COMBINATORICS

COMBINATIONS

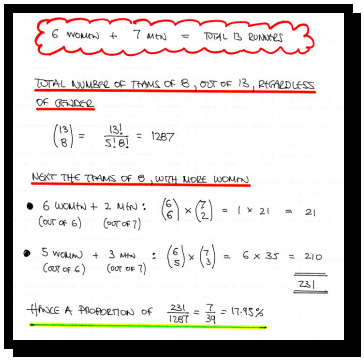
Question 1 (**)

The Oakwood Jogging Club consists of 7 men and 6 women who go for a 5 mile run every Thursday.

It is decided that a team of 8 runners would be picked at random out of the 13 runners, to represent the club at a larger meeting.

Determine the proportion of teams of 8, which have more women than men.

$\square$, $\boxed{\frac{7}{39}\approx 17.95\%}$



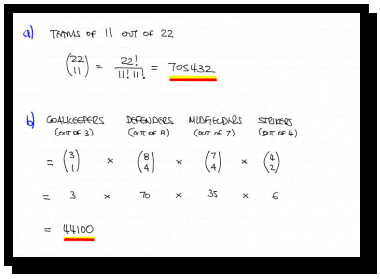
Question 2 (**)

A football manager has available for selection 3 goalkeepers, 8 defenders, 7 midfielders and 4 strikers.

a) Determine the number of possible teams of 11 he can select, assuming that all 22 players are equally likely to be picked up, and equally likely to play in any position.

b) Find the number of possible teams he can pick with 1 goalkeeper, 4 defenders, 4 midfielders and 2 strikers.

$\boxed{}$, $\boxed{44100}$, $\boxed{\dfrac{525}{8398} \approx 0.0625}$

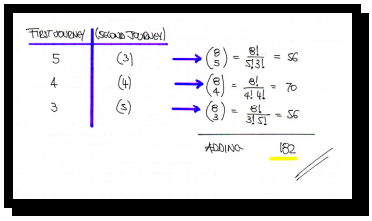


Question 3 (**)

A taxi which can carry at most 5 passengers on any journey, makes two journeys in transporting 8 passengers from their hotel to the airport.

Determine the number of different ways in which the people for the first journey may be selected.

$\square$, $\boxed{182}$



Question 4 (+)**

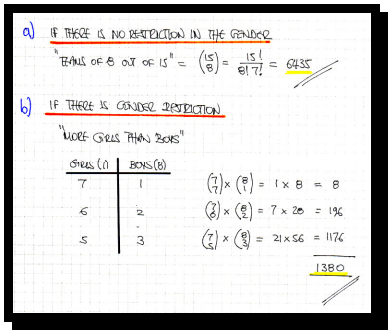
There are 8 boys and 7 girls in the student council of a school.

A committee of 8 people is to be selected from the members of this council to organize a sports day.

a) Find the number of different ways in which the committee can be selected if all the members are available.

b) Determine the number of different ways in which the committee can be selected if the committee is to have more girls than boys.

$\boxed{}$, $\boxed{6435}$, $\boxed{1380}$



Question 5 (+)**

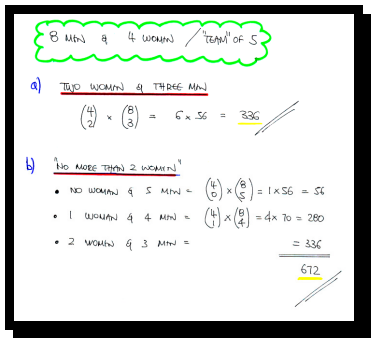
A **five** member committee is to be selected at random from a group consisting of 8 men and 4 women.

Find the number of possible committees which contain …

a) … exactly 2 women.

b) … no more than 2 women.

[ ] , [336] , [672]



Question 6 (+)**

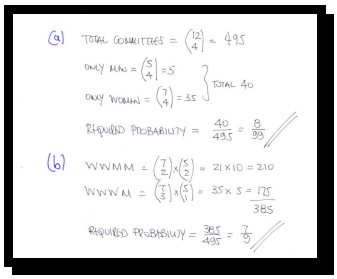
A committee of 4 people is to be chosen at random from a group of 5 men and 7 women.

Determine the probability that the committee will consist …

a) … of members of the same gender.

b) … of members of both genders but at least as many women as men.

$\boxed{\frac{8}{99}}$, $\boxed{\frac{7}{9}}$



Question 7 (*+)**

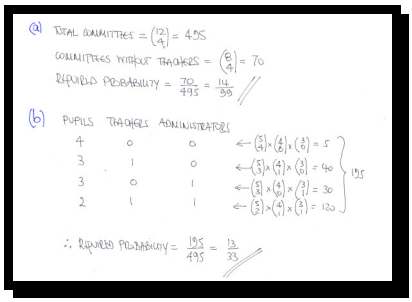
A committee of 4 people is to be chosen at random from the members of a school council which consists of 5 pupils, 4 teachers and 3 administrators.

Determine the probability that the committee will contain ...

a) … no teachers.

b) … at least 2 pupils, no more than 1 teacher and no more than 1 administrator.

$\boxed{\frac{14}{99}}$, $\boxed{\frac{13}{33}}$



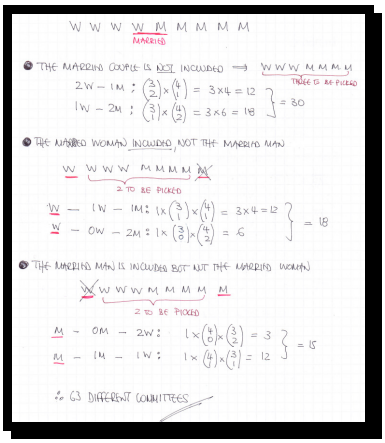
Question 8 (***+)

A committee of 3 people is to be picked from 9 individuals, of which 4 are women and 5 are men. One of the 4 women is married to one of the 5 men.

The selection rules state that the committee must have at least a member from each gender and no married couple can serve together in a committee.

Determine the number of possible committees which can be picked from these 9 individuals.

63

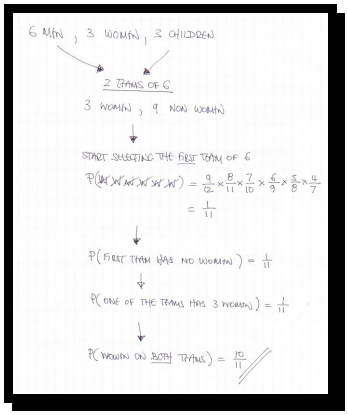


Question 9 (****)

From a total of 6 men, 3 women and 3 children, two teams of six people are selected at random.

Find the probability that both teams contain women.

$$\frac{10}{11}$$



PERMUTATIONS

Question 1 (+)**

The five letters of the word T-E-A-C-H are written on five separate pieces of card.

a) Find the number of arrangements that can be made using these five letters.

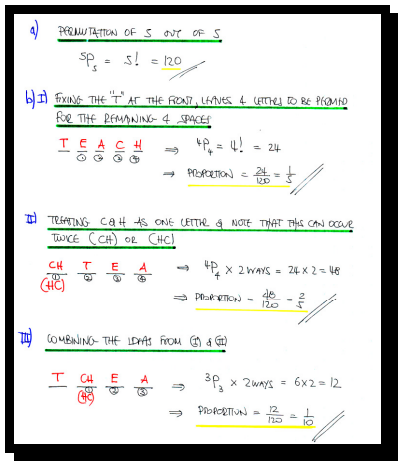
Find the proportion of five letter arrangements in which …

i. … the first letter is T.

ii. … the letters C and H are next to each other.

iii. … the first letter is T **and** the letters C and H are next to each other.

$\boxed{}$, $\boxed{120}$, $\boxed{\frac{1}{120}}$, $\boxed{\frac{1}{5}}$, $\boxed{\frac{2}{5}}$, $\boxed{\frac{1}{10}}$



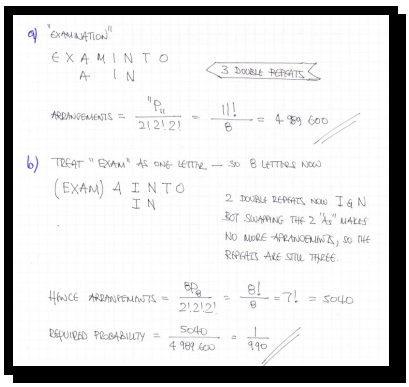
Question 2 (***)

The eleven letters of the word E-X-A-M-I-N-A-T-I-O-N are written on eleven separate pieces of card.

a) Find the number of arrangements that can be made using these eleven letters.

b) Find the probability that the four letter word E-X-A-M will appear in one of these eleven letter arrangements

$\boxed{4989600}$, $\boxed{\frac{1}{990}}$

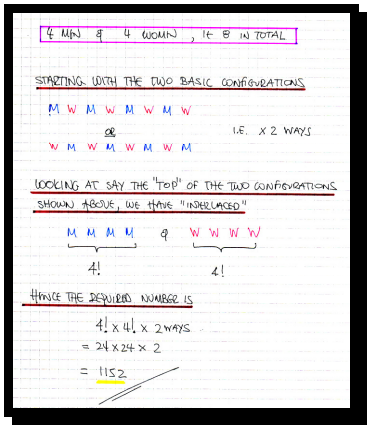


Question 3 (***)

4 men and 4 women are going to stand next to each other for a group photograph.

Given that the way they stand next to each other is completely random, determine the number of photographs that can be taken in which no 2 men and no 2 women stand next to each other.

$\square$, 1152



Question 4 (*+)**

Six books labelled as A, B, C, D, E and F are arranged at random on a shelf.

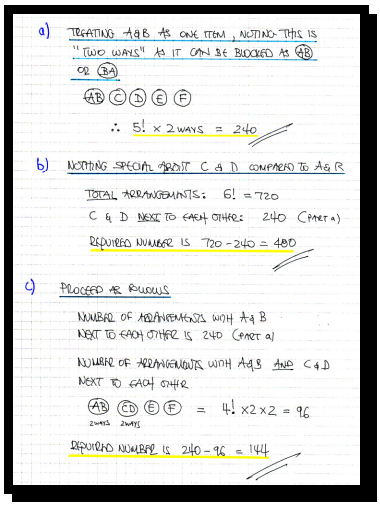
Determine the number of arrangements in which ...

a) ... A and B are placed next to each other.

b) ... C and D are **not** placed next to each other.

c) ... A and B are placed next to each other, and C and D are **not** placed next to each other.

$\boxed{}$, $\boxed{240}$, $\boxed{480}$, $\boxed{144}$



Question 5 (*+)**

A group of 7 pupils consists of 3 girls and 4 boys.

The names of two of the boys are Argi and Bargi.

All seven students sit in a random order on a bench.

a) Determine the number of sitting arrangements in which …

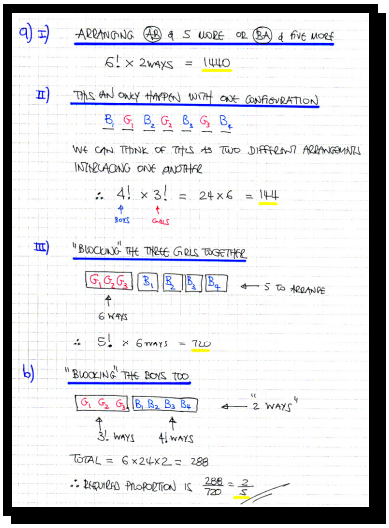
i. … Argi and Bargi sit next to each other.

ii. … no two boys sit next to each other.

iii. … the three girls sit next to each other.

b) Find the proportion of the sitting arrangements in which the three girls sat next to each other which include arrangements in which the four boys **also** sat next to each other.

$\boxed{}$, $\boxed{1440}$, $\boxed{144}$, $\boxed{720}$, $\boxed{\frac{2}{5}}$



Question 6 (*+)**

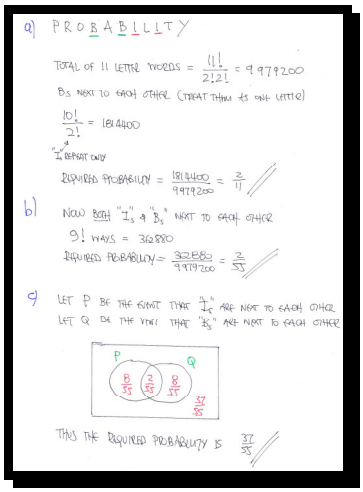
The 11 letters of the word *PROBABILITY* are written on 11 separate pieces of card. These cards are selected at random and arranged in a line next to each other.

a) Determine the probability that the two cards with the letter B will appear next to each other.

b) Find the probability that the two cards with the letter B will appear next to each other **and** the two cards with the letter I will appear next to each other.

c) Hence deduce the probability that the two cards with the letter B will **not** appear next to each other **and** the two cards with the letter I will **not** appear next to each other.

$$\boxed{\frac{2}{11}}, \quad \boxed{\frac{2}{55}}, \quad \boxed{\frac{37}{55}}$$



Question 7 (***+)

$$S\ S\ S\ T\ T\ T\ C\ I\ I\ A$$

The 10 letters above, are written on 10 separate pieces of card. These cards are selected at random and arranged in a line next to each other.

a) Find the probability that the 10 letter arrangement will spell *STATISTICS* .

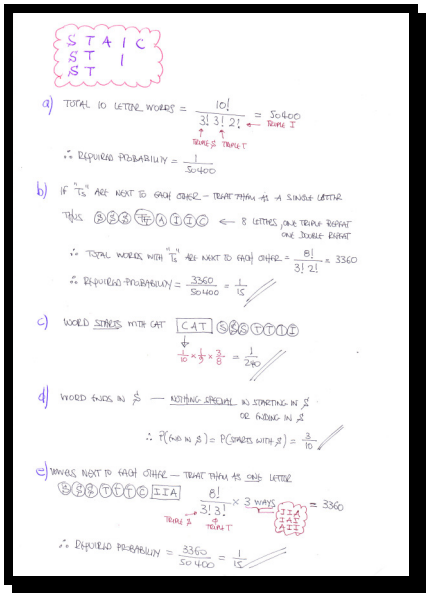
b) Determine the probability that in the 10 letter arrangement the 3 cards with the letter T will be next to one another.

c) Calculate the probability that the 10 letter arrangement will start with *CAT* , in that order.

d) Find the probability that the 10 letter arrangement will end with the letter S .

e) Determine the probability that in the 10 letter arrangement the 3 cards showing a vowel will be next to one another.

$$\boxed{\frac{1}{50400}}\ ,\ \boxed{\frac{1}{15}}\ ,\ \boxed{\frac{1}{240}}\ ,\ \boxed{\frac{3}{10}}\ ,\ \boxed{\frac{1}{15}}$$



Question 8 (*+)**

The 10 letters of the word

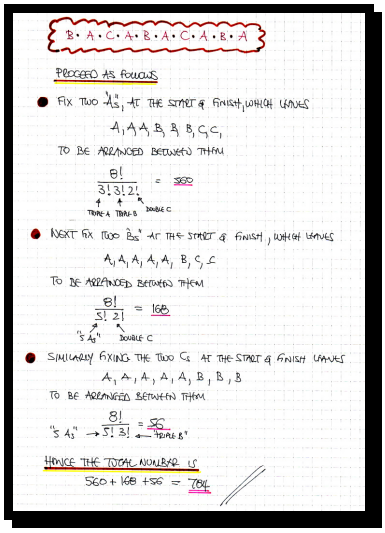
$$B\ A\ C\ A\ B\ A\ C\ A\ B\ A$$

are written on 10 separate pieces of card.

These cards are selected at random and arranged in a line next to each other.

Determine the number of arrangements which start and finish with the same letter.

$\square$, $\boxed{784}$



Question 9 (****)

Coloured pegs are to be placed in 4 holes which are drilled in a straight line, next to each other. These coloured pegs are identical in size and 2 of them are red, 2 of them are green, 2 of them are brown, 2 of them are orange, 2 of them are pink and 2 of them are blue.

6 pegs, one from each of the 6 colours, are picked from the 12 pegs and four are placed in the holes.

a) Determine the number of different arrangements which can be made.

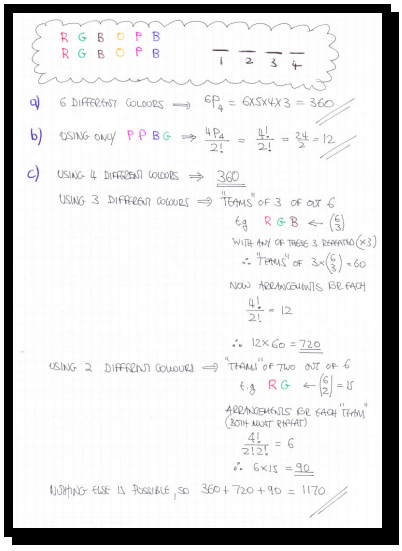
Next 4 pegs, 2 pink, 1 blue and 1 green are picked from the 12 pegs and are placed in the holes.

b) Find the number of different arrangements which can now be made.

Finally 4 pegs are picked at random from the total of 12 pegs and placed in the holes.

c) Determine the number of different arrangements which can be made on this occasion.

$\boxed{360}$, $\boxed{12}$, $\boxed{1170}$

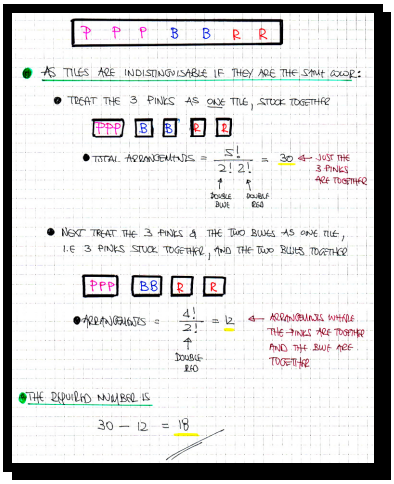


Question 10 (****)

Seven rectangular tiles, of which 3 are pink, 2 are blue and 2 are red, are placed in a straight line, next to each other.

Find the number of arrangements where the pink tiles are next to each other and the blue tiles are **not** next to each other.

$\boxed{}$, $\boxed{18}$



Question 11 (*****)

Five 1^{st} year students and three 2^{nd} year students are standing next to each other, for a photograph to be taken.

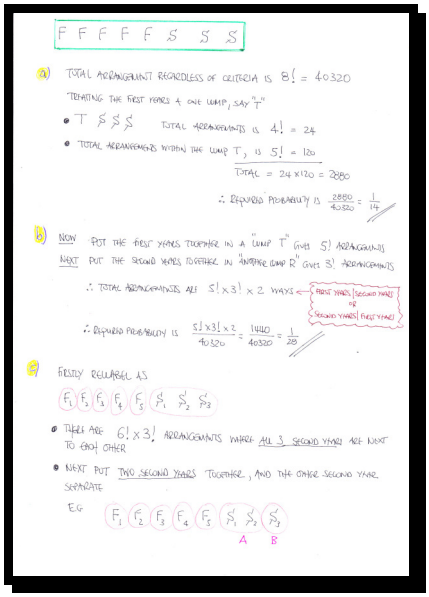
It assumed that the eight students positioned themselves at random.

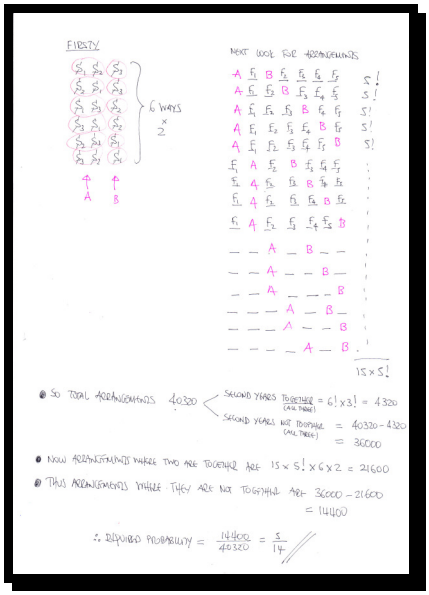
a) Find the probability that all the 1^{st} year students are standing next to each other.

b) Determine the probability that all the 1^{st} year students are standing next to each other and all the 2^{nd} year students are standing next to each other.

c) Find the probability that no 2^{nd} year students are standing next to each other.

$$\boxed{\frac{1}{14}}, \boxed{\frac{1}{28}}, \boxed{\frac{5}{14}}$$





Question 12 (*****)

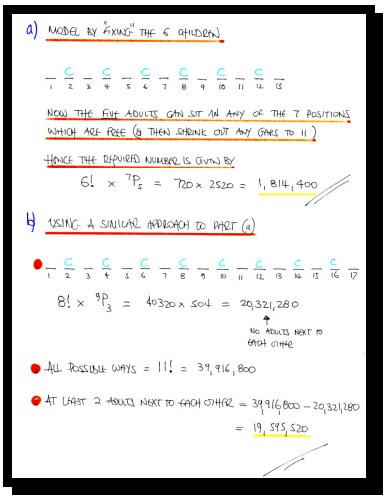
5 adults and 6 children go to the cinema and sit next to each other, in a row which contains 11 empty consecutive seats.

a) Determine the number of ways these 11 people can sit so that no two adults sit next to each other.

Another 3 adults and 8 children go to the cinema and sit next to each other, in a row which also contains 11 empty consecutive seats.

b) Find the number of ways these 11 people can sit so that at least two of the adults sit next to each other.

$\boxed{}$, $\boxed{1,814,400}$, $\boxed{19,595,520}$



MIXED COUNTING

Question 1 (*+)**

The numbers $1,\ 2,\ 3$ and 4 are to be used to make a four digit password.

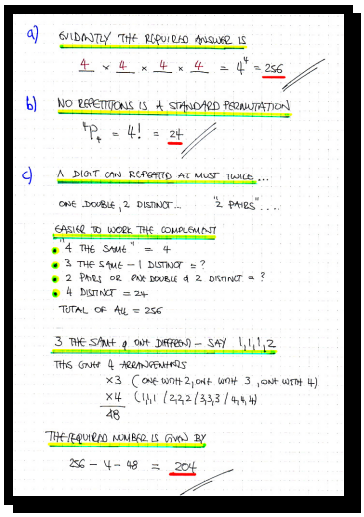
Calculate the number of the four digit passwords that can be created if …

a) ... any repetitions are allowed.

b) ... no repetitions are allowed

c) ... a digit can be repeated at most twice.

$\boxed{}$, $\boxed{4^4=256}$, $\boxed{4!=24}$, $\boxed{204}$



Question 2 (****)

Alex, Beth and Cain are 3 students in a class which consists of a total of 8 students.

a) Determine the number of selections of 4 students which contain both Alex and Beth but not Cain.

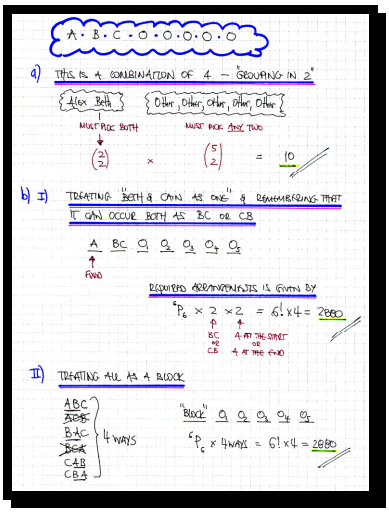
Next all 8 students are standing next to each for a group photo.

b) Determine the number of arrangements in which ...

i. ... Alex is standing at one end and Beth and Cain are standing next to each other.

ii. ... Alex and Beth are standing next to each other and Cain is standing next to them.

$\boxed{}$, $\boxed{10}$, $\boxed{2880}$, $\boxed{2880}$



Question 3 (**+)**

1, 2, 3, 4, 5, 6, 7, 8, 9

The above nine single digit numbers are written on nine separate pieces of card.

Four of these cards are picked at random and placed next to each other to form a four digit number.

Find the total different number of arrangements of …

a) … four digit numbers that can be formed.

b) … four digit **odd** numbers that can be formed.

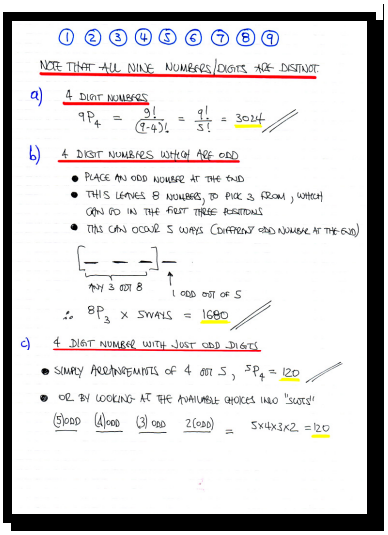
c) … four digit numbers that can be formed, whose all four digits are **odd**.

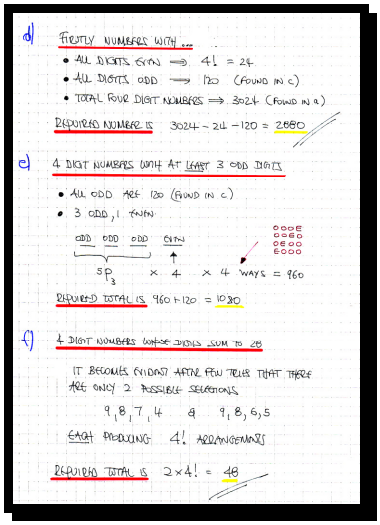
d) … four digit numbers that can be formed which have odd and even digits.

e) … four digit numbers that can be formed which have **at least** three odd digits.

f) … four digit numbers that can be formed whose **sum** of digits is 28.

$\boxed{}$, $\boxed{3024}$, $\boxed{1680}$, $\boxed{120}$, $\boxed{2880}$, $\boxed{1080}$, $\boxed{48}$





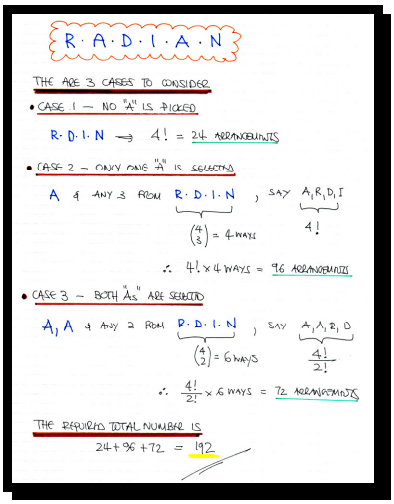
Question 4 (**+)**

The six letters of the word ***RADIAN*** are written on six separate pieces of card.

In an experiment, four cards are selected and placed next to each other, forming a four letter arrangement.

Calculate the number of different four letter arrangement.

$\boxed{}$, $\boxed{192}$



Question 5 (**+)**

$$\boxed{B},\ \boxed{A},\ \boxed{N},\ \boxed{A},\ \boxed{N},\ \boxed{A},\ \boxed{S}$$

The 7 letters shown above are written on separate pieces of card.

a) Find the number of arrangements which can be made if all 7 letters are used.

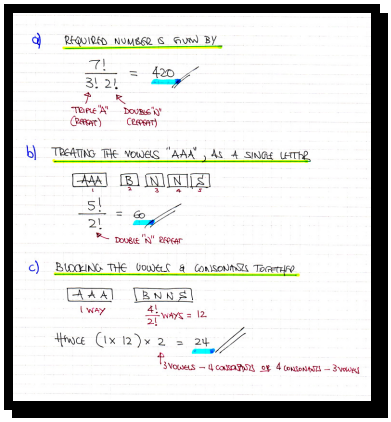
b) Find the number of arrangements which can be made if all 7 letters are used and the three vowels are together.

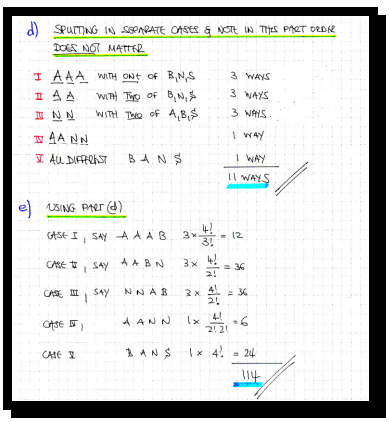
c) Find the number of arrangements which can be made if all 7 letters are used and the three vowels are together and the four consonants are together.

d) Determine the number of ways in which 4 letters can be picked from the total of 7 letters.

e) Calculate the number of arrangements of which 4 letters are used from the total of 7 letters.

$$\boxed{},\ \boxed{420},\ \boxed{60},\ \boxed{24},\ \boxed{11},\ \boxed{114}$$





Question 6 (**+)**

$$1,\ 1,\ 2,\ 2,\ 3,\ 3,\ 4,\ 4$$

The above 8 single digit numbers are written on 8 separate pieces of card.

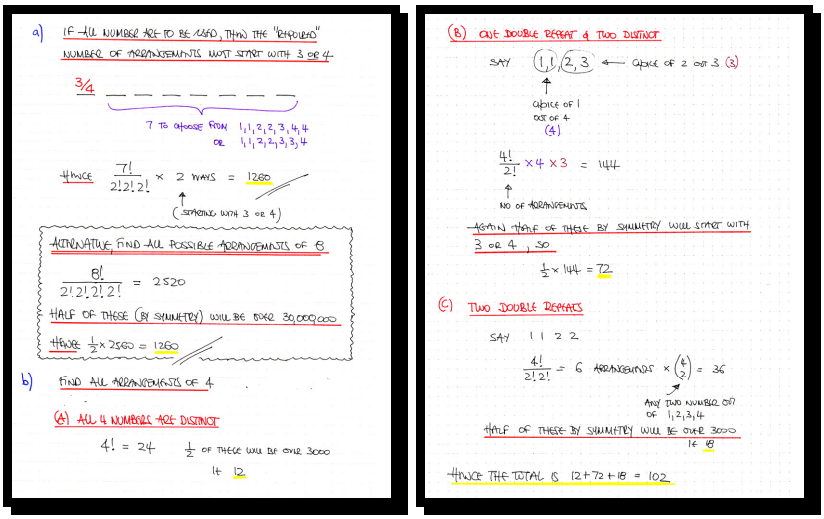
These cards are placed next to each other at random, forming an 8 digit number.

a) Determine the number of the 8 digit numbers that can be formed, which exceed $30,000,000$.

Next 4 cards are picked at random and placed next to each other to form a 4 digit number.

b) Find the number of 4 digit numbers that can be formed, which exceed 3000.

$\boxed{}$, $\boxed{1260}$, $\boxed{102}$



Question 7 (*****)

The 6 letters of the word *BUTTER* are written on 6 separate pieces of card.

In an experiment 4 cards are selected at random, forming a 4 letter **arrangement**.

a) Determine the number of 4 letter arrangements which ...

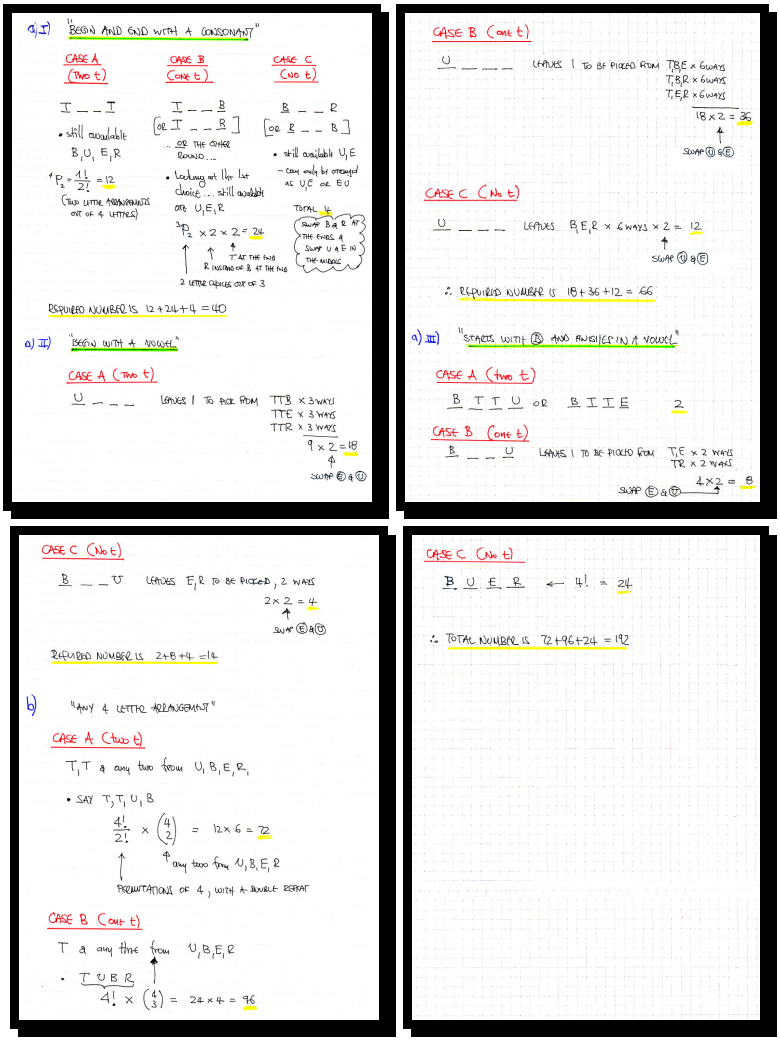
i. … will begin and end with a consonant.

ii. … will begin with a vowel.

iii. … will start with *B* and end with a vowel.

b) Find the total number of all 4 letter arrangements which can be formed.

[ ], [40], [66], [14], [192]



Question 8 (*****)

The 7 letters of the word *MINIMUM* are written on 7 separate pieces of card.

Four of these cards are picked at random, one after the other, and are arranged into a four letter word in the order they were picked.

Determine the number of the four letter words which can be formed.

[ ], [114]

