2^{nd} ORDER O.D.E.s PRACTICE

Question 1

Find a general solution for each of the following differential equations.

a) $2\dfrac{d^2y}{dx^2}+7\dfrac{dy}{dx}+3y=0$

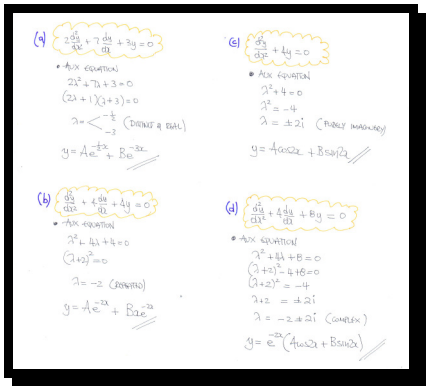
b) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+4y=0$

c) $\dfrac{d^2y}{dx^2}+4y=0$

d) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+8y=0$

$\boxed{y=A\mathrm{e}^{-3x}+B\mathrm{e}^{-\frac{1}{2}x}}$, $\boxed{y=A\mathrm{e}^{-2x}+Bx\mathrm{e}^{-2x}}$, $\boxed{y=A\cos 2x+B\sin 2x}$,

$\boxed{y=\mathrm{e}^{-2x}\left(A\cos 2x+B\sin 2x\right)}$



Question 2

Find a general solution for each of the following differential equations.

a) $4\dfrac{d^2y}{dx^2}-8\dfrac{dy}{dx}+5y=0$

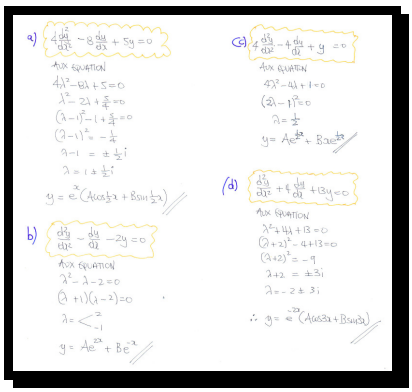
b) $\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}-2y=0$

c) $4\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+y=0$

d) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+13y=0$

$\boxed{y=\mathrm{e}^{x}\left(A\cos\frac{1}{2}x+B\sin\frac{1}{2}x\right)}$, $\boxed{y=A\mathrm{e}^{-x}+B\mathrm{e}^{2x}}$, $\boxed{y=A\mathrm{e}^{\frac{1}{2}x}+Bx\mathrm{e}^{\frac{1}{2}x}}$,

$\boxed{y=\mathrm{e}^{-2x}\left(A\cos 3x+B\sin 3x\right)}$



Question 3

Find a general solution for each of the following differential equations.

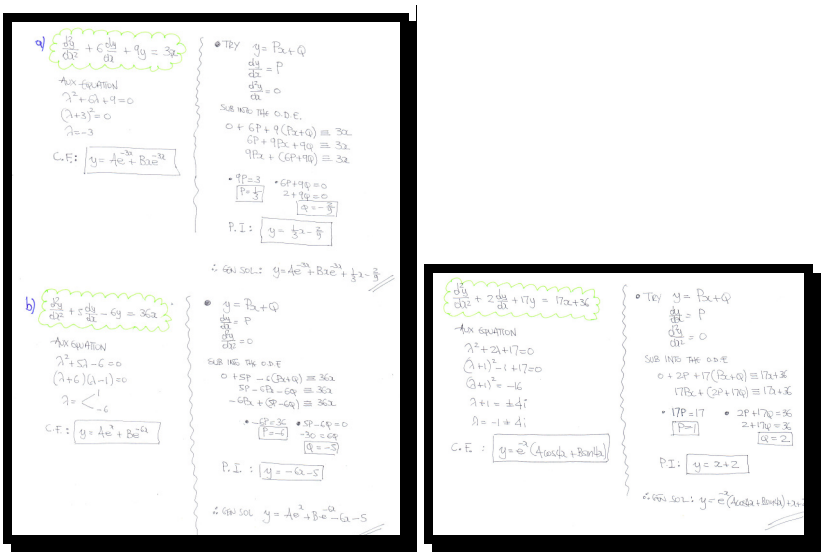
a) $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=3x$

b) $\dfrac{d^2y}{dx^2}+5\dfrac{dy}{dx}-6y=36x$

c) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+17y=17x+36$

$\boxed{y=Ae^{-3x}+Bxe^{-3x}+\frac{1}{3}x-\frac{2}{9}}$, $\boxed{y=Ae^{x}+Be^{-6x}-6x-5}$,

$\boxed{y=e^{-x}\left(A\cos 4x+B\sin 4x\right)+x+2}$



Question 4

Find a general solution for each of the following differential equations.

a) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+5y=10\sin x$

b) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+17y=17x+36$

c) $\dfrac{d^2y}{dx^2}+4y=12\cos x$

$\boxed{y=\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)+2\sin x-\cos x}$, $\boxed{y=\mathrm{e}^{-x}\left(A\cos 4x+B\sin 4x\right)+x+2}$,

$\boxed{y=A\cos 2x+B\sin 2x+4\cos x}$

Question 5

Find a general solution for each of the following differential equations.

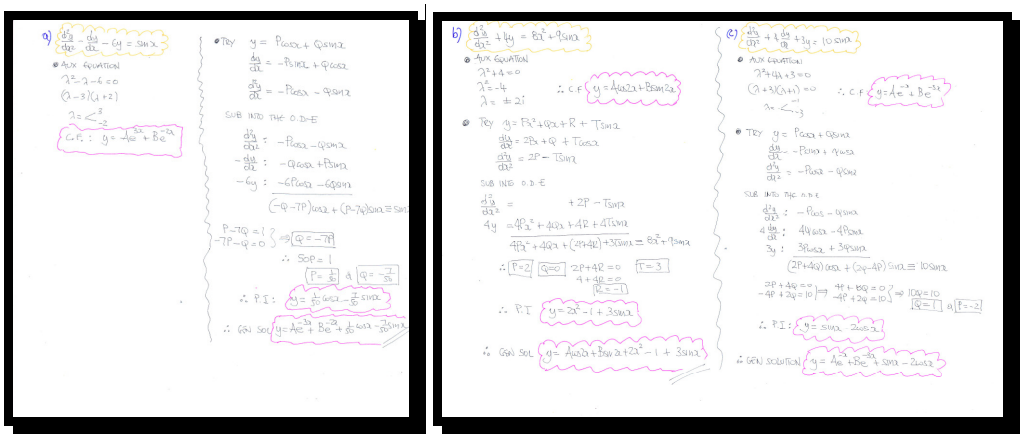
a) $\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}-6y=\sin x$

b) $\dfrac{d^2y}{dx^2}+4y=8x^2+9\sin x$

c) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+3y=10\sin x$

$\boxed{y=A\mathrm{e}^{3x}+B\mathrm{e}^{-2x}+\frac{1}{50}(\cos x-7\sin x)}$, $\boxed{y=A\cos 2x+B\sin 2x+3\sin x+2x^2-1}$,

$\boxed{y=A\mathrm{e}^{-x}+B\mathrm{e}^{-3x}+\sin x-2\cos x}$



Question 6

Find the solution of each of the following differential equations.

a) $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=\cos 3x$,

subject to the conditions $y=\dfrac{1}{2}$, $\dfrac{dy}{dx}=0$ at $x=0$.

b) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+5y=10$,

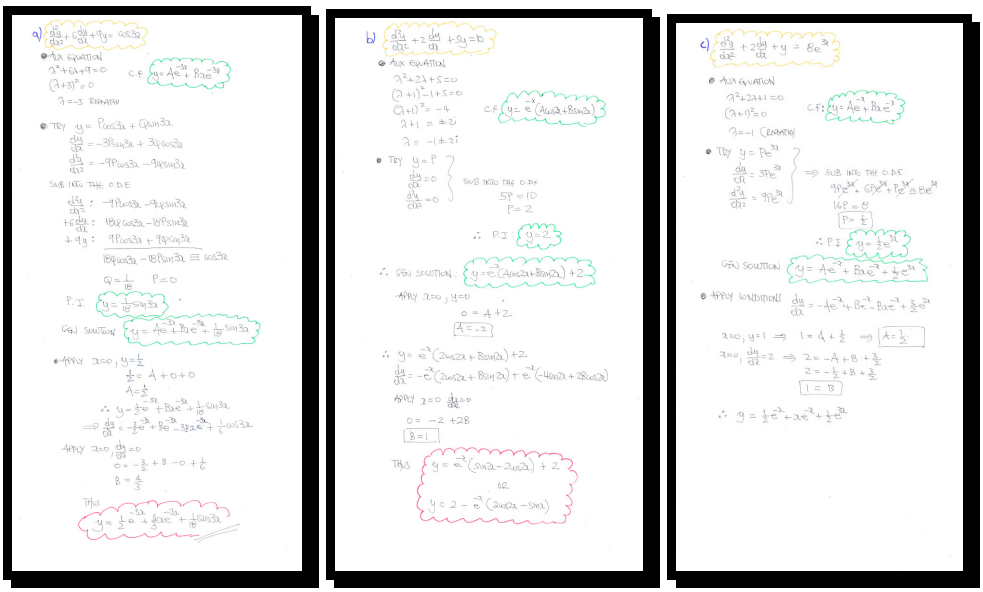
subject to the conditions $y=0$, $\dfrac{dy}{dx}=0$ at $x=0$.

c) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=8\mathrm{e}^{3x}$,

subject to the conditions $y=1$, $\dfrac{dy}{dx}=2$ at $x=0$.

$\boxed{y=\frac{1}{2}\mathrm{e}^{-3x}+\frac{4}{3}x\mathrm{e}^{-3x}+\frac{1}{18}\sin 3x}$, $\boxed{y=2-\mathrm{e}^{-x}\left(2\cos 2x-\sin 2x\right)}$,

$\boxed{y=\frac{1}{2}\mathrm{e}^{3x}+\frac{1}{2}\mathrm{e}^{-x}+x\mathrm{e}^{-x}}$



Question 7

Find the solution of each of the following differential equations.

a) $\dfrac{d^2y}{dx^2}-4y=10\mathrm{e}^{3x}$,

subject to the conditions $y=3$, $\dfrac{dy}{dx}=8$ at $x=0$.

b) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}-3y=15\mathrm{e}^{2x}$,

subject to the conditions $y=9$, $\dfrac{dy}{dx}=4$ at $x=0$.

c) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=8\mathrm{e}^{3x}$,

subject to the conditions $x=0$, $y=1$, $\dfrac{dy}{dx}=2$

$\boxed{y=\mathrm{e}^{2x}+2\mathrm{e}^{3x}}$, $\boxed{y=4\mathrm{e}^{x}+2\mathrm{e}^{-3x}+3\mathrm{e}^{2x}}$, $\boxed{y=2\mathrm{e}^{3x}-\mathrm{e}^{-x}-5x\mathrm{e}^{-x}}$

Question 8

Find the solution of each of the following differential equations.

a) $2\dfrac{d^2y}{dx^2}+5\dfrac{dy}{dx}+2y=2x+9$,

subject to the conditions $y=3$, $\dfrac{dy}{dx}=-1$ at $x=0$.

b) $\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+9y=27x$,

subject to the conditions $y=2$, $\dfrac{dy}{dx}=6$ at $x=0$.

c) $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=2x+3$,

subject to the conditions $y=2$, $\dfrac{dy}{dx}=-5$ at $x=0$.

$\boxed{y=\mathrm{e}^{-2x}+x+2}$, $\boxed{y=3x\left(1+\mathrm{e}^{3x}\right)+2}$, $\boxed{y=x^2+x-4+6\mathrm{e}^{-x}}$

Question 9

Find the solution of each of the following differential equations.

a) $2\dfrac{d^2y}{dx^2}-7\dfrac{dy}{dx}-4y=8\sin x-19\cos x$,

subject to the conditions $y=0$, $\dfrac{dy}{dx}=11$ at $x=0$.

b) $2\dfrac{d^2y}{dx^2}-7\dfrac{dy}{dx}-4y=8\sin x-19\cos x$,

subject to the conditions $y=0$, $\dfrac{dy}{dx}=11$ at $x=0$.

c) $2\dfrac{d^2y}{dx^2}-7\dfrac{dy}{dx}-4y=8\sin x-19\cos x$,

subject to the conditions $y=0$, $\dfrac{dy}{dx}=11$ at $x=0$.

$\boxed{y=\mathrm{e}^{-2x}+x+2}$, $\boxed{y=3x\left(1+\mathrm{e}^{3x}\right)+2}$, $\boxed{y=2\mathrm{e}^{4x}-4\mathrm{e}^{\frac{1}{2}x}+\sin x+2\cos x}$

Question 10 (harder PIs)

Find the solution of each of the following differential equations.

a) $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=2\mathrm{e}^{-2x}$,

subject to the conditions $y=2$, $\dfrac{dy}{dx}=0$ at $x=0$.

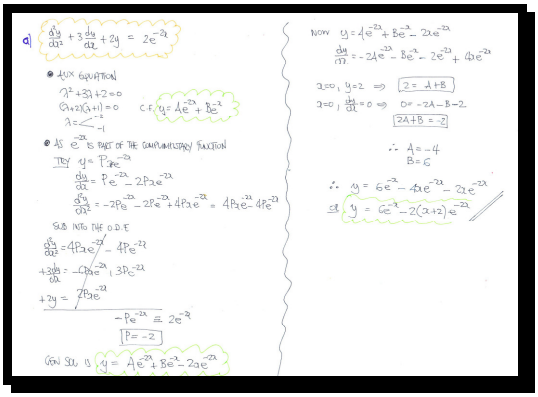
b) $\dfrac{d^2y}{dx^2}+9y=\cos 3x$,

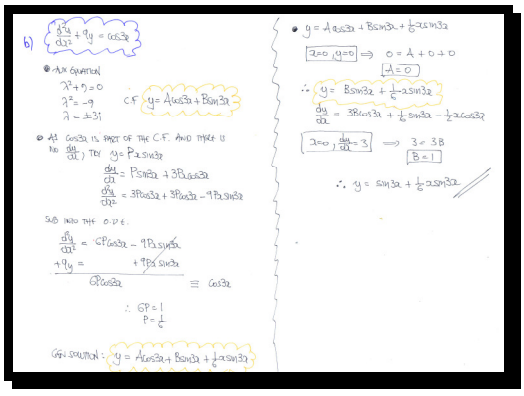
subject to the conditions $y=0$, $\dfrac{dy}{dx}=3$ at $x=0$.

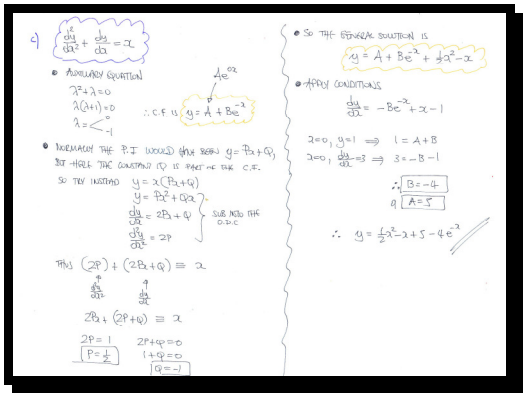
c) $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=x$,

subject to the conditions $y=1$, $\dfrac{dy}{dx}=3$ at $x=0$.

$\boxed{y=6\mathrm{e}^{-x}-2(x+2)\mathrm{e}^{-2x}}$, $\boxed{y=\sin 3x+\frac{1}{6}x\sin 3x}$, $\boxed{y=\frac{1}{2}x^2-x+5-4\mathrm{e}^{-x}}$







Question 11 (harder PIs)

Find the solution of each of the following differential equations.

a) $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}-4y=10\mathrm{e}^x$,

subject to the conditions $y=1$, $\dfrac{dy}{dx}=3$ at $x=0$.

b) $\dfrac{d^2y}{dx^2}+16y=8\cos 4x$,

subject to the conditions $y=2$, $\dfrac{dy}{dx}=0$ at $x=0$.

c) $\dfrac{d^2y}{dx^2}+100y=2\cos 10x$,

subject to the conditions $y=0$, $\dfrac{dy}{dx}=0$ at $x=0$.

$\boxed{y=\mathrm{e}^x\left(2x+1\right)}$, $\boxed{y=2\cos 4x+x\sin 4x}$, $\boxed{y=\frac{1}{10}x\sin 10x}$

Question 12 (harder PIs)

Find the solution of each of the following differential equations.

a) $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=\mathrm{e}^{x}$,

subject to the conditions $y=1$, $\dfrac{dy}{dx}=2$ at $x=0$.

b) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=\mathrm{e}^{-x}$,

subject to the conditions $y=1$, $\dfrac{dy}{dx}=-1$ at $x=0$.

c) $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}=100x$,

subject to the conditions $y=2$, $\dfrac{dy}{dx}=1$ at $x=0$.

$\boxed{y=\left(\frac{1}{2}x^2+x+1\right)\mathrm{e}^{x}}$, $\boxed{y=\left(\frac{1}{2}x^2+1\right)\mathrm{e}^{-x}}$, $\boxed{y=\mathrm{e}^{5x}+1-4x-10x^2}$

2^{nd} ORDER O.D.E.

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Question 1

Find the solution of each of the following differential equations.

a) $x^2\dfrac{d^2y}{dx^2}-6x\dfrac{dy}{dx}+12y=0$,

subject to the conditions $x=1$, $y=1$, $\dfrac{dy}{dx}=0$

b) $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}-4y=0$,

subject to the conditions $x=1$, $y=3$, $\dfrac{dy}{dx}=7$

c) $x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+4y=0$,

subject to the conditions $x=1$, $y=5$, $\dfrac{dy}{dx}=8$

$\boxed{y=4x^3-3x^4}$, $\boxed{y=\dfrac{1}{x}+2x^4}$, $\boxed{y=4x+x^4}$

Created by T. Madas