ADVANCED MENSURATION

Question 1 (**)

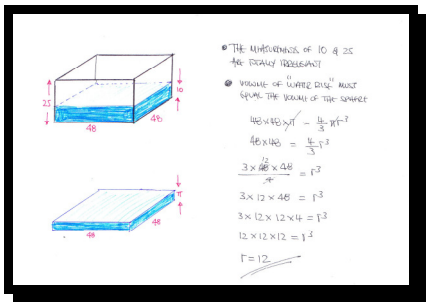
A water tank has a square base of length 48 cm and height 35 cm.

It is filled with water to a height of 25 cm.

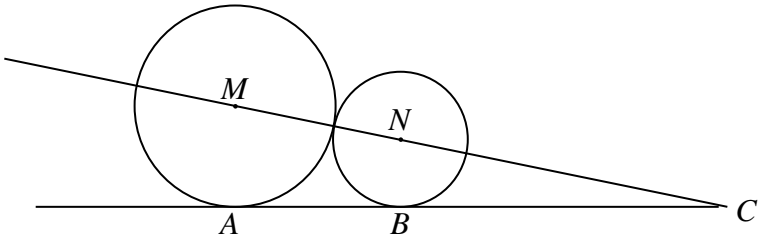
When a solid sphere is placed in the tank the water level rises by π cm.

Assuming the sphere is fully submerged determine the radius of the sphere.

$$\boxed{r = 12 \text{ cm}}$$



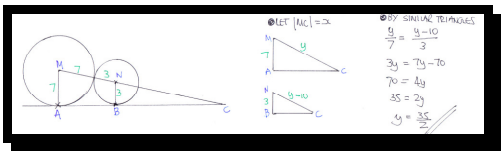
Question 2 (+)**



The figure above shows two circles, with centres at M and N, with respective radii 7 cm and 3 cm touch each other. The points M, N and C lie in a straight line and the straight line ABC is a common tangent to the circles.

Determine the length of MC.

$$\left|MC\right| = \frac{35}{2}$$



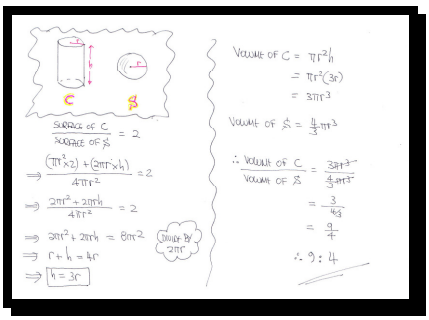
Question 3 (*+)**

A circular cylinder and a sphere both have radius r cm.

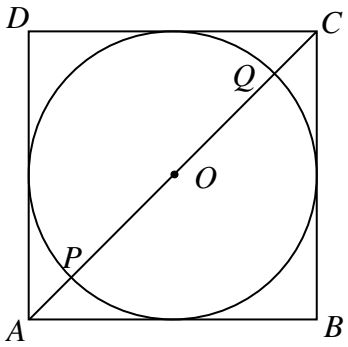
The total surface area of the cylinder is twice as large as the surface area of the sphere.

Determine the ratio of the volume of the cylinder to the volume of the sphere.

$$9:4$$



Question 4 (****)

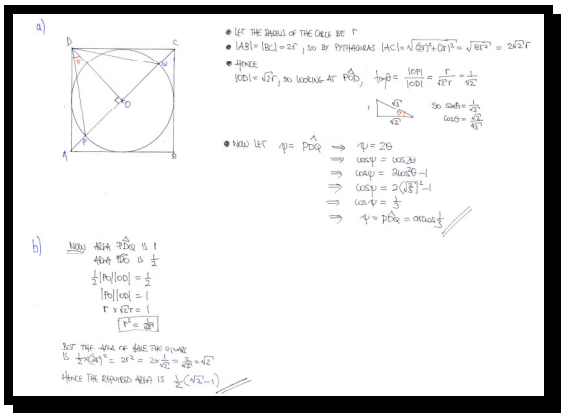


The figure above shows a circle with centre at O, inscribed in a square $ABCD$. The diagonal AC meets the circle at the points P and Q.

a) Show clearly that $\measuredangle PDQ = \arccos\frac{1}{3}$.

b) Given that the triangle PDQ is of unit area, determine the exact area of the triangle APD.

$$\boxed{\text{area} = \frac{1}{2}\left(\sqrt{2}-1\right)}$$



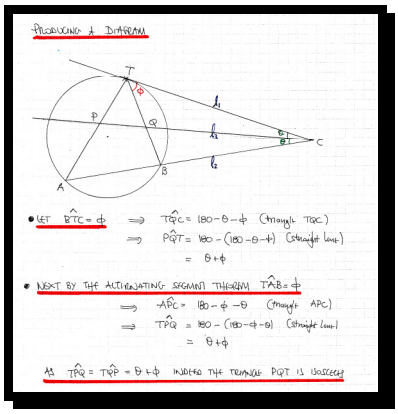
Question 5 (*****)

The following information is given.

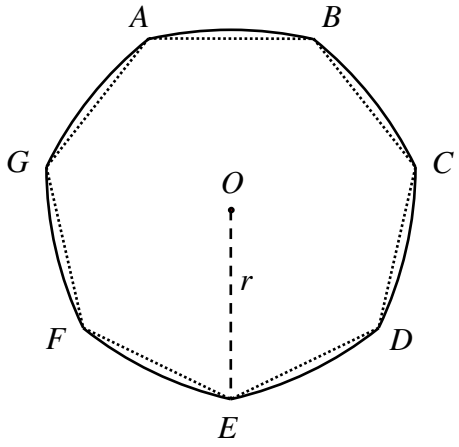
- The straight line l_1 is a tangent to a circle at the point T and the point C is another point on l_1.
- The straight line l_2 passes through C, intersecting the circle at two distinct points A and B.
- The straight line l_3 is the angle bisector of $\angle TCA$.

Given further that l_3 intersects TA and TB at the points P and Q respectively, prove that the triangle TPQ is isosceles.

, proof



Question 6 **(*****)**



The figure above shows a Reuleaux heptagon, $ABCDEFG$, constructed as follows.

Firstly a regular heptagon $ABCDEFG$ with centre at O and radius r is constructed. This is shown dotted in the figure.

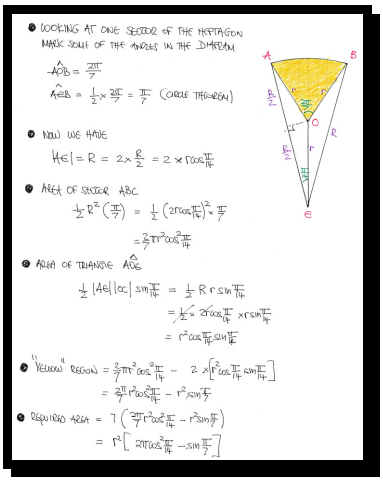
A circular arc $\overset{\frown}{AB}$ is drawn with centre at E and radius EA. A second circular arc $\overset{\frown}{BC}$ is drawn with centre at F and radius FB.

A third circular arc $\overset{\frown}{CD}$ is drawn with centre at G and radius GC and the process is repeated , forming a curved heptagon known as a Reuleaux heptagon.

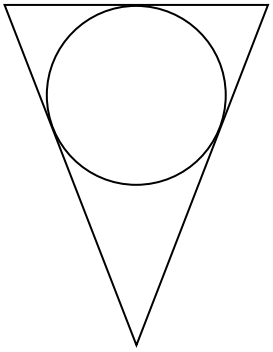
Show that the area of this Reuleaux heptagon is

$$r^2\left[2\pi\cos^2\left(\frac{\pi}{14}\right)-\sin\left(\frac{\pi}{7}\right)\right].$$

[ ], [proof]



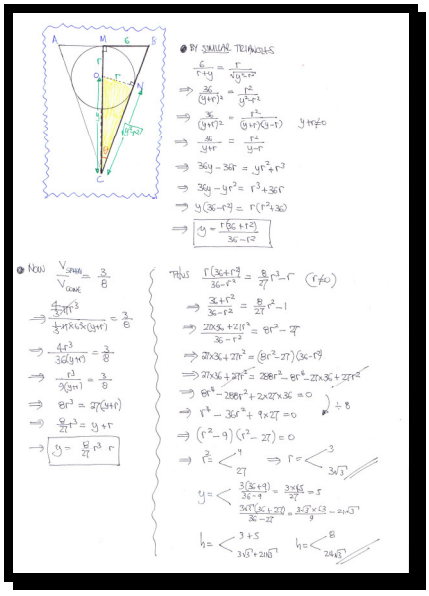
Question 7 (*****)



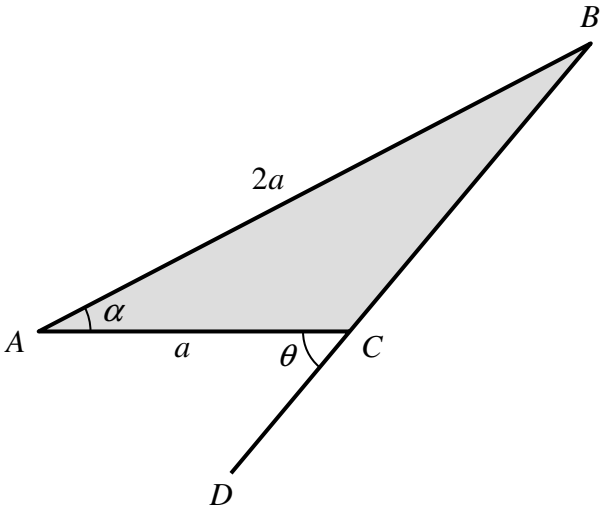
The figure above shows the cross sectional view of a solid sphere that **just fits** inside a right circular conical shell of radius 6 cm and height h cm.

If the sphere occupies $\frac{3}{8}$ of the volume of the conical shell determine the two possible values of h.

$$\boxed{h=8,\ 24\sqrt{3}}$$



Question 8 (*****)



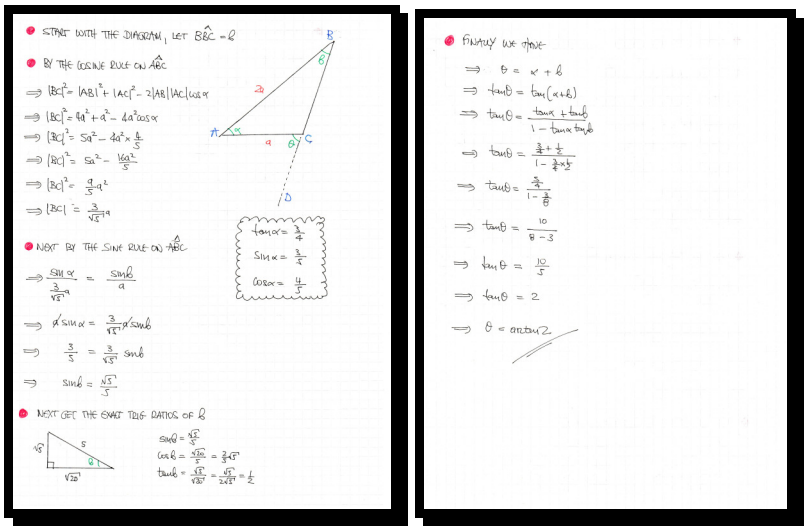
The figure above shows a triangle ABC, where $|AB| = a$ and $|AC| = 2a$.

The angle BAC is α, where $\tan\alpha = \frac{3}{4}$.

The side BC is extended to the point D so that the angle ACD is denoted by θ.

Show clearly that $\theta = \arctan 2$.

[ ], [proof]



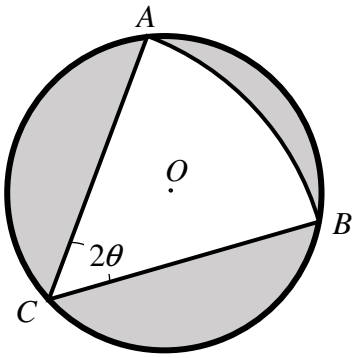
Question 9 (*****)

$$2x\tan x=1,\ x\neq\frac{1}{2}n\pi,\ n\in\mathbb{N}$$

a) Show that the above equation has a solution in the interval $(0.6,0.7)$.

b) Use the Newton Raphson method to find the solution of this equation, correct to 5 decimal places.

The figure below shows a circle, centre at O. The points A, B and C lie on the circumference of this circle. A circular sector ABC, subtending an angle of 2θ at C, is inscribed in this circle.



c) Determine the greatest proportion of the area of the circle, which can be covered by this sector.

You may give the answer as a percentage, correct to two decimal places

$\boxed{}$, $\boxed{x\approx 0.65327}$, $\boxed{\approx 52.45}$

