CONSISTENCY OF EQUATIONS

Question 1 (***)

The system of simultaneous equations

$$\begin{aligned} x+2y+z&=1\\ 2x+3y+z&=3\\ 3x+4y+z&=k \end{aligned}$$

where k is a scalar constant, does not have a unique solution, but is consistent.

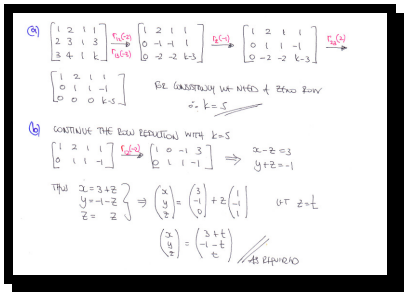
a) Determine the value of k.

b) Show that the general solution of the system can be written as

$$\begin{pmatrix} x\\ y\\ z \end{pmatrix}=\begin{pmatrix} t+3\\ -t-1\\ t \end{pmatrix},$$

where t is a scalar parameter.

$\boxed{k=5}$



Question 2 (***)

$$\begin{pmatrix} 2 & 5 & 3 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 2 \\ 4 \\ 10 \end{pmatrix}.$$

Show that the above simultaneous equations …

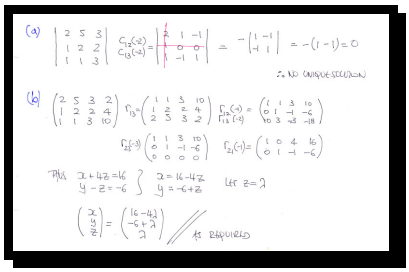
a) … do **not** have a unique solution.

b) … are **consistent** and their general solution can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 16-4\lambda \\ \lambda-6 \\ \lambda \end{pmatrix},$$

where λ is a scalar parameter.

proof



Question 3 (***)

The system of simultaneous equations

$$\begin{aligned} x+\ \ y+2z&=2\\ x+2y+\ \ z&=2\\ 2x+ay+5z&=b \end{aligned}$$

where a and b are constants, does **not** have a unique solution, but it is **consistent**.

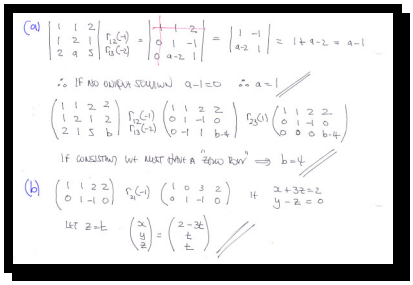
a) Determine the value of a and the value of b.

b) Show that the general solution of the system can be written as

$$\begin{pmatrix} x\\ y\\ z \end{pmatrix}=\begin{pmatrix} 2-3t\\ t\\ t \end{pmatrix},$$

where t is a parameter.

$\boxed{a=1}$, $\boxed{b=4}$



Question 4 (***)

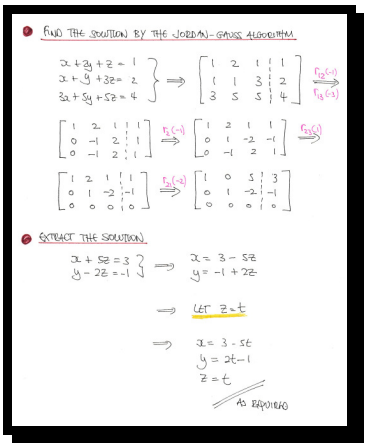
$$\begin{aligned} x+2y+z&=1\\ x+y+3z&=2\\ 3x+5y+5z&=4\end{aligned}$$

Show that the solution of the above simultaneous equations is

$$x=3-5t\,,\qquad y=2t-1\,,\qquad z=t$$

where t is a parameter.

[ ], [proof]



Question 5 **(***)**

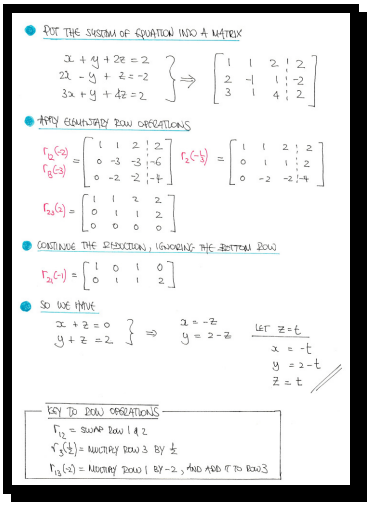
$$\begin{aligned} x+y+2z&=2\\ 2x-y+z&=-2\\ 3x+y+4z&=2 \end{aligned}$$

Show, by reducing the augmented matrix of the above system of equations into row echelon form, that the solution can be written as

$$x=-t\,,\qquad y=2-t\,,\qquad z=t$$

where t is a scalar parameter.

, proof



Question 6 (***)

A system of equation is given in matrix form below

$$\begin{pmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix},$$

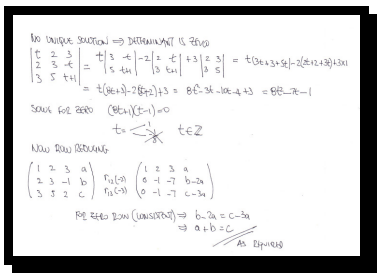
where t is an integer constant, and a, b and c are real constants.

The system of equations does not have a unique solution, but it is consistent.

Show clearly that

$$a+b=c\,.$$

proof



Question 7 **(***)**

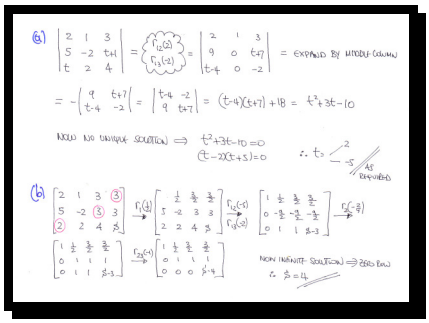
A system of equations is given below in terms of the scalar parameters t and s.

$$\begin{aligned} 2x &+ y &&+ 3z = t+1 \\ 5x &- 2y &&+ (t+1)z = 3 \\ tx &+ 2y &&+ 4z = s \end{aligned}$$

a) Show that if $t=-5$ or $t=2$, the system does not have a unique solution.

b) Determine the value of s is the system is to have infinite solutions with $t=2$.

$$\boxed{s=4}$$



Question 8 (***)

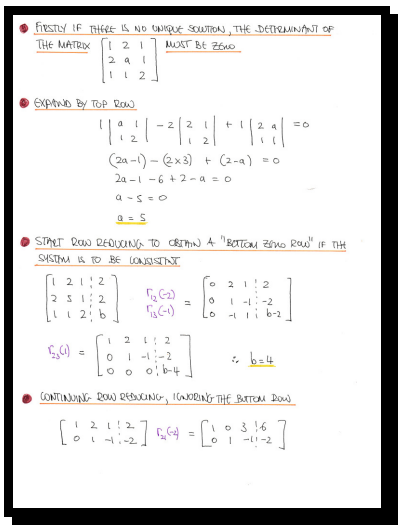
The three planes defined by the equations

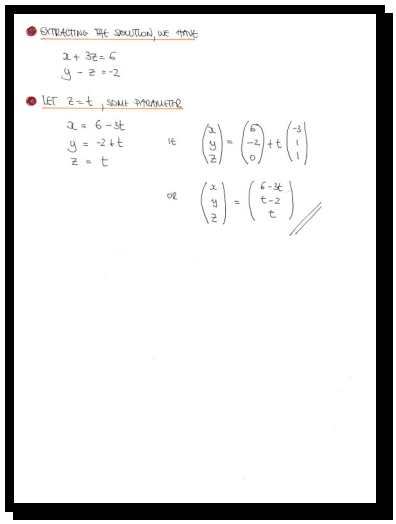
$$\begin{aligned} x+2y+\ z&=2\\ 2x+a\,y+\ z&=2\\ x+\ y+2z&=b \end{aligned}$$

where a and k are constants, intersect along a straight line L.

Determine an equation of L.

$\boxed{}$, $\boxed{\mathbf{r}=(6-3t)\mathbf{i}+(t-2)\mathbf{j}+t\mathbf{k}}$





Question 9 (***)

$$\begin{aligned} x+y-2z&=2\\ 3x-y+6z&=2\\ 6x+5y-9z&=k \end{aligned}$$

a) Show that the system of equations does not have a unique solution.

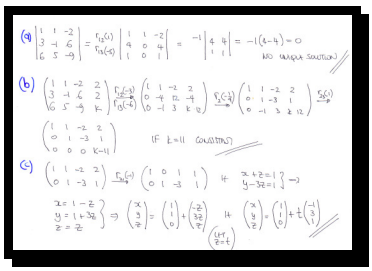
b) Show that there exists a value of k for which the system is consistent.

c) Show, by reducing the system into row echelon form, that the consistent solution of the system can be written as

$$x=1-t\,,\qquad y=3t+1\,,\qquad z=t$$

where t is a scalar parameter.

$\boxed{k=11}$



Question 10 (***)

$$\begin{aligned} 3x-y+5z&=5\\ 2x+y-5z&=10\\ x+y+kz&=7 \end{aligned}$$

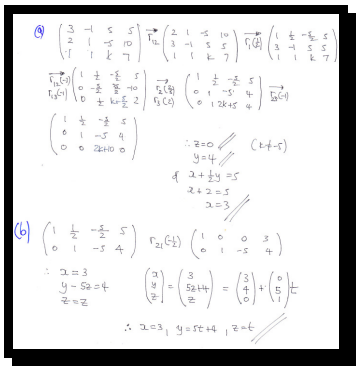
where k is a constant.

a) Given that $k\neq -5$ find the unique solution of the system of equations.

b) Given instead that $k=-5$ show, by reducing the system into row echelon form, that the consistent solution of the system can be written as

$$x=3\,,\quad y=5t+4\,,\quad z=t\,.$$

$$\boxed{x=3,\ \ y=4,\ \ z=0}$$



Question 11 (***)

A system of equations is given below

$$\begin{aligned} x+2y-z&=4\\ 2x-y+z&=5\\ 4x-7y+5z&=7 \end{aligned}$$

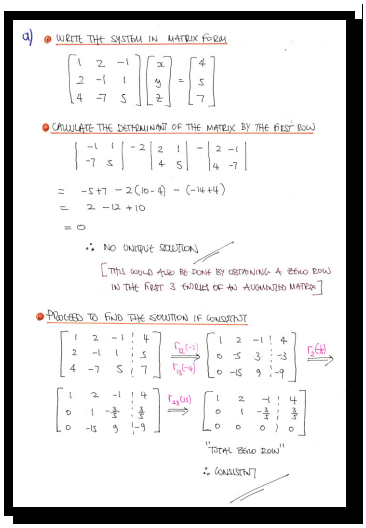
a) Show that the system does not have a unique solution but is consistent.

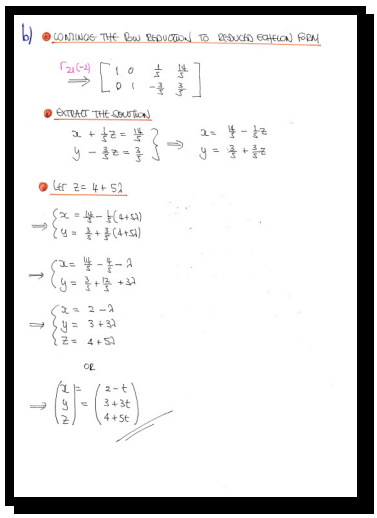
b) Show that the general solution of the system can be written as

$$\begin{pmatrix} x\\ y\\ z \end{pmatrix}=\begin{pmatrix} 2-t\\ 3+3t\\ 4+5t \end{pmatrix},$$

where t is a parameter.

[ ], [proof]





Question 12 (***)

$$\mathbf{A}=\begin{pmatrix} 9 & 2 & k \\ 1 & -1 & -3 \\ k-1 & 1 & 3 \end{pmatrix}, \quad \mathbf{x}=\begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

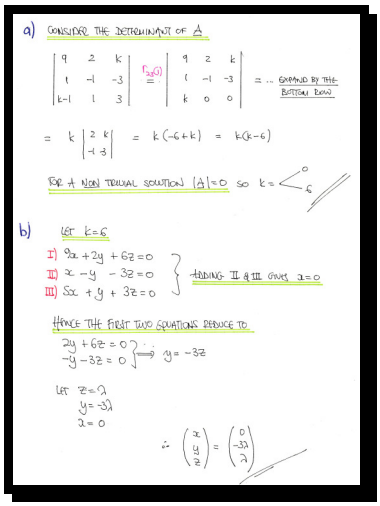
Consider the homogeneous system of simultaneous equations

$$\mathbf{Ax}=\mathbf{0}$$

a) Find the values of k for which the system has a **non trivial** solution.

b) If $k \neq 0$ find the general, non trivial solution of the system.

$\boxed{}$, $\boxed{k=0,\ 6}$, $\boxed{x=0,\ \ y=-3\lambda,\ \ z=\lambda}$



Question 13 (***)

Consider the system of simultaneous equations

$$\begin{aligned} kx+ky\ \ -z&=-1\\ ky+2z&=\ \ 2k\\ x+2y\ \ +z&=\ \ 1\end{aligned}$$

where the constant k can **only** take the values 0, 1 and 2.

Determine for each of the possible values of k whether the system …

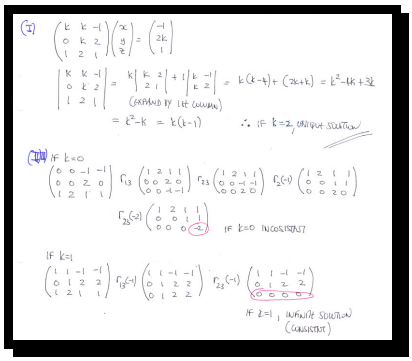
i. … has a unique solution

ii. … has no unique solution, but it is consistent.

iii. … is inconsistent.

$\boxed{k=0\Rightarrow \text{incosistent}}$, $\boxed{k=1\Rightarrow \text{no unique solution/consistent}}$,

$\boxed{k=2\Rightarrow \text{unique solution}}$



Question 14 **(***)**

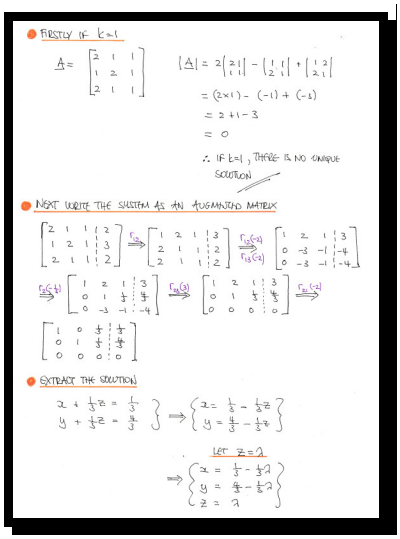
$$\mathbf{A}=\begin{pmatrix} k+1 & 1 & k \\ 1 & 2 & k \\ 2 & k & 1 \end{pmatrix}, \qquad \mathbf{x}=\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{b}=\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}.$$

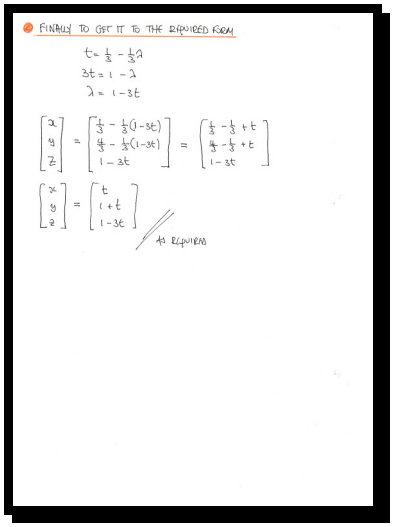
Show that the system of equations $\mathbf{Ax}=\mathbf{b}$ does not have a unique solution, but is consistent if $k=1$, and its general solution in this case can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} t \\ 1+t \\ 1-3t \end{pmatrix}.$$

where t is a scalar parameter.

$\boxed{}$, $\boxed{\text{proof}}$





Question 15 **(***)**

A system of equations is given below

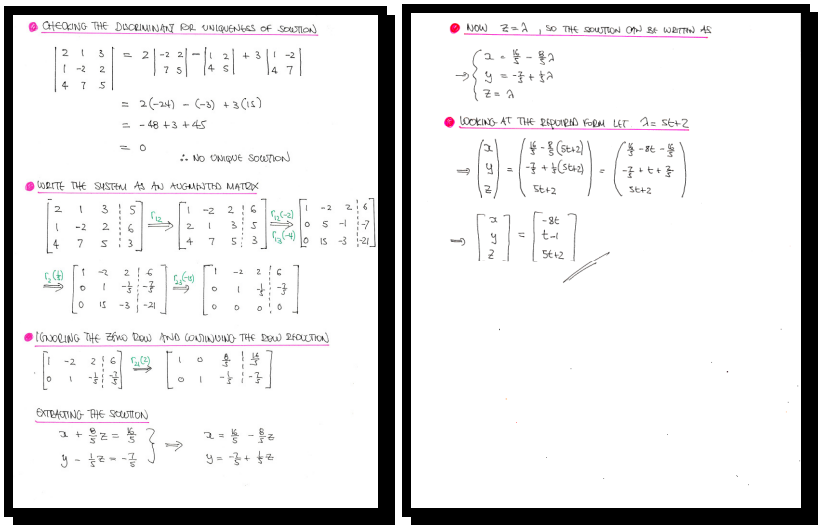
$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 4 & 7 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}.$$

Show that the system does not have a unique solution but is consistent and its general solution can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8t \\ t-1 \\ 5t+2 \end{pmatrix},$$

where t is a parameter.

, proof



Question 16 **(***)**

$$\begin{pmatrix} 3 & k & -1 \\ 2k & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

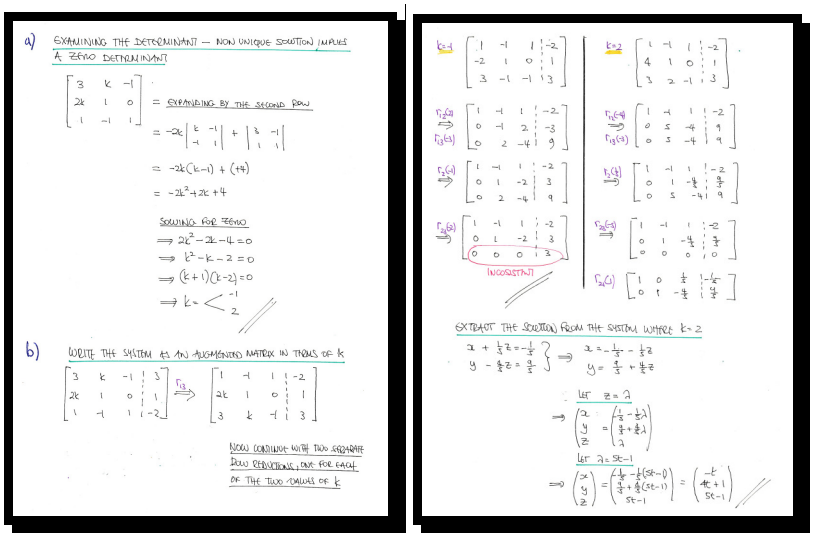
a) Determine the values of k for which the above system of equations does not have a unique solution.

b) Show, that one of these values of k leads to inconsistency and the other produces a general solution of the form

$$x=-t\,,\quad y=4t+1\,,\quad z=5t-1$$

where t is a scalar parameter.

$\boxed{}$, $\boxed{k=-1,\ 2}$



Question 17 **(***)**

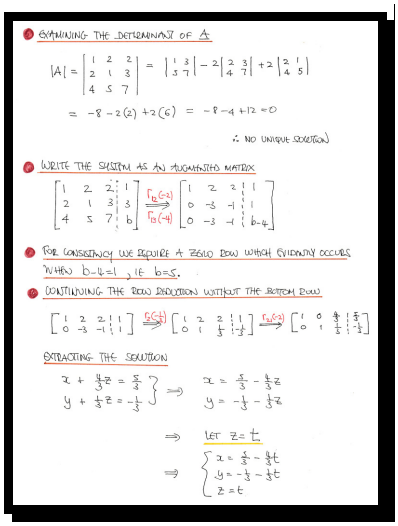
$$\mathbf{A}=\begin{pmatrix}1&2&2\\2&1&3\\4&5&7\end{pmatrix},\qquad \mathbf{x}=\begin{pmatrix}x\\y\\z\end{pmatrix},\qquad \mathbf{b}=\begin{pmatrix}1\\3\\b\end{pmatrix}$$

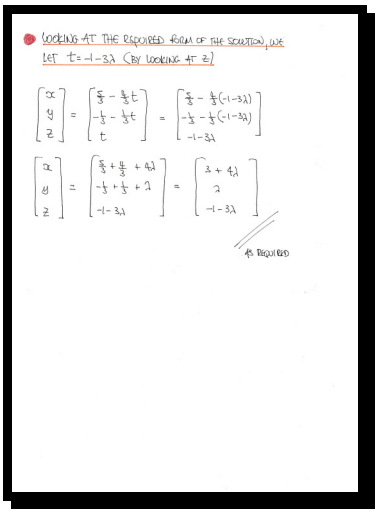
Show that the system of equations $\mathbf{Ax}=\mathbf{b}$ does not have a unique solution, but for a certain value of b is consistent and its general solution can be written as

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3+4\lambda\\\lambda\\-1-3\lambda\end{pmatrix}$$

where λ is a parameter.

$\boxed{}$, $\boxed{\text{proof}}$





Question 18 **(***+)**

Consider the following matrix equation

$$\begin{pmatrix} k & 1 & 0 \\ 3 & -2 & k-3 \\ 10k & 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} a \\ b \\ 15 \end{pmatrix},$$

where a, b and k are scalar constants.

a) Find the values of k for which the equation has a unique solution.

It is further asserted that $k=2$.

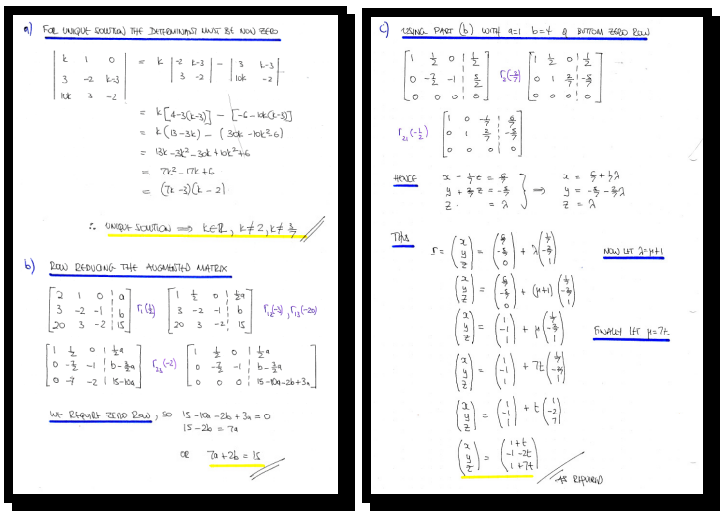
b) Express a in terms of b if the matrix equation is to be consistent.

c) Show that if $a=1$ and $b=4$, the solution of the matrix equation is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} t+1 \\ -2t-1 \\ 7t+1 \end{pmatrix},$$

where t is a scalar parameter.

$\boxed{}$, $\boxed{k\neq 2 \ \cup \ k\neq \frac{3}{7}}$, $\boxed{2a+7b=15}$



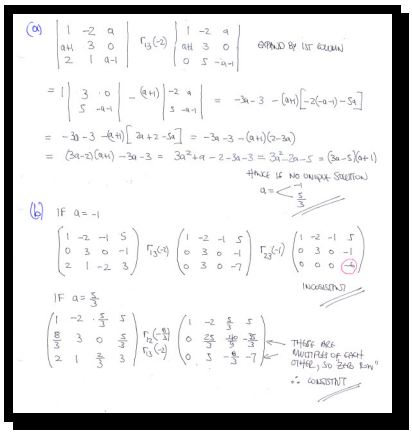
Question 19 **(***+)**

$$\begin{aligned} x\ -2y\ +\ az &= 5 \\ (a+1)x\ +3y &= a \\ 2x\ +\ y\ +(a-1)z &= 3 \end{aligned}$$

a) Determine the two values of the constant a for which the above system of equations does **not** have a unique solution.

b) Show clearly that the system is consistent for one of these values and inconsistent for the other.

$$\boxed{a=-1,\ \frac{5}{3}}$$



Question 20 (*+)**

A system of equations is given below

$$\begin{aligned} 3x+2y\ \ -z&=10\\ 5x\ \ -y-4z&=17\\ x+5y+pz&=q \end{aligned}$$

where p and q are constants.

a) Find the value of the constant p so that the system of equations does not have a unique solution.

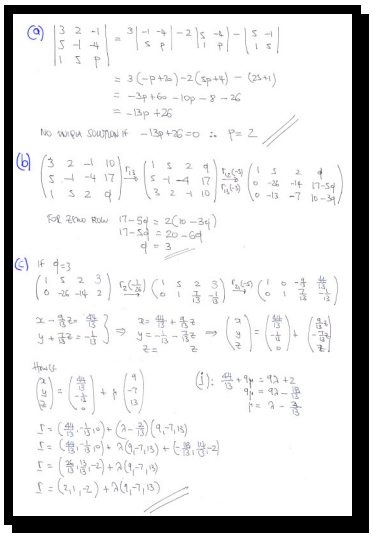
b) Show that for this value of p the system is consistent if $q=3$.

c) Show that the general solution of the system can be written as

$$\mathbf{r}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}+\lambda\left(9\mathbf{i}-7\mathbf{j}+13\mathbf{k}\right),$$

where λ is a scalar parameter.

$\boxed{p=2}$



Question 21 (****)

A system of equation is given below

$$\begin{aligned} 3x-2y-18z&=6\\ 2x+y-5z&=25 \end{aligned}$$

a) Show, by reducing the system into row echelon form, that the solution of the system can be written as

$$\mathbf{r}=8\mathbf{i}+9\mathbf{j}+\lambda\left(4\mathbf{i}-3\mathbf{j}+\mathbf{k}\right),$$

where λ is a scalar parameter.

A new system is now given

$$\begin{aligned} 3x-2y-18z&=6\\ 2x+y-5z&=25\\ 7x+ky+2z&=20 \end{aligned}$$

where k is a constant.

b) Determine if the system has solutions for different values of k.

$\boxed{k\neq 10\Rightarrow \text{unique, otherwise incosistent}}$

