y at $x=0.3$.

$\boxed{}$, $\boxed{y(0.1)\approx 0.1}$, $\boxed{y(0.3)\approx 0.3347}$, $\boxed{y(0.3)\approx 0.3388}$



Question 13 (***)

$$\frac{dy}{dx}=\frac{3x^2-y^2}{2xy},\quad y(k)=2,\quad k>0.$$

a) Use, in the standard notation, the approximation

$$y_n'\approx\frac{y_{n+1}-y_n}{h},\ h=0.1,$$

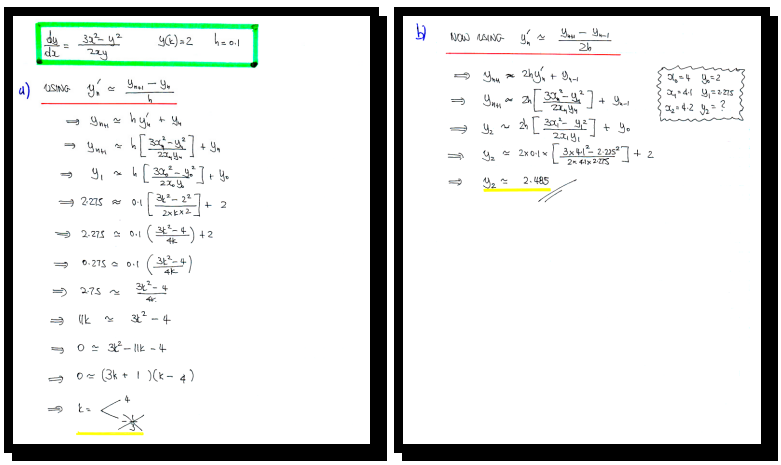
to find the value of k, given further that $y(k+h)\approx 2.275$.

b) Use the answer of part **(a)** and the approximation

$$y_n'\approx\frac{y_{n+1}-y_{n-1}}{2h},\ h=0.1,$$

with, to find, correct to 3 decimal places, the approximate value of $y(k+2h)$.

$\boxed{}$, $\boxed{k=4}$, $\boxed{y(k+2h)\approx 2.485}$



Question 14 (*+)**

The curve with equation $y=f(x)$, passes through the point $(0,1)$ and satisfies the following differential equation.

$$\frac{dy}{dx}=3x^2y+x^5.$$

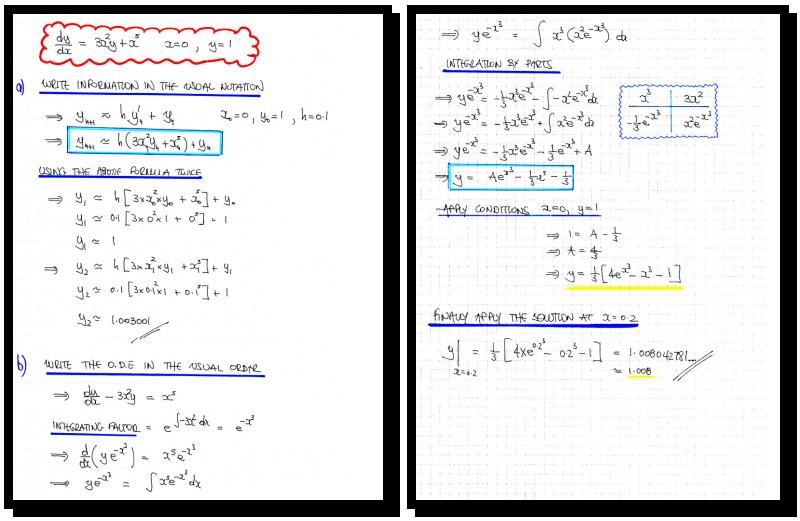
a) Use the approximation

$$\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1}-y_n}{h},$$

with $h=0.1$, to find, correct to 6 decimal places, the value of y at $x=0.2$.

b) Find the solution of the differential equation, and use it to obtain the value of y at $x=0.2$.

$\boxed{}$, $\boxed{y(0.2)\approx 1.003001}$, $\boxed{y(0.2)\approx 1.008}$



2^{nd} order O.D.E.s

Question 1 **(**+)**

The differential equation

$$\frac{d^2y}{dx^2}=\frac{x}{y^2}+\frac{1}{y},\ y\neq 0,$$

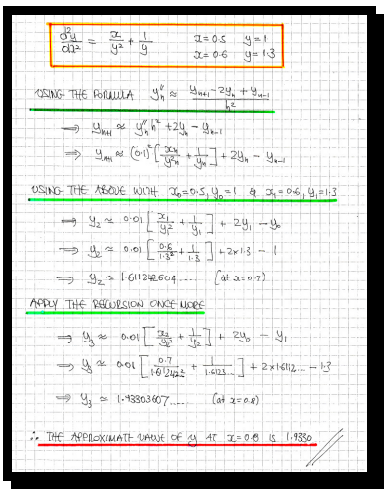
is to be solved numerically subject to the conditions $y(0.5)=1$ and $y(0.6)=1.3$.

Use the approximation

$$y_n'\approx\frac{y_{n+1}-2y_n+y_{n-1}}{h^2},\quad h=0.1,$$

to find, correct to 4 decimal places the value of y at $x=0.8$.

$\square$, $\boxed{y(0.8)\approx 1.9330}$



Question 2 **(***)**

The differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+y^3=0,\ y\neq 0,$$

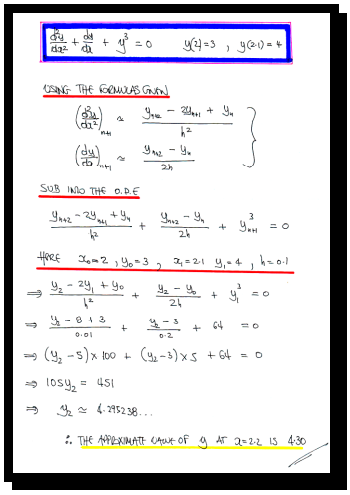
is to be solved numerically subject to the conditions $y(2)=3$ and $y(2.1)=4$.

Use the following approximations

$$\left(\frac{d^2y}{dx^2}\right)_{n+1}\approx\frac{y_{n+2}-2y_{n+1}+y_n}{h^2},\quad \left(\frac{dy}{dx}\right)_{n+1}\approx\frac{y_{n+2}-y_n}{2h},\qquad h=0.1,$$

to find, correct to 2 decimal places the value of y at $x=2.2$.

$\boxed{}$, $\boxed{y(2.2)\approx 4.30}$



Question 3 **(***+)**

The curve with equation $y = f(x)$, satisfies

$$\frac{d^2y}{dx^2} = 1 + x\sin y,$$

subject to the boundary conditions $y = 1$, $\frac{dy}{dx} = 2$, at $x = 1$.

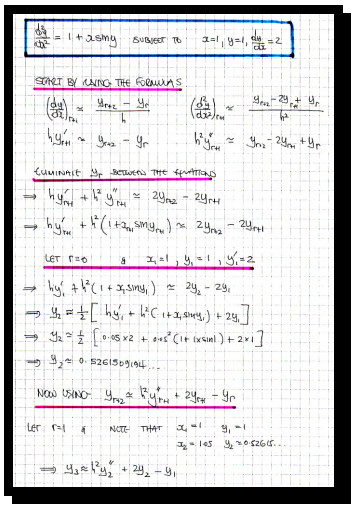
Use the approximations

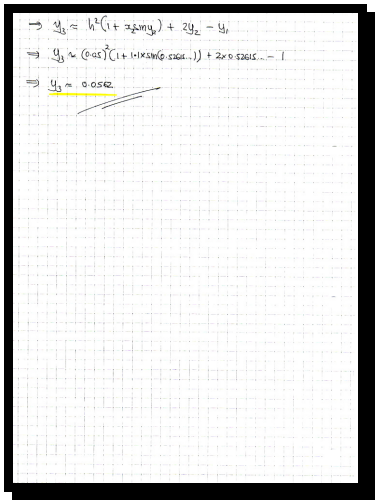
$$\left(\frac{d^2y}{dx^2}\right)_{r+1} \approx \frac{y_{r+2} - 2y_{r+1} + y_r}{h^2} \qquad \text{and} \qquad \left(\frac{dy}{dx}\right)_{r+1} \approx \frac{y_{r+2} - y_r}{2h},$$

to determine, correct to 4 decimal places, the value of y at $x = 1.1$.

Use $h = 0.05$ throughout this question.

$\square$, $\boxed{y(1.6) \approx 2.85}$





Question 4 **(***+)**

The curve with equation $y = f(x)$, satisfies

$$\frac{d^2y}{dx^2} = 4 + \sinh x \sinh y, \qquad y(1) = 1, \qquad \frac{dy}{dx}(1) = 1.$$

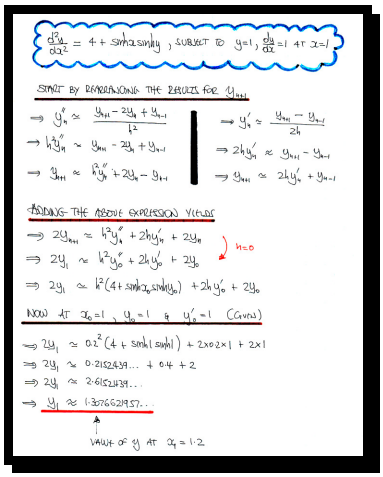
Use the approximations

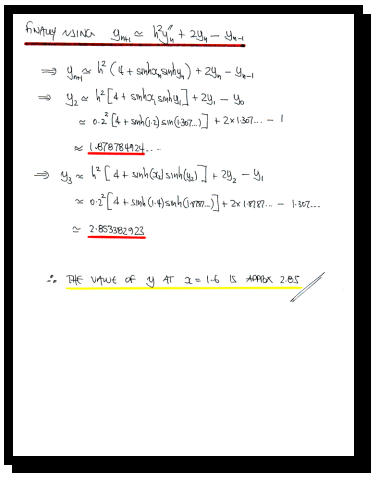
$$\left(\frac{d^2y}{dx^2}\right)_n \approx \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} \qquad \text{and} \qquad \left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_{n-1}}{2h},$$

to determine, correct to 2 decimal places, the value of y at $x = 1.6$.

Use $h = 0.2$ throughout this question.

$\boxed{}$, $\boxed{y(1.6) \approx 2.85}$





Question 5 **(***+)**

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+x^3=0, \qquad y(0)=1, \qquad \frac{dy}{dx}(0)=2.$$

a) Use the approximation formulae

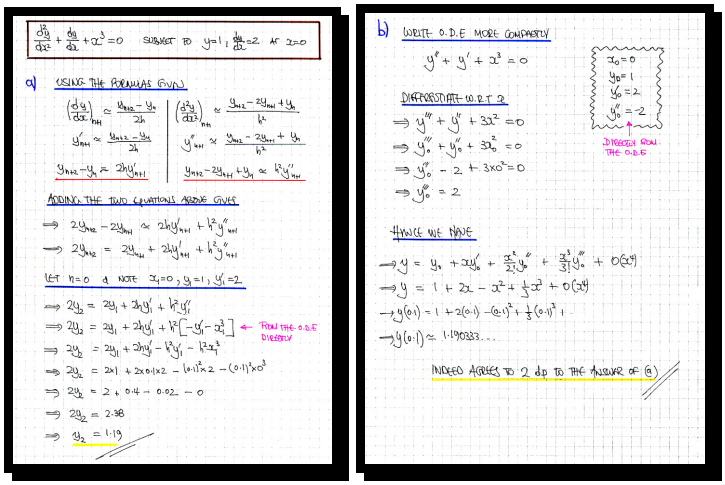
$$\left(\frac{d^2y}{dx^2}\right)_{n+1}\approx\frac{y_{n+2}-2y_{n+1}+y_n}{h^2} \qquad \text{and} \qquad \left(\frac{dy}{dx}\right)_{n+1}\approx\frac{y_{n+2}-y_n}{2h},$$

to determine, correct to 2 decimal places, the value of y at $x=0.1$.

Use $h=0.1$ throughout this part of the question.

b) By differentiating the differential equation given above, find the first 4 terms of the infinite convergent series expansion of y, in ascending powers of x, and use it to find, correct to 2 decimal places, another approximation for the value of y at $x=0.1$.

$\boxed{}$, $\boxed{y(0.1)\approx 1.19}$



Question 6 **(****)**

The curve with equation $y=f(x)$, satisfies

$$\frac{d^2y}{dx^2}=x+y+2, \qquad y(0)=0, \qquad \frac{dy}{dx}(0)=1.$$

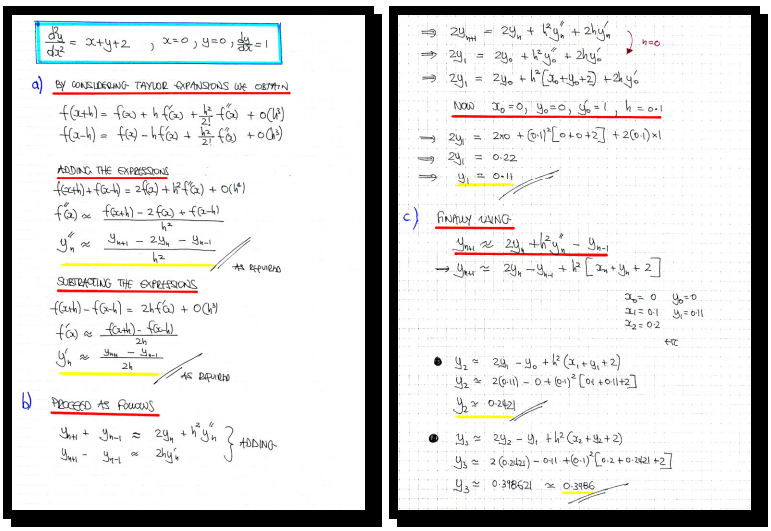
a) Use Taylor expansions to justify the validity of the following approximations.

$$\left(\frac{d^2y}{dx^2}\right)_n \approx \frac{y_{n+1}-2y_n+y_{n-1}}{h^2} \qquad \text{and} \qquad \left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1}-y_{n-1}}{2h}.$$

b) Hence show that $y(0.1)\approx 0.11$

c) Determine, correct to 4 decimal places, the value of $y(0.2)$ and $y(0.3)$.

$\boxed{}$, $\boxed{y(0.2)\approx 0.2421...}$, $\boxed{y(0.3)\approx 0.3986...}$



Question 7 **(***+)**

$$\frac{d^2y}{dx^2}=2+x^2y+y^2=0\,,\qquad y(0)=1,\qquad \frac{dy}{dx}(0)=1.$$

a) Use the approximation formulae

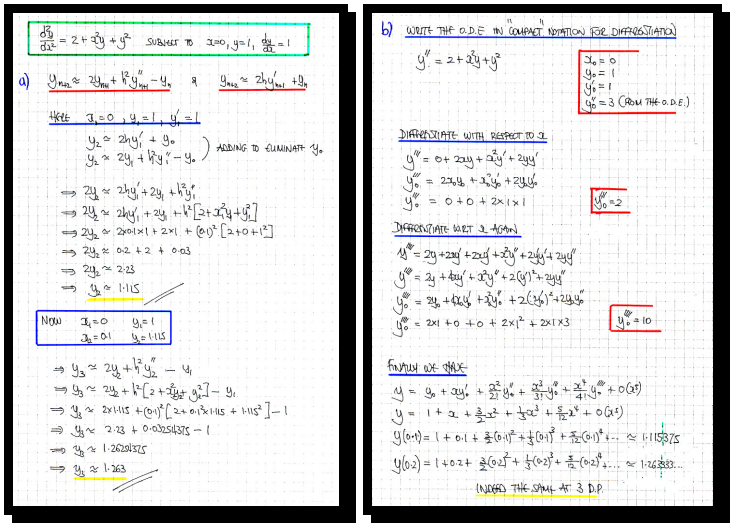
$$y_{n+2}\approx 2y_{n+1}+h^2y''_{n+1}-y_n \qquad \text{and} \qquad y_{n+2}\approx 2hy'_{n+1}+y_n\,,$$

to find, correct to 3 decimal places, the value of y at $x=0.1$ and $x=0.2$.

Use $h=0.1$ throughout this part of the question.

b) By differentiating the differential equation given above, determine the first 5 terms of the infinite convergent series expansion of y, in ascending powers of x, and use it to find, correct to 3 decimal places, approximations for the value of y at $x=0.1$ and $x=0.2$.

$\boxed{}$, $\boxed{y(0.1)\approx 1.115}$, $\boxed{y(0.2)\approx 1.263}$



Question 8 (****)

$$\frac{d^2y}{dx^2}=1+y\frac{dy}{dx}, \qquad y(0)=1, \qquad \frac{dy}{dx}(0.1)=1.1.$$

a) Use the approximation formulae

$$\left(\frac{d^2y}{dx^2}\right)_{r+1}\approx\frac{y_{r+2}-2y_{r+1}+y_r}{h^2} \qquad \text{and} \qquad \left(\frac{dy}{dx}\right)_{r+1}\approx\frac{y_{r+2}-y_r}{2h},$$

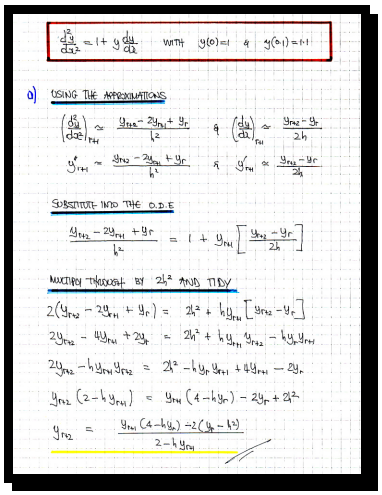
to show that

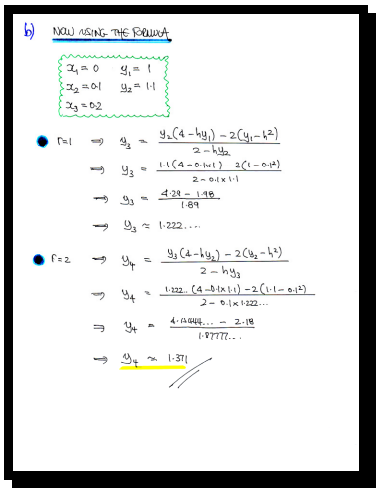
$$y_{r+2}\approx\frac{(4-hy_r)\,y_{r+1}-2\left(y_r-h^2\right)}{2-hy_{r+1}}.$$

determine, correct to 2 decimal places, the value of y at $x=0.1$.

b) Use the result shown in part **(a)**, with $h=0.1$, to find the value of y at $x=0.3$, correct to 3 decimal places.

$\boxed{}$, $\boxed{y(0.3)\approx 1.371}$





Question 9 (****)

The curve with equation $x = f(t)$, satisfies

$$\frac{d^2x}{dt^2} = -x, \qquad x(0) = 0, \qquad \frac{dx}{dt}(0) = 1.$$

Use Euler's method, with a step of 0.1, to find the approximate value of x at $t = 0.5$.

$\boxed{}$, $\boxed{x(0.5) \approx 0.480...}$

